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Research Paper

Scattering and Backscattering Study of Mechanical Plane Wave in Composite Materials Plates (Earth model 1066B and $LiNbO_3$)

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Abstract. Reflection and refraction phenomenon pattern of elastic plane wave at the interface between anisotropic monoclinic elastic half-space and isotropic elastic half-spaces is studied. Closed-form expression for phase velocity is obtained. Reflection and transmission coefficients are obtained using the method of Cramer's rule in determinant form. Also, the energy ratios are calculated in terms of reflection and transmission coefficients. Numerical examples are considered to exhibit all the findings graphically. The energy conservation law is implemented at each angle of incidence to validate the numerical results, and it is found that energy ratios are in good agreement with the energy conservation law.

Keywords: Reflection; Seismic wave transmission; Monoclinic; Amplitude ratio; Energy ratio; Composite materials.

1. Introduction

Earth is very complex in nature and contains various types of rocks and materials with amazing characteristics such as anisotropy, heterogeneity, sandiness, etc. When a wave propagates through the earth's interior, different mediums come in the way of wave motion. These waves incident on the interfaces between mediums and gives rise to different reflected and transmitted waves in the originating and continuing media, respectively. The signal of these reflected and refracted waves are not only helpful in providing information about the internal structure of the earth but also helpful in the exploration of valuable materials such as minerals, crystals, and metals. The study of anisotropic elasticity is essential for understanding the mechanical behaviour of materials. Anisotropy in these materials results from the presence of crystals of particular symmetry or thin laminates. Thus, the analytical studies on anisotropic propagation restrict the motion to fixed plane. A monoclinic medium possesses one plane of elastic symmetry in which crystals can be referred to three non-equal axes, two of which intersect at a sloping angle while the third is perpendicular to the other two. The monoclinic system is one of the most equilibrium systems with almost all raw materials belonging to one of its three classes. This motivates us to study the reflection and refraction phenomena of quasi waves in a monoclinic medium. Minerals like Lithium Tantalate, Lithium Niobate, and Orthoclase, etc. can be treated as monoclinic materials. The problem of reflection and refraction of plane harmonic elastic waves is of great importance in the field of rock dynamics, non-destructive testing, civil engineering, seismology, earthquake engineering, and geophysics. A large number of papers related to reflection and refraction of elastic waves in anisotropic media have appeared in the literature. Without going into the details of the research work in this field, we mention a few of the papers. Some representative papers in this field may be cited as Musgrave [1], Achenbach [2], Thapliyal [3], Daley and Hron [4], Keith and Crampin [5], Tolstoy [6], Pal and Chattopadhyay [7] and Borejko [8]. Nayfeh [9] derived analytical expressions for the reflection and transmission coefficients from the interfaces of liquid-anisotropic half-spaces possessing up to as low as monoclinic symmetry and the expressions for the distributions of stresses and displacements throughout the fluid-solid system. He has also deduced the variations of phase velocity and beam shifting parameters with azimuthal angles. Chattopadhyay and Choudhury [10] attempted a problem of reflection of P-waves at the plane boundary of a half-space of monoclinic type. Some problems of reflection and transmission of a plane wave in triclinic crystalline materials have been discussed by Chattopadhyay [11, 12]; Chattopadhyay and Rajneesh [13] obtained the reflection and transmission coefficients for different waves. Singh and Zorammuana [14] studied the reflection of the incident longitudinal wave at a plane free fibre-reinforced thermoelastic half-space. Some of the recent works in the field of reflection and transmission of plane waves in different types of anisotropic medium have been investigated by Singh [15], Chattopadhyay et al. [16], Dhua et al. [17], Chatterjee et al. [18, 19], Pal et al. [20], Kumar, Pal and Majhi [21], Kumar, Majhi, and Pal [22] and Verma [23]. Related to some best knowledge of angle reflection and about materials are available in Jianwen et al. [24] and Zhongxian et al. [25, 26].

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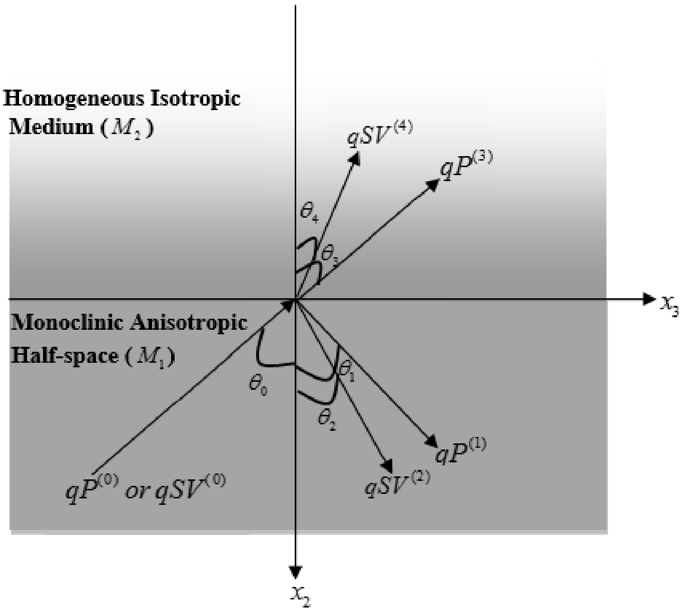

Fig. 1. Geometry of the problem

The study of reflection and refraction phenomena of elastic waves is become interesting in the field of seismology, in particular seismic prospecting, as the earth's surface might be considered to consist of different layers having different material properties. In this light, the main purpose of this paper is to discuss the reflection and transmission phenomenon of plane waves at the interface between monoclinic elastic half-space and isotropic medium. The incident wave is taken to be in the cause of the interface disturbance and reflected, and refracted waves must propagate away from the interface. The reflection and transmission coefficients have been discussed for various reflected and transmitted waves. The energy ratios for these waves are also calculated in terms of reflection and transmission coefficients. The rule of energy conservation at different angles of incidence has been verified. Elastic parameters for Lithium Niobate have been used for graphical representation of the present analytical study.

