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Research Paper

Role of Magnetic field on the Dynamical Analysis of Second Grade Fluid: An Optimal Solution subject to Non-integer Differentiable Operators

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Abstract. The dynamical analysis of MHD second grade fluid based on their physical properties has stronger resistance capabilities, low-frequency responses, lower energy consumption, and higher sensitivities; due to these facts externally applied magnetic field always takes the forms of diamagnetic, ferromagnetic and paramagnetic. The mathematical modeling based on the fractional treatment of governing equation subject to the temperature distribution, concentration, and velocity field is developed within a porous surfaced plate. Fractional differential operators with and without non-locality have been employed on the developed governing partial differential equations. The mathematical analysis of developed fractionalized governing partial differential equations has been established by means of systematic and powerful techniques of Laplace transform with its inversion. The fractionalized analytical solutions have been traced out separately through Atangana-Baleanu and Caputo-Fabrizio fractional differential operators. Our results suggest that the velocity profile decrease by increasing the value of the Prandtl number. The existence of a Prandtl number may reflect the control of the thickness of momentum and enlargement of thermal conductivity.

Keywords: Second grade fluid; Fractional differential operator; Magnetic field; Chemical reaction; non-Singular kernels.

1. Introduction

Convective flow is a self-sustained flow with effect of temperature gradient. The density is non- uniform due to variation of temperature. The effect of magnetic flux plays a major role in convective flow. The different industrial problems, the flow of fluid in permeable medium have achieved consideration in recent years. The atmospheric, oceanic circulation and emergency cooling system of advanced nuclear reactors is the most important application of free convection. The second order fluid with warmth transferring porous medium discussed by Tan and Masuoka [1]. A vertical plate with mixed MHD convection implanted on a permeable medium analyzed by Aldose et al. [2]. Rashidi et al. [3] investigating the difference between two normal kinds of liquid stream between clear liquid and permeable medium. Authors [4] investigated the precarious MHD stream of turning second grade fluid in permeable medium over a swaying plate. Ilyas et al. [5] discussed the precise answers for quickened flow of a pivoting second grade fluid in a permeable medium. Bilal et al. [6] investigate the flow of second grade fluid generated by a swaying plate. Farhad et al. [7] discussed the closed structure for second grade fluid over a swaying vertical plate. Literature shows more interest developed in the numerical and approximate solutions on convection flow of second grade fluids [8] – [15]. Exact solutions serve in multiple ways for the specialized relevance of the streams utilized the numerical solvers and to check the dependability of their solutions [16] – [18]. Generally the non-Newtonian fluids are divided into different categories such as differential, rate and integral fluids. The straightest forward kind of differential fluid which discussed the ordinary stresses known as the second grade fluid. Relatively to the Newtonian fluids a higher and complicated mathematical system exists for the non-Newtonian fluids. There are a variety of constitutive conditions to predict the non-Newtonian behavior. In 2010, the second grade fluid over a wavering plate discussed by Nazar et al. [19]. In this model the Laplace transform techniques helped to acquire the accurate solutions. Recently, Farhad et al. [20] discussed the MHD fluid to be electrically conducting and pass through the permeable medium. The procedure of heat transfer is expressed by using momentum, energy and continuity equation with stress and heat flux. The heat transfer phenomenon as the fins of heat exchanger, the tabulator inside the tube or plates, the convective physical

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transport phenomenon [21]. The study of flow in which fluids are electrically leading within the sight of attractive fields is known as MHD. The principle of MHD is helpful for the flow against laminar to turbulence. The MHD within the sight of diffusion and radiation are applications of mass and heat transfer [22].

The non-integer differential operators lie in the field of fractional calculus through which several different possibilities for defining complex number or real number powers of the differentiation and integration operators. In context with fractional differential operators, Abro [23] presented a comparative analysis of thermo-diffusion effects on unsteady-free convection flow using fractional versus non-fractional differential operator and investigated analytical calculation through Laplace and Fourier sine transform methods. From the past to the present, modeling of the different processes is handled through various types of fractional derivatives and fractal-fractional differential operators, such that Caputo (Power law), Atangana-Baleanu (Mittage-Leffler law), Caputo-Fabrizio (exponential law), Riemann-Liouville, modified Riemann-Liouville (Power law with boundaries) and few others [24] – [31]. Convective flow with ramped wall temperature for non-singular kernel analyzed by Bilal et al. [32]. Further, Bilal et al. [33] discuss MHD Maxwell fluid with heat effect using local and non-local operators. Moreover, authors [34] analyze the comparative study of heat transfer of MHD Maxwell fluid in view fractional operators. Raza et al. [35] discussed the fractional second grade fluid with time dependent shear stress. Additionally, the recent studies on modern fractional differential operators and second grade fluids can be traced out there in [36] – [45].

2. Problem Statement

Let us consider incompressible MHD Second grade fluid with constant temperature saturated with porous surface lying on the plate. Suppose γ is inclination angle of the plate and magnetic field is considered as B_o with y-axis as normal to the plate. A temperature T'_∞ and concentration C'_∞ are at initial time $\tau = 0$. At $\tau > 0$, plate started to move with velocity $u'(\eta', \tau')$ having its concentration and temperature depending upon time as $T'_\infty + T'_w h'(\tau')$ and $C'_\infty + C'_w g'(\tau')$. For such a flow, the pressure gradient is absent across the boundary layer. Under these assumptions and using Boussinesq approximation, the governing equations of MHD Second grade fluid are given as:

$$\frac{\partial u'}{\partial \tau'} = \upsilon \frac{\partial^2 u'}{\partial \eta'^2} + \frac{\alpha}{\rho}\frac{\partial^3 u'}{\partial \tau' \partial \eta'^2} + g\beta_T (T' - T'_\infty)\mathrm{Cos}\gamma + g\beta_C (C' - C'_\infty)\mathrm{Cos}\gamma - \frac{\sigma B_0^2}{\rho}u' - \frac{\mu}{k'_p}u' \tag{1}$$

$$\left(\rho C_p\right)\frac{\partial T'}{\partial \tau'} = K\frac{\partial^2 T'}{\partial \eta'^2} - \frac{\partial q_r}{\partial \eta'} + ST' - ST'_\infty \tag{2}$$

$$\frac{\partial C'}{\partial \tau'} = D\frac{\partial^2 C'}{\partial \eta'^2} + k_c C' - k_c C'_\infty \tag{3}$$

Subject to the initial and boundary condition as:

$$\begin{gathered} \tau' \le 0, \quad u'(\eta, 0) = 0, \quad T'(y,0) = T'_\infty, \quad C'(y,0) = C'_\infty, \qquad \eta' \ge 0, \\ \tau' > 0, \quad u'(\eta,\tau) = u_o f'(\tau'), \quad T'(\eta,\tau) = T'_\infty + T'_w h'(\tau'), \quad C'(\eta,\tau) = C'_\infty + C'_w g'(\tau'), \quad \eta' = 0, \\ \tau' > 0, \quad u'(\eta,\tau) \to 0, \quad T'(\eta,\tau) \to T'_\infty, \quad C'(\eta,\tau) \to C'_\infty, \quad \eta' \to \infty. \end{gathered} \tag{4}$$

For the simplification of governing equations (1 – 3), we introduce the dimensionless variable among the governing equations of MHD second grade fluid; we define them as described below:

$$\begin{gathered} \eta = \frac{u_0}{\upsilon}\eta', \quad u = \frac{u'}{u_0}, \quad \tau = \frac{u_0{}^2}{\upsilon}\tau', \quad T = \frac{T' - T'_\infty}{T'_w}, \quad C = \frac{C' - C'_\infty}{C'_w}, \quad K_r = \frac{\upsilon}{u_0{}^2 k'_c}, \quad M = \frac{\sigma B_o^2 \upsilon}{\rho u_o^2}, \\ Sc = \frac{\upsilon}{D}, \quad k_1 = \frac{\upsilon^4}{u_o^4 k_p}, \quad P_r = \frac{\mu C_p}{K}, \quad k_p = \frac{\upsilon^2}{u_o^2}k'_p. \end{gathered} \tag{5}$$

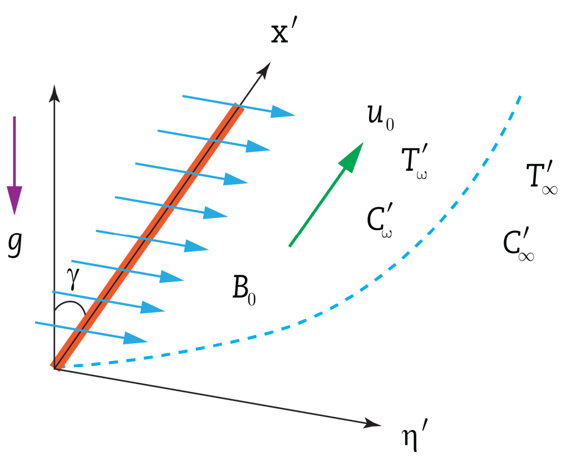

Fig. 1. Geometrical presentation of second grade model

After invoking the dimensionless quantities in equation (1 – 3), we have simplified governing equations of second grade fluid as:

$$\frac{\partial u}{\partial \tau}=\frac{\partial^2 u}{\partial \eta^2}+\alpha\frac{\partial^3 u}{\partial \tau\partial \eta^2}+\lambda Cos\gamma T+N_r Cos\gamma C-Mu-k_1 u, \tag{6}$$

$$\frac{\partial T}{\partial \tau}=\frac{1}{Pr}\left(1+\frac{4}{3}R_d\right)\frac{\partial^2 T}{\partial \eta^2}-ST, \tag{7}$$

$$\frac{\partial C}{\partial \tau}=\frac{1}{Sc}\frac{\partial^2 C}{\partial \eta^2}-K_r C, \tag{8}$$

with initial and boundary conditions:

$$\begin{array}{llll}\tau\le 0,\; u(\eta,0)=0, & T(\eta,0)=0, & C(\eta,0)=0, & \eta\ge 0,\\ \tau>0,\; u(0,\tau)=f(\tau), & T(0,\tau)=h(\tau), & C(0,\tau)=g(\tau), & \eta=0,\\ \tau>0,\; u(\infty,\tau)\to 0, & T(\infty,\tau)\to 0, & C(\infty,\tau)\to 0, & \eta\to\infty.\end{array} \tag{9}$$

