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Research Paper

Transient Response of Longitudinal Fins under Step Changes in Base Temperature and Heat Flux using Lattice Boltzmann Method

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Abstract. The present article reports the transient response of longitudinal fins having linear and non-linear temperature dependent thermal conductivity, convection coefficient and internal heat generation under two cases of base boundary condition, (i) step change in base temperature and (ii) step change in base heat flux. The fin tip is assumed to be adiabatic. Both, linear and non-linear, temperature dependency of thermo-physical properties is addressed in the mathematical formulation and the solution for the above cases is obtained using Lattice Boltzmann method (LBM) implemented in an in-house source code. LBM, being a dynamic method, simulates the macroscopic behavior by using a simple mesoscopic model and offers enormous advantages in terms of simple algorithm to handle even the most typical of boundary conditions that are easy and compact to program even in case of complicated geometries too. Although the transient response of longitudinal fins has been reported earlier, however power law variation of thermophysical properties for the above two base condition has not been reported till date. The present article first establishes the validity of LBM code with existing result and then extends the code for solving the transient response of the longitudinal fin under different sets of application-wise relevant conditions that have not been treated before. Results are reported for several combination of thermal parameter and are depicted in form of graphs.

Keywords: Transient response, Longitudinal fins, Lattice Boltzmann Method, Base heat flux, Base temperature.

1. Introduction

Although, Lattice Boltzmann method (LBM) came into light [1-3] in late eighties and since then it is used as an alternate approach for simulating the fluid flow problem, it is only in the recent past that it has been extended to solve the heat transfer problems [4-18]. Computational Fluid Dynamics (CFD) is based on continuum scale, which disregards the reality that matter is made up of individual molecules. In contrast, Lattice Boltzmann method is based on mesoscopic scale in which the collection of molecules is considered as single unit and is represented by a distribution function. In traditional approximation techniques, the nodal value of the field variable at any successive nodes is dependent on the nodal value at previous nodes. One may recall that in FDM, a value at a particular node in a grid is obtained by linear interpolation of the known value at the surrounding nodes of the grid. In FVM and FEM, the function interpolation might not be linear depending on the choice of basis function that are used to approximate the field variable. Such is not the case in LBM wherein the distribution functions, which represent a unit of collection of molecule, stream along a given direction (depending dimension of problem) and collide at successive node. These successive nodes are known as lattice sites wherein upon collision the distribution function further stream to different direction. It has been observed that in LBM, unlike CFD, the need to solve the energy equation at each time step is not essential and thus less computational effort/time is required to arrive at solution.

Under transient state, heat transfer through surfaces arise with step changes in boundary conditions till the field attains the steady state. In addition to these, under severely high temperature applications, the thermophysical properties vary non-linearly with temperature for many thermally conductive materials. In such high-temperature heat transfer related studies, fins arrest large interest among the researchers due to their wide and versatile applications under different operating conditions. These are extended surfaces for enhancing the heat transfer from heat sources/surfaces by exposing the larger surface area to convection. These extended surfaces are fabricated by extruding, welding, or wrapping a thin metal sheet on a surface. Numerous applications of fins are found in highly intricate installations of gas turbine engine, internal combustion engine, heat exchangers [19] and many more. For applications of fins in thermoelectric power generator, Zebarjadi [20] designed a fin model where convection coefficient of side wall were also addressed. Further, Dannelley and Baker [21] reported enhanced effectiveness of fractal fins due to structure obtained from the transformation of longitudinal fins. Such fractal fins find vast application in Micro-Electro-Mechanical-Systems [22]. Fins are used in majority of the applications, though steady state study of extended surface are justifiable, the transient study is very pertinent in certain application such as jet engines, reactors, space vehicles and, in the present digital era electronic component.

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Review of literature indicates that although LBM has been widely applied to solve conduction-radiation problems, its application in solving conduction-convection heat transfer problems have been rarely been reported. Mishra et al. [4] reported the solution of energy equation assuming negligible heat transfer through convection for a conduction-radiation problem with constant thermo-physical properties. In [4], the radiative information is calculated using DTM and then implemented in governing differential equation which is then solved using LBM. Mishra et al. [5] also reported the solution of energy equation of two-dimensional rectangular plate under conduction radiation heat transfer using LBM and FVM. Gupta et al. [6] extended these studies for solving heat transfer problems with temperature dependent thermal conductivity using LBM-FDM wherein the radiation information is obtained using DOM and variable thermal conductivity is incorporated with the help of modified relaxation time. In another work, Mishra et al. [7] used DTM to compute the radiation information for a conduction-radiation problem with variable refractive index. This is followed by implementation of LBM with variable relaxation time to solve the energy equation. Mahmoudi [8] and Mahmoudi et al. [9] also reported the solution of conduction-radiation problem wherein the radiation information is first obtained using LBM and is extended to solve the conduction-radiation problem with variable conductivity and variable refractive index. Hamila et al. [10] also used LBM to investigate the heat transfer problems with variable thermal conductivity by altering the equilibrium distribution function. Rahmati et al [11] reported the solution of 2-D non-Fourier heat conduction problem with variable thermal conductivity using LBM.

While a host of literature is available reporting different methods for solving the heat transfer problems applicable to different geometries, these solutions are quite broad and several attempts had been carried out to report the performances of typical geometries pertaining to different applications. One such application is that of heat transfer through extended surfaces and numerous articles are reported predicting their performance under a given set of conditions. Suryanarayna [23] and Donaldson [24] presented the analytical solution of longitudinal transient fins with insulated tip using Laplace transformation. In these studies fin base temperature or fin base heat flux is assumed to be a function of time while other properties treated as constant. Suryanaryana [25] further extended this work to analyse the transient performance of fin with time dependent base fluid temp. In another work, Assis and Kalman [26] reported the solution of fins of different profile with base flux and base transient boundary condition.

It is well established that in heat transfer problem, with large temperature differences between two extremities, the fin material properties become temperature dependent. In common practice this dependency is ignored as it renders non-linearity to the differential equation at the cost of accuracy of solution. In most of these practical applications, heat transfer coefficient varies non-linearly with the temperature (as a power law function of temperature) as reported by [28] and [29]. In recent times, researchers have reported advanced mathematical routines to handle such non-linearity and predict the fin performance more accurately. Malekzadeh et al. [30] used differential quadrature element method (DQM) to solve the non-linear steady and transient state fins with variable thermal conductivity. Mottsheki et al. [31] reported the transient behaviour of longitudinal fin with variable thermal conductivity and heat transfer coefficient for different profiles. In some situations, thermal conductivity of the material is considered as power law function of temperature [31]. Such variation of thermal conductivity in the application problem of fins is reported in [33] and [34]. However in these reports, heat generation is not considered whereas in some practical application of the fin, such as nuclear rod, jet engines or current carrying conductors, the performance is greatly affected by heat generation within the fin. Internal heat generation in thin fins were first reported by Minkler and Rouleau [35]. Further Unal [36] analytically determined the temperature distribution of straight fins with uniform and non-uniform heat generation along with the non-uniform heat transfer coefficient, in which non-uniform heat generation is treated with a third degree polynomial function of temperature. Razelos and Satyaprakash [37] optimized the convective trapezoidal pin fins under constant as well as spatially varying heat generation. In another work Kundu and Das [38] reported the optimum profile of thin fin under volumetric heat generation. In this context, study of the steady state longitudinal fin with temperature dependent thermal conductivity and internal heat generation has been reported by Sobamowo [39] using FDM and Ghasemi et al. [40] using DTM. Sobhan et al. [41] implemented the DTM-FDM technique to examine the performance of longitudinal fins of different profile with temperature dependent thermal conductivity, heat transfer coefficient and temperature dependent internal heat generation. Mhlonge et al. [42] studied the response of transient fin under step change in base temperature and step change in base heat flux with power law variation in thermo-physical properties. However, these mathematical approaches are quite involving and require substantial computational effort. Study of extended surface or fins with LBM has seldom being done and literature reviews indicate towards the scarcity of such studies. LBM, being a dynamic method, simulates the macroscopic behaviour by using a simple mesoscopic model and offers enormous advantages in terms of simple algorithm to handle even the most typical of boundary conditions that are easy and compact to program even in case of complicated geometries too. Bamdad et al. [14] used the inverse method to determine the temperature profile under transient condition and then used LBM and ACGM to determine the heat flux of longitudinal fin.