2. Definition of the Problem

A plane quasi-P (qP) wave or quasi-SV (qSV) is incident on the plane interface with anisotropic monoclinic elastic medium. A part of incident energy reflect back in originating media and remaining energy refract in the continuing media. The procedure to study the propagation of reflected and transmitted waves is explained as follows.

1. For propagation of plane wave in anisotropic monoclinic medium, the equation of motion in monoclinic medium is solved and obtained the phase velocity of SH- wave propagating in arbitrary direction of elastic symmetry of monoclinic medium. Also the phase velocities of qP and qSV waves are obtained in terms of polarization vector.
2. The stresses and displacement will be continuous at the perfect plane interface separating the monoclinic elastic media with isotropic medium. Using the boundary conditions, we found the reflection/transmission coefficients in the form of determinants using Cramer's rule.
3. Energy ratios are obtained for all the reflected and transmitted wave in terms of reflection and transmission coefficients.
4. Numerical examples are considered to exhibits the variations of all findings and energy share of different reflected and transmitted waves are calculated to verify the energy conservation law.

An incident quasi (P/SV) wave incident on the interface between monoclinic and isotropic medium. After incidence, two waves (quasi-P and quasi-SV) reflected in the lower monoclinic medium and transmit P and SV wave in isotropic medium.

3. Basic Equations

Using the generalised Hook's law into Euler's equations for elasticity, the equations of motions for wave propagating in (x_2, x_3) plane (plane of symmetry) in homogeneous anisotropic elastic medium of monoclinic type with 13 elastic constants are given by

$$C_{66}\frac{\partial^2 \upsilon_1}{\partial x_2^2} + 2C_{56}\frac{\partial^2 \upsilon_1}{\partial x_2 \partial x_3} + C_{55}\frac{\partial^2 \upsilon_1}{\partial x_3^2} = \rho \frac{\partial^2 \upsilon_1}{\partial t^2}, \tag{1}$$

$$C_{22}\frac{\partial^2 \upsilon_2}{\partial x_2^2} + C_{44}\frac{\partial^2 \upsilon_2}{\partial x_3^2} + C_{24}\frac{\partial^2 \upsilon_3}{\partial x_2^2} + C_{34}\frac{\partial^2 \upsilon_3}{\partial x_3^2} + 2C_{24}\frac{\partial^2 \upsilon_2}{\partial x_2 \partial x_3} + (C_{23}+C_{44})\frac{\partial^2 \upsilon_3}{\partial x_2 \partial x_3} = \rho \frac{\partial^2 \upsilon_2}{\partial t^2}, \tag{2}$$

$$C_{24}\frac{\partial^2 \upsilon_2}{\partial x_2^2} + C_{34}\frac{\partial^2 \upsilon_2}{\partial x_3^2} + C_{44}\frac{\partial^2 \upsilon_3}{\partial x_2^2} + C_{33}\frac{\partial^2 \upsilon_3}{\partial x_3^2} + 2C_{34}\frac{\partial^2 \upsilon_3}{\partial x_2 \partial x_3} + (C_{23}+C_{44})\frac{\partial^2 \upsilon_2}{\partial x_2 \partial x_3} = \rho \frac{\partial^2 \upsilon_3}{\partial t^2}, \tag{3}$$

where C_{ij} are elastic constant and symmetric in i and j, $\upsilon_i = \upsilon_i(x_2, x_3, t)$, $i = 1,2,3$ are the components of the displacement vector and ρ is the mass density.

Equation (1) represent the SH wave propagation because it is uncoupled in υ_1 while Eqs. (2) and (3) are coupled in υ_2 and υ_3 representing the qP and qSV waves. Let $p = (p_1, p_2, p_3)$ be the unit propagation vector, $d = (d_1, d_2, d_3)$ denote the unit displacement vector (polarization vector), β is the phase velocity and k is the wave number of plane wave propagating in the (x_2, x_3) plane. A solution to these wave equations of motion can be written as

$$\upsilon_j^{\,n}(x_2, x_3, t) = A_n d_j^{(n)} \exp\left(ik_n\left(x_2 p_2^{(n)} + x_3 p_3^{(n)} - \beta_n t\right)\right) \tag{4}$$

where $n = 0,1,2,3,4$ correspond to the incident qP wave, reflected qP wave, reflected qSV wave, refracted P wave and refracted SV wave, respectively.

For non-zero value of d_1, i.e. $d_1 \neq 0$, Eqs. (1) and (4) gives

$$C_{66} p_2^2 + 2C_{56} p_2 p_3 + C_{55} p_3^2 = \rho c^2 \tag{5}$$

Equation (5) represent the phase velocity of SH-wave propagating in an arbitrary direction in the plane of elastic symmetry of monoclinic medium.

