Unsteady Separated Stagnation Point Flow of Nanofluid past a Moving Flat Surface in the Presence of Buongiorno's Model

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Abstract. This paper explores energy and mass transport behavior of unstable separated stagnation point flow of nanofluid over a moving flat surface along with Buongiorno’s model. Characteristic of Brownian diffusion and thermophoresis are considered. Additionally, characteristics of chemical reaction is taken into account. A parametric investigation is performed to investigate the outcome of abundant parameters such as temperature, velocity and concentration. An appropriate equation is converting into a set of ODEs through employing appropriate transformation. The governing equations has been solved numerically by using the classical fourth-order Runge-Kutta integration technique combined with the conventional shooting procedure after adapting it into an initial value problem. Our findings depict that the temperature field \(\theta(\zeta)\) improves for augmenting values of thermophoresis parameter (Nt) with dual solutions of attached flow without inflection and flow with inflection. Also, the difference of Brownian motion parameter (Nb) with two different solutions of attached flow exists with energy profile. It can be found that an energy profile \(\theta(\zeta)\) elevates due to augmenting values of (Nb). It has been perceived that thermal boundary layer thickness elevates due to large amount of Brownian motion parameter (Nb).

Keywords: Stagnation point; Nanofluid; Buongiorno’s model.

1. Introduction

Recently, the study of stagnation point flow has an abundant application in many areas including food processing, paper production, polymer extrusion and heat exchanger. Bai et al. \([1]\) presented the MHD Maxwell nanofluid adjoining the stagnation point flow over a stretching surface in the presence of thermophoresis behavior. Wang \([2]\) considered the non-alignment of the stagnation flow towards shrinking sheet. He found that the convective heat transport declines for shrinking rate via elevates in the thickness of boundary layer. Rajappa \([3]\) addressed the unstable plane stagnation point flow along with mass transport. He concluded that the tough blowing at the wall deduced the rate of energy transport and the wall shear stress as well. Saif et al. \([4]\) analyzed the stagnation point flow of second grade nanomaterial towards a nonlinear stretching surface associate with variable surface thickness and energy transport manner is noticed due to the impact of mixed convection and melting heat. They concluded that the concentration and energy fields are hiked with greater thermophoresis parameter.

Nanofluid concept is very first initiated by Choi \([5]\), the mixture of nanometer sized particles suspended in a conventional liquids are ethylene glycol, water or oil. The advantage of this concept is less volume of nanoparticles dispersed in host fluid displays prominent enhancement on the thermal conductivity for energy transport manner. Few wide studies are related to nanofluid flows are available in Refs. \([6, 7]\). Buongiorno \([8]\) expanded a nanofluid model integrating the influence of thermophoresis and Brownian diffusion. Mustafa et al. \([9]\) addressed the nanofluid flow neighboring a stagnation-point towards a stretching surface by employing the Buongiorno model and analytically solved via homotopy analysis method (HAM). Recently, several researchers have analyzed the Stagnation-point flow in different geometries which can be found in Refs. \([10-15]\). Kohilavani et al. \([14]\) reported the similarity solutions of the unsteady stagnation-point flow and heat transport of a special third grade fluid over a permeable stretching/shrinking sheet. They found that the stability analysis portrays that the upper branch solution is stable meanwhile the lower branch solution is not stable. Yasin et al. \([15]\) examined the influence of Joule heating, viscous dissipation, partial velocity slip of a viscous and electrically conducting fluid neighborhood of the stagnation point on a stretching/shrinking sheet associated with effect of magnetic field. They found the results for the shrinking sheet there exist...
non-unique solutions (dual solutions) in particular range of parameters and for the stretching sheet the solution is unique. Numerical studies on different types of biological fluids have been investigated by several researchers [16-18]. An important objective of present work is dealing with an unsteady separated stagnation point flow of nanofluid past a moving flat surface in the presence of energy and mass transport. Furthermore, the effects of chemical reaction are examined. Here, we employed Buongiorno’s nanofluid model which includes the thermophoresis and Brownian motion parameter. An appropriate equation is converted from PDEs to ODEs via utilizing proper transformation. Finally, the different parameters’ effect on temperature, velocity and concentration are shown in detail.