An extensive survey of literature reveals that the transient response of longitudinal fins in high temperature applications wherein the thermo-physical properties vary with temperature (and time), subject to step change in base temperature/step change in base heat flux with internal heat generation is not yet been reported. The complexity of the problem presents a hindrance to applicability of common and popular approximation and hence calls for the ingenuity of Lattice Boltzmann Method and is proposed in the present work.

2. Mathematical formulation of the transient response of longitudinal fin

A longitudinal fin of rectangular cross-section (width along z-axis, w and thickness along y-axis, t) of length (along x-axis), l, as shown in Fig 1(a-e), with adiabatic tip is studied under different base boundary conditions. The unsteady (transient) state is formulated using one dimensional energy balance with heat generation for fin materials having temperature depended thermophysical properties. Schematic representation of energy balance equation is shown in Fig 1(f) wherein it is assumed the fin material is homogeneous and isotropic and thermal condition at the tip of the fin does not vary along the width and thickness of the fin. At the contact surface of the fin to the base wall of the heat source, thermal resistance due to contact is assumed to be negligible.

The different base boundary conditions, as mentioned above, treated in the mathematical formulation are as follows:

Case 1: Step change in base temperature

Case 2: Step change in base heat flux

In high temperature applications, under transient conditions, when substantial heat dissipates through the fin, the above-stated two cases of base boundary conditions mathematically simulate the practical scenario in the most apposite manner. Particularly in the application of the reactor, air cooled engine or combustion chamber etc., the transient state of heat flow is an important area of research wherein, in the transient state, the base boundary condition of the extended surface or fins could be best handled mathematically by assuming any one the two case motioned above. Further, it is observed that under high temperature application the thermal conductivity of different fin material exhibit a dependency on the temperature. Although literature review indicates that most of the studies assume linear dependence of thermal conductivity on temperature however, for most of the fin materials, this dependency is nonlinear at high operating temperature ranges [32]. As a result, the present study proposes the mathematical formulation of longitudinal rectangular fins under heat generating conditions with non-linear temperature dependency of thermophysical properties. Further the formulation is generalized for variable heat transfer coefficient and varying internal heat generation. The mathematical formulation is derived using energy balance applied to element shown in Fig 1(f) as follows (Eq. 1-2):

$$Q_x^{cond} - Q_{x+dx}^{cond} + Q_g = Q_E + Q^{conv} \tag{1}$$

Here, Q_x^{cond} and Q_{x+dx}^{cond} is the rate of heat conducted at x and $x+dx$ respectively, Q^{conv} is the rate of heat convection from the surface of the element, Q_g is the rate of heat generation inside the element and Q_E is the rate of change of energy content of the element

$$A_c\left[\frac{d}{dx}\left(k\frac{dT}{dx}\right)\right]dx + Q_g A_c dx = hP(T-T_A)dx + \rho C A_c \frac{dT}{dt}dx \tag{2}$$

Eq. 2 is the governing equation for heat transfer through structures and is treated with temperature dependent thermophysical properties, k and h under heat generation Q_g in the following discussion. Various categories of temperature dependence of thermophysical properties are addressed in the present article and each is solved with both possible base boundary conditions as mentioned above. The normalized variables representing space, time and fin parameter are as follows:

$$X = \frac{x}{L}; \quad t^* = \frac{\alpha t}{L^2}; \quad N^2 = \frac{h_0 P L^2}{k_a A_c}$$

2.1 Transient response under step change in base temperature

The heat transfer coefficient (h) is assumed to be a power law function of temperature as follows:

$$h = h_0\left(\frac{T-T_A}{T_0-T_A}\right)^{\varepsilon_t}$$

Excess temperature is normalized using the following expression:

$$\theta_t = \left(\frac{T-T_A}{T_0-T_A}\right)$$

Further, the normalized heat generation is expressed as:

$$Q_t = \frac{Q_{g0}A_c}{h_0 P(T_0-T_A)}$$

For determining the transient response, the boundary conditions in normalized coordinates of fins having step change in base temperature are treated as:

$$\theta_t = 1\,|_{X=0}$$
$$\frac{d\theta_t}{dX} = 0\,|_{X=1}$$

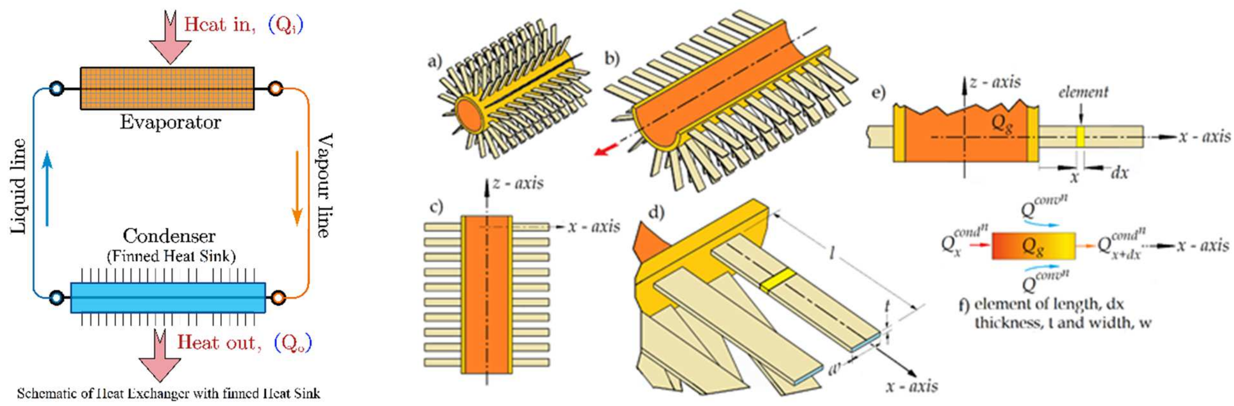

Fig. 1. Longitudinal rectangular fin: (a) Condenser pipe (finned heat sink) with array of longitudinal fins, (b) Half section of pipe carrying fins, (c) Sectional top view of pipe carrying fins, (d) Isometric view of removed section of pipe and fins with fin geometry, (e) Removed section of pipe and fin (top view) and (f) Heat balance over an element within the fin.

2.1.1 Conductivity and heat generation being linearly dependent on temperature

The following equations are derived for longitudinal fin with thermal conductivity (k) and heat generation (Q_g) assumed to be a linear function of temperature as defined in Eq. 3(a-b).

$$k = k_a\left[1+\xi_t(T-T_A)\right] \tag{3a}$$

$$Q_g = Q_{go}\left[1+\gamma_t(T-T_A)\right] \tag{3b}$$

Upon substituting Eq. 3(a-b), the governing equation is obtained as follows:

$$\left[k_a\left[1+\xi_t(T-T_A)\right]\frac{d^2T}{dx^2}\right]+Q_{go}\left[1+\gamma_t(T-T_A)\right]=h_0\left(\frac{T-T_A}{T_0-T_A}\right)^{\varepsilon_t}\frac{P}{A_c}(T-T_A)+\rho C\frac{dT}{dt} \tag{4}$$

In the above equation, T_A is the ambient temperature and T_0 is the suddenly imposed base temperature (at $t = 0$). In terms of thermal parameters, the normalized coefficient of heat generation variation and normalized coefficient of thermal conductivity variation for the first case (Eq. 4) are as follows:

$$\lambda_t = \gamma_t(T-T_A)$$
$$\beta_t = \xi_t(T-T_A)$$

Substituting the normalized variables in Eq. 4, the governing equation for first case is obtained as follows:

$$(1+\beta_t\theta_t)\frac{d^2\theta_t}{dX^2}+Q_tN^2(1+\lambda_t\theta_t)-N^2\theta_t^{\varepsilon_t+1}=\frac{d\theta_t}{dt^*} \tag{5}$$

2.1.2 Conductivity and heat generation varying with power law function of temperature

In this case, the mathematical formulation is reported with step change in base temperature boundary condition for fin material having thermal conductivity and internal heat generation varying as a power law function of temperature as given in Eq. 6 (a-b). The variation of heat transfer coefficient is given in Eq. 3(a).

