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Research Paper

Nonlinear Primary Frequency Response Analysis of Self-Sustaining Nanobeam Considering Surface Elasticity

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Abstract. This paper is focused to investigate the effects of nonlinear sources, including viscoelastic foundation and geometrical nonlinearity along with the surface elasticity and residual surface stress effects on the primary frequency response of a harmonically excited nanoscale Bernoulli-Euler beam. Due to large surface-area-to-volume ratio, the theory of surface elasticity as well as residual surface stress effects are taken into account within the beam models. The Galerkin approach accompanied by trigonometric shape functions is utilized to reduce the governing PDEs of the system to ODEs. The multiple scales perturbation method theory is applied to compute the nonlinear frequency response of nanobeam. The effects of linear and nonlinear viscoelastic damping coefficient of the medium, crystallographic directions of [100] and [111] of anodic alumina, geometrical nonlinear term and geometrical property on the nonlinear primary frequency response of nanoscale beam are investigated. The results show that theses parameters have a significant effect on the nonlinear frequency response of nanobeams in the case of primary resonance.

Keywords: Self-Sustaining Nanobeam, Primary Resonance, Surface Elasticity, Viscoelastic Medium, Crystallographic Directions.

1. Introduction

In recent decades, many research efforts have been conducted on the perception of the dynamical behavior of micro- and nano-sized structures on account of their superior physical properties. Among them, microbeams have remarkable applications, such as high precision measurement devices, Micro-Electro-Mechanical Systems (MEMS) transducers, chemical and biological sensors. Since performing controlled experiments at the nanoscale is very difficult and prohibitively expensive, the development of theoretical tools for modeling nanostructured materials has attracted the attention of many researchers. Two main categories of theoretical tools including atomistic approaches and continuum mechanics approaches were employed to investigate the bending, buckling and vibrational behaviors of nanobeams. The linear and nonlinear vibration analysis of nanobeams was carried out in time and frequency domains. Many vibrational phenomena can be detected only in nonlinear approaches such as; chaotic vibration, bifurcation of Poincare map, continuous power spectrum, irregular phase plan, backward and forward jumping in primary and secondary frequency response, dependency of natural frequency to amplitude. Accordingly, many investigations were focused to study the effect of mechanical and geometrical nonlinear sources on the vibrational behavior of nanobeams.

Many researches were employed different tools including bifurcation diagrams, largest Lyapunov exponents, Melnikov method, etc. to detect the chaotic parameter of nanoscale structure [1-13]. Ge and Yi [1] implemented a method for the approximation of the fractional derivative to study the chaotic vibration of a nonlinear damped Mathieu and nano resonator system. Using a bifurcation diagram, Phase portrait, Poincaré section, and time history, Mayoof and Hawwa [2] investigated the chaotic conditions of the carbon nanotubes (CNTs) with waviness along its axis. Patel and Joshi [9] investigated the nonlinear chaotic vibration of a double-walled CNT-based mass sensor subjected to harmonic excitation using time response, Poincaré maps and Fast Fourier transformation diagrams. The nonlinear chaotic characteristics of single-walled carbon nanotubes (SWCNTs) with different boundary conditions were carried out by Hu et al. [11] by implementing the translational and rotational springs. The diagrams of bifurcation were employed to detect the chaotic parameters. Ramezannejad Azarboni et al. [12] studied the chaotic vibration of a harmonically excited curved SWCNT by implementing the bifurcation diagram and largest Lyapunov exponents. The chaotic and periodic responses of CSWCNT were presented in phase portrait and Poincare map. Ramezannejad Azarboni and Edalatpanah [13] investigated the thermomagnetic chaotic vibration of SWCNT subjected to harmonic excitation by considering the surface effects. The amplitude and frequency of harmonic external load were selected as controlling parameters to detect the critical chaotic values using a bifurcation diagram and the largest Lyapunov exponents. The dynamic behaviors of the system in the chaotic and periodic regime were illustrated in time history, phase plane and Poincare map figures.

Another category of researches was focused to investigate the nonlinear dynamic response of nano stretchers in the

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frequency domain and the effect analysis of different mechanical and geometrical parameters on the behavior of nanostructures [14-38]. Based on the Bernoulli–Euler beam model, Fu et al. [14] computed the nonlinear frequency of single- and double-walled nanotubes by applying the incremental harmonic balance method (IHBM). The nonlinear frequency response of nanobeam with the axial initial load under linear viscoelastic medium was studied by Wang and Li [16] based on the nonlocal continuum Bernoulli–Euler beam theory. The effects of nonlocal parameter, axial initial load, number of modes, elastic term of Winkler foundation and the geometrical parameter on the primary resonance of nanobeam were analyzed. El-Borgi et al. [17] investigated the nonlinear free and forced vibration of functionally graded (FG) nanobeams under a nonlinear elastic medium. Applying the variational Iteration Method (VIM) and the multiple scales time method, the effects of nonlocal coefficient, FG parameters and the nonlinear coefficient of the elastic medium on the primary resonance or frequency response were studied. The nonlinear dynamic behaviors of bistable micro/nano-electro-mechanical of an initially curved arch shaped MEMS resonator subjected to a combined DC and AC electrostatic parallel plate field were investigated by Tajaddodianfar et al. [18]. Dai et al. [19] performed numerically the nonlinear forced vibration of a cantilevered nanobeam by considering the surface elasticity effect. The influences of positive and negative residual surface stress for different aspect ratios on the primary resonance response and occurrence of jumping were analyzed. Based on the Euler Bernoulli beam model, Caruntu and Luo [20] carried out the frequency response of a cantilevered SWCNT subjected to electrostatic force and viscoelastic foundation with nonlinear elastic terms. The effects of van der Waals force, damping of foundation and frequency on the primary resonance of the voltage response of SWCNT were studied. Sari [21] analyzed the thermomagnetic nonlinear frequency response of nanobeam under to nonlinear elastic medium in the case of superharmonic resonance by employing the nonlocal Euler–Bernoulli beam theory. Applying the multiple scales method, the effects of nonlocal parameter, thermomagnetic filed, and the nonlinear quadratic and cubic coefficient of elastic foundation on the superharmonic resonance of SWCNT were studied. Ramezannejad Azarboni [22] analyzed the nonlinear frequency analysis of SWCNT resting to linear viscoelastic medium under to axial thermomagnetic load and lateral external harmonic excitation in the case of primary resonance. The effect of geometrical nonlinearity, thermomagnetic field, surface elasticity and residual stress effects and different boundary conditions, including simply supported, clamped-clamped and simply supported-clamped at two ends of nanobeam on the primary resonance were investigated.

