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Research Paper

Analysis and Optimization of Truss Structures, Constrained Handling using Genetic Algorithm

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Abstract. In this study, an attempt is made to minimize the weight of Howe roof and ten member-6 Node trusses, separately. Two constraints, maximum allowable deflection and maximum allowable member stresses have been considered. For the first truss, permissible deflection is not known from the literature; therefore, it is determined using the exhaustive search method. Once magnitudes of the constraints are identified, member cross-sectional areas are varied to get the optimal weight. Both the exhaustive search method and the genetic algorithm have been implemented for this purpose. During the optimization, members tending to form a string may be eliminated from the structure. Doing this, we could further reduce the weights of the trusses and even less than the minimum available in the literature. The second truss is an indeterminate structure, and Maxwell Betti reciprocal theorem is applied to calculate the member forces. Also, further reduction of members is made for this truss, keeping in mind that the truss becomes determinate with the decrease in the member(s).

Keywords: Trusses, Constrained optimization, Exhaustive Search, Genetic Algorithm, Maxwell Betti Theorem.

1. Introduction

Structural optimization of truss structure has always been a matter of concern for low material consumption and ease of transportations to on-site locations. Some optimization techniques are used for this purpose like; particle swarm optimization, simulated annealing, ant colony optimization, genetic algorithm (GA), etc. Here, we shall discuss the constrained weight optimization of Howe Roof Truss and ten member-6 Node truss.

Schmit and Miura [1] developed software ACCESS 1 (a combination of FEM and mathematical programming) to analyze the structural members. Optimality criteria method has been proposed by Rizzi [2]. It focusses on eliminating non-active constraints using the Gauss-Seidel iterative method. Stress and displacement constraints were imposed on the structure. Topology optimization of trusses was carried out by Ringertz [3]. FEM was implemented on initial configuration, and its results are used to formulate a nonlinear programming problem to get the optimal solutions. Later on, Harmony search (HS) algorithm has been proposed by Lee and Geem [4] and Lee et al. [5]. The big Bang-Crunch algorithm has been suggested by Camp [6]. This methodology deals with both discrete and continuous variable optimization. ACO has been proposed by Kaveh and Shojaee [7] and Luh and Lin [8]. Here, the primary purpose was to eliminate the weaker sections. Differential evolution (DE) has been proposed by Wu and Tseng [9]. It implies penalty-based, self-adaptive strategy for reduction of infeasible solutions. A combination of cellular automata and linear programming was used by Faramarzi and Afshar [10]. It is a two-phase algorithm. The first phase deals with the topological aspect and second phase deal with the sizing element.

The PSO has been proposed by Li et al. [11] for pin-connected trusses involving discrete variables. Later on, Li et al. [12] conglomerated PSO with HS and applied to both 2D and 3D truss members. A hybrid PSO and swallow swarm optimization (SSO) algorithm has been proposed by Kaveh et al. [13] and divided the entire population into sub-colonies. Recently, a new technique called water evaporation optimization has been proposed by Kaveh and Bakshpoori [14] for structural optimization. Grammatical evolution has been proposed by Fenton et al. [15]; it is an extension of genetic programming (GP). Apart from finding the minimum cross-sectional area, it also focuses on knowledge of section geometry and orientation. GP alone has been proposed by Assimi et al. [16] for simultaneous optimization of sizing and topology of trusses and eliminate the redundant members and joints.

GA has been used by a number of investigators [17-24]. Rajeev and Krishnamoorthy [17] proposed a penalty-based transformation of GA, in which the penalty parameter depends on the degree of constraint violation. They emphasized only on the discrete member areas. Hajela and Lee [18] considered kinematic stability at the beginning to generate a stable structure. Later on, they reduced the member sizes to minimize the weight using GA. Coello et al. [19] used GA to generate discrete values of the cross-sectional areas of truss members. Erbatur et al. [20] used GA for the optimal design of planar and space structures. They transformed the constrained optimization problem to unconstrained one with the help of penalty terms. Real-coded GA has been put forward by Deb and Gulati [21] and applied on 2D planes and 3D space trusses. Penalty based approach was used to formulate

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the problem and the concept of basic and non-basic nodes have been suggested. Basic nodes are those that are required in the structure. Non-basic nodes are those whose existence can be neglected. As a result, it has reduced the computational time compared to FE based analysis and avoided solutions excluding the duplicate members. However, they have not neglected the negative cross-sectional areas of the members, which is quite an unrealistic one. Two processes are mentioned in research carried out by Togan and Daloglu [22] using GA. In order to reduce the size of the problem, they have grouped some of the members and adaptive penalty function approach was used. Bi-population based GA has been proposed by Talaslioglu [23]. It is a modified form of GA and avoids the complexity of multi-population search strategy. Another model has been suggested by Dede et al. [24]. Real-coded GA with restricted range approach has been applied, 25 and 72 bars space trusses, as well as 200 and 940 bars plane trusses, have been considered for analysis. Real-coded, as well as binary-coded GA, have been proposed in this paper. The computational time was found to be less when RRA is used.