$$Q_g = Q_{go}\left(\frac{T-T_A}{T_0-T_A}\right)^{\zeta_t} \tag{6a}$$

$$k = k_a\left(\frac{T-T_A}{T_0-T_A}\right)^{\kappa_t} \tag{6b}$$

The governing equation for transient response of longitudinal fin having power law variation of thermal conductivity, heat transfer coefficient and heat generation under step change in base temperature is, thus, obtained as (Eq. 7):

$$\theta_t^{\kappa_t}\frac{d^2\theta_t}{dX^2}+Q_tN^2\theta_t^{\zeta_t}-N^2\theta_t^{\varepsilon_t+1}=\frac{d\theta_t}{dt^*} \tag{7}$$

2.2 Transient response under step change in base heat flux

Several thermal phenomenon are recoded in real life applications wherein the transient condition is initiated (at time t = 0) due to sudden variation in base heat flux. The base boundary condition, in this case, is step change in heat flux at the base of the fin. The formulation is carried out for two different possible variations of thermophysical properties. Similar to the derivation reported in the preceding section, excess temperature and heat generation is normalized as follows:

$$\theta_q = \frac{k_aA_c(T-T_A)}{Q_0L} \tag{8a}$$

$$Q_q = \left(\frac{Q_{go}A_cL}{Q_0}\right) \tag{8b}$$

The heat transfer coefficient (h) varies with temperature according to power law as follows:

$$h = h_0\left(\frac{k_aA_c(T-T_A)}{Q_0L}\right)^{\varepsilon_q} \tag{9}$$

The transient response for longitudinal fins is obtained under the following boundary conditions in normalized coordinates for step change in base heat flux:

$$\frac{d\theta_q}{dX}=1\,|_{X=0}$$
$$\frac{d\theta_q}{dX}=0\,|_{X=1}$$

2.2.1 Conductivity and heat generation being linearly dependent on temperature

Thermal conductivity (k) and internal heat generation (Q_g) are assumed to be linearly dependent on temperature (Eq. 10 a-b)) and power law variation of heat transfer coefficient is considered according to Eq. 9.

$$k = k_a\left[1+\xi_q\left(T-T_A\right)\right] \tag{10a}$$

$$Q_g = Q_{go}\left[1+\gamma_q\left(T-T_A\right)\right] \tag{10b}$$

Upon substituting Eq. 10(a-b), the governing equation in non-dimensional coordinates is obtained as follows:

$$\left(1+\beta_q\theta_q\right)\frac{d^2\theta_q}{dX^2}+Q_q\left(1+\lambda_q\theta_q\right)-N^2\theta_q^{\,\varepsilon_q+1}=\frac{d\theta_q}{dt^*} \tag{11}$$

In Eq. 11,

$$\lambda_q=\gamma_q\left(\frac{Q_0L}{k_aA_c}\right);\quad \beta_q=\xi_q\left(\frac{Q_0L}{k_aA_c}\right);$$

2.2.2 Conductivity and heat generation varying with power law function of temperature

The thermal conductivity and internal heat generation are assumed to vary non-linearly according to power law function of temperature as given by Eq. 12 (a-b). Heat transfer coefficient follows the relation given in Eq. 9. Upon substituting the parameters, the normalized governing differential equation is obtained as given in Eq. 13.

$$k = k_a\left[\frac{k_aA_c\left(T-T_A\right)}{Q_0L}\right]^{\kappa_q} \tag{12a}$$

$$Q_g = Q_{go}\left[\frac{k_aA_c\left(T-T_A\right)}{Q_0L}\right]^{\zeta_q} \tag{12b}$$

$$\theta_q^{\kappa_q}\frac{d^2\theta_q}{dX^2}+Q_q\theta_q^{\zeta_q}-N^2\theta_q^{\,\varepsilon_q+1}=\frac{d\theta_q}{dt^*} \tag{13}$$

3. Lattice Boltzmann Solver

In the preceding section, the governing differential equation modeling the transient response of longitudinal fins with temperature dependent thermal conductivity, internal heat generation and heat transfer coefficient is derived for step change in base temperature (Eq. 5 and Eq.7) and step change in base heat flux (Eq. 11 and Eq.13). The non-linear governing differential equations (Eqs. 5, 7, 11 and 13) are then solved using LB-Solver, an in-house MATLAB® source code (flowchart shown in Fig 3). Although, in recent years, as an alternative to CFD, Lattice Boltzmann Equations have paved deeply into the sphere of numerical schemes based on Methods of Approximations, these have proved equally efficient to simulate transport equations involving heat and thermal energy. In one of the popularly adopted approaches, the mathematical model of transport equations, being partial differential equations (PDEs), are treated by different discretization based numerical schemes at macroscopic scales wherein these are converted into set of linear algebraic equations. In another approach for solving the transport equations, the tiny particles are simulated at microscopic scale and treated with Hamilton's equation. However, the drawback of the first approach is the difficulty to arrive at a solution in presence of non-linearity and other complexities, while the second approach is simply too large to handle if applied at problems of macroscopic dimensions. Lattice Boltzmann equation exists at the mesoscopic scale and has been reported widely to have the ability to close the gap between simulations at macroscopic and microscopic levels. This is possible due to the setting up of simplified mesoscopic kinetic models based on the microscopic processes such that the macroscopic averaged properties comply with conservation laws. LB equation is spatially and temporally second order accurate, sufficient in terms of precision requirement in most applications. In LBM, spatial discretization is carried out using blocks called lattices and at each lattice site, a complete set of particle distribution functions are located. The dimension of the problem (n) and the number of velocity vectors (m) generate the nomenclature (DnQm) for the types of lattice node arrangements (Fig. 2. a-c). In general, for one dimensional problems, the commonly used lattice models are D1Q2, D1Q3 and D1Q5 (Fig. 2.a-c).

3.1 Discretized Boltzmann equation with Bhatnagar-Gross-Krook-Welander (BGKW) approximation

The discretized form of Boltzmann Equation, in absence of external forces is reported as follows [1]:

$$\frac{\partial f_i}{\partial t}+c_i.\nabla f_i=\Omega \tag{14}$$

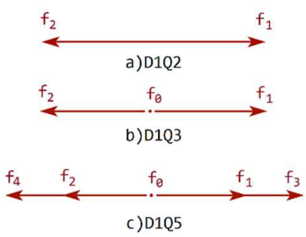

Fig. 2(a-c). Lattice models commonly used in 1D formulation with respective nomenclatures (f_i are the distribution functions).

Table 1: Normalized variable relaxation time (τ^*) and source terms (S) for discretized LB equations

S.N0.	Case	Governing Equation	τ^*	S
1	Step change in base temperature	Eq. 5	$\frac{(1+\beta_t\theta_t)}{(\Delta X/\Delta t^*)^2}+\frac{\Delta t^*}{2}$	$Q_t N^2(1+\lambda_t\theta_t)-N^2\theta_t^{\varepsilon_t+1}$
		Eq. 7	$\frac{\theta_t^{\kappa_t}}{(\Delta X/\Delta t^*)^2}+\frac{\Delta t^*}{2}$	$Q_t N^2\theta_t^{\zeta_t}-N^2\theta_t^{\varepsilon_t+1}$
2	Step change in base heat flux	Eq. 11	$\frac{(1+\beta_q\theta_q)}{(\Delta X/\Delta t^*)^2}+\frac{\Delta t^*}{2}$	$Q_q(1+\lambda_q\theta_q)-N^2\theta_q^{\varepsilon_q+1}$
		Eq. 13	$\frac{\theta_q^{\kappa_q}}{(\Delta X/\Delta t^*)^2}+\frac{\Delta t^*}{2}$	$Q_q\theta_q^{\zeta_q}-N^2\theta_q^{\varepsilon_q+1}$

In Eq. 14, f_i are the distribution functions at the lattice nodes of the Boltzmann model, c_i are the discretized velocities for streaming of the distribution functions from one node to the subsequent node and Ω is the collision operator in terms of distribution function, f. Eq. 14 is difficult to solve due to the presence of the collision term Ω and hence a simple term based on BGKW approximation is used to replace the collision term without introducing significant error. Upon substituting Ω with BGKW approximation in Eq. 14:

$$\frac{\partial f_i}{\partial t}+c_i.\nabla f_i=\omega(f_i^{eq}-f_i)=\frac{(f_i^{eq}-f_i)}{\tau} \tag{15}$$