So far, the vast majority of previously conducted researches on the nonlinear frequency analysis have been limited of CNTs resting to the linear and nonlinear viscoelastic medium with nonlinear elastic terms. The main objective of this work is to investigate the effects of nonlinear viscoelastic medium with nonlinear damping term along with the geometrical nonlinearity due to large deformation, residual stress and surface elasticity, crystallographic directions of [100] and [111] of anodic alumina, and geometrical parameter on the primary resonance of nanobeam with a rectangular cross section. The Galerkin decomposition method and multiple scale technique are implemented to compute the nonlinear primary frequency response of nanobeam. The results show that the nonlinear damping term can be produced a closed curve domain in addition to an asymptotic curve domain on nonlinear primary frequency response of nanobeams.

2. Governing equation

2.1 Surface elasticity and residual surface stress theory

Figure 1 shows a simply-supported nanobeam with a distributed elastic foundation. To describe governing equations, a Cartesian reference coordinate (x,y,z) is attached at the center of the left side surface of the beam. Beam dimensions can be represented by length L, width b, and thickness h, as shown in Figure 1, while its Young's modulus, shear modulus, and mass density are E, G and ρ respectively.

In micron and sub-micron scales, the mechanical properties of surface layers are not the same as those of bulk environment, because of different local environment types experienced by atoms in bulk and those within near surface layers. In order to consider the surface energy effects, it is presumed that upper and lower surfaces of the beam have identical surface elastic modulus E_s and constant residual surface stress τ. However, different loading schemes have been applied to model residual surface stress as an external load. The Laplace-Young equation is utilized here to take the effect of residual surface stress into account. The curvature tensor $\kappa_{\alpha\beta}$ of the surface can be related to the stress jump, denoted by $\langle\sigma_{ij}^{+}-\sigma_{ij}^{-}\rangle$, through:

$$\langle\sigma_{ij}^{+}-\sigma_{ij}^{-}\rangle n_i n_j=\sigma_{\alpha\beta}^{s}\kappa_{\alpha\beta} \tag{1}$$

where σ_{ij}^{+} and $\sigma_{ij}^{-}(i,j=1,2,3)$ denote the stress above and below the surface, respectively, n_i and n_j are the normal unit vectors of the surface and $\sigma_{\alpha\beta}^{s}(\alpha,\beta=1,2)$ is the surface stress. Note that conventional Einstein's summation rules apply to above Latin and Greek indices. Eq. (1) implies that the residual surface stress acts on a deformed beam as a distributed load exerted on the surface. The curvature of the bending beam can be considered as

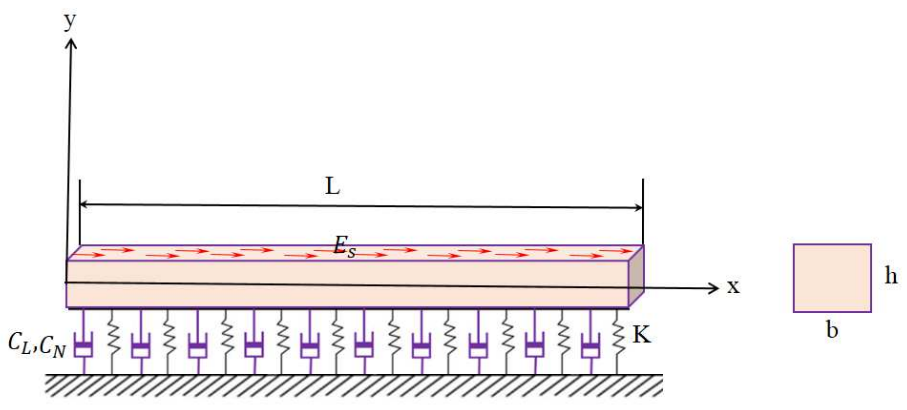

Fig. 1. Schematic of nanobeam with rectangular cross section resting on nonlinear viscoelastic medium

$$\kappa = \frac{\partial^2 y}{\partial x^2} \tag{2}$$

By substituting Eq. (2) into Eq. (1) and assuming that upper and lower residual surface stresses are denoted by τ_u and τ_b respectively, approximated distributed loads on two surfaces become.

$$P_u(x) = -\tau_u b \frac{\partial^2 y}{\partial x^2} \tag{3}$$

$$P_b(x) = -\tau_b b \frac{\partial^2 y}{\partial x^2} \tag{4}$$

Moreover, the distributed force acting on the beam by the nonlinear viscoelastic support can be described by

$$P_e(x) = -ky(x) + C_L \frac{\partial y(x)}{\partial t} + C_N \left(\frac{\partial y(x)}{\partial t}\right)^3 \tag{5}$$

where k, C_L, C_N are elastic constant, linear damping and nonlinear damping coefficient of the viscoelastic foundation and $y(x)$ is transverse deflection of the beam at distance x from the reference origin. From the energy point of view, it should be noted that in consequence of considering the effect of surface elasticity and existing the viscoelastic foundation, additional elastic strain energy and damping energy terms arise in calculating the total potential energy of the system. Furthermore, the work of the residual surface stress acting as an external distributed loading must be considered.