There is a number of optimization methods that have been applied for the structural optimization of trusses. Some of the important ones are Bee algorithm [25], vibrating particle system [26], element removal algorithm [27]. Lieu et al. [28] presented adaptive hybrid evolutionary firefly algorithm (AHEFA) for shape and size optimization of truss structures. A mutation scheme has been depicted to differentiate between local and global search criteria.

We are also aiming to minimize the weight of different trusses, maintaining the stress and deflection constraints. Two different trusses have been considered, Howe roof truss and 6-member 10-node truss. The reason behind the choice of these two trusses is of their popularity and usability. Howe roof truss is modelled and analyzed in STAAD. Pro.Vi8 by Parekh et al. [29]. Different span lengths ranging from 7 to 28 metres have been considered. However, no such mathematical problem formulation has been mentioned. Rest of the paper is structured as follows: optimization of weight both the trusses are explained along with the obtained results in Section 2. Some conclusions are made, and future scopes are indicated in Section 3.

2. Weight Minimization of Truss Structure

Weight minimization of two useful trusses (Howe Roof Truss and ten member-6 Node truss) has been carried out in this study. Initially, an analysis will be made to find out the maximum deflection that the structure would be able to withstand under a specific loading pattern. After that, the problem will be formulated as constrained optimization one and solutions will be searched using two methods: Exhaustive search approach and Genetic Algorithm. Further reduction in weights will be made eliminating the redundant/unimportant members and nodes of the structure.

2.1 Optimization of Howe Roof Truss

A practical example of Howe Roof truss is seen in Sealdah Railway station, India. The model used for this study is shown in Fig. 1. Structural steel grade A36 is used for Howe Roof Truss, and its properties are mentioned below:

- σ_{ut} 400-550 MPa
- E 200 GPa
- σ_{yt} 250 MPa
- σ_{yc} 152 MPa
- FOS 20

The process followed to achieve the minimized weight is summarized in Fig. 2.

Step 1: Determination of Allowable Deflection

From a 3-D arrangement, as shown in Fig. 1, we have converted to 2-D configuration, as shown in Fig. 3. There exist six such Howe roof trusses usually (refer to Fig. 1), and they run parallel to each other. The concrete is assumed to have 4-4-2 metres tributary load distribution. The density of the concrete is considered to be $\gamma = 23.6$ kN/m^3. Slab thickness (t) is taken to be 0.127m. Each tributary rests on a beam, exerting UDL on it. By action-reaction force, reactions provided by beam acts as a load on the truss. The beam is continuous and hence becomes indeterminate. Clapeyron's theorem of three moments is applied to calculate reactions forces. The same forces act as a point load on the truss joints. Later, the load is assumed to vary in the proportion of weight factors x_1 and x_2, as shown in Fig.3. With variations in x_1 and x_2, load P also varies, such that total weight is conserved. Maximum deflection occurs if the load is concentrated towards the nodes U, P’ and W’ as shown in Fig. 3.

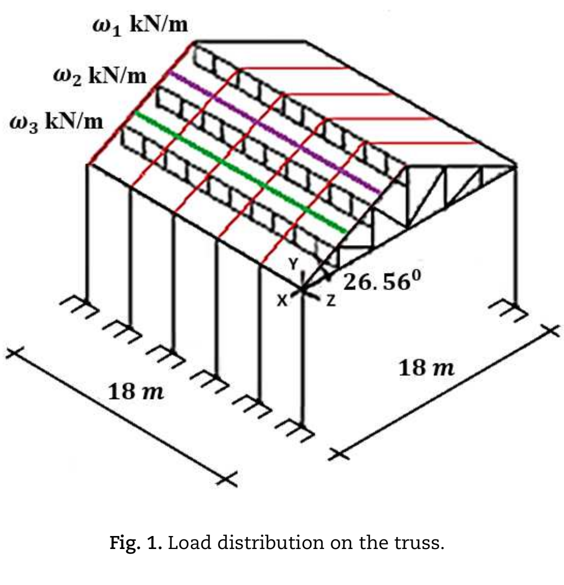


Fig. 1. Load distribution on the truss.