In discretized form, Eq. 15 is written as:

$$f_i(x+c_i dt, t+dt)-f_i(x,t)=\frac{\Delta t\left[f_i^{eq}(x,t)-f_i(x,t)\right]}{\tau} \tag{16}$$

In Eq. 14 and 15, f_i^{eq} is the (equilibrium) distribution function at relaxation time τ when equilibrium is attained and is determined by the type of problem or application. In Eq. 16, f_i^{eq} is defined as:

$$f_i^{eq}=(\sum f_i)w_i\left(1+\frac{uc_i}{c_s^2}+\frac{(uc_i)^2}{2c_s^4}-\frac{u.u}{2c_s^2}\right) \tag{17}$$

In diffusion problems, macroscopic velocity, u in Eq. 17 is zero. Eq. 16 is a linear PDE wherein the expression at LHS represents streaming meaning that the distribution function advects along the lattice link with velocity c_i (and weightage, w_i) and the expression at the RHS represents collision. The completion of these two steps completes one time step. Relaxation time, τ is related to diffusion coefficient by Chapman-Enskong expansion as follows [1]:

$$\alpha=\frac{\Delta x^2}{\Delta t\times D}\left(\frac{\tau}{\Delta t}-0.5\right)=\frac{\Delta x^2}{\Delta t\times D}\left(\frac{1}{\omega}-0.5\right)$$

Accuracy and stability Guidelines for LBM-BGKW approximation have been reported by [2, 43], wherein the necessary stability condition is reported match for $\tau/\Delta t>0.5$. Hereby, following the same, the constraint of non-negativity of distribution function is also fulfilled.

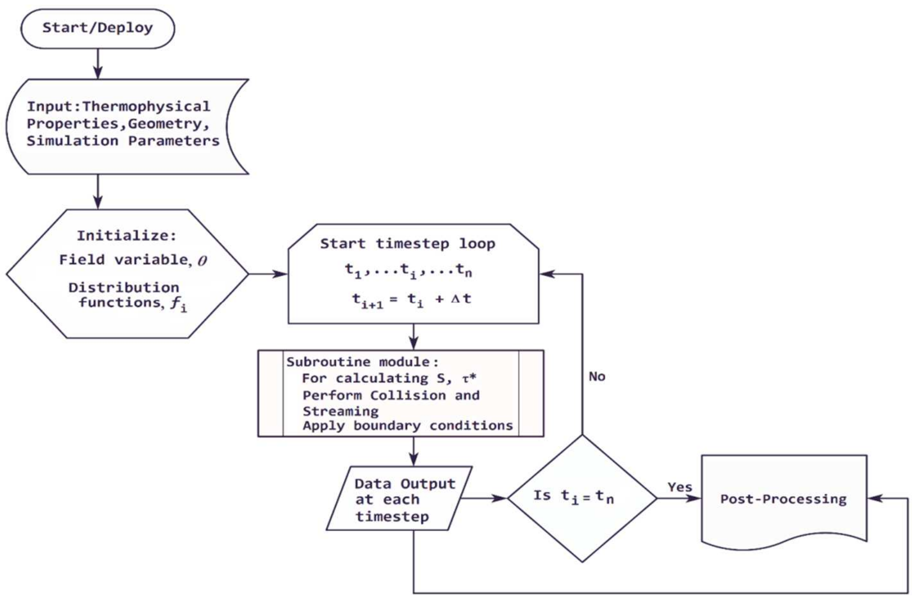

Fig. 3. Solution algorithm of Lattice Boltzmann Solver in-house code.

3.2 Lattice Boltzmann equations for simulating transient response of longitudinal fins

For the present study, normalized Boltzmann equation for determining the transient response of longitudinal fins subject to cases described in the preceding sections is derived using D1Q3 lattice model and reported as follows:

$$f_i^*(X + dX, t^* + dt^*) - f_i(X, t^*) = \frac{\Delta t^* \left[f_i^{*eq}(X, t^*) - f_i^*(X, t^*) \right]}{\tau^*} + w_i S \Delta t^* \tag{19}$$

In Eq. 19, the last term of the expression in the RHS is the source term (S) for the heat diffusion problems arising out of the convection, heat generation and other similar terms present in the governing differential equation. The normalized field variable and normalized equilibrium distribution function are converted to an equivalent terms in Boltzmann equation and are as follows:

$$\theta(X, t^*) = \sum f_i^*(X, t^*)$$

$$f_i^{*eq} = w_i \theta$$

Eq. 19 is the governing equation in LB discretized form. When compared to the governing differential equations (Eq. 5, 7, 11 and 13), the normalized relaxation time (τ^*) and source terms (S) for each case is given in Table 1. Corresponding to eq. 7 and eq. 13 in Table 1, the relaxation time (τ^*) is taken as a nonlinear term thereby taking into account the nonlinear material properties prevailing in most of the application of heat transfer. Also, the value of relaxation time varies according to the value of temperature at each node. Kruger et al. [2] have reported the accuracy and stability for BGKW approximation, in terms of τ^*.

4. Results and validations

In this section, results obtained from the solution of governing equations (Eq. 5, 7, 9 and 13) using LBM solvers are reported under different boundary condition respectively. As reported in the previous section, the variation of heat transfer coefficient is assumed to be a power law function for all the cases. The power law index (ε), has been assumed to be -0.25, 0, 0.25, 0.33, 2 and 3 throughout the study. The assumed values of power law indices represent most of the practical applications such as boiling and condensation (laminar), turbulent natural convection, nucleate boiling, radiation heat transfer [27] and many more. The normalized fin parameter, N is taken in the range of 0.1 to 4.0. Larger value of N helps in attaining the steady state readily [23]. Normalized heat generation parameter, Q, is varied from 0.0 to 1.0. The scalar multipliers, β and λ, used to define the linear dependence of thermal conductivity and heat generation on temperature (Eq. 5 and 11) respectively are varied from 0.0 to 1.0 while in case of power law variation of thermal conductivity (κ) and heat generation (ζ) with temperature (Eq. 7 and 13), the normalized indices are varied from 0.0 to 2.0 and 0.0 to 3.0 respectively. The proposed formulation is validated as shown in Figs 4(a-b). The effects of thermo-physical parameters (N, Q, β and λ) and indices (ε, κ and ζ) on temperature distribution along the fin length are investigated and reported in Figs. 5-24(a-b). The variation of normalized fin temperature (at t^* = 1.0) and instantaneous fin tip temperature (at X = 1.0) with different combinations of thermo-physical parameters are reported in Figs 5-9(a-b) and Figs 10-14(a-b) for linear and power law varying properties of fins subject to step change in base temperature while similar plots are reported in Figs 15-19(a-b) and 20-24(a-b) for fins having linear and power law varying properties subject to step change in base heat flux. The combination of parameter values are given in the legends of the respective figures. In case of high temperature applications, the value of thermal conductivity and heat generation may increase or decrease due to temperature dependence, these variation are assumed to be linear or nonlinear, depending upon the value β, κ, λ *and* ζ. Values of $\beta > 0$ and $\kappa < 0$ represent increase in thermal conductivity with temperature and vice-versa. Similar behaviours were found for λ and ζ.

4.1 Validation

Temperature profile obtained along the fin length for the step change in base temperature and step change in base heat flux using Lattice Boltzmann solver is compared with results obtained from Moitsheki and Harley [31] and Mhlonge et al. [42] respectively and plotted in Fig 4(a) and 4(b). In both cases, [31] and [42], the governing differential equations were solved using lie symmetry analysis wherein depending upon the symmetry of differential equation, exact solution and numerical solution are obtained. The insights of lie principle are available in the texts [44-45]. The obtained results are found to be in good agreement with the benchmarks results. Fig. 4 (a-b) also depicts the effect of normalized time (t^*) on normalized temperature (θ_t). The normalized time (t^*) is the final time up to which the fin behaviour is explored in the code. In this case, t^*=1.0. It is to be noted from the figures that at each time step, the temperature recorded along the fin length is the *instantaneous temperature*.