2.2 Bernoulli-Euler nanobeam

The von Karman strain for Euler Bernoulli beam can be considered;

$$\varepsilon_{xx} = -z\frac{\partial^2 y_m(x,t)}{\partial x^2} + \frac{1}{2}\left(\frac{\partial y_m(x,t)}{\partial x}\right)^2 \tag{6}$$

where $y_m(x,t)$ is the lateral deflection of the beam. According to constitutive relation, the strain energy is calculated as follows;

$$U_b = \frac{1}{2}\int_0^L \left[\int_A \sigma_{xx}\varepsilon_{xx} dA\right] = \frac{1}{2}\int_0^L \left[\int_A \sigma_{xx}\left(-z\frac{\partial^2 y_m(x,t)}{\partial x^2} + \frac{1}{2}\left(\frac{\partial y_m(x,t)}{\partial x}\right)^2\right) dA\right] = \frac{1}{2}\int_0^L \left\{\frac{1}{4}EA\left(\frac{\partial y_m(x,t)}{\partial x}\right)^4 + EI\left(\frac{\partial^2 y_m(x,t)}{\partial x^2}\right)^2\right\} dx \tag{7}$$

Considering the surface effect, a nanobeam can be modeled as three layered beams consisting of two surface layers (upper and lower layers) and a bulk layer (core layer). The corresponding strain energy can be written as follows;

$$U_b = \int_0^L \left\{\frac{1}{2}(EI)^*\left(\frac{\partial^2 y_m(x,t)}{\partial x^2}\right)^2\right\} dx \tag{8}$$

where

$$(EI)^* = EI + 2E_s bh^2 \tag{9}$$

The kinetic energy of the beam can be indicated as:

$$T = \frac{1}{2}\int_0^L \left\{\int_A \rho\left(\frac{\partial y_m(x,t)}{\partial t}\right)^2 dA\right\} dx = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial y_m(x,t)}{\partial t}\right)^2 dx \tag{10}$$

Work done with surface stress forces can be written as:

$$W_F = -\int_0^L F_s y_m(x,t) dx \tag{11}$$

where

$$F_s = -(\tau_u + \tau_b) b \frac{\partial^2 y_m(x,t)}{\partial x^2} \tag{12}$$

Applying the Hamilton's principle leads to

$$\delta\int_{t_1}^{t_2} (T - U_b + W_F) dt = 0 \tag{13}$$

Considering the nonlinear viscoelastic foundation and the external harmonic excitation, the equation of motion of nanobeam can be derived as;

$$\left(EI+2E_s bh^2\right)\frac{\partial^4 y_m(x,t)}{\partial x^4}-\left(\tau_u+\tau_b\right)b\frac{\partial^2 y_m(x,t)}{\partial x^2}+ky_m(x,t)+C_L\frac{\partial y_m(x,t)}{\partial t}-C_N\left(\frac{\partial y_m(x,t)}{\partial t}\right)^3+\rho A\frac{\partial^2 y_m(x,t)}{\partial t^2}$$
$$=\frac{EA}{2L}\left(\int_0^L\left(\frac{\partial y_m(x,t)}{\partial x}\right)^2dx\right)\frac{\partial^2 y_m(x,t)}{\partial x^2}+F\cos\omega t \tag{14}$$

For nanobeam with a simply supported boundary condition at two ends the polynomial admissible functions can be implemented as

$$y_m(x,t)=Y_m(t)\sin\frac{m\pi x}{L} \tag{15}$$

Substituting polynomial admissible functions for simply supported boundary conditions in the Eq. (14) and on can be derived the ODEs respect to time by applying the Galerkin decomposition method over the length of nanobeam.

$$\frac{d^2Y_m(t)}{dt^2}+\frac{C_L}{\rho A}\frac{dY_m(t)}{dt}+\frac{1}{\rho A}\left[\left(\frac{m\pi}{L}\right)^4 EI+2E_s bh^2+\left(\frac{m\pi}{L}\right)^2\left(\tau_u+\tau_b\right)b+k\right]Y_m(t)-\frac{3}{4}\frac{C_N}{\rho A}\left(\frac{dY_m(t)}{dt}\right)^3+\frac{1}{4}\frac{E}{\rho}\left(\frac{m\pi}{L}\right)^4Y_m^3(t)=\frac{F}{\rho A}\cos\omega t \tag{16}$$

The following dimensionless parameters can be employed to present a convenient format of ODE of vibration.

$$r=\sqrt{\frac{I}{A}},u_m=\frac{Y_m}{r} \tag{17}$$

The ODE of motion for nanobeam is derived as follows.

$$\frac{d^2u_m(t)}{dt^2}+\frac{C_L}{\rho A}\frac{du_m(t)}{dt}+\omega_0^2u_m(t)-\frac{3}{4}\frac{C_N r^2}{\rho A}\left(\frac{du_m(t)}{dt}\right)^3+\frac{1}{4}\frac{Er^2}{\rho}\left(\frac{m\pi}{L}\right)^4u_m^3(t)=\frac{F}{\rho A}\cos\omega t \tag{18}$$

where

$$\frac{1}{\rho A}\left[\left(\frac{m\pi}{L}\right)^4 EI+2E_s bh^2+\left(\frac{m\pi}{L}\right)^2\left(\tau_u+\tau_b\right)b+k\right] \tag{19}$$

3. Applying the technique of multiple scale

In accordance with the multiple times scale technique, one can be rewritten Eq. (18) as:

$$\ddot{u}_m+\omega_0^2u_m=-\varepsilon\eta_1\dot{u}_m+\varepsilon\eta_2\dot{u}_m^3-\varepsilon\eta_3u_m^3+\eta_4\cos\omega t \tag{20}$$

where

$$\eta_1=\frac{C_L}{\rho A},\eta_2=\frac{3}{4}\frac{C_N r^2}{\rho A},\eta_3=\frac{1}{4}\frac{Er^2}{\rho}\left(\frac{m\pi}{L}\right)^4,\eta_4=\frac{F}{\rho A} \tag{21}$$