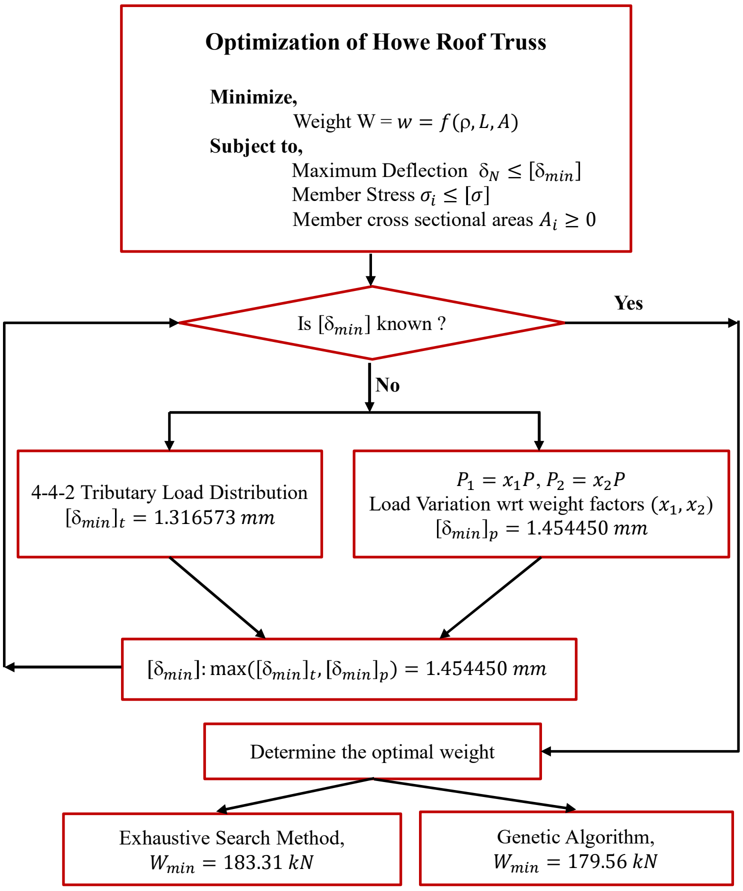


Fig. 2. Flowchart for optimization of Howe roof truss.

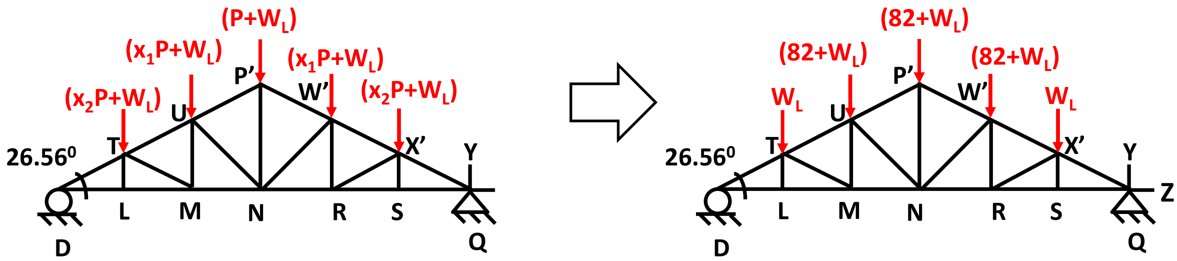


Fig. 3. Point Loads acting on the truss (all values are kN) wrt x_1 and x_2.

Wind load (W_L) has also been considered in the design analysis of Howe roof truss. IS (Indian Standards): 875-1987 Part-3 has been refereed to calculate the wind load. As per the IS standards, the Howe roof truss used in the present study falls under Category 4, Class A domain. The plan area of the truss structure, as shown in Fig. 1 is calculated to be 324 m^2. The basic wind speed as per clause 5.2 and wind pressure as per clause 5.4 has been calculated to be as 50 m/s and 960 N/m^{2,} respectively. The external and internal pressure coefficients are (-0.8) and (-0.2), respectively. It is to be noted that there are a total of 6 truss structures that are parallel to each other, as shown in Fig. 1. As a result, the wind load that acts on each node of the truss is calculated to be 6.2208 kN. Wind load inherently acts in an upward direction. Hence, fasteners or holding down bolts are designed to provide an equal and opposite reaction force on the truss, as shown in Fig. 3 so that the wind load can be resisted.

For a given load configuration, maximum allowable deflection is [δ] = 1.454450 mm.

Step 2: Minimization of the weight of the structure

The structure has ten diagonal members, six horizontal members and five vertical members. Cross-sectional areas of diagonal, horizontal and vertical members are denoted by A_1, A_2 and A_3 respectively. The problem is to,

$$\text{Minimize, } W = 2.702A_1 + 1.377A_2 + 1.032A_3 \quad MN. \tag{1}$$

Subjected to,
From Allowable Deflection Criteria:

$$\left(\frac{2.788}{A_1} + \frac{2.328}{A_2} + \frac{0.249}{A_3}\right)10^{-2} \leq 1.454450\ \ mm \tag{2}$$

From Allowable stress Criteria:

$$\left.\begin{aligned} &\frac{0.309866}{A_1} \le 7.6\ kPa \ \ \text{(Diagonal member DT: Compression)} \\ &\frac{0.277165}{A_2} \le 12.5\ kPa \ \ \text{(Horizontal member DL: Tension)} \\ &\frac{0.110849}{A_3} \le 12.5\ kPa \ \ \text{(Vertical member NP: Tension)} \end{aligned}\right\} \tag{3}$$

and $A_1, A_2, A_3 \ge 0$

From equation (3), we get $[A_1, A_2, A_3] \ge [0.040771, 0.022173, 0.008867]$ m^2. However, considering the limiting area values as derived from eq (3), does not satisfy the eq (2) and demands to increase them. Firstly, we have considered all the areas are the same and equals to the maximum of all the values derived from equation (3). It has been observed that since all areas are equal, then they should be greater than 0.040771 m^2. Keeping the area on the conservative side, we assume the maximum area should be A_{max}= 0.0408 m^2, and the minimum weight corresponding to the same is W_{min}=208.60 kN.