Similarly, the phase velocities of quasi-P and quasi-SV waves can be obtained with the help of Eqs. (2) to (4) which gives

$$\begin{aligned} &\left(A - \rho\beta^2\right) d_2 + B d_3 = 0 \\ &B d_2 + \left(C - \rho\beta^2\right) d_3 = 0 \end{aligned} \tag{6}$$

where

$$A(p_2, p_3) = C_{22} p_2^2 + C_{44} p_3^2 + 2C_{24} p_2 p_3, \tag{7}$$

$$B(p_2, p_3) = C_{24} p_2^2 + C_{34} p_3^2 + (C_{23} + C_{44}) p_2 p_3, \tag{8}$$

$$C(p_2, p_3) = C_{44} p_2^2 + C_{33} p_3^2 + 2C_{34} p_2 p_3. \tag{9}$$

From Eq. (6), we get

$$\frac{d_2}{d_3} = \frac{B}{\rho\beta^2 - A} = \frac{\rho\beta^2 - C}{B} \tag{10}$$

Eliminating $\rho\beta^2$ from Eq. (8) and using Eq. (7), we get

$$\left[C_{24}\left(d_3^2 - d_2^2\right) + (C_{22} - C_{44}) d_2 d_3\right] p_2^2 + \left[C_{34}\left(d_3^2 - d_2^2\right) + (C_{44} - C_{33}) d_2 d_3\right] p_3^2 + \left[(C_{22} + C_{44})\left(d_3^2 - d_2^2\right) + 2(C_{24} - C_{34}) d_2 d_3\right] p_2 p_3 = 0 \tag{11}$$

A plane wave whose displacement vector d is parallel to propagation vector then it represent longitudinal waves i.e. Eq. (4) represent the longitudinal waves if $d_2 = p_2, d_3 = p_3$.

In that case, Eq. (9) gives

$$C_{24} p_2^4 + (C_{23} - C_{22} + 2C_{44}) p_2^3 p_3 - 3(C_{24} - C_{34}) p_2^2 p_3^2 - (C_{23} - C_{33} + 2C_{44}) p_2 p_3^2 - C_{34} p_3^4 = 0 \tag{12}$$

Equation (12) represent the directions of propagation for which quasi-P waves are purely longitudinal. Eq. (10) gives the phase velocity of quasi-P wave as

$$\rho\beta_{qP}^2 = A + \left(\frac{p_3}{p_2}\right) B = \left(\frac{p_2}{p_3}\right) B + C \tag{13}$$

Similarly, Eq. (4) represent a transverse wave when the displacement vector of plane wave is perpendicular to the propagation vector i.e. $d.p = d_2 p_2 + d_3 p_3 = 0$. In this case Eq. (11) also leads to Eq. (12). The phase velocity of transverse qSV wave is given by

$$\rho\beta_{qSP}^2 = A - \left(\frac{p_3}{p_2}\right) B = C - \left(\frac{p_2}{p_3}\right) B \tag{14}$$

Thus Eq. (12) represents the direction of propagation for which quasi-P wave are purely longitudinal and quasi-SV wave are purely transverse. Using Eqs. (13) and (14), one can determine the phase velocity of quasi-P and quasi-SV waves, respectively if the propagation angle, stiffness coefficient and mass density of the monoclinic medium are known through they are propagating.

For the case of isotropic medium, we consider

$$\begin{aligned} &C_{11} = C_{22} = C_{33} = \lambda + 2\mu, \\ &C_{12} = C_{13} = C_{23} = \lambda, \\ &C_{44} = C_{55} = C_{66} = \mu, \\ &C_{14} = C_{24} = C_{34} = 0. \end{aligned} \tag{15}$$

where λ and μ are Lame's parameters.

Eq. (15) in Eqs. (13) and (14) gives $\beta_{qP} = \sqrt{(\lambda+2\mu)/\rho}$ and $\beta_{qSV} = \sqrt{\mu/\rho}$, thus qP and qSV waves reduces to P and SV waves, respectively. Thus, the phase velocity of various reflected and refracted waves depends on the angle of propagation, elastic properties and density of the medium.

4. Boundary Condition and Solution of the Problem

Consider a two dimensional plane quasi-(P/SV) wave with propagation direction θ_0 propagating through monoclinic medium and incident on the plane interface between monoclinic elastic half-space and isotropic medium [Fig.(1)]. This incident quasi wave will produce two reflected qP and qSV waves in the monoclinic medium and transmit P and SV waves in the isotropic medium (S). Let $p = (0, p_2, p_3)$ be the propagation vector for different reflected and transmitted waves and is given by

$$\begin{aligned}
&p_2^{(0)} = \cos\theta_0, p_3^{(0)} = \sin\theta_0, p_2^{(1)} = -\cos\theta_1, p_3^{(1)} = \sin\theta_1,\\
&p_2^{(2)} = -\cos\theta_2, p_3^{(2)} = \sin\theta_2, p_2^{(3)} = \cos\theta_3, p_3^{(3)} = \sin\theta_3,\\
&p_2^{(4)} = \cos\theta_4, p_3^{(4)} = \sin\theta_4, d_2^{(0)} = \cos\theta_0, d_3^{(0)} = \sin\theta_0,\\
&d_2^{(1)} = -\cos\theta_1, d_3^{(1)} = \sin\theta_1, d_2^{(2)} = \sin\theta_2, d_3^{(2)} = \cos\theta_2,\\
&d_2^{(3)} = \cos\theta_3, d_3^{(3)} = \sin\theta_3, d_2^{(4)} = \sin\theta_4, d_3^{(4)} = \cos\theta_4,
\end{aligned} \tag{16}$$