The frequency of excitation can be considered as $\omega\approx\omega_0$ to study the nonlinear primary resonance analysis of nanobeam. Accordingly,

$$\omega=\omega_0+\varepsilon\sigma \tag{22}$$

where σ is the detuning parameter which defines the nearness of linear natural frequency of nanobeam. Based on the perturbation method of multiple scale, the following time scales are produced.

$$T_i=\varepsilon^i t \qquad i=0,1,2,\cdots \tag{23}$$

Applying the first and second derivatives with respect to t ,

$$\frac{d}{dt}=\frac{dT_0}{dt}\frac{\partial}{\partial T_0}+\frac{dT_1}{dt}\frac{\partial}{\partial T_1}+\cdots=D_0+\varepsilon D_1\cdots \tag{24}$$

$$\frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0D_1+\varepsilon^2\left(D_1^2+2D_0D_2\right)+\cdots \tag{25}$$

Considering the nonlinearity of dynamic behavior of nanobeam and in order to apply the method of multiple times scale, the following asymptotic expansions are assumed for the solution of nonlinear vibration of system.

$$u=u_0+\varepsilon u_1 \tag{26}$$

Subsequently, the excitation term is also written as

$$\eta_4\cos\omega t=\varepsilon\eta_4\cos\left(\omega_0T_0+\sigma T_1\right) \tag{27}$$

Substituting Eq. (22) through (27) into Eq. (20), the following equation is obtained.

$$\begin{aligned}&\left[D_0^2+2\varepsilon D_0D_1+\varepsilon^2\left(D_1^2+2D_0D_2\right)\right](u_0+\varepsilon u_1)+\omega_0^2(u_0+\varepsilon u_1)\\&=-\varepsilon\eta_3(u_0+\varepsilon u_1)^3-\varepsilon\eta_1[D_0+\varepsilon D_1](u_0+\varepsilon u_1)-\varepsilon\eta_2[D_0+\varepsilon D_1](u_0+\varepsilon u_1)^3+\varepsilon\eta_4\cos(\omega_0T_0+\sigma T_1)\end{aligned} \tag{28}$$

Equating same power of ε^0 and ε, the following equations are found.

$$D_0^2u_0+\omega_0^2u_0=0 \tag{29}$$

$$D_0^2u_1+\omega_0^2u_1=2D_0D_1u_0-\eta_1D_0u_0-\eta_2(D_0u_0)^3-\eta_3u_0^3+\eta_4\cos\omega_0T_0 \tag{30}$$

The general solution of Eq. (29) can be stated as,

$$u_0=R(T_1)\exp(i\omega_0T_0)+\bar{R}(T_1)\exp(-i\omega_0T_0) \tag{31}$$

where $R(T_1)$ is an unknown complex function and $\bar{R}(T_1)$ is the complex conjugate $R(T_1)$. Substituting Eq. (31) into Eq. (30) leads

$$D_0^2u_1+\omega_0^2u_1=-\left[i\left(2\omega_0\frac{dR}{dT_1}+\omega_0\eta_1R\right)-i\eta_2\omega_0^3R^2\bar{R}+3\eta_3R^2\bar{R}\right]\exp(i\omega_0T_0)-\eta_3R^3\exp(3i\omega_0T_0)-i\eta_2R^3\omega_0^3\exp(3i\omega_0T_0)+\frac{1}{2}\eta_4\exp(i\sigma T_1)+C.C \tag{32}$$

where C.C stands for the complex conjugate terms. To investigate the non-resonance analysis of harmonically excited nanobeam, the secular terms of Eq. (32) should be eliminated and the coefficient of $\exp(i\omega_0T_0)$ are set to be zero as;

$$i\left(2\omega_0\frac{dR}{dT_1}+\omega_0\eta_1R\right)-i\eta_2\omega_0^3R^2\bar{R}+3\eta_3R^2\bar{R}=+\frac{1}{2}\eta_4\exp(i\sigma T_1) \tag{33}$$

The polar representation in the form of $R=0.5a\exp(i\theta)$ is applied to separate the real and imaginary parts of Eq. (33),

$$\frac{da}{dT_1}=-\frac{1}{2}\eta_1a+\frac{1}{8}\eta_2\omega_0^2a^3+\left(\frac{1}{2\omega_0}\eta_4\right)\sin\gamma \tag{34}$$

$$\frac{d\gamma}{dT_1}=\sigma a-\frac{3}{8\omega_0}\eta_3a^3-\left(\frac{1}{2\omega_0}\eta_4\right)\cos\gamma \tag{35}$$

where $\gamma=\sigma T_1-\theta$. The steady state motion of nanobeam leads when da/dT_1 and $d\gamma/dT_1$ consequently,

$$\frac{1}{2}\eta_1a-\frac{1}{8}\eta_2\omega_0^2a^3=\left(\frac{1}{2\omega_0}\eta_4\right)\sin\gamma \tag{36}$$

$$\sigma a-\frac{3}{8\omega_0}\eta_3a^3=\left(\frac{1}{2\omega_0}\eta_4\right)\cos\gamma \tag{37}$$

After some mathematical manipulations, the implicit expression of the primary resonance response of nanobeam can be derived as follows.

$$\left[\left(\frac{1}{2}\eta_1a-\frac{1}{8}\eta_2\omega_0^2a^3\right)^2+\left(\sigma a-\frac{3}{8\omega_0}\eta_3a^3\right)^2\right]=\left(\frac{1}{2\omega_0}\eta_4\right)^2 \tag{38}$$

The Eq. (38) can be rewritten as follows;

$$\left[\left(\frac{1}{2}\lambda_La-\frac{1}{8}\lambda_Na^3\right)^2+\left(\sigma a-\lambda_Ga^3\right)^2\right]=\left(\frac{1}{2}\lambda_F\right)^2 \tag{39}$$

where

$$\lambda_L=\eta_1,\lambda_N=\eta_2\omega_0^2,\lambda_G=\frac{\eta_3}{\omega_0},\lambda_F=\frac{\eta_4}{\omega_0} \tag{40}$$

4. Results and discussion

Depending on the crystallographic structure, anodic alumina may possess positive or negative elastic constant and different residual surface stresses. The material and geometric parameters of the nanobeams are taken to be $E=70GPa$, $\upsilon=0.3$, $\rho=2700kg/m^3$ the length $L=15nm$, the width $b=1nm$, the height $h=0.5nm$. The surface related properties of the anodic alumina with crystallographic directions of [100] and [111] are $E_s=-7.9253N/m$, $\tau=0.5689\ N/m$ and $E_s=5.1882N/m$, $\tau=09108N/m$, respectively.