3. Concentration Field

3.1 CF – Fractional Operator

Fractional operators are quite flexible for describing the behaviors of concentration field for MHD Second grade fluid through the characterization of governing equations. Generating fractional governing equation of concentration (8) via CF-fractional operator by exchanging the partial derivative with fractional derivative κ,

$${}^{CF}D_\tau^\kappa C(\eta,\tau)=\frac{1}{Sc}\frac{\partial^2 C(\eta,\tau)}{\partial \eta^2}-K_r C(\eta,\tau), \tag{10}$$

where, ${}^{CF}D_\tau^\kappa$ is known as CF-fractional derivative defined as [27]:

$${}^{CF}D_\tau^\kappa N(\zeta,\tau)=\frac{1}{1-\kappa}\int_0^\tau \exp\left(-\frac{\kappa(\tau-\xi)}{1-\kappa}\right)N'(\xi)d\xi,\quad 0<\kappa<1. \tag{11}$$

Solving the uncoupled and fractionalized governing equation of concentration (10) by Laplace transform method. One has to need the following typical property of Caputo-Fabrizio fractional operator defined in equation (11),

$$L\left({}^{CF}D_\tau^\kappa N(\zeta,\tau)\right)=\frac{sL\left(N(\zeta,\tau)\right)-N(\zeta,0)}{(1-\kappa)s+\kappa}, \tag{12}$$

employing Laplace transformation on (10) with the help of (12), we explored second order partial differential equation,

$$\frac{\partial^2 \bar{C}_{cf}(\eta,s)}{\partial \eta^2}-Sc\left(\frac{s}{(1-\kappa)s+\kappa}+K_r\right)\bar{C}_{cf}(\eta,s)=0. \tag{13}$$

More suitable form of concentration field is,

$$\frac{\partial^2 \bar{C}_{cf}(\eta,s)}{\partial \eta^2}-Sc\left(\frac{g_1 s+g_2}{g_3 s+g_4}\right)\bar{C}_{cf}(\eta,s)=0. \tag{14}$$

The required solution of Eq. (14) is,

$$\bar{C}_{cf}(\eta,s)=c_1 e^{\eta\sqrt{Sc\left(\frac{g_1 s+g_2}{g_3 s+g_4}\right)}}+c_2 e^{-\eta\sqrt{Sc\left(\frac{g_1 s+g_2}{g_3 s+g_4}\right)}}, \tag{15}$$

with the help of Eq. (9), we find out the arbitrary constants for concentration equation:

$$\bar{C}_{cf}(\eta,s)=G(s)e^{-\eta\sqrt{Sc\left(\frac{g_1 s+g_2}{g_3 s+g_4}\right)}}. \tag{16}$$

Inverting Eq. (16) by means of Laplace transform and using Appendix (A1)-(A2), we get suitable expression as:

$$C_{cf}(\eta,\tau)=\int_0^\tau g'(\tau-s)\Pi\left(\eta\sqrt{Sc},\tau,0,g_1,g_2,g_3,g_4\right). \tag{17}$$

3.2 ABC – Fractional Operator

Fractional operators are quite flexible for describing the behaviors of concentration field for MHD Second grade fluid through the characterization of governing equations. Generating fractional governing equation of concentration (8) via ABC-fractional operator by exchanging the partial derivative with fractional derivative κ,

$$^{ABC}D_{\tau}^{\kappa}C(\eta,\tau)=\frac{1}{Sc}\frac{\partial^2 C(\eta,\tau)}{\partial\eta^2}-K_r C(\eta,\tau), \tag{18}$$

where, $^{ABC}D_{\tau}^{\alpha}$ is known as ABC-fractional derivative defined as [25]:

$$^{ABC}D_{\tau}^{\alpha}f(\xi,\tau)=\frac{M(\alpha)}{1-\kappa}\int_0^{\tau}E_{\alpha}\left(-\frac{\alpha(t-\tau)^{\alpha}}{1-\alpha}\right)f'(\tau)d\tau, \quad \text{with} \quad \sum_{m=0}^{\infty}\frac{(-t)^{\alpha m}}{\Gamma(1+\alpha m)}=E_{\alpha}(-t)^{\alpha}, \tag{19}$$

where $M(\alpha)$ denotes a normalization function obeying $M(0)=M(1)=1.$ Solving the uncoupled and fractionalized governing equation of concentration (18) by Laplace transform method. One has to need the following typical property of Caputo-Fabrizio fractional operator defined in equation (19),

$$L\left(^{ABC}D_{\tau}^{\alpha}f(\xi,\tau)\right)=\frac{s^{\alpha}L\left(f(\xi,\tau)\right)-s^{\alpha-1}f(\xi,0)}{(1-\alpha)s^{\alpha}+\alpha}, \tag{20}$$

employing Laplace transformation on (18) with the help of (20), we explored second order partial differential equation:

$$\frac{\partial^2\bar{C}_{abc}(\eta,s)}{\partial\eta^2}-Sc\left(\frac{s^{\kappa}}{(1-\kappa)s^{\kappa}+\kappa}+K_r\right)\bar{C}_{abc}(\eta,s)=0. \tag{21}$$

More suitable form of concentration field is,

$$\frac{\partial^2\bar{C}_{abc}(\eta,s)}{\partial\eta^2}-Sc\left(\frac{g_1 s^{\kappa}+g_2}{g_3 s^{\kappa}+g_4}\right)\bar{C}_{abc}(\eta,s)=0. \tag{22}$$

The required solution of Eq. (22) is,

$$\bar{C}_{abc}(\eta,s)=c_1 e^{\eta\sqrt{Sc\left(\frac{g_1 s^{\kappa}+g_2}{g_3 s^{\kappa}+g_4}\right)}}+c_2 e^{-\eta\sqrt{Sc\left(\frac{g_1 s^{\kappa}+g_2}{g_3 s^{\kappa}+g_4}\right)}}, \tag{23}$$

with the help of Eq. (9), we find out the arbitrary constants for concentration equation:

$$\bar{C}_{abc}(\eta,s)=G(s)e^{-\eta\sqrt{Sc\left(\frac{g_1 s^{\kappa}+g_2}{g_3 s^{\kappa}+g_4}\right)}}. \tag{24}$$

3.2.1 Special Case

Now the required result discuss by considering $G(s)=1/s$ as a boundary condition on concentration,

$$\bar{C}_{abc}(\eta,s)=\frac{1}{s}e^{-\eta\sqrt{Sc\left(\frac{g_1 s^{\kappa}+g_2}{g_3 s^{\kappa}+g_4}\right)}}. \tag{25}$$

Inverting Eq. (25) by means of Laplace transform and using Appendix (A3)-(A6), we get suitable expression as:

$$C_{abc}(\eta,\tau)=M(\tau,\beta)\chi\left(\eta\sqrt{Sc},\tau,0,g_1,g_2,g_3,g_4\right). \tag{26}$$

4. Temperature Field

4.1 CF – Fractional Operator

Fractional operators are quite flexible for describing the behaviors of temperature field for MHD Second grade fluid through the characterization of governing equations. Generating fractional governing equation of temperature (7) via CF-fractional operator by exchanging the partial derivative with fractional derivative κ,

$$^{CF}D_{\tau}^{\kappa}T(\eta,\tau)=\frac{1}{P_r}\left(1+\frac{4}{3}R_d\right)\frac{\partial^2 T(\eta,\tau)}{\partial\eta^2}-ST(\eta,\tau), \tag{27}$$

employing Laplace transformation on (27) with the help of (12), we explored second order partial differential equation

$$\frac{\partial^2\bar{T}_{cf}(\eta,s)}{\partial\eta^2}-P_{ro}\left(\frac{s}{(1-\kappa)s+\kappa}+S\right)\bar{T}_{cf}(\eta,s)=0. \tag{28}$$

More suitable form of concentration field is,

$$\frac{\partial^2\bar{T}_{cf}(\eta,s)}{\partial\eta^2}-P_{ro}\left(\frac{g_5 s+g_6}{g_3 s+g_4}\right)\bar{T}_{cf}(\eta,s)=0. \tag{29}$$

The required solution of Eq. (29) is,

$$\bar{T}_{cf}(\eta,s)=c_1 e^{\eta\sqrt{P_{ro}\left(\frac{g_5 s+g_6}{g_3 s+g_4}\right)}}+c_2 e^{-\eta\sqrt{P_{ro}\left(\frac{g_5 s+g_6}{g_3 s+g_4}\right)}}, \tag{30}$$

with the help of Eq. (9), we find out the arbitrary constants for temperature equation:

$$\overline{T}_{cf}(\eta,s)=H(s)e^{-\eta\sqrt{P_{ro}\left(\frac{g_5 s+g_6}{g_3 s+g_4}\right)}}. \tag{31}$$

Inverting Eq. (16) by means of Laplace transform and using Appendix (A1)-(A2), we get suitable expression as:

$$T_{cf}(\eta,\tau)=\int_0^\tau h'(\tau-s)\Pi\left(\eta\sqrt{P_{ro}},\tau,0,g_5,g_6,g_3,g_4\right). \tag{32}$$