For the physically most significant case of perfect contact, the displacement and stresses are continuous at the interface i.e. at $x_2 = 0$. Boundary conditions are

$$v_2^{(0)} + v_2^{(1)} + v_2^{(2)} = v_2^{(3)} + v_2^{(4)}, \tag{17}$$

$$v_3^{(0)} + v_3^{(1)} + v_3^{(2)} = v_3^{(3)} + v_3^{(4)}, \tag{18}$$

$$\tau_{23}^{(0)} + \tau_{23}^{(1)} + \tau_{23}^{(2)} = \tau_{23}^{(3)} + \tau_{23}^{(4)}, \tag{19}$$

$$\tau_{22}^{(0)} + \tau_{22}^{(1)} + \tau_{22}^{(2)} = \tau_{22}^{(3)} + \tau_{22}^{(4)}, \tag{20}$$

Applying the boundary conditions, we get the following equations

$$A_0\cos\theta_0\exp(i\eta_0) - A_1\cos\theta_1\exp(i\eta_1) + A_2\sin\theta_2\exp(i\eta_2) = A_3\cos\theta_3\exp(i\eta_3) + A_4\sin\theta_4\exp(i\eta_4), \tag{21}$$

$$A_0\sin\theta_0\exp(i\eta_0) + A_1\sin\theta_1\exp(i\eta_1) + A_2\cos\theta_2\exp(i\eta_2) = A_3\sin\theta_3\exp(i\eta_3) + A_4\cos\theta_4\exp(i\eta_4), \tag{22}$$

$$k_0A_0X_0\exp(i\eta_0) + k_1A_1X_1\exp(i\eta_1) + k_2A_2X_2\exp(i\eta_2) = k_3A_3X_3\exp(i\eta_3) + k_4A_4X_4\exp(i\eta_4), \tag{23}$$

$$k_0A_0Y_0\exp(i\eta_0) + k_1A_1Y_1\exp(i\eta_1) + k_2A_2Y_2\exp(i\eta_2) = k_3A_3Y_3\exp(i\eta_3) + k_4A_4Y_4\exp(i\eta_4), \tag{24}$$

where $\eta_n = k_n\left(x_2p_2^{(n)} + x_3p_3^{(n)} - \beta_n t\right)$, $n = 0,1,2,3,4$ is the phase function of plane wave. The above Eqs. (21) to (24) must be valid for all values of x_3 and t, we have

$$\eta_0 = \eta_1 = \eta_2 = \eta_3 = \eta_4$$

which gives

$$\begin{aligned}
&k_0\sin\theta_0 = k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3 = k_4\sin\theta_4 = k,\\
&k_0\beta_0 = k_1\beta_1 = k_2\beta_2 = k_3\beta_3 = k_4\beta_4 = \omega.
\end{aligned}$$

where k and ω are the wave number and circular frequency, respectively.
Equations (21) to (24) may be written as

$$A_0\cos\theta_0 - A_1\cos\theta_1 + A_2\sin\theta_2 = A_3\cos\theta_3 + A_4\sin\theta_4 \tag{25}$$

$$A_0\sin\theta_0 + A_1\sin\theta_1 + A_2\cos\theta_2 = A_3\sin\theta_3 + A_4\cos\theta_4, \tag{26}$$

$$k_0A_0X_0 + k_1A_1X_1 + k_2A_2X_2 = k_3A_3X_3 + k_4A_4X_4, \tag{27}$$

$$k_0A_0Y_0 + k_1A_1Y_1 + k_2A_2Y_2 = k_3A_3Y_3 + k_4A_4Y_4, \tag{28}$$

Solving the above Eqs. (25) to (28), we obtain the reflection and transmission coefficients for different reflected and refracted waves as

$$\frac{A_1}{A_0} = \frac{D_1}{D_0}, \frac{A_2}{A_0} = \frac{D_2}{D_0}, \frac{A_3}{A_0} = \frac{D_3}{D_0}, \frac{A_4}{A_0} = \frac{D_4}{D_0}, \tag{29}$$

where the values of $X_0, X_1, X_2, X_3, X_4, Y_0, Y_1, Y_2, Y_3, Y_4, D_0, D_1, D_2, D_3$ and D_4 are given in appendix. Thus, it can be seen that the reflection and transmission coefficients are function of incident angle and the material's properties through which they are propagating.

5. Energy Conservation

In order to validate the numerical results, we will implement energy conservation law at each angle of incident. If the sum of all the energy ratios at each angle of incident becomes almost unity then energy conservation law holds good. Consider the energy partition of various reflected and transmitted wave due to incident quasi-P (qP) wave at the interface $x_2 = 0$. The rate of energy transmission is given by

$$p^{*} = \tau_{22}\frac{dv_2}{dt} + \tau_{23}\frac{dv_3}{dt} \tag{30}$$

The energy equation due to incident wave is given by

$$E_0 = L_0\omega A_0^2 \exp\left(2i\left(x_2\cos\theta_0 + x_3\sin\theta_0 - \beta_0 t\right)\right) \tag{31}$$

where $L_0 = -ik_0\left(Y_0\cos\theta_0 + X_0\sin\theta_0\right)$ and ω is the angular frequency.