2.1.1. Approach 1: Exhaustive Search Method

The problem stated in Section 2 is a constrained nonlinear optimization problem. It has one objective function as expressed in equation (1), one subjective constraint expressed in equation (2), subjective constraints as expressed in equation (3) and three variables A_1, A_2 and A_3. Equation (3) gives rise to limiting value of all the areas, and maximum of them are considered to be A_{max}. However, we want to reduce them further for horizontal and vertical members. Here two weight factors x_1 and x_2 have been considered and varied between [0, 1] in a way that, $Ai=xi \times A_{max}$. Therefore, the present problem will have only two variables x1 and x2 and both of them are varied systematically from 0 to 1 with an increment of 0.0001. Weights and deflection values for all those values have been noted. The solutions that satisfy equation (2) are considered as feasible solutions, and they are sorted according to the descending order of weight values to find the optimal solution.

The optimal result is obtained as follows: W_{min}=183.31 kN, $[A_1, A_2, A_3]$ = [0.0408, 0.0408, 0.0163] m^2.

2.1.2. Approach 2: Genetic Algorithm for Howe Roof Truss

Accuracy of the exhaustive search method depends on its increment value. If incremental values of the variables are decreased, accuracy will be less, but the increase will cause an enormous increase in the computational time. Therefore, in the present method, a global search technique has been applied.

The genetic algorithm starts with a couple of initial solutions named as populations. Each population is a candidate solution and goodness of each population is made equal to the objective function value. The present problem has one objective (minimize the weight). However, GA can only solve in principle the maximization problem. Therefore, objective was to maximize the inverse of the truss weight. Also, there are four subjective constraints, violation of those subjective constraints are assigned a penalty. If constraints are violated, objective function value will be reduced. The objective function for GA is expressed in equation (4). Once the goodness of each function is evaluated, they are then modified with three operators namely reproduction, crossover and mutation. Goodness evaluation followed by reproduction, crossover and mutation is named as one generation. GA progresses generation after generation until the termination criteria are reached. In the present study, termination criteria are considered as a pre-specified high value of generation.

$$\text{Maximize,} \quad (1/W) + C_1(\delta_{max} - \delta) + C_2(\sigma_{max1} - \sigma_1) + C_3(\sigma_{max} - \sigma_2) + C_4(\sigma_{max} - \sigma_3) \tag{4}$$

where,

δ_{max}=1.454450 mm; σ_{max1}=7.6 kPa; σ_{max2}= σ_{max3}=12.5 kPa

$$\left(\frac{2.788}{A_1} + \frac{2.328}{A_2} + \frac{0.249}{A_3}\right)10^{-2} \le 1.454450\ mm$$

$$\sigma_1 = \frac{0.309866}{A_1}\ kPa \qquad \sigma_2 = \frac{0.277165}{A_2}\ kPa \qquad \sigma_3 = \frac{0.110849}{A_3}\ kPa$$

and C_1, C_2, C_3, C_4 are constants.

The performance of GA depends on its parameters. Therefore, a parametric study is carried out, and a binary coded GA with 10 bits for each variable is used to minimize the weight of the truss structure.

Best results are obtained with the following parameters (refer to Fig. 4):

Population size =100	Mutation rate = 0.014	Max iterations =300
A_1= 0.041038 m^2	A_2= 0.038109 m^2	A_3= 0.015648 m^2

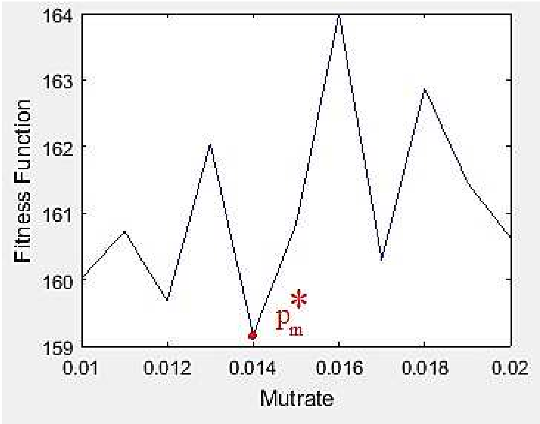
Minimum weight = 179.56 kN. As compared to the exhaustive search method, the weight has reduced **by 2.045 %.**