2. Model Development and Problem Formulation

An unsteady 2-D incompressible flow of nanofluid on the adjoining of stagnation point past a consecutively moving surface along with time dependent external potential flow is considered. The coordinate system and schematic model described in Fig. 1. The schematic system is considered as plate surface in X-axis and Y-axis normal to previous one. The velocity elements \((U^*, V^*)\) are considered in the Cartesian coordinates \((X, Y)\) also time indicated by \(t\). An outer fluid is associated with strain rate volume \(\dot{s}(t) > 0\) effects on this plate surface in Y-axis. An appropriate equation of a nanofluid along with energy and mass transport under the above assumptions are

\[
\frac{\partial U^*}{\partial X} + \frac{\partial V^*}{\partial Y} = 0 \tag{1}
\]

\[
\frac{\partial U^*}{\partial t} + U^* \frac{\partial U^*}{\partial X} + V^* \frac{\partial U^*}{\partial Y} + \frac{1}{\rho} \frac{\partial \rho}{\partial X} = \nu \left( \frac{\partial^2 U^*}{\partial X^2} + \frac{\partial^2 U^*}{\partial Y^2} \right) \tag{2}
\]

\[
\frac{\partial V^*}{\partial t} + U^* \frac{\partial V^*}{\partial X} + V^* \frac{\partial V^*}{\partial Y} + \frac{1}{\rho} \frac{\partial \rho}{\partial Y} = \nu \left( \frac{\partial^2 V^*}{\partial X^2} + \frac{\partial^2 V^*}{\partial Y^2} \right) \tag{3}
\]

\[
\frac{\partial C^*}{\partial t} + U^* \frac{\partial C^*}{\partial X} + V^* \frac{\partial C^*}{\partial Y} = D_{\text{a}} \left( \frac{\partial C^*}{\partial X} + \frac{\partial C^*}{\partial Y} \right) + \rho \frac{\partial^2 C^*}{\partial X^2} + \frac{D_{\text{a}}}{T_c} \left( \frac{\partial^2 C^*}{\partial X^2} + \frac{\partial^2 C^*}{\partial Y^2} \right) - R(C - C_{\text{in}}) \tag{4}
\]

and also boundary conditions are Dholey [19]:
The boundary conditions due to the consideration that the plate is moving in its own plane with a velocity \(U_\infty^*\) which is \(\lambda\) times of the free stream velocity \(U_\infty^*(X,t)\) are given by

\[
U_\infty^* = \lambda U_\infty^*, T^* = T^*\infty, V^* = 0, C^* = C^*\infty \quad \text{at} \quad Y = 0, \tag{6}
\]

\[
U^* = U^*_\infty, T^* = T^*\infty, C^* = C^*\infty, \quad \text{as} \quad Y \rightarrow \infty.
\]

The host equations and conditions are rewritten in the non-dimensional forms, and the used non-dimensional quantities are followed by

\[
\begin{align*}
X &= \frac{X}{L}, & \dot{X} &= \frac{X'}{L}; \\
Y &= \frac{X'}{L}, & \dot{Y} &= \frac{U^*_L}{\nu}; \\
\ddot{X} &= \frac{U''_L}{\nu}, & \ddot{Y} &= \frac{V''_L}{\nu}; \\
\dot{t} &= \frac{t'}{L}, & \dot{F} &= \frac{F'_L}{\rho \nu}; \\
\ddot{t} &= \frac{\dot{t}'}{L}, & \ddot{\psi} &= \frac{\psi'}{\nu}, & \ddot{\psi}_1 &= \frac{\psi_1'}{L}.
\end{align*}
\tag{7}
\]

Fig. 1. Schematic configuration of the considered system.
We employed the appropriate similarity variables which can be written as
\[ U' = a s(t) F'(\zeta), \quad V' = a \sqrt{t} F(\zeta), \quad \zeta = \frac{a}{\sqrt{t(1+y)}}, \quad \theta(\zeta) = \frac{T - T_s}{T_s - T_c}, \quad \phi(\zeta) = \frac{C_s - C}{C_s - C_c}. \]  
(8)