The energy ratios of the various reflected and transmitted waves are determined as the ratios of energy of reflected and transmitted wave to the energy of incident wave. The modulus of energy ratios of different reflected and transmitted waves are given as

$$\begin{aligned} \left|\frac{E_1}{E_0}\right| &= \left|\frac{L_1}{L_0}\right|\left|\frac{A_1}{A_0}\right|^2, \left|\frac{E_2}{E_0}\right| = \left|\frac{L_2}{L_0}\right|\left|\frac{A_2}{A_0}\right|^2, \\ \left|\frac{E_3}{E_0}\right| &= \left|\frac{L_3}{L_0}\right|\left|\frac{A_3}{A_0}\right|^2, \left|\frac{E_4}{E_0}\right| = \left|\frac{L_4}{L_0}\right|\left|\frac{A_4}{A_0}\right|^2. \end{aligned} \tag{32}$$

where

$$\begin{aligned} L_1 &= -ik_1\left(-Y_1\cos\theta_1 + X_1\sin\theta_1\right), \\ L_2 &= -ik_2\left(Y_2\sin\theta_2 + X_2\cos\theta_2\right), \\ L_3 &= -ik_3\left(Y_3\cos\theta_3 + X_3\sin\theta_3\right), \\ L_4 &= -ik_4\left(Y_4\sin\theta_4 + X_4\cos\theta_4\right). \end{aligned} \tag{33}$$

Equation (32) shows the energy ratios as a function of reflection and transmission coefficients, material's constants and angle of incident wave. The total energy associated with incident wave partitioned into different reflected and transmitted waves but the total energy remains conserved.

6. Numerical Example and Discussions

The procedure discussed in this study, solves a general mathematical model for reflection and transmission of plane harmonic waves at plane boundary between anisotropic monoclinic and isotropic medium. The expressions derived in the study, define the effect of various parameters on reflection/transmission coefficients, phase velocity and energy partition at the interface. These expressions are implicit and not very easy. Hence the effects of various parameters can only be analyzed through numerical examples. For the numerical simulation of the analytical results, parameters in the monoclinic media have been chosen to match material like Lithium Niobate ($LiNbO_3$) which is a compound of niobium, lithium and oxygen and is mainly used in optical wave guides, mobile phones and piezoelectric sensor etc. Values of the elastic parameters have been taken as

$$\begin{aligned} &C_{22} = 20.3\text{GPa}, C_{23} = 7.5\text{GPa}, C_{24} = -0.9\text{GPa}, C_{33} = 24.5\text{GPa}, \\ &C_{34} = 0\text{GPa}, C_{44} = 0.6\text{GPa}, \rho = 4700\,\text{KG/M}^3. \end{aligned}$$

For isotropic medium, elastic parameters have been taken from earth model 1066B, based on free oscillation frequencies as

$$\begin{aligned} &\lambda = 9.79\text{GPa}, \mu = 6.34\text{GPa}, \rho = 3364\,\text{KG/M}^3. \\ &C_{11}' = C_{22}' = C_{33}' = \lambda + 2\mu, \\ &C_{12}' = C_{13}' = C_{23}' = \lambda, \\ &C_{44}' = C_{55}' = C_{66}' = \mu. \end{aligned}$$

7. Graphical Interpretation

Figures (2) to (5) represent the variations of reflection and transmission coefficients with incident angle. From these figures, it is clear that the behaviour of anisotropic monoclinic elastic materials is absolutely different from isotropic materials. In Fig. (2), the reflection coefficient of reflected qP wave increases rapidly between 2^0 to 5^0. After 5^0 it represents almost sinusoidal nature. The maximum value of reflection coefficient for reflected qP and qSV waves are 2.4 and 1.98 at almost same angle of incidence. The maximum values of transmission coefficients for P and SV waves are 2.36 and 1.81 respectively. These maximum values for transmitted P and SV waves exist when the incident angle becomes almost 30^0 and 25^0 respectively. Fig. (6) exhibits the variation of energy ratios with incident angle. The energy ratios E_1/E_0 and E_2/E_0 represent almost same behaviour. The energy ratio E_3/E_0 decreases between 2^0 to 10^0 and then increases upto 20^0. The value of energy ratio E_3/E_0 becomes maximum and minimum when incident angle becomes about 20^0 and 30^0. Also the energy ratio of refracted SV wave shows almost equal experience between 2^0 to 25^0. The maximum value 1.31 exist when the angle of incident become almost 30^0. In order to show that how the nature of incident wave influence the reflection and transmission phenomena, we have shown the variation of reflection/transmission angle and phase velocity with incident angle for incidence of qP and qSV wave. Fig. (7) and Fig. (10) represents the variation of angle of reflection and transmission with incident angle for incident qP and qSV wave, respectively. From Figs. (7) and (10), it is clear that changes in reflection and refraction angles depends upon the nature of incident wave. Similarly, Figs. (8) and (11) exhibits the variation of phase velocities with incident angle due to incident of qP and qSV wave respectively. The phase velocities of various waves depend upon the nature of incident wave. Figure (9) gives the variation of reflection and transmission coefficients with dimensionless wave number. It is clear that the reflection/transmission angle coefficient remains constant for particular wave as the wave number increases.

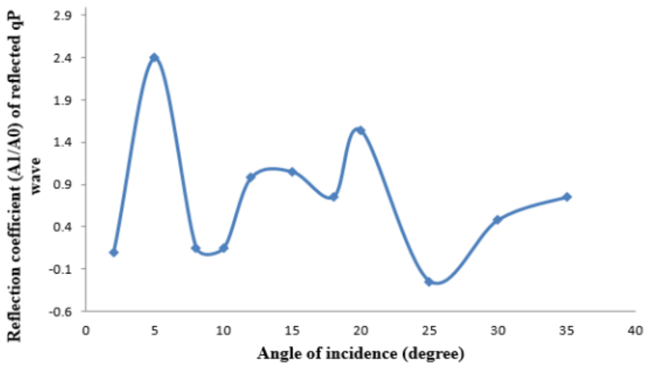

Fig. 2. Variation of reflection coefficient with incident angle for reflected qP wave.