4.2 ABC – Fractional Operator

Fractional operators are quite flexible for describing the behaviors of temperature field for MHD Second grade fluid through the characterization of governing equations. Generating fractional governing equation of temperature (7) via ABC-fractional operator by exchanging the partial derivative with fractional derivative κ,

$${}^{ABC}D_\tau^\kappa T(\eta,\tau)=\frac{1}{P_r}\left(1+\frac{4}{3}R_d\right)\frac{\partial^2 T(\eta,\tau)}{\partial\eta^2}-ST(\eta,\tau), \tag{33}$$

employing Laplace transformation on (33) with the help of (20), we explored second order partial differential equation

$$\frac{\partial^2\overline{T}_{abc}(\eta,s)}{\partial\eta^2}-P_{ro}\left(\frac{s^\kappa}{(1-\kappa)s^\kappa+\kappa}+S\right)\overline{T}_{abc}(\eta,s)=0. \tag{34}$$

More suitable form of concentration field is,

$$\frac{\partial^2\overline{T}_{abc}(\eta,s)}{\partial\eta^2}-P_{ro}\left(\frac{g_5 s^\kappa+g_6}{g_3 s^\kappa+g_4}\right)\overline{T}_{abc}(\eta,s)=0. \tag{35}$$

The required solution of Eq. (35) is,

$$\overline{T}_{abc}(\eta,s)=c_1 e^{\eta\sqrt{P_{ro}\left(\frac{g_5 s^\kappa+g_6}{g_3 s^\kappa+g_4}\right)}}+c_2 e^{-\eta\sqrt{P_{ro}\left(\frac{g_5 s^\kappa+g_6}{g_3 s^\kappa+g_4}\right)}}, \tag{36}$$

with the help of Eq. (9), we find out arbitrary constants for temperature equation:

$$\overline{T}_{abc}(\eta,s)=H(s)e^{-\eta\sqrt{P_{ro}\left(\frac{g_5 s^\kappa+g_6}{g_3 s^\kappa+g_4}\right)}}. \tag{37}$$

4.2.1 Special Case

Now the required result discuss by considering $H(s)=1/s$ as a boundary condition on temperature,

$$\overline{T}_{abc}(\eta,s)=\frac{1}{s}e^{-\eta\sqrt{P_{ro}\left(\frac{g_5 s^\kappa+g_6}{g_3 s^\kappa+g_4}\right)}}. \tag{38}$$

Inverting Eq. (25) by means of Laplace transform and using Appendix (A3)-(A6), we get suitable expression as:

$$T_{abc}(\eta,\tau)=M(\tau,\beta)\chi\left(\eta\sqrt{P_{ro}},\tau,0,g_5,g_6,g_3,g_4\right). \tag{39}$$

5. Velocity Field

5.1 CF – Fractional Operator

Fractional operators are quite flexible for describing the behaviors of velocity field for MHD Second grade fluid through the characterization of governing equations. Generating fractional governing equation of velocity (6) via CF-fractional operator by exchanging the partial derivative with fractional derivative κ,

$${}^{CF}D_\tau^\kappa u(\eta,\tau)=\frac{\partial^2 u}{\partial\eta^2}+\alpha\,{}^{CF}D_\tau^\kappa u(\eta,\tau)\frac{\partial^2 u}{\partial\eta^2}+\lambda\mathrm{Cos}\gamma T+N_r\mathrm{Cos}\gamma C-Mu-k_1u, \tag{40}$$

employing Laplace transformation on (40) with the help of (12), we explored second order partial differential equation

$$\left(\frac{s}{(1-\kappa)s+\kappa}+b_o\right)\overline{u}_{cf}(\eta,s)=\left(1+\frac{\alpha s}{(1-\kappa)s+\kappa}\right)\frac{\partial^2\overline{u}_{cf}(\eta,s)}{\partial\eta^2}+\lambda\cos\gamma\overline{T}+\lambda N_r\cos\gamma\overline{C}. \tag{41}$$

The required solution of homogenous part of Eq. (41) is,

$$\overline{u}_c(\eta,s)=c_1 e^{\eta\sqrt{\left(\frac{g_7 s+g_8}{g_9 s+g_4}\right)}}+c_2 e^{-\eta\sqrt{\left(\frac{g_7 s+g_8}{g_9 s+g_4}\right)}}, \tag{42}$$

and particular solution can be given as follow after making use of Eqs. (16) and (31),

$$\bar{u}_p(\eta,s)=-\frac{-g_3^2\lambda\cos\gamma H(s)}{g_{10}}\left\{1+\frac{B_1}{s+A_1}+\frac{B_2}{s+A_2}\right\}e^{-\eta\sqrt{P_{ro}\frac{g_5s+g_6}{g_3s+g_4}}}-\frac{g_3^2\lambda N_r\cos\gamma G(s)}{g_{20}}\times\left\{1+\frac{B_3}{s+A_3}+\frac{B_4}{s+A_4}\right\}e^{-\eta\sqrt{Sc\frac{g_1s+g_2}{g_3s+g_4}}} \tag{43}$$

and solution of Eq.(41) can be given as:

$$\bar{u}_{cf}(\eta,s)=c_1e^{\eta\sqrt{\frac{g_7s+g_8}{g_9s+g_4}}}+c_2e^{-\eta\sqrt{\frac{g_7s+g_8}{g_9s+g_4}}}-\frac{g_3^2\lambda\cos\gamma H(s)}{g_{10}}\left\{1+\frac{B_1}{s+A_1}+\frac{B_2}{s+A_2}\right\}e^{-\eta\sqrt{P_{ro}\frac{g_5s+g_6}{g_3s+g_4}}}-\frac{g_3^2\lambda N_r\cos\gamma G(s)}{g_{20}}\times\left\{1+\frac{B_3}{s+A_3}+\frac{B_4}{s+A_4}\right\}e^{-\eta\sqrt{Sc\frac{g_1s+g_2}{g_3s+g_4}}} \tag{44}$$

using Eq. (9), we find out the arbitrary parameter c_1 and c_2 as follow:

$$\begin{aligned}\bar{u}_{cf}(\eta,s)=F(s)e^{\eta\sqrt{\frac{g_7s+g_8}{g_9s+g_4}}}+\frac{g_3^2\lambda\cos\gamma H(s)}{g_{10}}\left\{1+\frac{B_1}{s+A_1}+\frac{B_2}{s+A_2}\right\}\left\{e^{-\eta\sqrt{\frac{g_7s+g_8}{g_9s+g_4}}}-e^{-\eta\sqrt{P_{ro}\frac{g_5s+g_6}{g_3s+g_4}}}\right\}\\+\frac{g_3^2\lambda N_r\cos\gamma G(s)}{g_{20}}\left\{1+\frac{B_3}{s+A_3}+\frac{B_4}{s+A_4}\right\}\left\{e^{-\eta\sqrt{\frac{g_7s+g_8}{g_9s+g_4}}}-e^{-\eta\sqrt{Sc\frac{g_1s+g_2}{g_3s+g_4}}}\right\}\end{aligned} \tag{45}$$

Inverting Eq. (45) by means of Laplace transform and using Appendix (A1)-(A2), we get suitable expression as:

$$\begin{aligned}u_{cf}(\eta,t)=\Bigg\{&\int_0^\tau F'(\tau-s)\Pi(\eta,\tau,0,g_7,g_8,g_9,g_4)+\Lambda_1\Big[H'(\tau-s)\Big(\Pi(\eta,\tau,0,g_7,g_8,g_9,g_4)-\Pi\big(\eta\sqrt{P_{ro}},\tau,0,g_5,g_6,g_3,g_4\big)\Big)\Big]\\&+\Lambda_2\Big[H(\tau-s)\Big(\Pi(\eta,\tau,A_1,g_7,g_8,g_9,g_4)-\Pi\big(\eta\sqrt{P_{ro}},\tau,A_1,g_5,g_6,g_3,g_4\big)\Big)\Big]+\Lambda_3\Big[H(\tau-s)\Big(\Pi(\eta,\tau,A_2,g_7,g_8,g_9,g_4)-\Pi\big(\eta\sqrt{P_{ro}},\tau,A_2,g_5,g_6,g_3,g_4\big)\Big)\Big]\\&+\Lambda_4\Big[G'(\tau-s)\Big(\Pi(\eta,\tau,0,g_7,g_8,g_9,g_4)-\Pi\big(\eta\sqrt{Sc},\tau,0,g_1,g_2,g_3,g_4\big)\Big)\Big]+\Lambda_5\Big[G(\tau-s)\Big(\Pi(\eta,\tau,A_3,g_7,g_8,g_9,g_4)-\Pi\big(\eta\sqrt{Sc},\tau,A_3,g_1,g_2,g_3,g_4\big)\Big)\Big]\\&+\Lambda_6\Big[G(\tau-s)\Big(\Pi(\eta,\tau,A_4,g_7,g_8,g_9,g_4)-\Pi\big(\eta\sqrt{Sc},\tau,A_4,g_1,g_2,g_3,g_4\big)\Big)\Big]\Bigg\}\end{aligned} \tag{46}$$

5.2 ABC – Fractional Operator

Fractional operators are quite flexible for describing the behaviors of velocity field for MHD Second grade fluid through the characterization of governing equations. Generating fractional governing equation of velocity (6) via CF-fractional operator by exchanging the partial derivative with fractional derivative κ,

$${}^{ABC}D_\tau^\kappa u(\eta,\tau)=\frac{\partial^2u}{\partial\eta^2}+\alpha\,{}^{ABC}D_\tau^\kappa u(\eta,\tau)\frac{\partial^2u}{\partial\eta^2}+\lambda\mathrm{Cos}\gamma T+N_r\mathrm{Cos}\gamma C-Mu-k_1u, \tag{47}$$

employing Laplace transformation on (47) with the help of (20), we explored second order partial differential equation

$$\left(\frac{s^\kappa}{(1-\kappa)s^\kappa+\kappa}+b_o\right)\bar{u}_{abc}(\eta,s)=\left(1+\frac{\alpha s^\kappa}{(1-\kappa)s^\kappa+\kappa}\right)\frac{\partial^2\bar{u}_{abc}(\eta,s)}{\partial\eta^2}+\lambda\cos\gamma\bar{T}+\lambda N_r\cos\gamma\bar{C}. \tag{48}$$