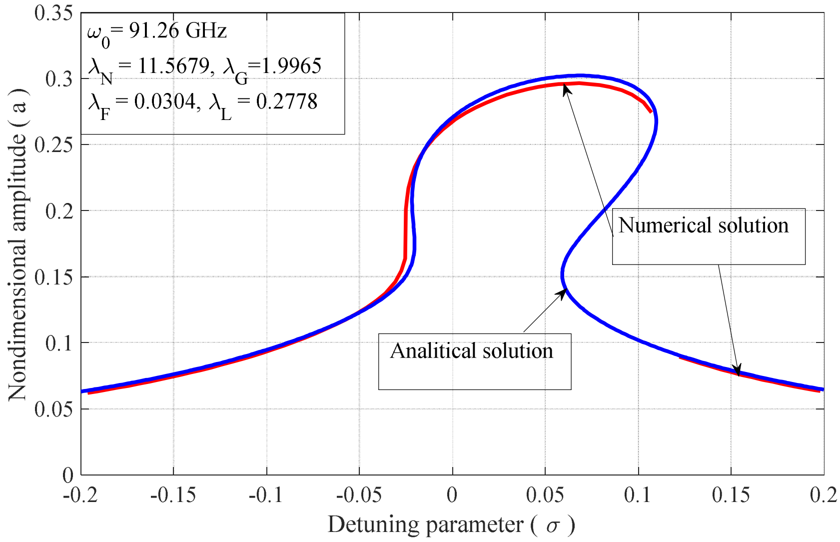

Fig. 2. Comparison of numerical and analytical results for validation

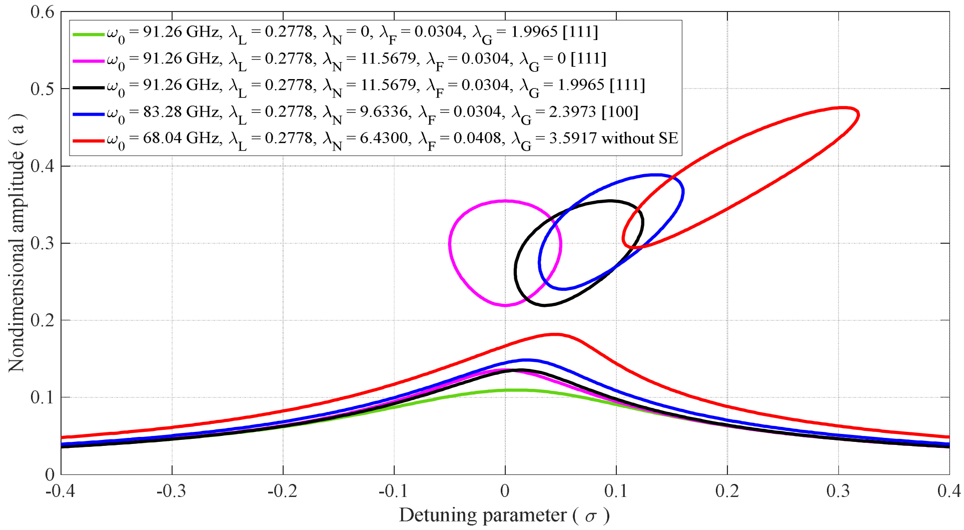

Fig. 3. Comparison of different parameter on the nonlinear frequency response of nanobeam

At first, Numerical simulations have been carried out for the nonlinear system to verify the reliability of the analytical results as shown in figure 2 Solutions have been done by numerical integration via a fourth-order Runge-Kutta integration algorithm. As depicted in figure 2 one can be observed a satisfactory accuracy between numerical results and the analytical results obtained in this study.

Comparing the effects of crystallographic directions of [111] and [100], the nonlinearity of viscoelastic foundation, surface elasticity and geometrical nonlinearity on the nonlinear primary resonance of simply supported Bernoulli-Euler nanobeam are illustrated in Fig 3. Ignoring the nonlinear term of the viscoelastic medium, the frequency response of nanobeam consists of an asymptotic curve with a horizontal axis without a closed curve domain $\lambda_N = 0$. The second domain with a closed curve domain is appeared by considering the nonlinearity of the viscoelastic medium. In this case, the nonlinear primary frequency response of nanobeam includes two subdomains that are symmetric without geometrical nonlinearity, $\lambda_G = 0$. The effect of geometrical nonlinearity, $\lambda_G \neq 0$, is to bent the primary frequency response curve with two separate domains. Comparing the crystallographic directions of [111] and [100] effects on the frequency response show that in the case of [100] the non-dimensional amplitude of frequency is larger than of [111] ones. Moreover, neglecting the surface elasticity effect, the frequency response of nanobeam is developed in both domains.