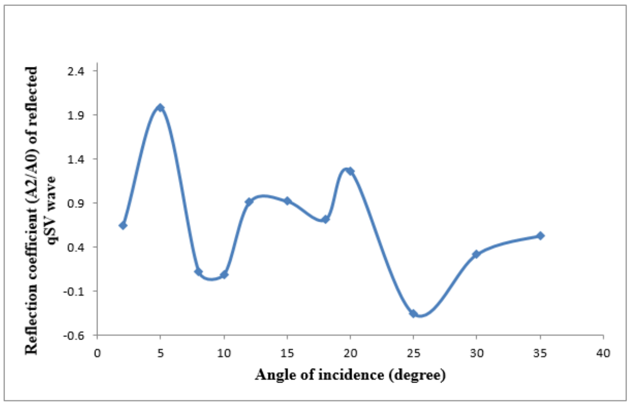

Fig. 3. Variation of reflection coefficient with incident angle for reflected qSV wave.

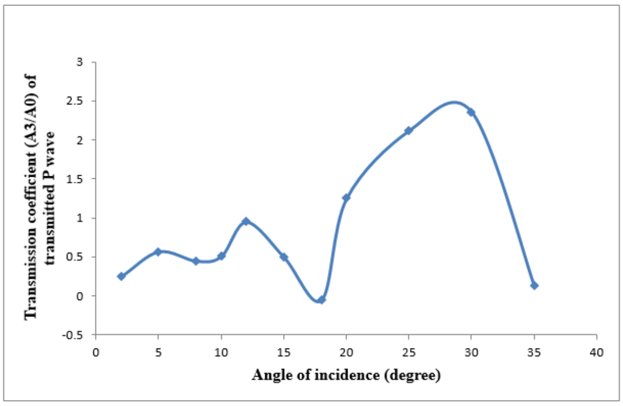

Fig. 4. Variation of transmission coefficient with incident angle for refracted P wave.

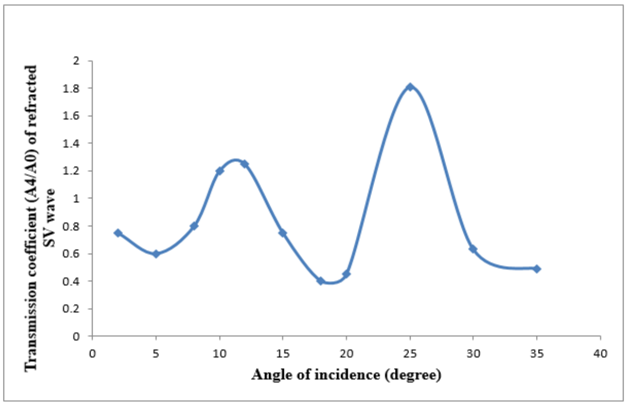

Fig. 5. Variation of transmission coefficient with incident angle for refracted SV wave.

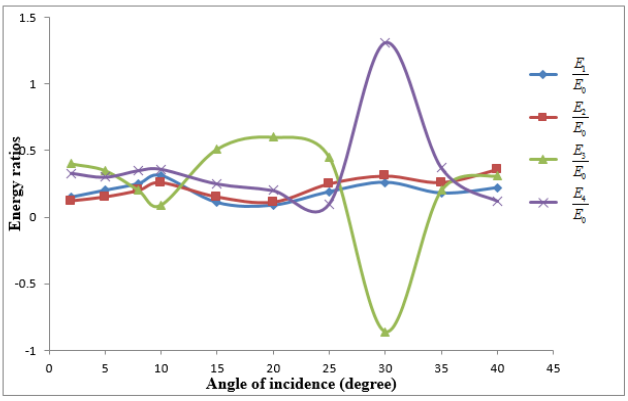

Fig. 6. Variation of energy ratios of different reflected and refracted waves with incident angle.

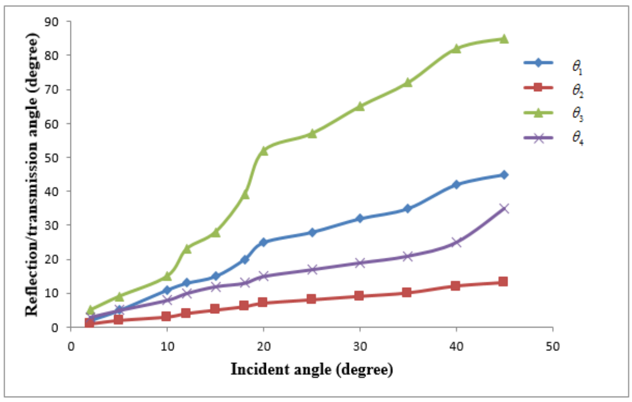

Fig. 7. Variation of reflection/transmission angle with incident angle.

8. Validation of the Study

For validation, energies ratios given below in Table 1 are in well agreement with the Singh and Arora [27].

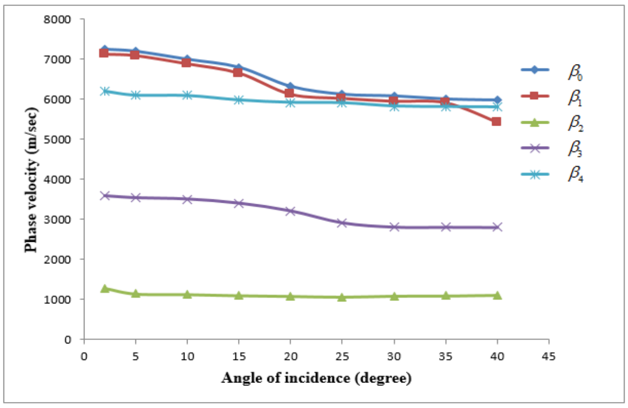

Fig. 8. Variation of phase velocity with incident angle.