The required solution of homogenous part of Eq. (48) is,

$$\bar{u}_c(\eta,s)=c_1e^{\eta\sqrt{\left(\frac{g_7s^\kappa+g_8}{g_9s^\kappa+g_4}\right)}}+c_2e^{-\eta\sqrt{\left(\frac{g_7s^\kappa+g_8}{g_9s^\kappa+g_4}\right)}}, \tag{49}$$

and particular solution can be given as follow after making use of Eqs. (16) and (31),

$$\bar{u}_p(\eta,s)=-\frac{-g_3^2\lambda\cos\gamma H(s)}{g_{10}}\left\{1+\frac{B_1}{s^\kappa+A_1}+\frac{B_2}{s^\kappa+A_2}\right\}e^{-\eta\sqrt{P_{ro}\frac{g_5s^\kappa+g_6}{g_3s^\kappa+g_4}}}-\frac{g_3^2\lambda N_r\cos\gamma G(s)}{g_{20}}\left\{1+\frac{B_3}{s^\kappa+A_3}+\frac{B_4}{s^\kappa+A_4}\right\}e^{-\eta\sqrt{Sc\frac{g_1s^\kappa+g_2}{g_3s^\kappa+g_4}}} \tag{50}$$

and solution of Eq.(41) can be given as:

$$\bar{u}_{abc}(\eta,s)=c_1e^{\eta\sqrt{\frac{g_7s^\kappa+g_8}{g_9s^\kappa+g_4}}}+c_2e^{-\eta\sqrt{\frac{g_7s^\kappa+g_8}{g_9s^\kappa+g_4}}}-\frac{g_3^2\lambda\cos\gamma H(s)}{g_{10}}\left\{1+\frac{B_1}{s^\kappa+A_1}+\frac{B_2}{s^\kappa+A_2}\right\}e^{-\eta\sqrt{P_{ro}\frac{g_5s^\kappa+g_6}{g_3s^\kappa+g_4}}}-\frac{g_3^2\lambda N_r\cos\gamma G(s)}{g_{20}}\times\left\{1+\frac{B_3}{s^\kappa+A_3}+\frac{B_4}{s^\kappa+A_4}\right\}e^{-\eta\sqrt{Sc\frac{g_1s^\kappa+g_2}{g_3s^\kappa+g_4}}} \tag{51}$$

using Eq. (9), we find out the arbitrary parameter c_1 and c_2 as follow:

$$\begin{aligned}\bar{u}_{abc}(\eta,s)=F(s)e^{\eta\sqrt{\frac{g_7s^\kappa+g_8}{g_9s^\kappa+g_4}}}+\frac{g_3^2\lambda\cos\gamma H(s)}{g_{10}}\left\{1+\frac{B_1}{s^\kappa+A_1}+\frac{B_2}{s^\kappa+A_2}\right\}\left\{e^{-\eta\sqrt{\frac{g_7s^\kappa+g_8}{g_9s^\kappa+g_4}}}-e^{-\eta\sqrt{P_{ro}\frac{g_5s^\kappa+g_6}{g_3s^\kappa+g_4}}}\right\}\\+\frac{g_3^2\lambda N_r\cos\gamma G(s)}{g_{20}}\left\{1+\frac{B_3}{s^\kappa+A_3}+\frac{B_4}{s^\kappa+A_4}\right\}\left\{e^{-\eta\sqrt{\frac{g_7s^\kappa+g_8}{g_9s^\kappa+g_4}}}-e^{-\eta\sqrt{Sc\frac{g_1s^\kappa+g_2}{g_3s^\kappa+g_4}}}\right\}\end{aligned} \tag{52}$$

5.2.1 Special Case

Now the required result discuss by considering $F(s)=G(s)=H(s)=1/s$, as a boundary condition on velocity,