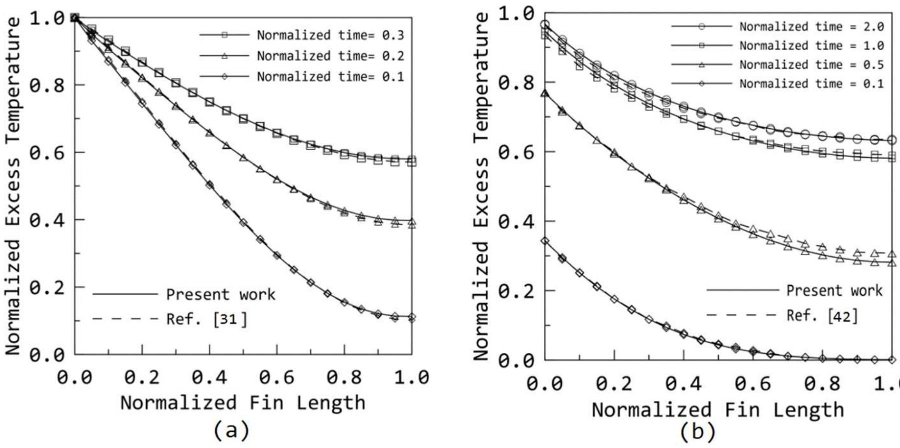

Fig. 4. Normalized temperature distribution in transient fin (a) for constant base temperature having $Q_t = \lambda_t = 0.0$, N = 1.0, $\beta_t = 1.0$ and $\varepsilon_t = 1.0$ and 4(b) for constant base flux having $Q_q = 0.0$, N = 2.0, $k_q = 0.1$ and $\varepsilon_q = 4.0$.

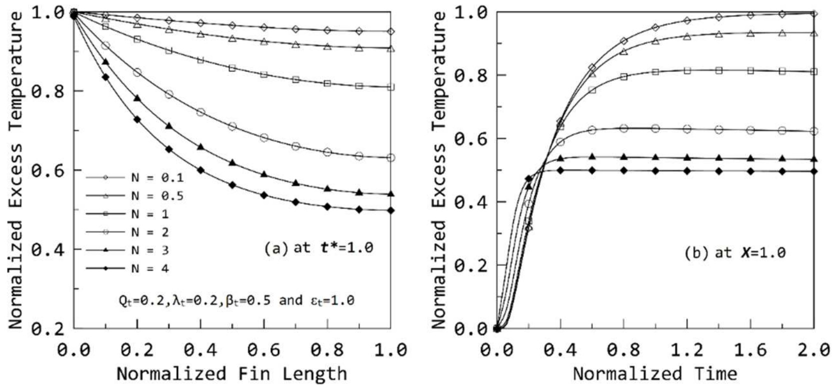

Fig. 5. Effect of N on temperature for fin with linear property variation under step change in base temperature.

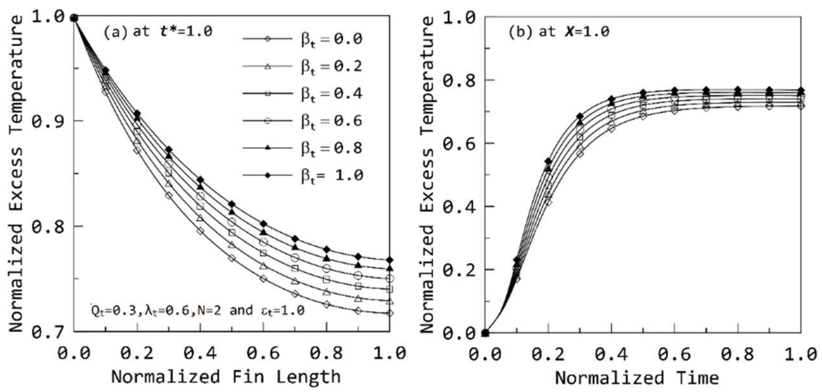

Fig. 6. Effect of β on temperature for fin with linear property variation under step change in base temperature.

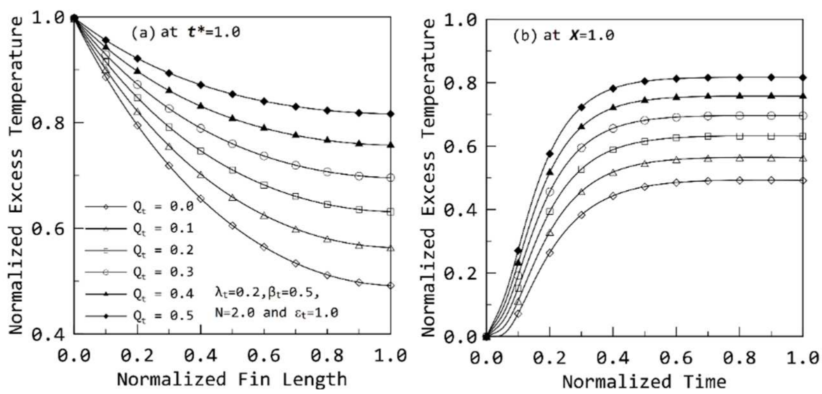

Fig. 7. Effect of Q on temperature for fin with linear property variation under step change in base temperature.

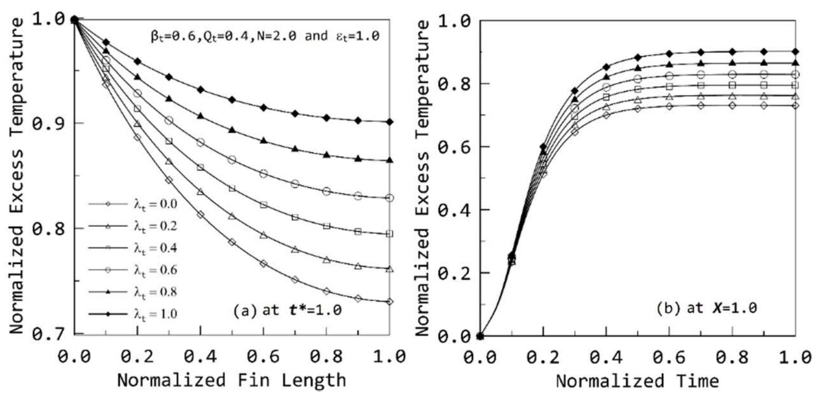

Fig. 8. Effect of λ on temperature for fin with linear property variation under step change in base temperature.

4.2 Response of fin temperature due to step change in base temperature

In this case, transient response within the fin is initiated by sudden change in base temperature. The temperature response is reported for both cases of linear and power law variation of thermal conductivity and heat generation with temperature. In Figs. 5(a)-9(a), first, the temperature distribution is plotted along the normalized fin length at $t^* = 1.0$ followed by the plot of instantaneous temperature of the fin tip (at X = 1.0) throughout the runtime in Figs. 5(b)-9(b).

In Fig 5, the effect of fin parameter (*N*) on normalized temperature (θ_t) is first plotted along the normalized length (*X*) followed by the transient response of instantaneous fin tip temperature till the steady state is attained. It is evident that with increase in fin parameter, *N*, more heat is transferred to environment and normalized temperature decreases along the length of fin. Fig 6 reports the effect of normalized thermal conductivity parameter (β_t) on normalized temperature distribution for given *N*, Q_t, ε_t and λ_t. It is observed that as β_t increases, heat conduction through fin increases and consequently the heat transfer rate through the fin increases. In Fig 7 and Fig 8, the effect of normalized heat generation parameter (Q_t and λ_t) on normalized temperature profile for constant value of ε_t, *N* and β_t respectively is plotted. It is observed that with increase in normalized internal heat generation parameter, temperature gradient of fin decreases. Fig 9 shows the effect of ε_t on fin temperature distribution for given values of *N*, Q_t, β_t and λ_t. It is to be noted that in subsequent plots, the temperature at fin tip (*X* = 1.0) in Figs 5-9(a) corroborates with the *instantaneous temperature* recorded in Figs 5-9(b) at $t^* = 1.0$.