2.2 Optimization of 10 Member-6 Node Truss

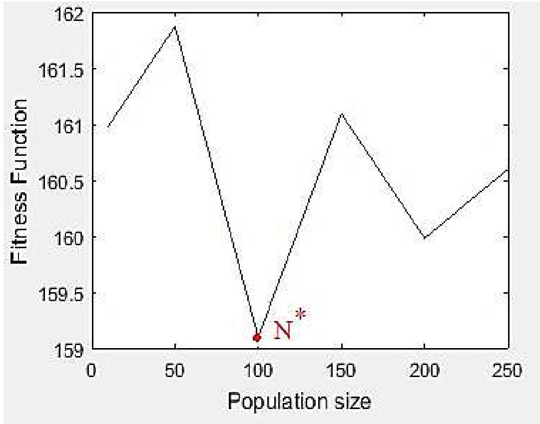
The optimized weight of this structure is available in the literature. The material is considered to be Aluminium along with the following properties:

- [σ] 172.36 MPa (25 ksi)
- E 68.94 GPa (10000 ksi)
- [δ] 50.8 mm (2 inches)
- ρ 2768 kg/m^3 (0.1 $lb/inch^3$)

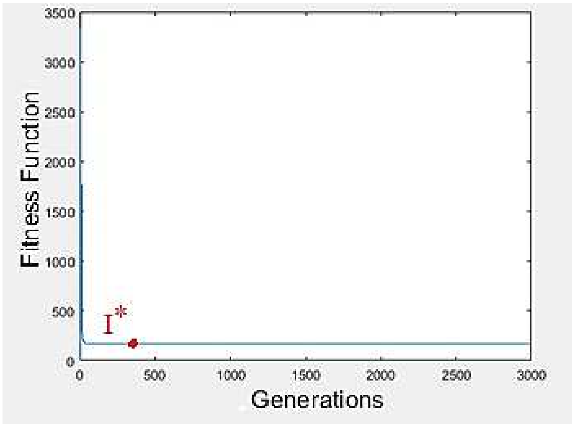
This is an indeterminate structure with a degree of indeterminacy being two. Maxwell Betti reciprocal theorem (force method) has been applied to calculate the support reactions. The schematic of this structure is as shown in Fig. 5.



(a) Fitness vs Probability of mutation



(b) Fitness vs Population size



(c) Fitness vs Generations

Fig. 4. Results of a parametric study of GA for Howe Roof Truss.

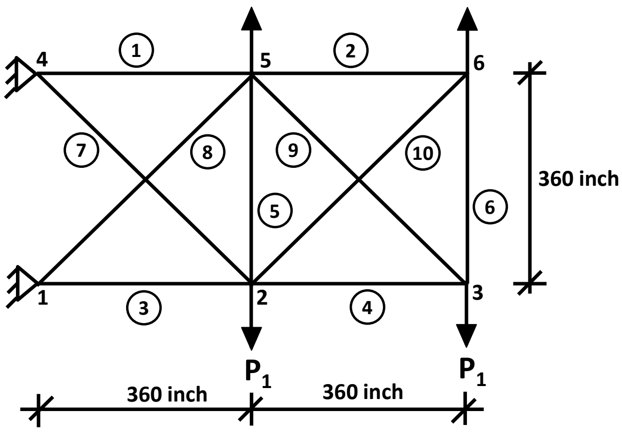


Fig. 5. A 10 member-6 node truss statically indeterminate truss.

The problem can be formulated as follows:

Minimize, (5)

Weight $W = f(\rho, L, A)$

Subject to

Maximum deflection $\delta_N \leq [\delta_{\min}]$

Member Stress $\sigma_i \leq [\sigma]$

Member cross sectional areas $A_i \geq 0$

Two different methods, exhaustive search and genetic algorithm have been applied to obtain the lowest weight under two loading conditions:

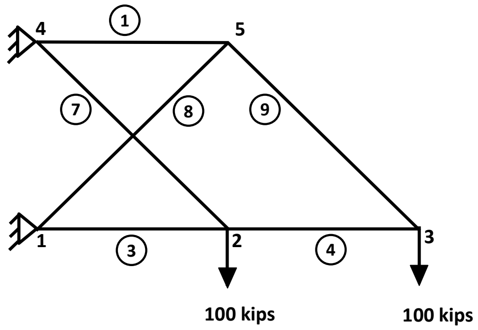


Fig. 6. Reduced truss structure for load case 1.

2.2.1. Load Case 1: P_1=444.822kN, P_2=0

Initially, considering equal cross-sectional areas, we get $A_{max} \geq 0.0127$ m^2 and weight for this situation =3749.46 kg. Later, using exhaustive search method and considering different cross-sectional areas for various members, lowest weight = 3387.602 kg. It has been noted that weight of truss reduced by 9.65% as compared to the condition where all the members have equal cross-sectional areas. Further reduction in weight is achieved by applying GA. The best result is achieved by following GA parameters: Population size =500, Max iterations =800, Mutation rate =1e-05, which gave Minimized weight = **2269.676 kg** with the member cross-sectional areas mentioned below:

- A_1=0.0186
- A_2=6.4516e-9
- A_3=0.01614
- A_4=0.009081
- A_5=6.4516e-9
- A_6=6.4516e-9
- A_7=0.00491
- A_8=0.01336
- A_9=0.0141
- A_{10}=6.4516e-9

The areas mentioned above are in m^2. During this study, areas are varied between 6.4516e-9 to 0.0225806 m^2 (1e-05 to 35 inch^2). It has been observed that irrespective of population size, iteration numbers and mutation rate, members A_2, A_5, A_6 and A_{10} always hit the lower limit. It indicates that the stresses induced are almost negligible in those members, and they tend to form a string. They are redundant members and may be removed from the truss structure.