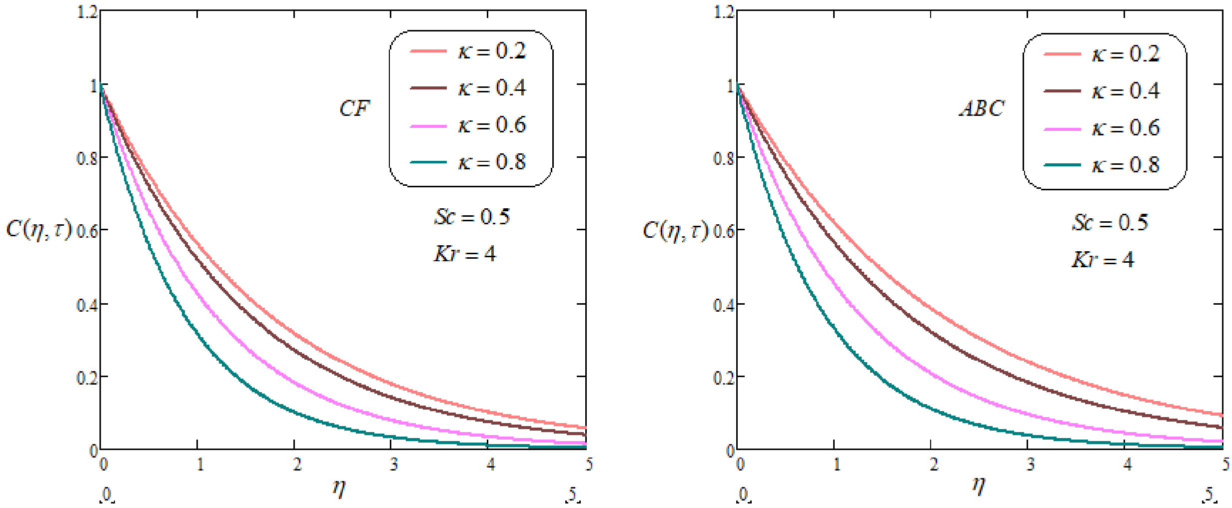

Fig. 2. Behavior of concentration curve for κ via CF and AB-approaches

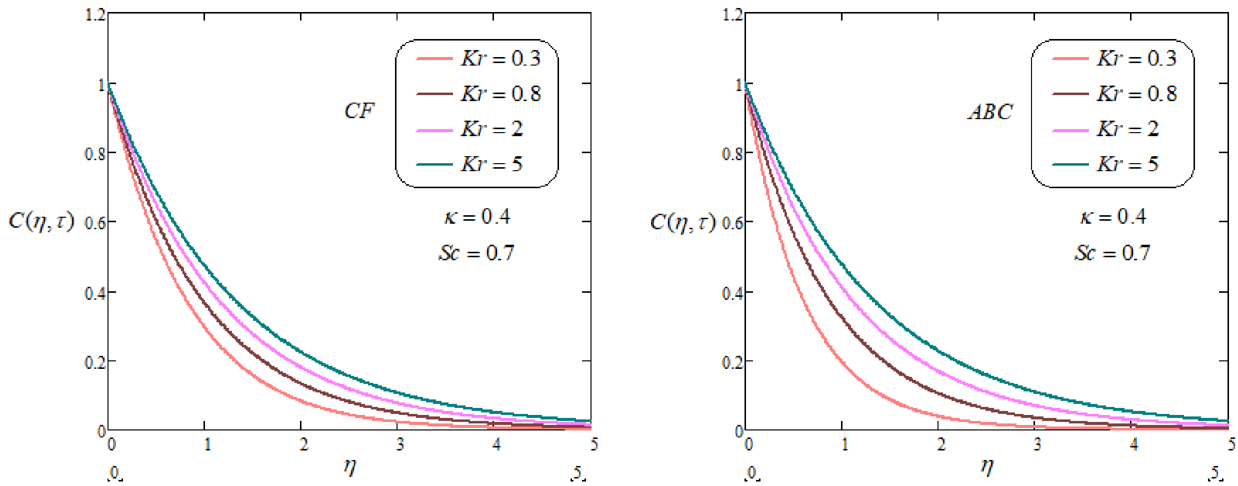

Fig. 3. Behavior of concentration curve for Kr via CF and AB-approaches

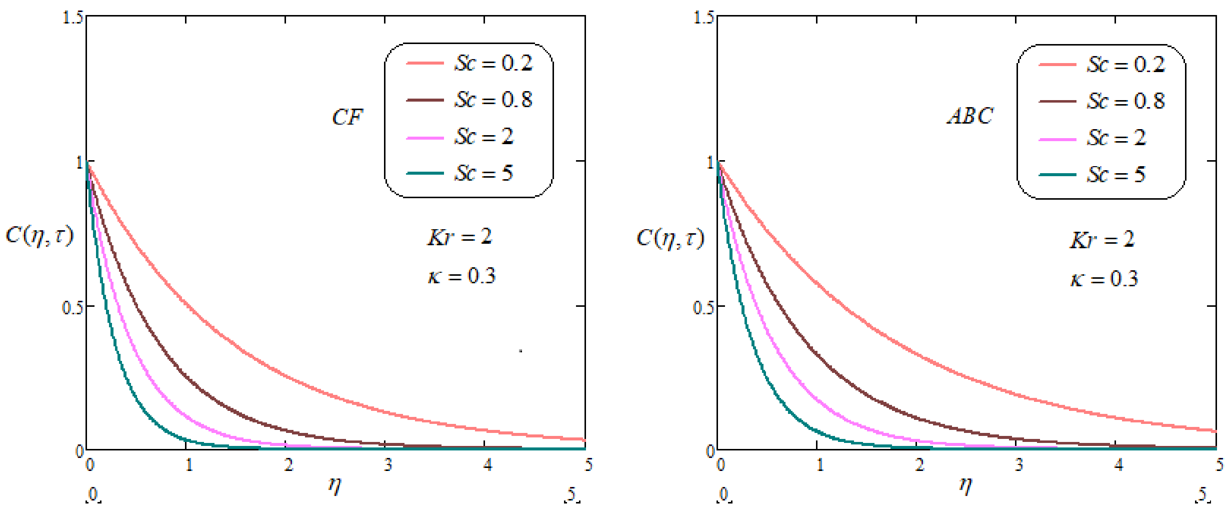

Fig. 4. Behavior of concentration curve for Sc via CF and AB-approaches

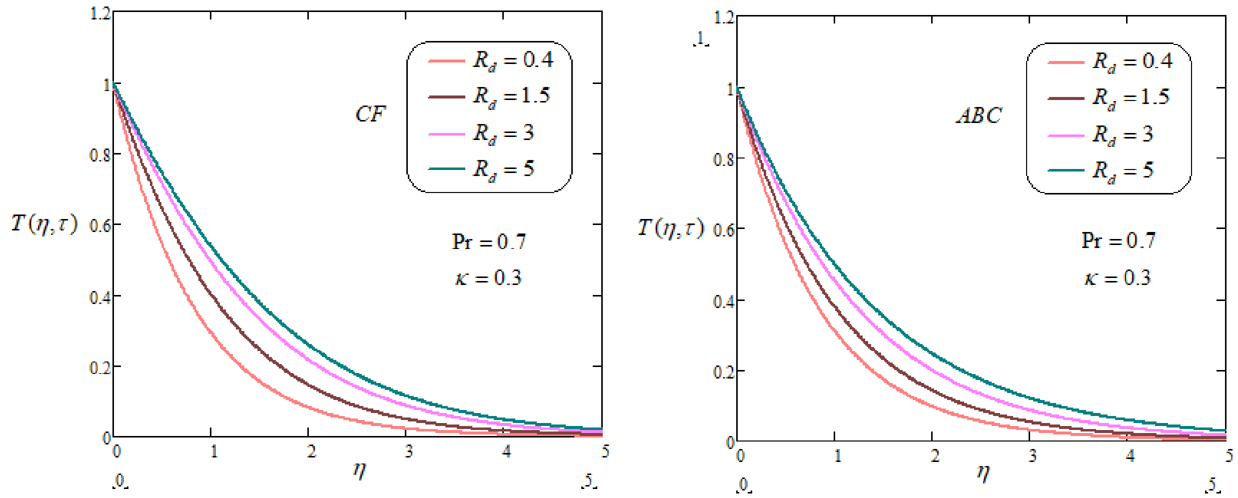

Fig. 5. Behavior of temperature curve for R_d via CF and AB-approaches

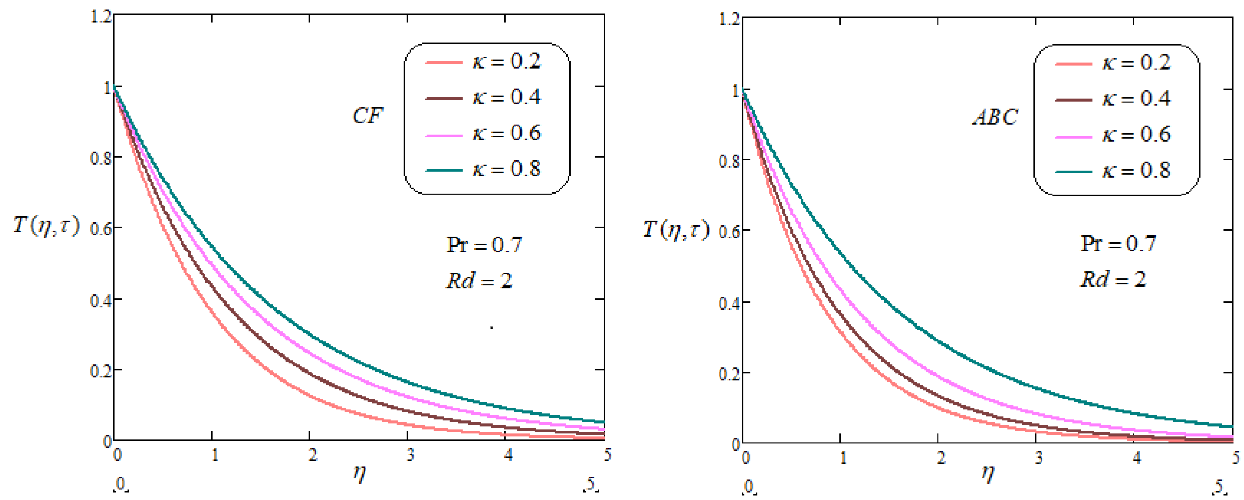

Fig. 6. Behavior of temperature curve for κ via CF and AB-approaches

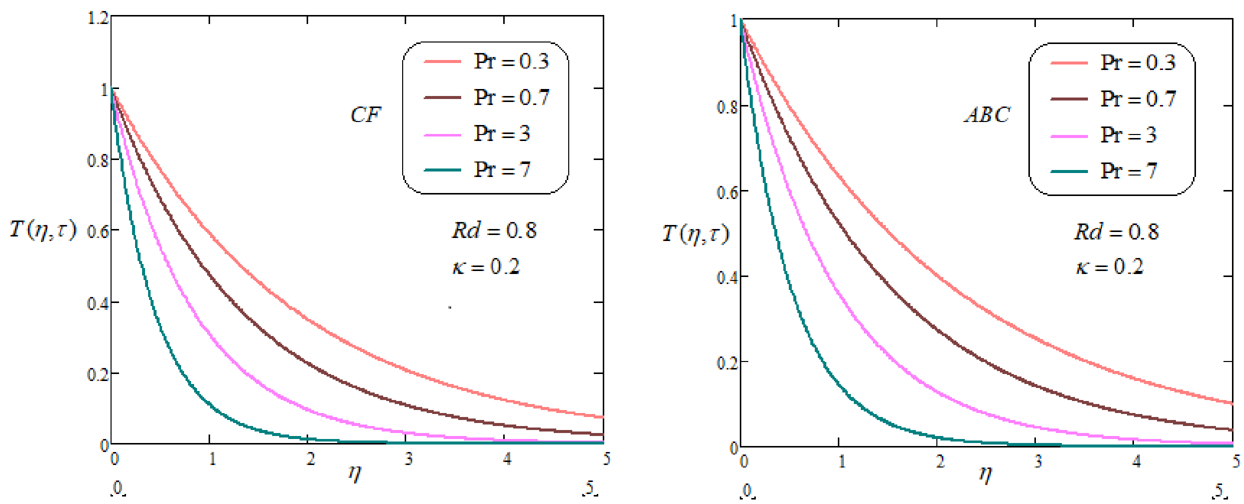

Fig. 7. Behavior of temperature curve for Pr via CF and AB-approaches

$$\begin{aligned}\bar{u}_{abc}(\eta,s) &= \frac{1}{s}e^{\eta\sqrt{\frac{g_7 s^\kappa + g_8}{g_9 s^\kappa + g_4}}} + \frac{g_3^2\lambda\cos\gamma}{s g_{10}}\left\{1+\frac{B_1}{s^\kappa + A_1}+\frac{B_2}{s^\kappa + A_2}\right\}\left\{e^{-\eta\sqrt{\frac{g_7 s^\kappa + g_8}{g_9 s^\kappa + g_4}}} - e^{-\eta\sqrt{P_{ro}\frac{g_5 s^\kappa + g_6}{g_3 s^\kappa + g_4}}}\right\} \\ &+ \frac{g_3^2\lambda N_r\cos\gamma}{s g_{20}}\left\{1+\frac{B_3}{s^\kappa + A_3}+\frac{B_4}{s^\kappa + A_4}\right\}\left\{e^{-\eta\sqrt{\frac{g_7 s^\kappa + g_8}{g_9 s^\kappa + g_4}}} - e^{-\eta\sqrt{Sc\frac{g_1 s^\kappa + g_2}{g_3 s^\kappa + g_4}}}\right\}\end{aligned} \tag{53}$$

Inverting Eq. (53) by means of Laplace transform and using Appendix (A3)-(A6), we get suitable expression as:

$$\begin{aligned}u_{abc}(\eta,t) = \Big\{ & h(\tau,\beta)\chi(\eta,\tau,0,g_7,g_8,g_9,g_4) + \Lambda_1 h(\tau,\beta)\Big[\Big(\chi(\eta,\tau,0,g_7,g_8,g_9,g_4) - \chi\big(\eta\sqrt{P_{ro}},\tau,0,g_5,g_6,g_3,g_4\big)\Big)\Big] \\ & + \Lambda_2 h(\tau,0)\Big[\Big(\chi(\eta,\tau,A_1,g_7,g_8,g_9,g_4) - \chi\big(\eta\sqrt{P_{ro}},\tau,A_1,g_5,g_6,g_3,g_4\big)\Big)\Big] + \Lambda_3 h(\tau,0)\Big[\Big(\chi(\eta,\tau,A_2,g_7,g_8,g_9,g_4) - \chi\big(\eta\sqrt{P_{ro}},\tau,A_2,g_5,g_6,g_3,g_4\big)\Big)\Big] \\ & + \Lambda_4 h(\tau,\beta)\Big[\Big(\chi(\eta,\tau,0,g_7,g_8,g_9,g_4) - \chi\big(\eta\sqrt{Sc},\tau,0,g_1,g_2,g_3,g_4\big)\Big)\Big] + \Lambda_5 h(\tau,0)\Big[\Big(\chi(\eta,\tau,A_3,g_7,g_8,g_9,g_4) - \chi\big(\eta\sqrt{Sc},\tau,A_3,g_1,g_2,g_3,g_4\big)\Big)\Big] \\ & + \Lambda_6 h(\tau,0)\Big[\Big(\chi(\eta,\tau,A_4,g_7,g_8,g_9,g_4) - \chi\big(\eta\sqrt{Sc},\tau,A_4,g_1,g_2,g_3,g_4\big)\Big)\Big]\Big\}\end{aligned} \tag{54}$$