Similar results are plotted in Figs 10-14 (a-b) for power law dependent heat generation parameter and thermal conductivity. Temperature decreases along the fin length with increase in normalized fin parameter (N) since more heat is transferred to environment as shown in Fig. 10 (a-b). The effect of non-linear heat generation is clearly evident from Fig. 10(b) wherein the instantaneous temperature of fin tip is plotted with normalized time (t^*). It is observed that with increase in normalized fin parameter, *N*, the steady state is attained at a faster rate. Interestingly, it is also observed that for smaller *N* (< 1.0), the temperature at fin tip rises after a time lag. This, in other words, could be termed as a response delay of the fin tip to experience rise in temperature. The power dependent thermal conductivity, heat transfer coefficient, along with the geometry parameters of the fin play a significant role in determining *N* and subsequently in determining the response delay. In other words, the delay in response could be explained as the dissipation of entire heat flowing through the fin before reaching the tip. This phenomenon is termed as response delay and the normalized time till response delay could be termed as time lag.

To further investigate this phenomenon, the effect of thermal conductivity (κ_t) on response delay is studied and reported in Fig 11(a-b) for *N* = 0.5. It is observed that as the value κ_t increases, heat conduction through fin decreases and consequently the heat transfer rate through the fin decreases. In Fig 12(a), the temperature at X = 1.0 represents the fin tip temperature at $t^* = 1.0$ and points at the occurrence of response delay resulting into inevitable time lag. This is corroborated in Fig 12(b) wherein the instantaneous temperature is plotted against normalized time. Fig 12 and Fig 13 shows the effect of normalized heat generation parameter (Q_t and ζ_t) on normalized temperature for constant value of ε_t, *N* and κ_t respectively and similar observations are plotted. It is observed that with increase in normalized internal heat generation parameter (Q_t), temperature gradient of fin decreases and with increase in normalized internal heat generation parameter (ζ_t) temperature gradient of fin increases. In Fig 14, the effect of ε_t on temperature profile with constant value of *N*, Q_t, κ_t and ζ_t is plotted.

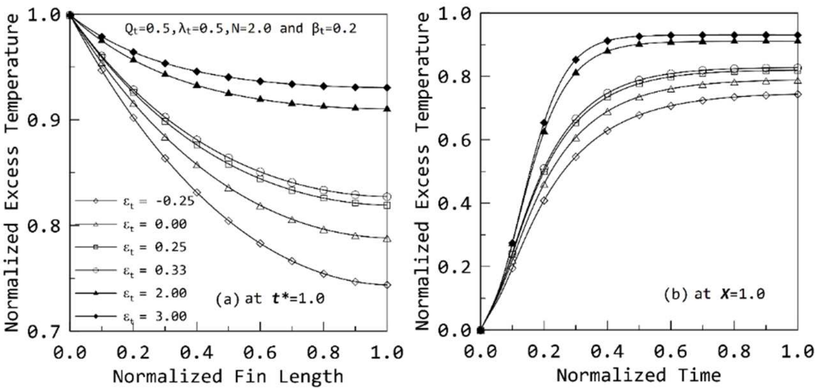

Fig. 9. Effect of ε on temperature for fin with linear property variation under step change in base temperature.

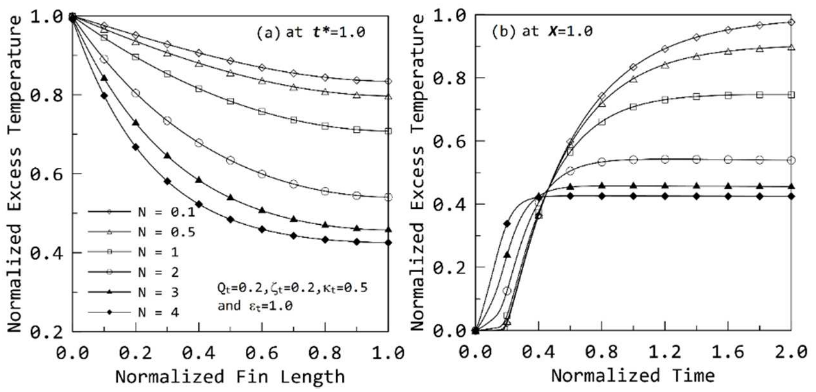

Fig. 10. Effect of *N* on temperature for fin with power law variation under step change in base temperature.

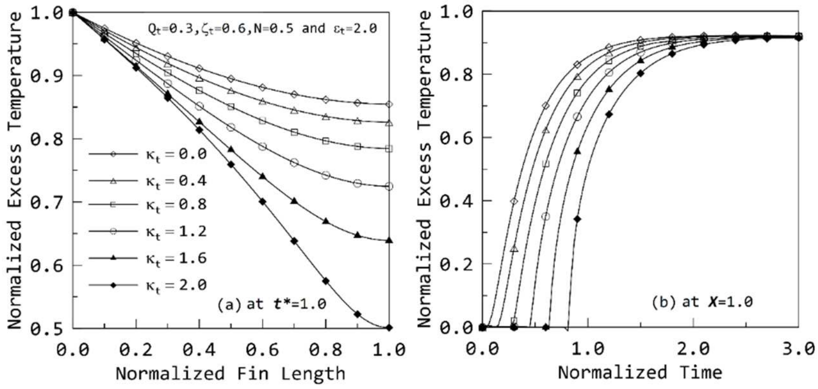

Fig. 11. Effect of k on temperature for fin with power law variation under step change in base temperature.

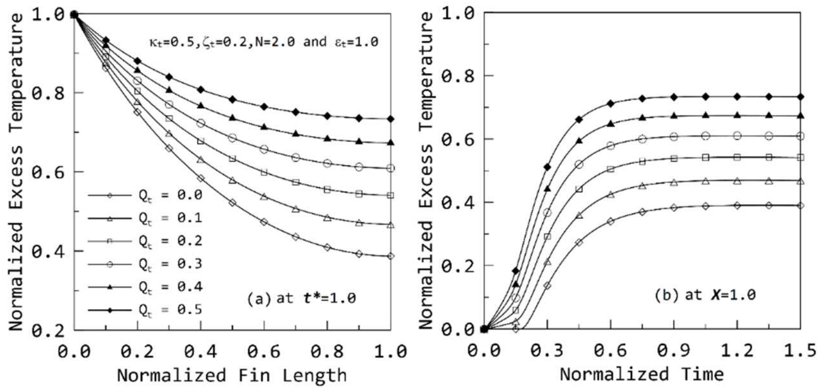

Fig. 12. Effect of Q on temperature for fin with power law variation under step change in base temperature.

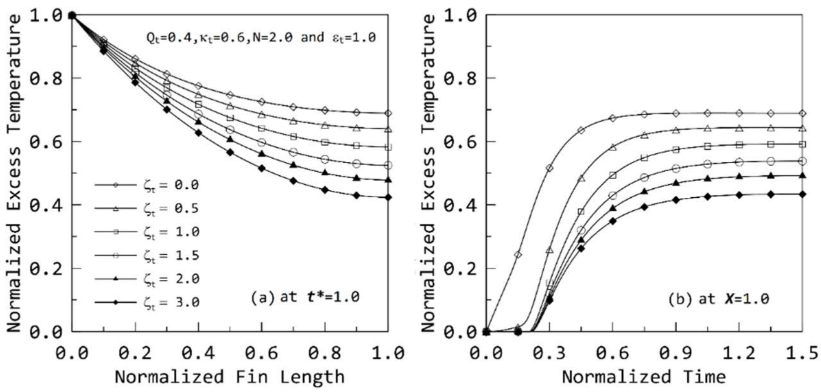

Fig. 13. Effect of ζ on temperature for fin with power law variation under step change in base temperature.

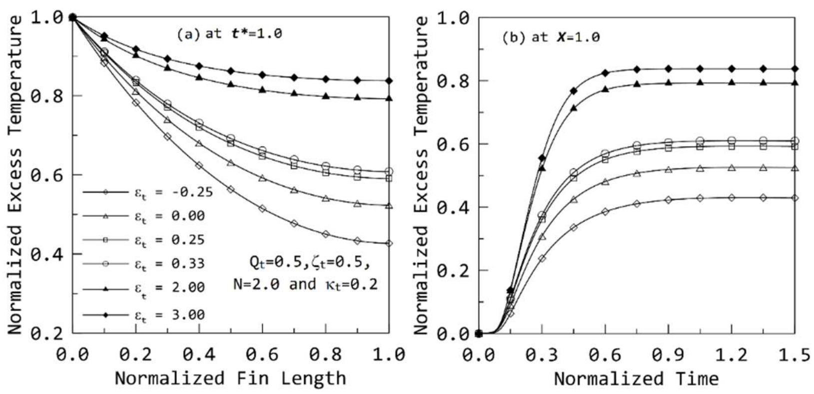

Fig. 14. Effect of ε on temperature for fin with power law variation under step change in base temperature.