Now the problem formulations deal with six variables. The overhead truss (refer to Fig. 6) is a determinate structure. Once again, the method of joints is applied to calculate the member forces. The virtual work method is applied to calculate the displacement at nodes. By applying GA following results are achieved, Population size =200, Max iterations =1500, Mutation rate =0.011 for which Minimized weight **= 2216.4117 kg**.

- A_1=0.01895
- A_3=0.01316
- A_4=0.00983
- A_7=0.003688
- A_8=0.01401
- A_9=0.01454

The areas mentioned above are in m^2. Here, the result obtained has crossed the existing benchmark and comparison is presented in Table 1.

Unit of areas is used as Inch^2 and weight as lbs since comparisons mentioned in the literature is in the fps system. The corresponding stresses induced in the members and the displacement values at joints are determined and presented in Table 2.

2.2.2. Load Case 2: P_1=667.2332 kN, P_2=222.411 kN

Correspondingly, results for load case 2 (refer to Fig. 7) have been derived by the same methodology as in Case 1.

Table 1. Optimized cross-sectional areas (sq. inch) for Load Case 1

Element	Schimt and Miura	Rizzi	Ringretz	Lee and Geem	Luh and Lin	Wu and Tseng	Faram-arzi and Afshar	L.J.Li et al.	Kaveh et al.	Kaveh and Bakhsh-poori	Fenton et al.	Fenton et al.	Assimi et al.	Haj-ela, Lee & Lin	Deb and Gulati	Present Study
	AM	OCM	MP	HS	ACO	AMPDE	CA-LP	HPSO	HPSSO	WEO	GE	DO-GE	SOGP	GA	GA	
	[1]	[2]	[3]	[4]	[8]	[9]	[10]	[11]	[13]	[14]	[15]	[15]	[16]	[18]	[21]	
A_1	33.43	30.73	30.1	30.15	29.81	30.378	30.0953	30.3704	30.5384	30.5755	30.5	29.5	30.0996	28	29.68	29.3862
A_2	0.1	0.1	----	0.102	----	0.1	----	0.1	0.1	0.1	0.2	----	----	----	----	----
A_3	24.26	23.93	22.07	22.71	22.24	23.468	22.1321	23.167	23.151	23.3368	23.8	23.6	22.0346	24	22.07	20.4133
A_4	14.26	14.73	15	15.27	15.3	15.196	15.0476	15.183	15.2057	15.1497	17.4	16.8	15.2892	16	15.3	15.24
A_5	0.1	0.1	----	0.102	----	0.1	----	0.1	0.1	0.1	0.1	----	----	----	----	----
A_6	0.1	0.1	----	0.544	----	0.533	----	0.551	0.5489	0.5276	0.2	----	----	----	----	----
A_7	8.388	8.542	6.08	7.541	6.09	7.437	6.0802	7.46	7.4653	7.4458	7.7	6.1	6.0854	6	6.09	5.71173
A_8	20.74	20.95	21.3	21.56	21.44	21.084	21.2806	20.978	21.0644	20.9892	23.1	21	21.2318	21	21.44	21.7285
A_9	19.69	21.84	21.3	21.45	21.24	21.433	21.2806	21.508	21.5294	21.5236	21.9	22.8	21.2227	22	21.29	22.5469
A_{10}	0.1	0.1	----	0.1	----	0.1	----	0.1	0.1	0.1	0.1	----	----	----	----	----
Weight (lbs)	5089	5076.66	4900	5057.88	4899.11	5060.45	4898.31	5060.92	5060.86	5060.99	5287	5056.88	4898.49	4942.7	4899	4886.351

Table 2. Member Stresses and Nodal Displacement in optimized structure for load case 1

Member	Stress (MPa)	Member	Stress (MPa)
1	46.9251	7	170.7128
3	-67.5516	8	-44.8749
4	-45.2411	9	43.2461

Node	Nodal Displacement (mm)	
	x-direction	y-direction
2	-8.9588	-45.7487
3	-14.9588	-50.7791
5	6.2233	-11.901

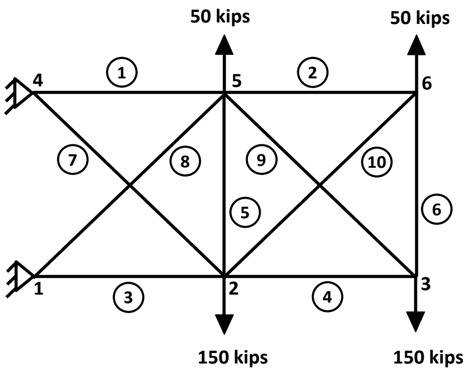


Fig. 7. Schematic for load case 2.