where

$$\begin{gathered} g_1 = 1+K_r-\kappa K_r, g_2=\kappa K_r, g_3 = 1-\kappa, g_4=\kappa, g_5 = 1+S-\kappa S, g_6=\kappa S, g_7 = 1+b_o-\kappa b_o, g_8=\kappa b_o, \\ g_9 = 1+\alpha-\kappa, g_{10} = P_{ro}g_5g_9 - g_7g_3, g_{11} = P_{ro}g_6g_9 + P_{ro}g_5g_4 - g_7g_4 - g_8g_3, g_{12} = P_{ro}g_6g_4 - g_8g_4, \\ g_{13} = \frac{2g_4}{g_3}, g_{14} = \frac{g_4^2}{g_3^2}, g_{15} = \frac{g_{11}}{g_{10}}, g_{16} = \frac{g_{12}}{g_{10}}, g_{17} = g_{13}-g_{15}, g_{18} = g_{14}-g_{16}, g_{19} = \sqrt{\frac{g_{15}^2-4g_{16}}{4}}, \\ g_{20} = Scg_1g_9 - g_7g_3, g_{21} = Scg_1g_4 + Scg_2g_9 - g_7g_4 - g_8g_3, g_{22} = Scg_2g_4 - g_8g_4, g_{23} = \frac{g_{21}}{g_{20}}, g_{24} = \frac{g_{22}}{g_{20}}, \\ g_{25} = g_{13}-g_{23}, g_{26} = g_{14}-g_{24}, g_{27} = \sqrt{\frac{g_{23}^2-4g_{24}}{4}}, B_1 = \frac{g_{15}g_{17}+2g_{17}g_{19}-2g_{18}}{4g_{19}}, B_2 = \frac{-g_{15}g_{17}+2g_{17}g_{19}+2g_{18}}{4g_{19}}, \\ B_3 = \frac{g_{23}g_{25}+2g_{25}g_{27}-2g_{26}}{4g_{26}}, B_4 = \frac{-g_{23}g_{25}+2g_{25}g_{27}+2g_{26}}{4g_{26}}, A_1 = \frac{g_{15}}{2}+g_{19}, A_2 = \frac{g_{15}}{2}-g_{19}, A_3 = \frac{g_{23}}{2}+g_{29}, A_3 = \frac{g_{23}}{2}-g_{29}, \\ \Lambda_1 = \frac{g_3^2\lambda\cos\gamma}{g_{10}}, \Lambda_2 = \frac{g_3^2\lambda\cos\gamma B_1}{g_{10}}, \Lambda_3 = \frac{g_3^2\lambda\cos\gamma B_2}{g_{10}}, \Lambda_4 = \frac{g_3^2\lambda\cos\gamma N_r}{g_{20}}, \Lambda_5 = \frac{g_3^2\lambda N_r\cos\gamma B_3}{g_{20}}, \Lambda_6 = \frac{g_3^2\lambda N_r\cos\gamma B_4}{g_{20}}. \end{gathered} \tag{55}$$

6. Results & Discussion

This section is dedicated to present physical interpretation of the obtained results via CF and AB differential operators under generalized boundary conditions on MHD Second grade model. Results are investigated via Laplace transformation for velocity, temperature and concentration based on singular verses non-singular and local verses non-local kernels. The graphical representations are depicted for showing the influences of different physical parameters such $R_d, P_r, \kappa, K_r, Sc, M, \lambda, k_1,$ and angle of inclination γ on velocity, temperature and concentration profile using the package of MATHCAD-15. In this continuity, figure (2) discuss the variation of κ on the concentration profile via CF and AB approach. The concentration profile reduce by enhance the value of fractional parameter κ. As $\kappa \to 1$, fractional models convert into ordinary model. Figure (3) represent the influence of K_r on the concentration profile with altered value of time. As shown in figure (3), concentration field increase as K_r increase. The influence of Sc on concentration profile shown in figure (4). Physically, Schmidt number is used to characterize fluid motion and it relates the thickness of hydro-dynamic layers and mass transfer boundary layers. It is observed that by larger value of Sc, the concentration profile becomes decreases. Temperature profile enhance by increasing the value of R_d as shown in figure (5). Figure (6) shown the effect of κ on temperature profile. It is show that temperature increase as increase in κ. Further, it is noted that temperature enhanced for both fractional models.

Figure (7) show the effect of P_r on temperature profile. It is noted that temperature and thermal reduce by enhance the value of P_r. The behavior of temperature profile in ABC differential operator is more suitable as compared to other fractional operators. Figure (8) analyze the impact on Sc for velocity field versus time. It is defined as "the ratio between viscous diffusion rate and mass diffusion rate. Physically, Sc is used to characterize fluid motion and it relates the thickness of hydro-dynamic layers and mass transfer boundary layers. The velocity and concentration boundary layers in Sc almost coincide with each other, because diffusion of mass and momentum are comparable. It is observed that by larger value of Sc, the velocity becomes decreases. Figure (9) discuss the variation of R_d on velocity with help of time. The large value of radiation parameter R_d, causes to increase in the fluid flow. The rate of energy transport of fluid increase due to increase the intensity of radiation parameter and decrease the viscosity. Due to such behavior fluid moves faster and enhance the fluid velocities. Figure (10) discuss the behavior of P_r. Specific heat and conductivity are depend on P_r. It is ratio of kinematic viscosity and thermal diffusivity. The thickness of momentum and boundary layer is control by prandtl number. It seen from the graph, decreasing the velocity, observed by increase the value of P_r. The lower prandtl number enhance the thermal conductivity and increase the boundary layer. Figure (11) discuss the influence of magnetic field M on velocity components. These graphical representation indicates that an increase in M, the velocity reduce due to Lorentz force. It behave as a drag force. By increasing the parameter of magnetic field, the Lorentz force also increase. Fluid flow on the boundary layer is slow down due to this force. Figure (12) shows the behavior of velocity curves for λ. It is observed that velocity enhance with the increase in λ for all fractional models. The velocity behavior is also observed for variation of time. Clearly, ABC model achieved maximum velocity as compared to other fractional models. The effect of boundary layer thickness are discussed by changing the inclination angle γ on fluid velocity in figure (13). It show that with increasing the value of γ, the velocity and boundary layer thickness becomes decrease when fluid is moving. Figure (14) discuss the behavior of fractional parameter κ. The absolute velocity decrease as increase in the value of parameter κ. The behavior clearly show that the large value of κ have greater stability as compared to lower value. The permeability parameter of porous medium k_1 increase the velocities because the large value of k_1 reduce the resistance and increase the momentum which accelerates the fluid motion. The behavior of k_1 shown in figure (15). The impact of N_r on velocity curve is deliberate in figure (16). The behavior of velocity curve increases with an increasing value of $N_r > 0$.