The influences of linear and nonlinear damping term, λ_L and λ_G , of viscoelastic foundation with and without considering the geometrical nonlinearity of nanobeam with crystallographic direction of [111] is depicted in figure 4 to 7. For small values of linear damping term, the frequency response of nanobeam has an asymptotic curve with a horizontal axis without a closed curve domain. In increasing the λ_L , the second closed curve domain is appeared for a critical value of λ_L . As λ_L is increased further, the second closed curve domain produces with an increasing amplitude of response as shown in figure 4. In the presence of geometrical nonlinearity, λ_G , the forward frequency response of nanobeam in the case of primary resonance can be observed as depicted in figure 6, but in the absence of λ_G the frequency response is symmetric. Similarly, the results can be observed by changing the nonlinear damping term, λ_N , of viscoelastic foundation except that increasing of λ_N has a positive effect on the elimination of development of multivalued frequency response of nanobeam as illustrated in Figures 6 and 7.

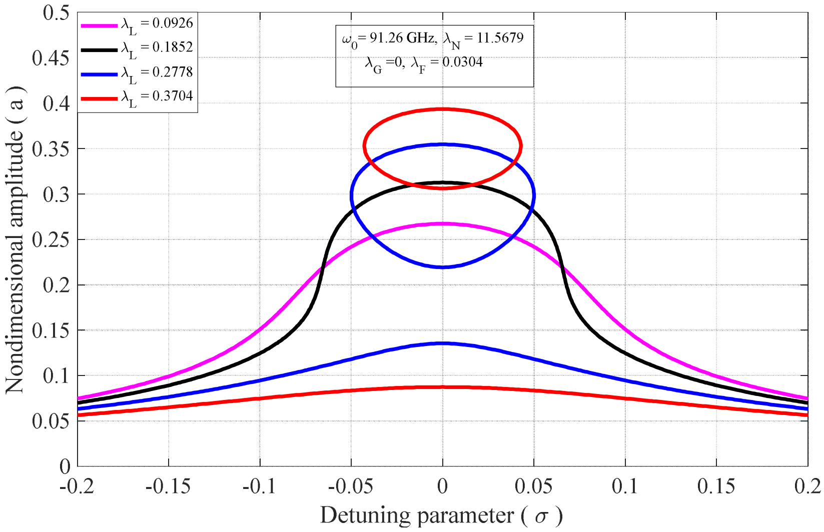

Fig. 4. Effect of the linear parameter of viscoelastic foundation on nonlinear frequency response for [111] model without geometrical nonlinearity

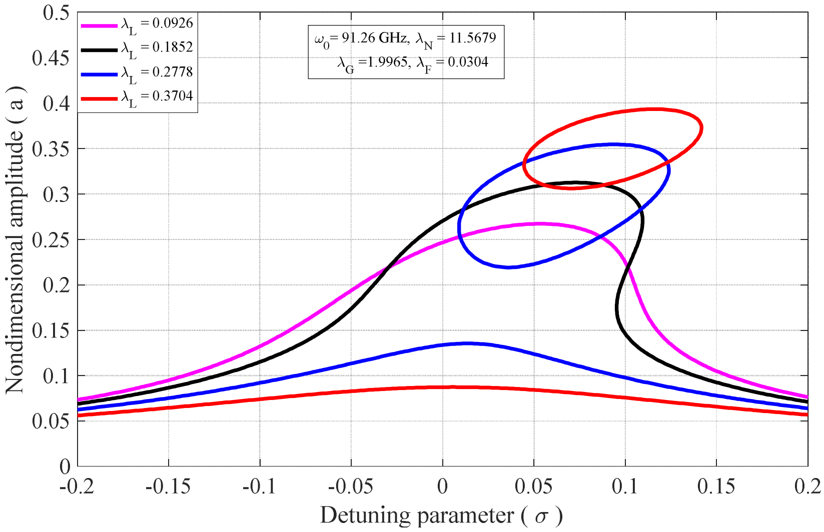

Fig. 5. Effect of the linear parameter of viscoelastic foundation on nonlinear frequency response for [111] model with geometrical nonlinearity

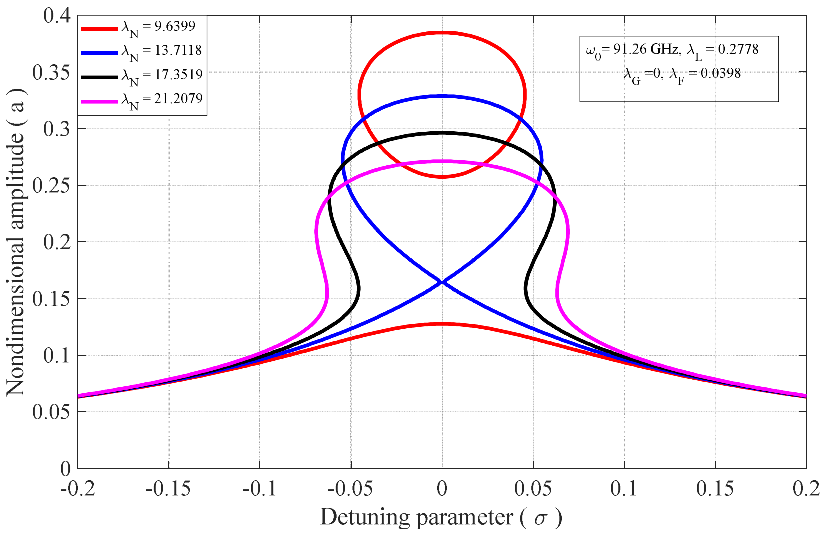

Fig. 6. Effect of the nonlinear parameter of viscoelastic foundation on nonlinear frequency response for [111] model without geometrical nonlinearity