Table 3. Elimination of any two members at a time

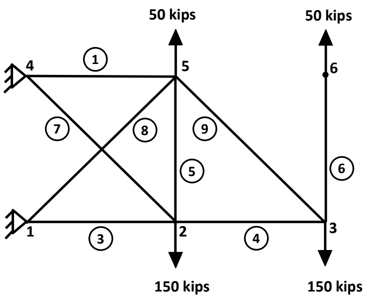
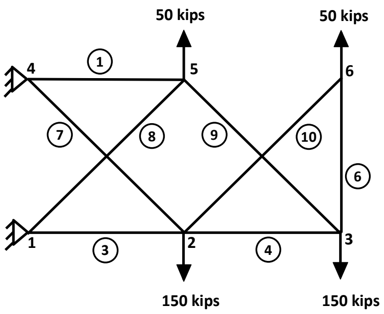
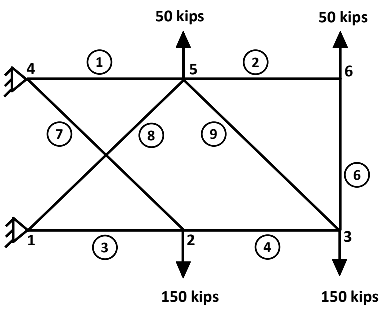
Members Eliminated	Figure	Optimal weight (kg)	Cross-sectional Areas of the members (sq. m.)
2nd and 10th		At node 6, the member is merely an overhanging bar. Because of this force transmission is not possible along with the members.	Not Calculated
2nd and 5th		2041.462	• A_1=0.01538 • A_3=0.015 • A_4=0.00971 • A_6=0.00157 • A_7=0.006116 • A_8=0.008764 • A_9=0.01216 • A_{10}=0.00006451
5th and 10th		2019.180	• A_1=0.015 • A_2=0.00006451 • A_3=0.01519 • A_4=0.009142 • A_6=0.001577 • A_7=0.006116 • A_8=0.009142 • A_9=0.01216

Table 4. Optimized cross-sectional areas (sq. inch) for Load Case 2

Element	Schimt and Miura	Rizzi	Lee and Geem	L.J.Li et al.	Kaveh et al.	Kaveh and Bakhshpoori	Fenton et al.	Fenton et al.	Assimi et al.	Present Study
	AM [1]	OCM [2]	HS [4]	HPSO [11]	HPSSO [13]	WEO [14]	GE [15]	DO-GE [15]	SOGP [16]	
A_1	24.29	23.53	23.25	23.353	23.5238	23.5804	24.7	23.4	24.3983	23.2594
A_2	0.1	0.1	0.102	0.1	0.1	0.1003	0.1	----	0.119	0.1
A_3	23.35	25.29	25.73	25.502	25.3686	25.1582	29	24.8	23.5008	23.5526
A_4	13.66	14.37	14.51	12.25	14.378	14.1801	15.4	15	14.4777	14.1716
A_5	0.1	0.1	0.1	0.1	0.1	0.1002	0.3	----	----	----
A_6	1.969	1.97	1.977	1.972	1.9697	1.9708	2	2	2.0176	2.44526
A_7	12.67	12.39	12.21	12.363	12.3678	12.4511	11.1	9.5	9.4458	9.48104
A_8	12.54	12.83	12.61	12.894	12.7972	12.9349	14.1	14.3	15.0801	14.1716
A_9	21.97	20.33	20.36	20.356	20.3258	20.3595	20.7	20.6	17.3136	18.8621
A_{10}	0.1	0.1	0.1	0.101	0.1	0.1001	0.1	0.1	----	----
Weight (lbs)	4691.84	4676.92	4668.81	4677.29	4676.95	4677.31	4919.5	4612.8	4452.6	4451.5314

Table 5. Member Stresses and Nodal Displacement in the optimized structure for load case 2.

Member	Stress (MPa)	Member	Stress (MPa)
1	44.4643	6	140.982
2	0	7	154.2656
3	-73.1846	8	-34.4021
4	-48.6519	9	51.6944

Node	Nodal Displacement (mm)	
	x-direction	y-direction
2	-16.1582	-50.7889
3	-9.7059	-50.6241
5	5.8969	-15.0219
6	5.8969	-32.0915

(a) Initially, GA has been applied, considering all ten members and varied.
Population size =600, Max iterations =1000, Mutation rate =0.014
Minimum weight=**2644.869 kgs**

- A_1=0.01589
- A_2=0.001702
- A_3=0.020811
- A_4=0.00908
- A_5=0.001513
- A_6=0.003216
- A_7=0.01532
- A_8=0.00586
- A_9=0.014
- A_{10}=0.002081

The areas mentioned above are in m^2.