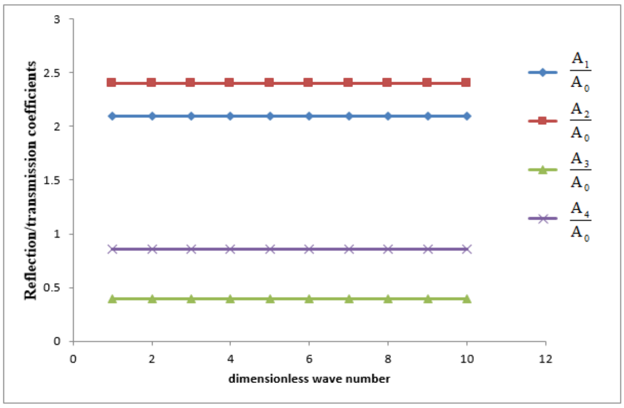

Fig. 9. Variation of reflection/transmission coefficients with wave number.

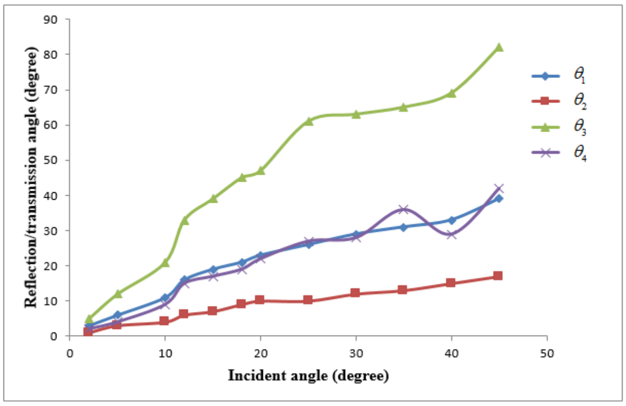

Fig. 10. Variation of reflection/transmission angle with incident angle in case of incident qSV wave.

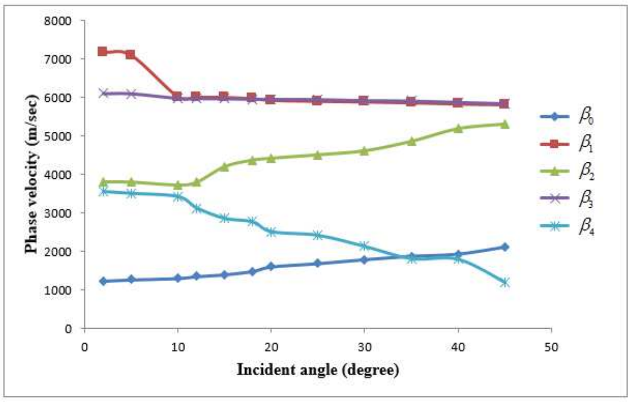

Fig. 11. Variation of phase velocity with incident angle in case of incident qSV wave.

Table 1. Comparison between energy ratios

Incident angle	Energy ratios			
	P_1/P_0	P_2/P_0	P_3/P_0	P_4/P_0
20	0.2	0.18	0.19	0.44
35	0.25	0.19	0.22	0.36
50	0.3	0.3	0.3	0.12
65	0.5	0.2	-0.2	0.6

Summing all these energy ratios at 35∘ ≈ 1. We have seen that the sum of all the energy ratios at 20∘ and 35∘ is approximately unity which validates the law of conservation of energy. In a similar manner, we can show that the energy conservation law holds good at all other angle of incidence

9. Conclusions

Reflection seismology is a tool for investigating the internal structure of the earth as well as for the exploration of valuable materials buried under the earth's surface. These reflected and refracted waves carry much information, which help study the character of the material present on the earth among several other important information. Therefore, the study of reflection and transmission of elastic/seismic waves is of great practical importance in theoretical as well as in observational seismology. In this paper, a closed-form expression for phase velocity of wave propagation in anisotropic monoclinic elastic half-space was obtained. It was observed that the phase velocity of these waves in the monoclinic medium is different from the velocity of waves in isotropic medium. The phase velocity of various reflected and refracted waves depends not only upon the incident angle, elastic constants, and density of the medium as well as the nature of the incident wave. The reflection and transmission coefficients were obtained in the form of a determinant. Also, energy ratios were calculated as a function of reflection and transmission coefficients, and it was found that these energy ratios are directly proportional to the square of reflection and transmission coefficients. Energy conservation law holds good at each angle of incidence. The present study may be useful to geophysicists and metallurgists for the analysis of rock and material structures through Non-Destructive Testing (NDT). Thus, modeling of reflection and transmission of plane waves plays a significantly important role in the exploration of petroleum, earthquake disaster prevention, geophysics, civil engineering, and signal processing.