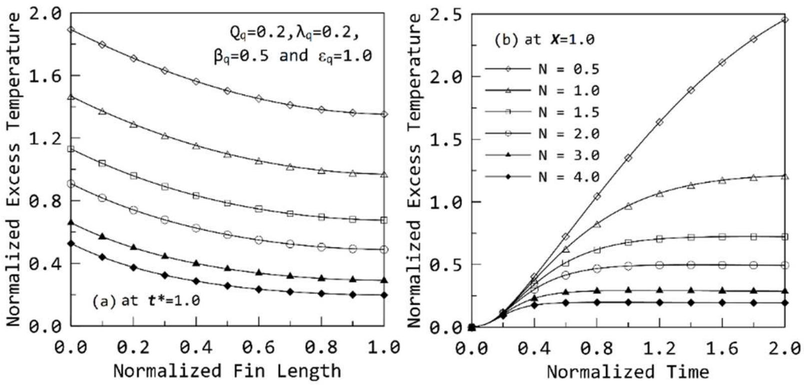

Fig. 15. Effect of N on temperature for fin with linear property variation under step change in base heat flux.

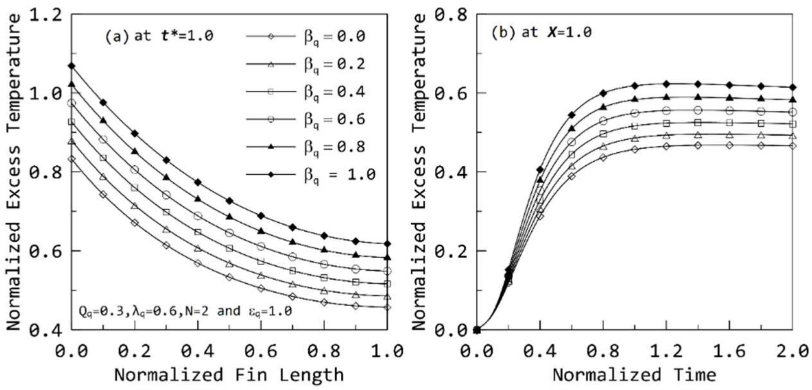

Fig. 16. Effect of β on temperature for fin with linear property variation under step change in base heat flux

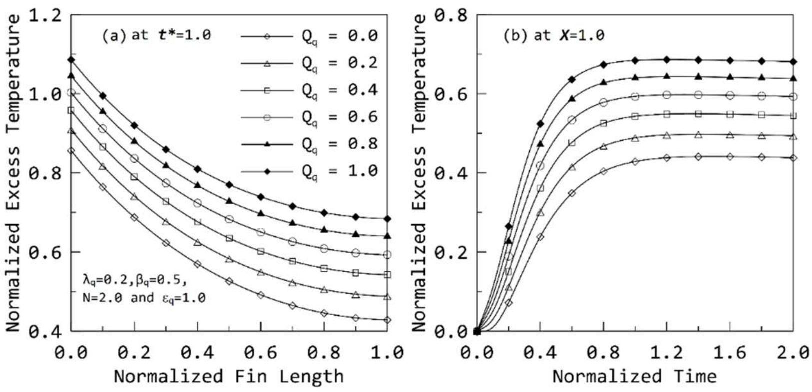

Fig. 17. Effect of Q on temperature for fin with linear property variation under step change in base heat flux.

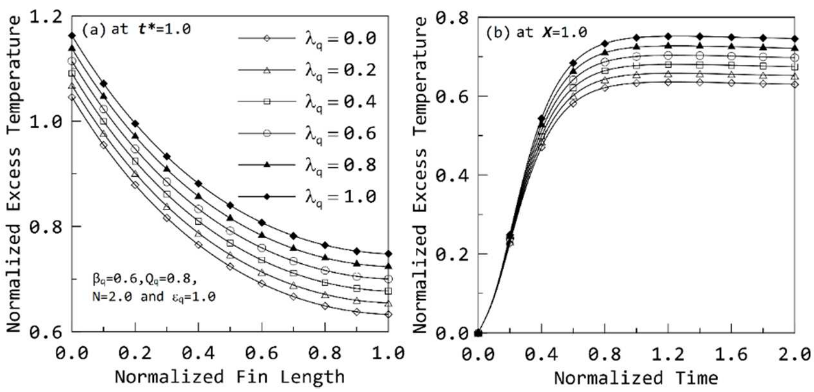

Fig. 18. Effect of λ on temperature for fin with linear property variation under step change in base heat flux

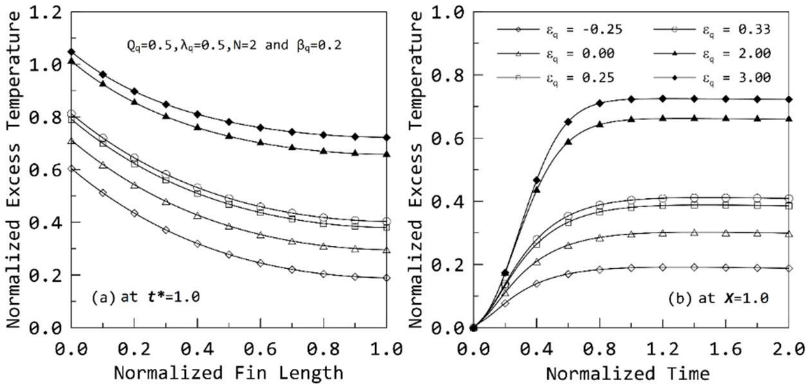

Fig. 19. Effect of ε on temperature for fin with linear property variation under step change in base heat flux.

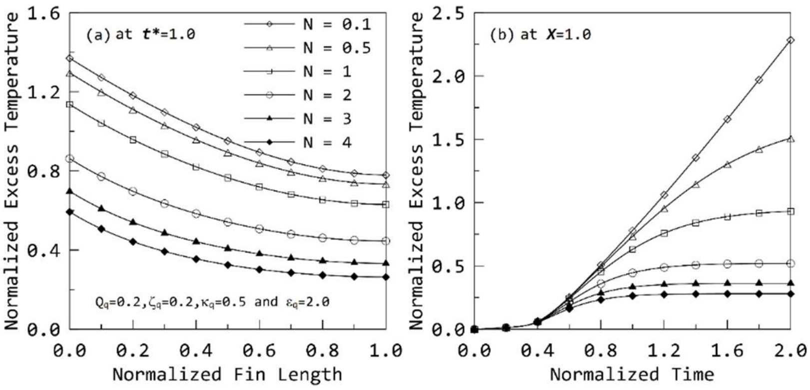

Fig. 20. Effect of N on temperature for fin with power law variation under step change in base heat flux.