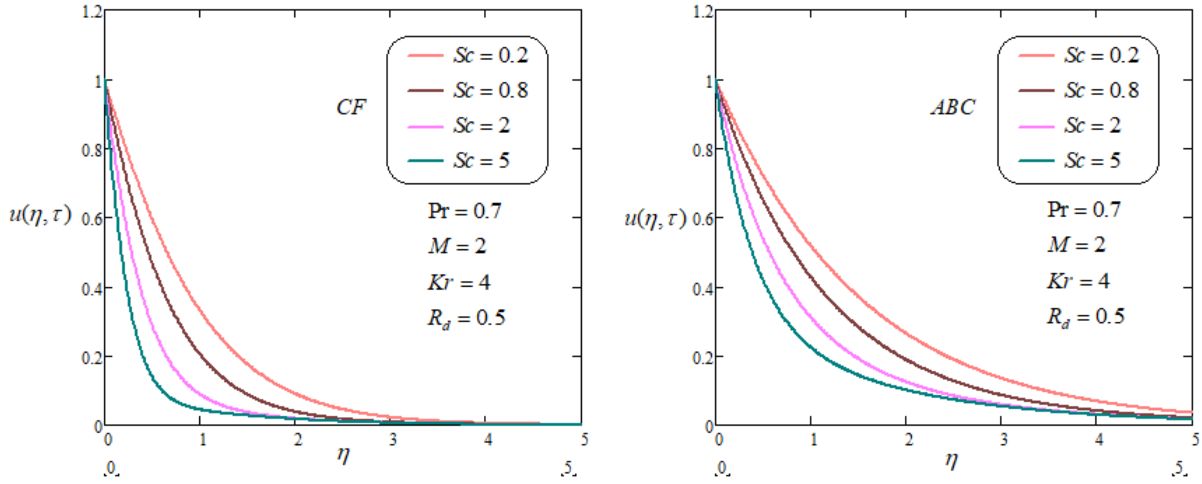

Fig. 8. Behavior of velocity curve for Sc via CF and AB-approaches

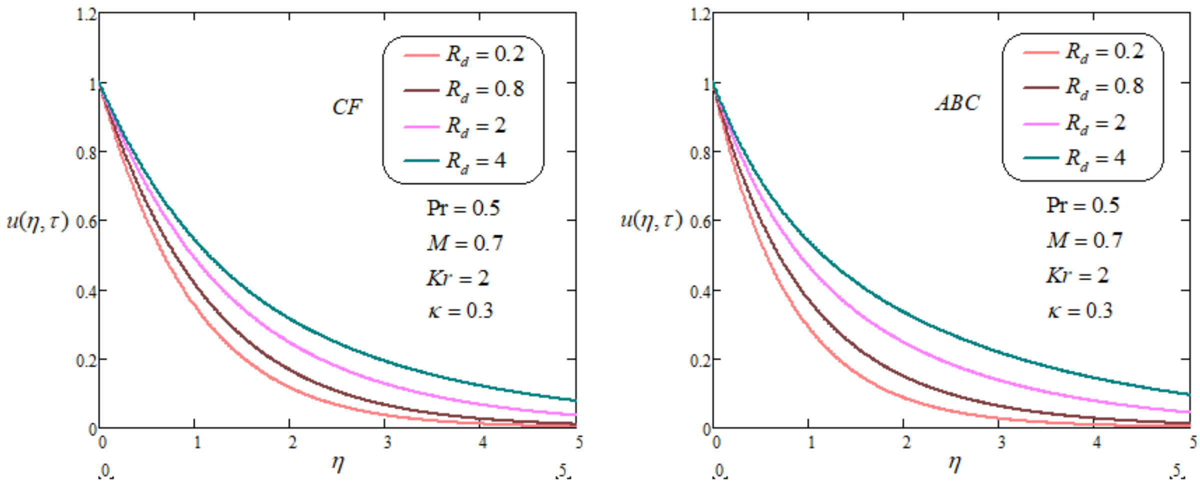

Fig. 9. Behavior of velocity curve for R_d via CF and AB-approaches

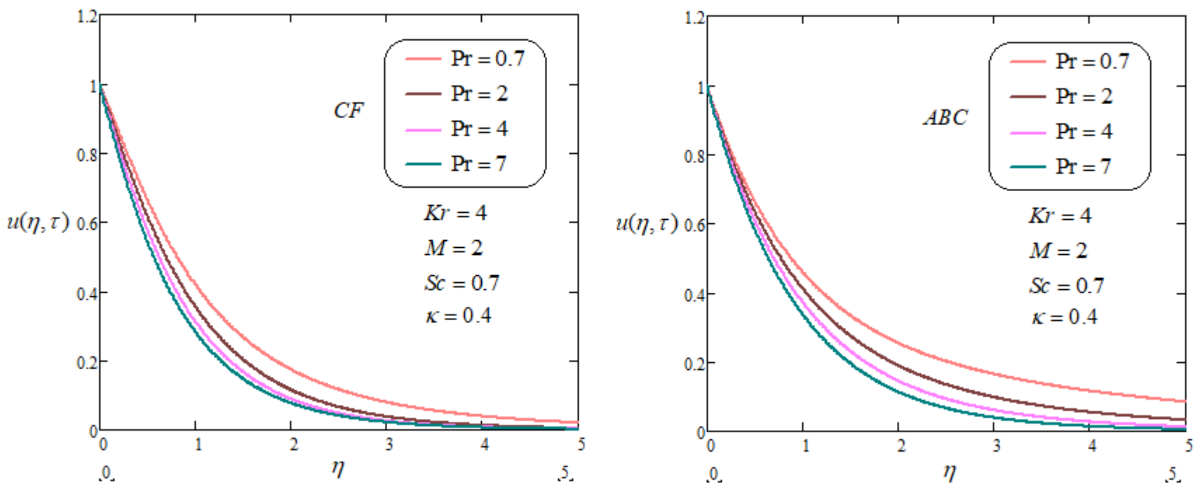

Fig. 10. Behavior of velocity curve for Pr via CF and AB-approaches

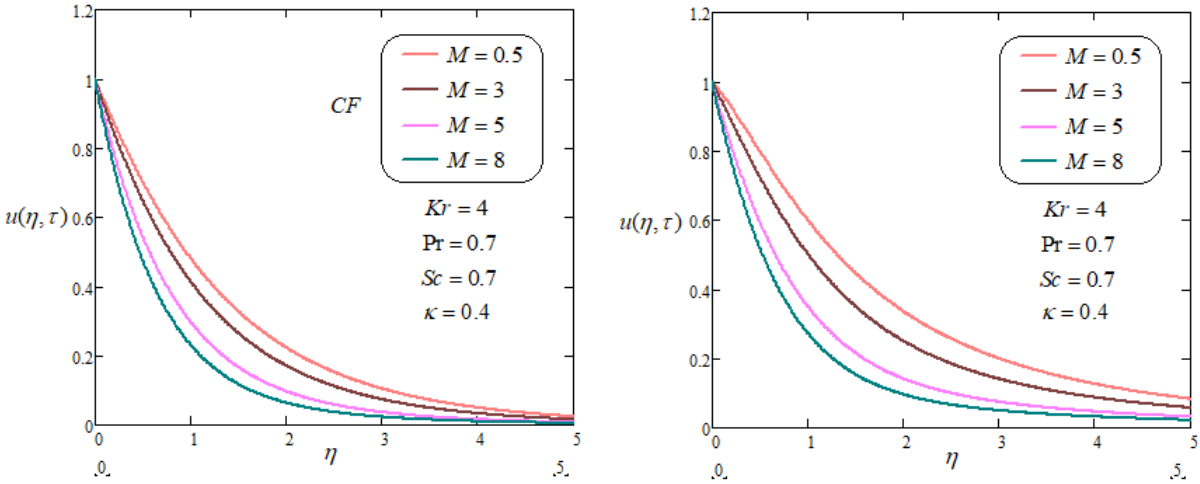

Fig. 11. Behavior of velocity curve for M via CF and AB-approaches

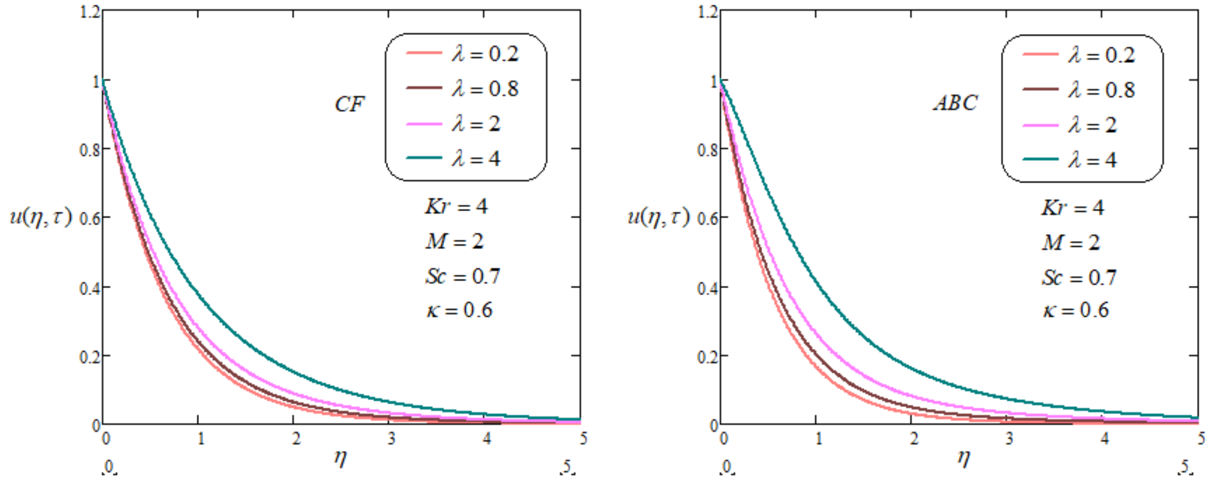

Fig. 12. Behavior of velocity curve for λ via CF and AB-approaches

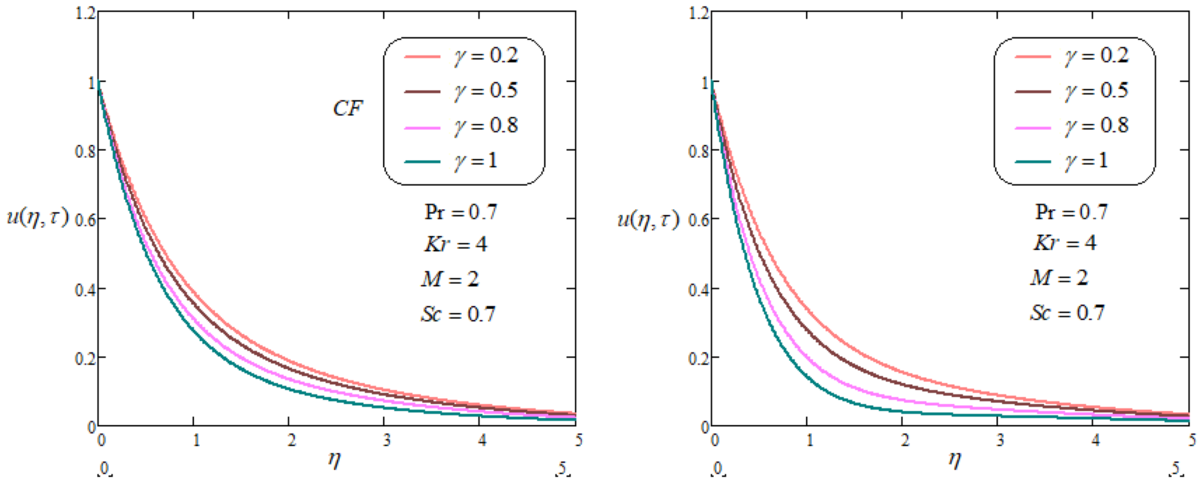

Fig. 13. Behavior of velocity curve for γ via CF and AB-approaches

7. Conclusion

This section is dedicated to present physical interpretation of the obtained results via CF and AB differential operators under radiation effect and chemical reaction parameter on MHD second grade fluid with generalized boundary conditions. Results are investigated via Laplace transformation with inversion algorithm for velocity, temperature and concentration based on local verses non-local kernels. The graphical representations are depicted for showing the influences of different physical parameters such as fractional parameters κ, magnetic effect M, Prandtl number Pr, Schmidt number Sc, chemical reaction parameter Kr on velocity, temperature and concentration profile using the package of MATHCAD-15. The following points are extracted from the instant study:

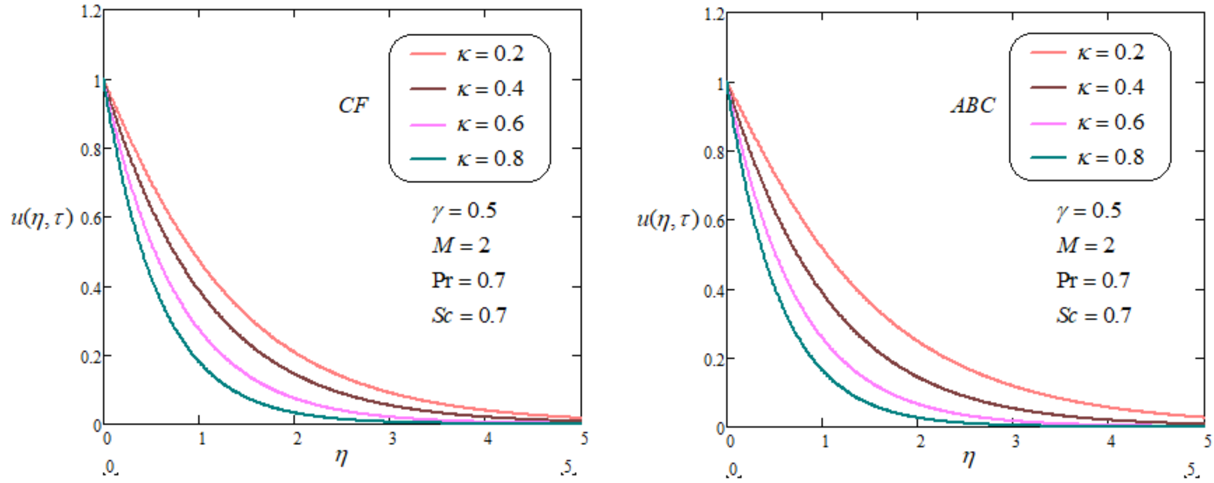

Fig. 14. Behavior of velocity curve for κ via CF and AB-approaches

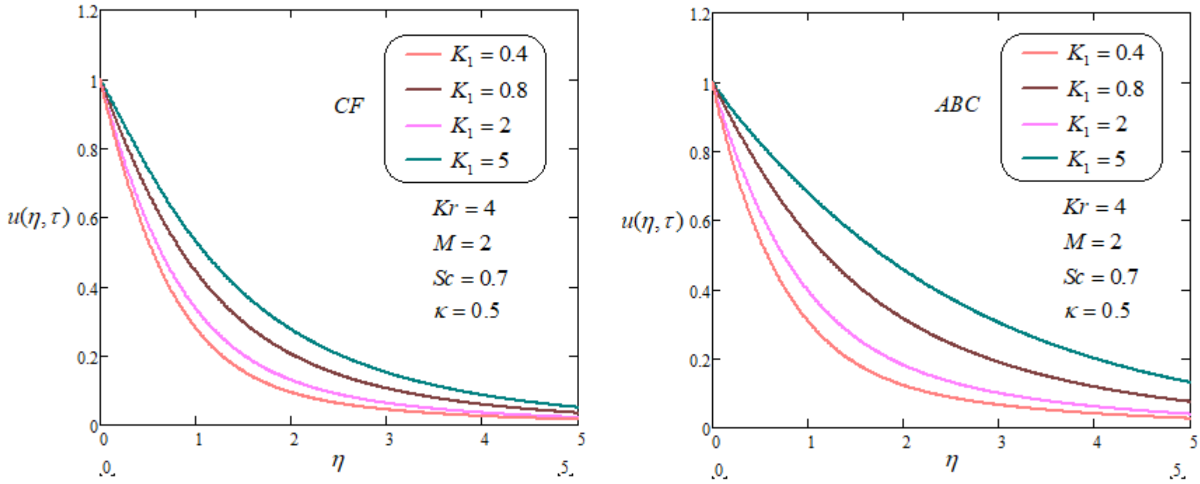

Fig. 15. Behavior of velocity curve for k_1 via CF and AB-approaches

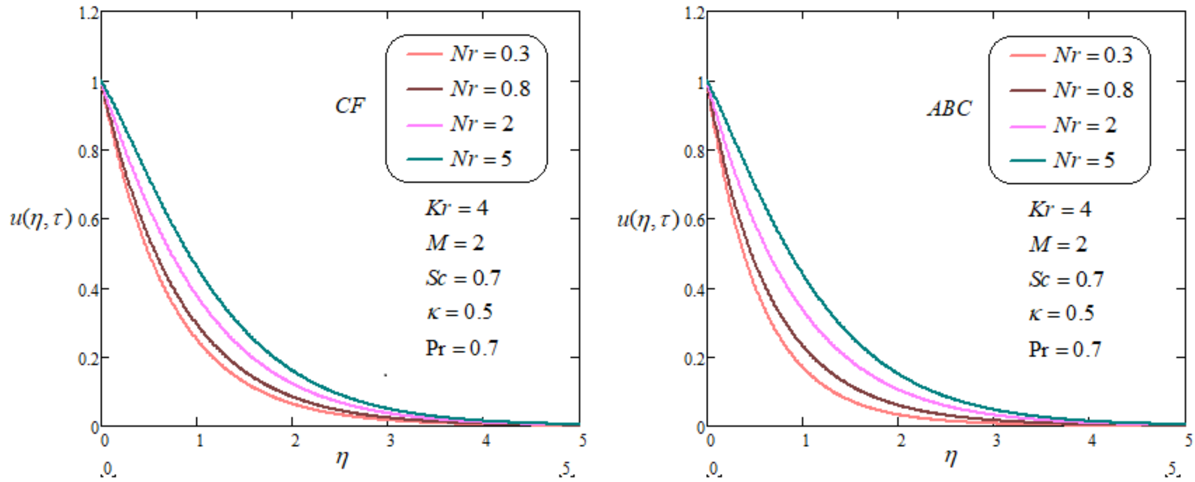

Fig. 16. Behavior of velocity curve for N_r via CF and AB-approaches

- The velocity field decrease by increasing the value of $\Pr$.
- The behavior of fluid curve is same in case of fractional parameter κ. The behavior of velocity curve decreases by increasing the value of κ.
- By large value of M, the velocity profile decrease.
- Temperature increases as enhancement of radiation parameter.
- The fluid velocity profile increases with an increasing value of Nr.
- The behavior of velocity curve increases by increasing the value of porosity.
- The behavior of velocity curve increases by increasing Kr for both non-integer differentiable operator.
- The behavior of velocity curve decreases by increasing Sc.
- Enhance the fluid motion by existence of free convection.
- ABC fractional operator is more considerable as compared to the CF operator.

Appendix

$$\Pi(\varsigma,s,\lambda,a,b,c,d)=\frac{1}{s-\lambda}\exp\left(-\varsigma\sqrt{\frac{as+b}{cs+d}}\right), \tag{A1}$$

$$\Pi(\varsigma,t,\lambda,a,b,c,d)=e^{-\lambda t-\varsigma\sqrt{e_1}}-\frac{\varsigma\sqrt{e_2}}{2\pi}\int_0^\infty\int_0^t\frac{e^{-\lambda t}}{\sqrt{t}}\exp\left(\lambda t-e_3t-\frac{\varsigma^2}{4u}-e_1u\right)I_1(2\sqrt{e_2ut})dtdu, \tag{A2}$$

$$\bar{\chi}_1(\varsigma,s,a,b,c)=\frac{1}{s^\alpha+a}\exp\left(-\varsigma\sqrt{\frac{s^\alpha+b}{s^\alpha+c}}\right), \tag{A3}$$

$$\chi_1(\varsigma,t,a,b,c)=\int_0^\infty\chi(\varsigma,t,a,b,c)g(u,t)du, \tag{A4}$$

$$\chi(\varsigma,t,a,b,c)=e^{-at-\varsigma}-\frac{\varsigma\sqrt{b-c}}{2\sqrt{\pi}}\int_0^\infty\int_0^t\frac{e^{-at}}{\sqrt{t}}\exp\left(at-ct-\frac{\varsigma^2}{4u}-u\right)I_1(2\sqrt{(b-c)ut})dtdu, \tag{A5}$$

$$h(t,\varepsilon)=\frac{1}{t^\varepsilon\Gamma(1-\varepsilon)},g(u,t)=t^{-1}\aleph(0,-\varepsilon,-ut^{-\varepsilon}),\aleph(k,-\varepsilon,t)=\sum_o^\infty\frac{t^n}{n!\Gamma(k-n\varepsilon)}. \tag{A6}$$

Author Contributions

M.B. Riaz proposed the problem, initiated the conceptualization, developed the mathematical modeling and examined the theory valiadation; S.T. Saeed solved the proposed mathematical model and verify the required results; D. Baleanu ploted the graphs using software and verify the results present in literature. Final review, editing and writing done by all the authors and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Nomenclature

υ	Kinematic viscosity [m^2s^{-1}]	s	Laplace transform
g	Acceleration due to gravity [ms^{-2}]	τ	Time (s)
ρ	Fluid denisty [kgm^{-3}]	Pr	Prandtl number
C_p	Specific heat [$Jkg^{-1}K^{-1}$]	Sc	Schmidt number
k	Thermal conductivity [$Wm^{-1}K^{-1}$]	λ	Relaxation time (s)
β	Thermal expansion coefficient [K^{-1}]	k_1	Permeability [m^2]
μ	Dynamic viscosity [$kgms^{-1}$]	M	Magnetic field

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