Author Contributions

The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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Appendix

$$\begin{aligned} X_0 &= C_{42}\cos^2\theta_0 + C_{43}\sin^2\theta_0 + 2C_{44}\sin\theta_0\cos\theta_0, \\ X_1 &= C_{42}\cos^2\theta_1 + C_{43}\sin^2\theta_1 - 2C_{44}\sin\theta_1\cos\theta_1, \\ X_2 &= (C_{43} - C_{42})\sin\theta_2\cos\theta_2 + C_{44}\left(\sin^2\theta_2 - \cos^2\theta_2\right), \\ X_3 &= 2C_{44}'\sin\theta_3\cos\theta_3, X_4 = C_{44}'\left(\sin^2\theta_4 - \cos^2\theta_4\right), \end{aligned} \tag{A.1}$$

$$\begin{aligned}
&Y_0 = C_{22}\cos^2\theta_0 + C_{23}\sin^2\theta_0 + 2C_{24}\sin\theta_0\cos\theta_0,\\
&Y_1 = C_{22}\cos^2\theta_1 + C_{23}\sin^2\theta_1 + 2C_{24}\sin\theta_1\cos\theta_1,\\
&Y_2 = (C_{23} - C_{22})\sin\theta_2\cos\theta_2 + C_{24}\left(\sin^2\theta_2 - \cos^2\theta_2\right),\\
&Y_3 = C_{22}'\cos^2\theta_3 + C_{23}'\sin^2\theta_3, Y_3 = \left(C_{22}' - C_{23}'\right)\sin\theta_4\cos\theta_4,
\end{aligned} \tag{A.1}$$

$$D_0 = \begin{vmatrix} \frac{\cos\theta_1}{\cos\theta_0} & -\frac{\sin\theta_2}{\cos\theta_0} & \frac{\cos\theta_3}{\cos\theta_0} & \frac{\sin\theta_4}{\cos\theta_0} \\ \frac{\sin\theta_1}{\sin\theta_0} & \frac{\cos\theta_2}{\sin\theta_0} & -\frac{\sin\theta_3}{\sin\theta_0} & \frac{\cos\theta_4}{\sin\theta_0} \\ \frac{k_1X_1}{k_0X_0} & \frac{k_2X_2}{k_0X_0} & -\frac{k_3X_3}{k_0X_0} & -\frac{k_4X_4}{k_0X_0} \\ \frac{k_1Y_1}{k_0Y_0} & \frac{k_2Y_2}{k_0Y_0} & -\frac{k_3Y_3}{k_0Y_0} & -\frac{k_4Y_4}{k_0Y_0} \end{vmatrix}, \tag{A.2}$$

$$D_1 = \begin{vmatrix} 1 & -\frac{\sin\theta_2}{\cos\theta_0} & \frac{\cos\theta_3}{\cos\theta_0} & \frac{\sin\theta_4}{\cos\theta_0} \\ -1 & -\frac{\cos\theta_2}{\sin\theta_0} & -\frac{\sin\theta_3}{\sin\theta_0} & \frac{\cos\theta_4}{\sin\theta_0} \\ -1 & \frac{k_2X_2}{k_0X_0} & -\frac{k_3X_3}{k_0X_0} & -\frac{k_4X_4}{k_0X_0} \\ -1 & \frac{k_2Y_2}{k_0Y_0} & -\frac{k_3Y_3}{k_0Y_0} & -\frac{k_4Y_4}{k_0Y_0} \end{vmatrix}, \tag{A.3}$$

$$D_2 = \begin{vmatrix} \frac{\cos\theta_1}{\cos\theta_0} & 1 & \frac{\cos\theta_3}{\cos\theta_0} & \frac{\sin\theta_4}{\cos\theta_0} \\ \frac{\sin\theta_1}{\sin\theta_0} & -1 & -\frac{\sin\theta_3}{\sin\theta_0} & \frac{\cos\theta_4}{\sin\theta_0} \\ \frac{k_1X_1}{k_0X_0} & -1 & -\frac{k_3X_3}{k_0X_0} & -\frac{k_4X_4}{k_0X_0} \\ \frac{k_1Y_1}{k_0Y_0} & -1 & -\frac{k_3Y_3}{k_0Y_0} & -\frac{k_4Y_4}{k_0Y_0} \end{vmatrix}, \tag{A.4}$$

$$D_3 = \begin{vmatrix} \frac{\cos\theta_1}{\cos\theta_0} & -\frac{\sin\theta_2}{\cos\theta_0} & 1 & \frac{\sin\theta_4}{\cos\theta_0} \\ \frac{\sin\theta_1}{\sin\theta_0} & \frac{\cos\theta_2}{\sin\theta_0} & -1 & \frac{\cos\theta_4}{\sin\theta_0} \\ \frac{k_1X_1}{k_0X_0} & \frac{k_2X_2}{k_0X_0} & -1 & -\frac{k_4X_4}{k_0X_0} \\ \frac{k_1Y_1}{k_0Y_0} & \frac{k_2Y_2}{k_0Y_0} & -1 & -\frac{k_4Y_4}{k_0Y_0} \end{vmatrix}, \tag{A.5}$$

$$D_4 = \begin{vmatrix} \frac{\cos\theta_1}{\cos\theta_0} & -\frac{\sin\theta_2}{\cos\theta_0} & \frac{\cos\theta_3}{\cos\theta_0} & 1 \\ \frac{\sin\theta_1}{\sin\theta_0} & \frac{\cos\theta_2}{\sin\theta_0} & -\frac{\sin\theta_3}{\sin\theta_0} & -1 \\ \frac{k_1X_1}{k_0X_0} & \frac{k_2X_2}{k_0X_0} & -\frac{k_3X_3}{k_0X_0} & -1 \\ \frac{k_1Y_1}{k_0Y_0} & \frac{k_2Y_2}{k_0Y_0} & -\frac{k_3Y_3}{k_0Y_0} & -1 \end{vmatrix}. \tag{A.6}$$

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