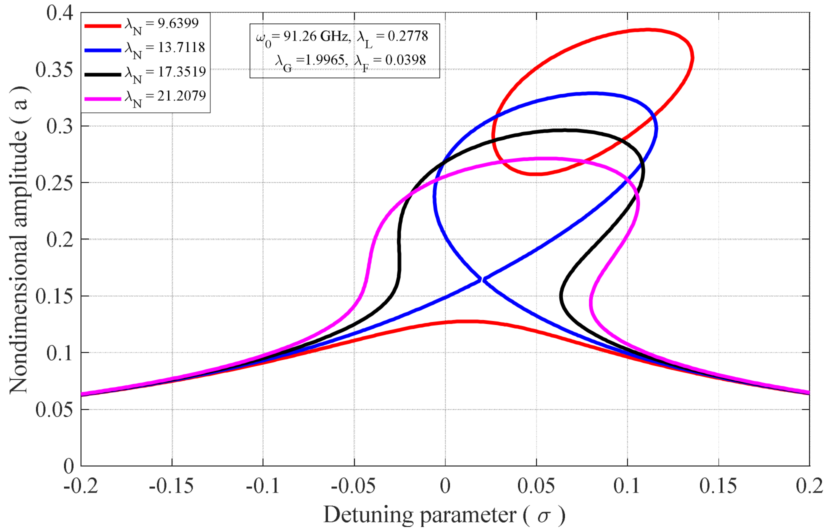

Fig. 7. Effect of the nonlinear parameter of viscoelastic foundation on nonlinear frequency response for [111] model without geometrical nonlinearity

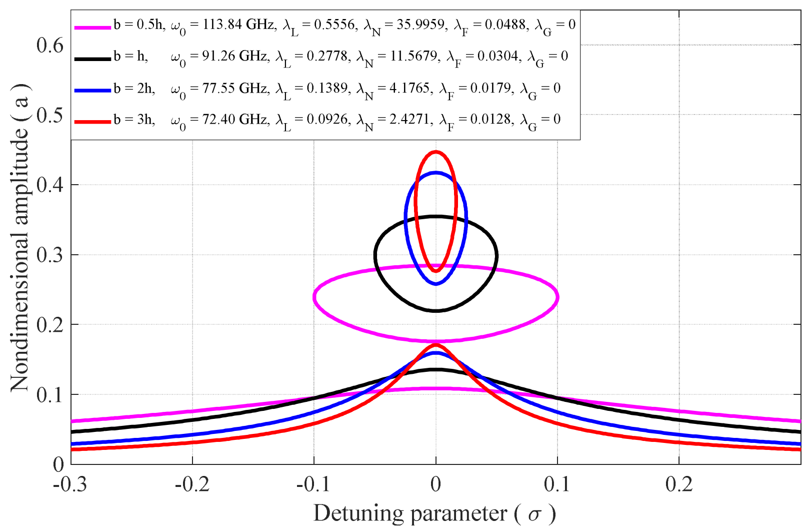

Fig. 8. Effect of change of cross section on nonlinear frequency response for [111] model without geometrical nonlinearity

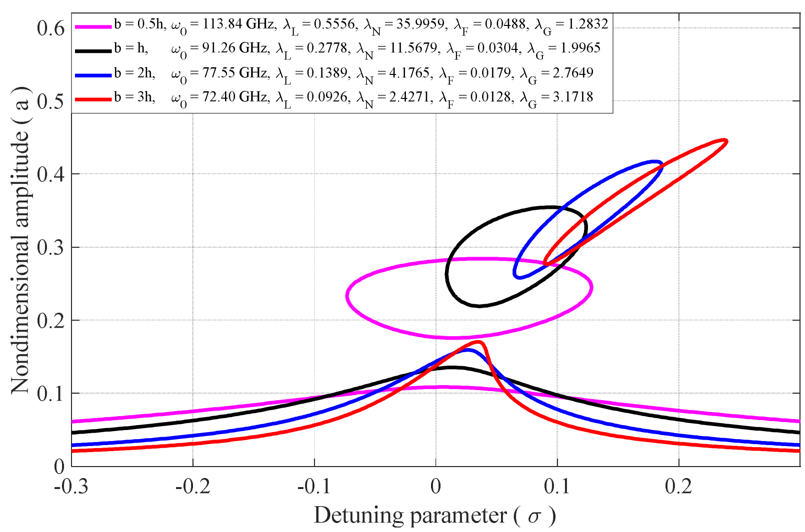

Fig. 9. Effect of change of cross section on nonlinear frequency response for [111] model with geometrical nonlinearity

The effect of the cross section area of nanobeam on the primary resonance response with and without geometrical nonlinearity are presented in figures 8 and 9. As depicted in figures 8 and 9, on cam be concluded that by increasing the cross section area the surface effect increase and the closed curve domain get constricted. Moreover, the amplitude of frequency response is increased by increasing the cross section area. The symmetric and forward asymmetric behavior can be found in the absence and presence of geometrical nonlinearity in figures 8 and 9, respectively.

The effect of length of nanobeam on the frequency response on nanobeam in the case of the primary resonance with and without the nonlinear term of geometry are plotted in figures 10 and 11. Increasing in length has a negative sensitivity effect on the frequency response of nanobeam because the closed curve domain of frequency response expands with multivalued frequency response especially in the presence of geometrical nonlinearity as illustrated in figure 11.

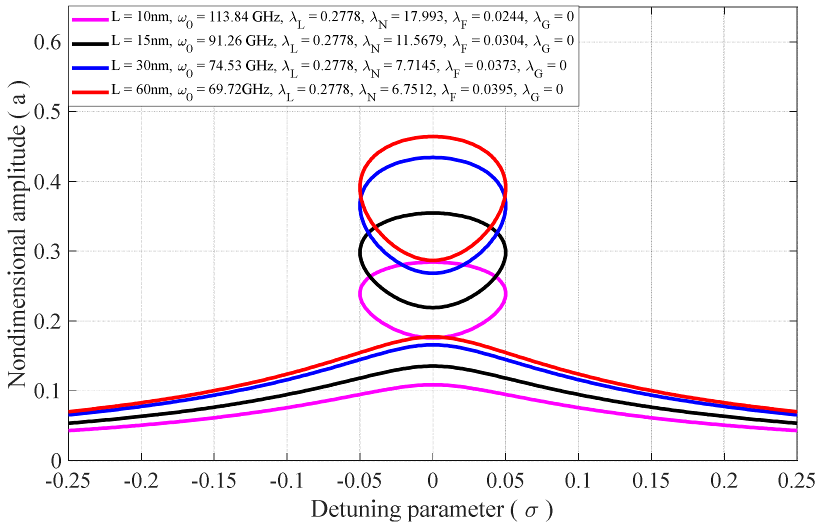

Fig. 10. Effect of length of nanobeam on nonlinear frequency response for [111] model without geometrical nonlinearity