Here, stagnation flow strength indicates \( a > 0 \). Moreover, an external potential flow point difference manipulated fluid flow inside the boundary layer and also it denotes the time function \( t \). Substitute eq. (7) into the governing equations (1)-(5) along with the boundary conditions (eq. (6)) and using above similarity variables, (eq. (8)), the following equations will be obtained:

\[ F''' + aFF'' - \frac{s}{s'} \left[ \frac{1}{2} F''' + \frac{3}{2} F' \right] = 0, \]  
(9)

\[ \theta'' + Pr \left[ aF\theta' - \frac{\lambda}{2} \theta' + N t \theta'' + N b \phi' \right] = 0, \]  
(10)

\[ \phi'' + Sc \left( \frac{\beta}{2} \phi' - aF\phi' + \frac{N t}{N b} \theta'' - ScCr \phi \right) = 0, \]  
(11)

and

\[ F(0) = 0, \quad \theta(0) = 1, \quad F'(0) = \lambda, \quad \phi(0) = 1, \quad F'(\infty) \to 1, \quad F(\infty) \to (\zeta - \delta), \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0. \]  
(12)

From eq. (9), \( \frac{s}{s'} = \beta \),  
(13)

Let us consider \( \beta \neq 0 \), necessary condition for comparison of the current study and also integrating above equation as follows

\[ s(t) = \frac{1}{\beta (-t + \tau)} \]  
(14)

Here \( \tau \) is the steady reference value of time \( t \) such that \( s(0) = (\tau)^{-1} > 0 \). If \( \beta = 0 \), eq. (14) produces \( s(t) \) such as unvarying \((\pi 1)\) and also non-dimensional elements of velocity is mentioned in eq. (9) are deduced for stable stagnation-point flow to an inflexible stationary plate highlighted via Hiemenz [13] while \( a = 1 \). Consequently, the dimensionless parameter \( \beta \) calculates the instability power of this current problem and unstable boundary layer flow detaches from stable flow. Employing eq. (13) into eq. (9)

\[ F''' + aFF'' - \beta \left[ \frac{1}{2} F''' - 1 + F' \right] = 0 \]  
(15)

where \( c = -(a + \beta) \) by using the boundary condition and applying in above one

\[ F''' + aFF'' + a - aF'' - \beta \left[ \frac{1}{2} F''' - 1 + F' \right] = 0 \]  
(16)

Consider the analytical solution for \( \beta = 2a \) through employing a function \( G(\zeta) = F(\zeta) - \zeta \) transformation such that

\[ F(\zeta) = \zeta + \frac{1 - \lambda}{\sqrt{3} a + \lambda} \left( e^{\sqrt{3} a \zeta - 1} \right), \quad \text{if} \quad \sqrt{3} a = \lambda > 0. \]  
(17)

For more details refer this article [19] and [20]. The non-dimensional pressure \( P(X, \zeta, t) \) distribution is attained by:

\[ P(X, \zeta, t) = - \left( \frac{a + \beta}{2 b^2} \right) X^2 + \frac{a}{b(t - \tau)} \left[ F'' + \frac{a}{2} F' - \frac{\beta}{2} \sqrt{t - \tau} \right] + P_0(t) \]  
(18)

where \( P_0(t) \) is called stagnation pressure. The stream function \( \psi \) is defined as

\[ \psi = a \frac{X}{\sqrt{b(t - \tau)}} F(\zeta), \quad \zeta = \frac{Y}{\sqrt{b(t - \tau)}}. \]  
(19)

The engineering interest of in heat and mass transfer flow, local skin friction \( C_f \), local Nusselt number \( N_u \) and local Sherwood number \( S_h \) are defined by

\[ C_f = \frac{2}{U'(X, Y)} \frac{\partial U}{\partial Y} \bigg|_{Y=0} = \frac{2}{a X} \sqrt{b(t - \tau)} F''(0), \]  
(20)

\[ N_u = \frac{X}{T_c - T_s} \frac{\partial U}{\partial Y} \bigg|_{Y=0} = -\frac{X}{\sqrt{b(t - \tau)}} \theta'(0), \]  
(21)

\[ S_h = \frac{X}{C_s - C_c} \frac{\partial U}{\partial Y} \bigg|_{Y=0} = -\frac{X}{\sqrt{b(t - \tau)}} \phi'(0), \]  
(22)
3. Result and Discussion