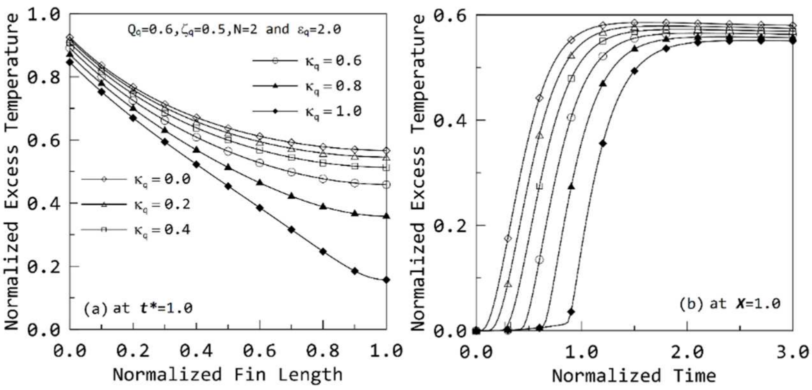

Fig. 21. Effect of k on temperature for fin with power law variation under step change in base heat flux

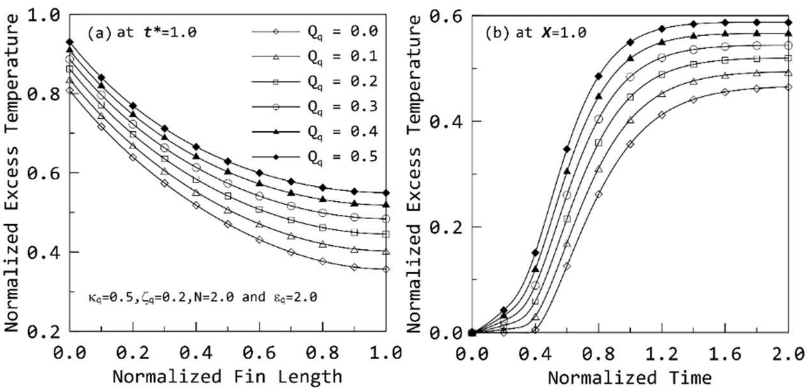

Fig. 22. Effect of Q on temperature for fin with power law variation under step change in base heat flux

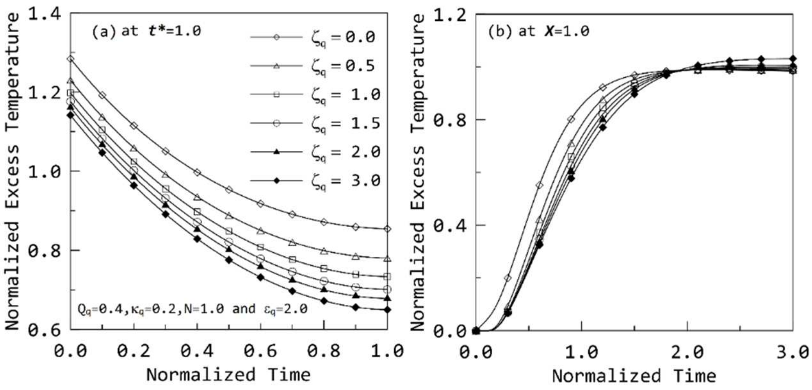

Fig. 23. Effect of ζ on temperature for fin with power law variation under step change in base heat flux

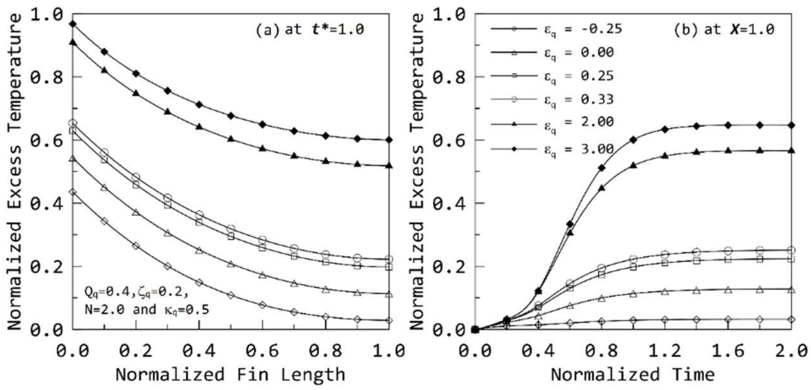

Fig. 24. Effect of ε on temperature for fin with power law variation under step change in base heat flux

4.3 Response of fin temperature due to step change in base heat flux

In this case, the transient state within the fin is initiated by sudden change in base heat flux (q_0). The distribution of normalized temperature θ_q along the normalized length (X) and variation of normalized fin tip temperature with normalized time (t^*) for linear and power law variation of thermal conductivity and heat generation parameter is reported. In Fig. 15 (a-b), the effect of normalized fin parameter on temperature distribution along the fin length and instantaneous fin tip temperature is reported for given values of heat generation and thermal conductivity parameters. From Fig 15 (b) it is observed that for higher value of *N* the fin tip attains steady state readily at relatively low temperature compared to smaller values of N. In Fig 16(a) the temperature distribution is plotted at normalized time $t^* = 1.0$.

Similar plots are shown in Figs. 16-19 (a-b). In these Figs the effect of variation of different thermal parameters on fin temperatures is reported like the effect of normalized thermal conductivity parameter (β_q), normalized heat generation parameter (Q_q and λ_q) and ε_q on temperature profile is reported in Fig. 16(a-b)-19(a-b) respectively.

Similarly Fig. 20(a-b)-24 (a-b) reports the effect of variation of different thermo-physical parameters on temperature distribution along the fin length and instantaneous tip temperature for power law dependent conductivity and heat generation parameters. It is observed from Fig. 20 that normalized temperature decreases along the length of fin with increase in normalized fin parameter (*N*) as more heat is transferred to environment for higher values of *N*. Fig. 21 shows the effect of normalized thermal conductivity parameter (κ_q) on normalized temperature distribution for constant value of *N*, Q_q, ε_q and ζ_q. It is observed that as the value κ_q increases heat conduction through fin decreases consequently the heat transfer rate through the fin decreases. Fig. 22 and Fig. 23 shows the effect of normalized heat generation parameter (Q_q and ζ_q) on normalized temperature profile for constant value of ε_q, N and κ_q respectively. It is observed from this Fig that fin temperature varies inversely with Q_q and directly with ζ_q. Fig. 24 shows the effect of ε_q on temperature profile with constant value of *N*, Q_q, κ_q and ζ_q.

5. Conclusion

A newly developed method, Lattice Boltzmann method is successfully implemented to determine the transient response of a longitudinal fin having temperature dependent thermo-physical properties. In this work fin temperature along the fin length and instantaneous fin tip temperature are reported for fin with step change in base temperature and step change in base heat flux respectively. The results obtained using LBM are successfully validated with available literature and are found to be in good agreement. The temperature dependence of thermo-physical parameters treated in this article are based on linear as well as power law function. It is observed that the fin temperature attains its steady state readily for larger values of fin parameter N. In case of power law dependent thermal conductivity and heat generation, an interesting occurrence of response delay in fins is reported. The reason for this is attributed to the fact that in such cases the heat transfer takes place entirely before reaching the fin tip. The non-linearity of thermal conductivity, heat generation and convection coefficient, all taken together in the mathematical model, physically represents the practical application scenario of heat transfer through fins under extreme temperature conditions as the material property behaviour becomes non-linear at excessively high temperature conditions. The methods readily reported in available literature so far could not easily deal with these accumulated non-linearity arising in high-temperature applications. The proposed solution based on LBM is more suitable, in this respect, for high temperature application because the inherent nature of LBM model deals with the accumulated non-linearity in a linear manner by means of manipulations using the local streaming and collision process. Moreover non-linear geometry arising out of variable profile of fin can also solved by this method.

Author Contributions

The work presented in this paper has been offered to A. Sahu (Scholar) by S. Bhowmick (Supervisor) as a part of his research work. A. Sahu and S. Bhowmick have mathematically formulated the problem to establish a framework for development of in-house source code. A. Sahu developed the code in MATLAB under the guidance of S. Bhowmick, conducted the simulation experiments and established the validation of the code. The subsequent simulation runs by A. Sahu produced the data presented in the paper. The draft manuscript was written by A. Sahu. The authors, together, discussed the results and thereupon prepared the final manuscript.

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Not applicable.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

A_c	Cross-section area	N^2	Normalized thermo-physical properties
c_s	Speed of sound	P	Fin Perimeter
D	Dimension of problem	Q_0	Heat transfer at base of fin
f_i	Particle distribution function	t^*	Normalized time
f_i^*	Normalized particle distribution function	T_0	Temperature at the base of fin
c_i	Particle velocity	T_A	Temperature of ambient air
h	Convective heat transfer coefficient	$X(x/L)$	Normalized length
h_0	Convective heat transfer coefficient at fin base	α	Thermal diffusivity
L	Length of Fin	τ	Relaxation time
k	Thermal conductivity of material	ω	Relaxation frequency
k_a	Thermal conductivity of fin material at ambient temperature		**Subscripts** t and q: for the cases of step change in base temperature and step change in heat flux respectively.

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