Here it has been observed that areas A_2, A_5, and A_{10} have meagre value compared to the others. However, the reduction of all those three members leads to the indeterminacy of the structure. Therefore, it will be wiser to eliminate any two members out of them, and the resulting structure is converted to a determinate form without affecting its stability.

Here, the result achieved has again crossed the existing benchmark (refer to Table 4).

Unit of areas is used as $Inch^2$ and weight as lbs since comparisons mentioned in the literature is in the fps system. The corresponding stresses induced in the members and the displacement values at joints are determined and presented in Table 5.

3. Conclusion

Structural optimization of truss structure is an age-old problem. We have attempted the same to check whether further improvement can be made. Fewer researchers tried to identify a low weight truss that can withstand high deflection. Initially, we have formulated the problem as structural optimization problem subjected to two significant constraints. The two limitations are mentioned below.

- The truss must be able to withstand a minimum amount of deflection.
- The maximum value of member stress must be less than their allowable stress in compression or tension.

Both the constraints are equally important. However, the determination of limiting value of deflection of a truss structure is challenging to identify, and it is also necessary to obtain the location where this deflection should occur. We have systematically derived the mathematical formulation and obtained the limiting value of deflection under different load condition.

The tributary length of the concrete slab is indirectly changed by varying the load concerning weight factors x_i. These weight factors take value from [0, 1]. The load is varied along the span of the truss symmetrically. It is observed that when the load is concentrated towards the central region of the structure, deflection increases. For a specific combination of x_i, the configuration obtained is such, that we set the benchmark for limiting value of deflection. During this study, some of the critical observations are mentioned below.

- Once the constraints are known, the weights of two different trusses are optimized using two different methods.

- Howe roof truss is a determinate structure, and hence it is a simple model to analyze. Satisfactory results were obtained by applying the exhaustive search approach. More optimized results were found by using GA. It could even found to be better compared to the existing results.
- On the contrary, ten-member 6-node trusses is an indeterminate structure. Maxwell Betti Reciprocal theorem is applied to calculate the member forces when all ten members were considered. This involves matrix inversions. Preliminary results were obtained by exhaustive search approach. However, as the number of steps increased, it required enormously large computational time. Hence, we had to switch to the Genetic Algorithm.
- We observed that cross-sectional areas of a few members of the second truss were almost negligible compared to others, and hence their existence was meaningless. It was converted to the determinate structure by eliminating a few members, maintaining the truss stability.
- The results obtained were satisfactory, and we had been able to cross the benchmark.

3.1. Future Scope

Present work can be extended in some ways. Some of them are mentioned below.

- This procedure can also be applied to other truss models that have been discussed in the literature.
- Different nature of the cross-sectional area can be studied. This can be characterized by shape optimization. Just like in case of beams, I-section is most preferred since it provides a maximum moment of resistance. Similarly, the effect of shape (circular, square, rectangle, I-section, etc.) of truss member can be analyzed.
- Shape optimization of truss structure can also be carried out. At locations, where space is our constrained, we can apply this concept (refer to Fig. 8).
- Topological optimization of the truss is also an area of interest that can be carried out in future. Connectivity of the nodes can affect the truss-weight, without causing instability of the structure (refer to Fig. 9).

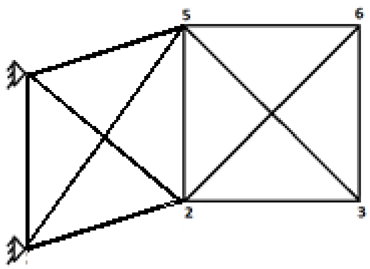


Fig. 8. An Alternate shape of 10 member-6 Node truss.

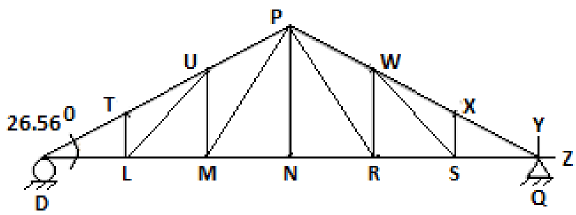


Fig. 9. the alternate topology of Howe roof truss.

Author Contributions

P.R. Sasti Charan developed the mathematical model and examined the validation through simulation; N.B. Hui analyzed the empirical results; J. Paulo Davim examined the theory validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

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Nomenclature

γ	Specific weight	A_1	Area of the diagonal member
δ_i	Deflection for i^{th} node	A_2	Area of the horizontal member
$[\delta]$	Allowable deflection	A_3	Area of the vertical member
ρ	Material Density	A_i	Area of the i^{th} member

$[\sigma]$	Allowable Stress	l_i	Length of the ith member
σ_i	Stress in ith member	W	Weight of the truss
ω_i	UDL on ith beam	t	Slab thickness

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