In the current study, the dimensionless ODEs (9)-(11) associated with conditions (eq. (12)) are elucidated numerically via employing RK-method with shooting system. In the current study, our main aim is to notice the impact of different parameters such as velocity ratio parameter $\lambda$, unsteadiness parameter $\beta$, Brownian motion parameter $Nb$, chemical reaction parameter $Cr$, thermophoresis parameter $Nt$, Schmidt number $Sc$, Prandtl number $Pr$ on various field of temperature $\theta(\zeta)$, concentration $\phi(\zeta)$ and velocity $F(\zeta)$ respectively.

Figs. 2(a) and 2(b) show the distinct values of unsteadiness in parameter $\beta$ for an effect of velocity ratio parameter $\lambda$ on velocity profile $F(\zeta)$ and $F'(\zeta)$, respectively. There is formation of an inverted boundary layer $\lambda>1$ along with $\lambda=0$ which denotes the boundary layer having no formation neighboring the surface of plate. The perpendicular element of velocity $F(\zeta)$ is computed for velocity ratio parameter $\lambda$ when $\beta=1$ and $\beta=0$. It is mainly focused the boundary value flow structure done with the traveling flat surface will have increasing $\lambda$. Thus, the velocity $F(\zeta)$ elevates in each Fig. 2(a) and Fig. 2(b). Additionally, $F'(\zeta)$ represented the horizontal velocity elevates along with augmenting values of $\lambda=0$ and $F'(\zeta)$ occur to have an inverted the structure of boundary layer meanwhile $\lambda>1$ considered for attached flow solution.
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Fig. 4. Difference of $\theta(\zeta)$ for dual fixed values of $Pr=0.72$ and 7 when $\beta=0$ and $\lambda=-1.2$.

Fig. 5. Difference of $\phi(\zeta)$ for dual fixed values of $Pr=0.72$ and 7 when $\beta=0$ and $\lambda=-1.2$.

Fig. 6. Difference of $\phi(\zeta)$ for dual fixed values of $Pr=0.72$ and 7 when $\beta=0$ and $\lambda=-1.2$.

Figs. 3 and 4 display the temperature distribution of the characteristics of Brownian motion parameter (Nb) and thermophoresis parameter (Nt) for dual constant value of Prandtl number $Pr$ (= 7 for water at 60°F and 0.72 for air) associated to certain amount of $\beta=0$ and $\lambda=-1.2$ for different attached flow solutions occur. Since, the boundary layer of thermal is higher for flow with an inflection region than the flow without an inflection region. Moreover, the volume of non-dimensional energy slope neighboring the wall, it is larger for flow without an inflection region than the flow with an inflection region. Initially, it can be noticed that thermophoresis parameter have dissimilar values and its leads to different rate of heat transport are explored in Fig. 3. An energy profile $\theta(\zeta)$ improves with augmenting amount of thermophoresis parameter (Nt) with dual solutions of attached flow without inflection and flow with inflection. It is perceived that the fluid particles and greater thermophoretic force transports from extreme high (warm) to extreme low (cool) temperature core. Secondly, the difference of Brownian motion parameter (Nb) with two different solutions of attached flow exist on temperature profile. It can be found that an energy field $\theta(\zeta)$ elevates due to augmenting values of Nb. It has been perceived that thermal boundary layer thickness elevates via large amount of Brownian motion parameter (Nb).
Figs. 5 and 6 demonstrate the concentration distribution of the behavior of chemical reaction parameter (Cr) and Schmidt number (Sc) for dual constant value of Prandtl number Pr (= 7 for water at 60°F and 0.72 for air) associated to certain amount of β=0 and λ =−1.2 for different attached flow solutions occur. We noticed that the enhancement of concentration field by increasing value of Schmidt number is presented in Fig. 5. It is led to the concentration decrement and the mass boundary layer thickness increment. Moreover, chemical reaction parameter Cr effect on mass distribution profile is portrayed in Fig. 6. While increasing Cr, the mass profile improves and gives the thinner mass boundary layer thickness.