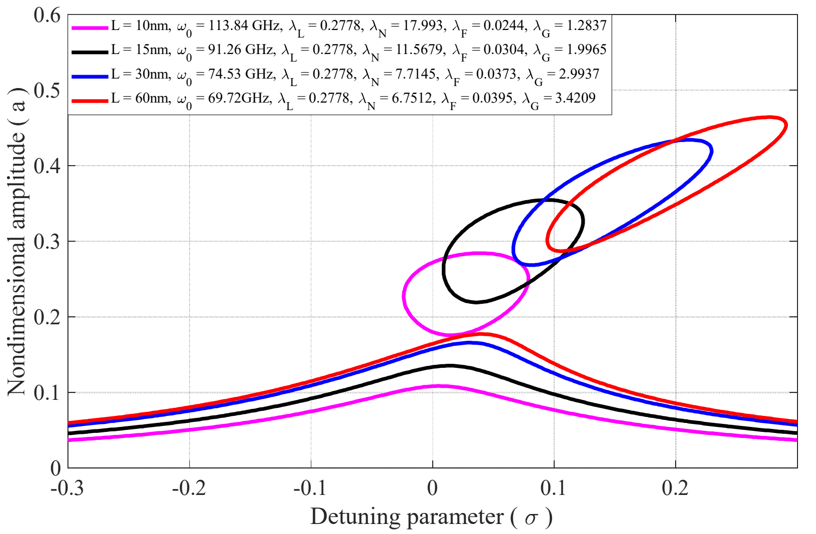

Fig. 11. Effect of length of nanobeam on nonlinear frequency response for [111] model with geometrical nonlinearity

5. Conclusion

In this study, the nonlinear nanoscale Bernoulli-Euler theory has been successfully applied to analyze the primary resonance response of the self-sustaining nanobeam rested on a nonlinear viscoelastic foundation and subjected to harmonic lateral loads. The surface elasticity and residual surface stress are considered to derive the governing equations of the system. Galerkin's method is used to transform the partial differential equations into the ordinary ones. The effects of linear and nonlinear damping terms of foundation constant, crystallographic directions of anodic alumina of [100] and [111], geometrical nonlinearity and geometrical parameters on the nonlinear primary resonance of nanobeam are investigated. Useful conclusions can be indicated as follows:

1- The surrounding viscoelastic foundation has stabilizing effect on the frequency response characteristics of nanoscale beams.
2- Due to increase of stiffness, the crystallographic direction of [111] is more stable than the crystallographic directions of [100].
3- Ignoring the geometrical nonlinearity, the symmetric frequency responses are observed of nanobeam.
4- The forward asymmetric frequency response curves are found by considering the effect of geometrical nonlinearity.

5- The nonlinear damping term produces a closed curve domain that is expanded by decreasing λ_N .
6- The cross section area and length of nanobeam have significant effects on the frequency response characteristics of nanoscale beams in the case of primary resonance.

Author Contributions

H. Ramezannejad Azarboni conceived of the presented idea and developed the theory and performed the computations. H. Heidari discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

L	Length of nanobeam [nm]	$\kappa_{\alpha\beta}$	Curvature tensor [$1/m$]
h	Thickness of nonobeam [nm]	n_i	Normal unit vector of the surface
b	Width of nanobeam [nm]	n_j	Normal unit vector of the surface
E	Young's modulus [Tpa]		Curvature of the bending beam [$1/m$]
G	Shear modulus [TPa]	U_b	Strain energy [J]
ρ	Mass density [kg/m^3]	A	Cross section area [nm^2]
E_s	Surface elastic modulus [N/m]	I	Moment of inertia [kgm^2]
τ	Constant residual surface stress [N/m]	T	Kinetic energy [J]
σ_{ij}^{+}	Stress above surface [N/m^2]	W_F	Work done with surface stress forces [J]
σ_{ij}^{-}	Stress below the surface [N/m^2]	F	Amplitude of external force [N]
$\sigma_{\alpha\beta}^{s}$	Surface stress [N/m^2]	ω	Frequency of external force [rad/s]
τ_u	Upper residual surface stress [N/m^2]	Y_m	Deflection of nanobeam [nm]
τ_b	Lower residual surface stress[N/m^2]	η_1	Non-dimensional parameter
$P_u(x)$	Upper distributed loads [N/m]	η_2	Non-dimensional parameter
$P_b(x)$	Lower distributed loads [N/m]	η_3	Non-dimensional parameter
$P_e(x)$	Distributed force of support [N/m]	η_4	Non-dimensional parameter
C_L	Linear damping of foundation [Ns/m^2]	ω_0	Natural frequency of nanobeam [rad/s]
C_N	Nonlinear damping of foundation [Ns^3/m^4]	σ	Detuning parameter
k	Elastic constant of foundation [N/m^2]	T_i	Time scale
ε_{xx}	von Karman strain	D_0	Derivative operator
y_m	Lateral deflection of beam [nm]	D_1	Derivative operator
$R(T_1)$	Unknown complex function	D_2	Derivative operator
$\bar{R}(T_1)$	Unknown complex conjugate function	a	Amplitude of complex function
γ	Non-dimensional phase	θ	Phase of complex function
λ_L	Non-dimensional parameter	λ_N	Non-dimensional parameter
λ_G	Non-dimensional parameter	λ_F	Non-dimensional parameter

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