Figs. 7 and 8 characterize the streamlines flow patterns for different λ over the moving flat surface when a=0.5, 1 and 1.5. The flow pattern reveals the contour of the stream function ψ is mentioned in (19) for different λ when a=0.5 and (t−t) = 1. It can be found that the streamlines with higher values of λ are minor than the consistent streamlines of λ=0 and decreasing values of λ. Actually, the streamlines are shifted gradually toward the plate surface with an increasing value of λ, and this trend persists even after the value of λ=1 for which no boundary layer is formed above the plate surface since F″(0) = 0. This indicates the strong attachment of the momentum boundary layer flows to the plate surface which leads to the reduction of the boundary layer thickness as well as to yield the attached flow solution of this flow problem for any given value of λ>0. Moreover, no non-zero saddle point exists within the flow for a positive value of λ. On the other hand, a non-zero saddle point is found inside the flow for a negative value of λ (see Figs. 8(c) and 8(d)). It is noticeable that this saddle point together with the horizontal dividing streamline (ψ = 0) moves away from the plate surface with the increase of |λ| when λ < 0.

In fact, this result has a close relation with the velocity F(t) which decreases continuously with the increase in |λ| when λ < 0 but up to the critical value of λc. As a result, the reverse flow zone of F(t) increases so that F(t) becomes zero far away from the plate surface which ensures the detachment of the boundary layer flow to the plate surface. Thus, we see that the detachment of the boundary layer flow enhances with an increasing value of |λ| when λ < 0 and ultimately it must be off the plate surface after a certain negative value of λ depending on the values of β. Finally, we conclude that a positive value of λ, irrespective of the value of β, has a strong stabilizing influence on the present unsteady flow dynamics as there exists no non-zero saddle point in this case. This set of graphs indicate the attached flow with greater λ<0, although it is disconnected from the surface of plate associate to lower λ<0 comparative to the stable stagnation-point flow λ=0. Finally, the streamlines are isolated continuously with every values of velocity ratio parameter λ is displayed from Figs. 7 and 8, respectively.

4. Conclusion

The current study deals with the unsteady stagnation point flow of nanofluid past a moving flat surface along with time dependent outer inviscid flow velocity for fixed surface temperature. Characteristics of thermophoresis, Brownian motion and chemical reaction parameter are taken under the consideration. The governing system converted into a system of ODEs employing similarity transformations and the results are carried out through using RK-method along with shooting technique. An important outcome is highlighted:

- F’(t) represented the horizontal velocity elevates along with augmenting values of λ>0 and F’(t) occur to have an inverted the structure of boundary layer meanwhile velocity ratio parameter λ >1 considered for attached flow solution.
- Streamlines pattern represents the flow remains attached with greater λ>0, and it is disconnected from the surface of plate for lower λ<0 then comparative to the stable stagnation-point flow λ=0.
- An energy profile θ(t) improves while augmenting thermophoresis parameter (Nt) with dual solutions of attached flow without inflection and flow with inflection.
- Concentration distribution are increasing the function of chemical reaction parameter Cr and decreasing functions of Schmidt number Sc.
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(a) $\lambda = 3, F''(0) = -5.477225576$

(b) $\lambda = 0, F''(0) = 2.121320344$

(c) $\lambda = -0.1, F''(0) = 2.307379466$

(d) $\lambda = -0.5, F''(0) = 3.0$

Fig. 8. Streamlines depicting attached flow for $\lambda \geq 0$ and reverse flow for $\lambda < 0$ when $\beta = (r-1) = 1$ and $\alpha = 1.5$. Streamline values change from $-2$ to $2$ in unit increments.

Author Contributions

The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Nomenclature

- $\alpha$: Thermal diffusivity
- $\nu$: Kinematic viscosity
- $R$: Chemical reaction rate
- $D_T$: Thermophoresis diffusion coefficient
- $T_w$ and $T_\infty$: Wall and ambient fluid temperature
- $\delta_1$: Measure of the displacement thickness
- $\psi$: Stream function
- $L$: Length scale
- $r$: Propositional of capacity of nanoparticle heat and capacity of fluid heat
- $C_w$ and $C_\infty$: Wall and ambient fluid nanoparticle concentration

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