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Research Paper

Comparative Study of Plane Poiseuille Flow of Non-isothermal Couple Stress Fluid of Reynold Viscosity Model using Optimal Homotopy Asymptotic Method and New Iterative Method

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Abstract. In this paper, we have explored the steady Poiseuille flow of couple stress fluid between two parallel plates under the influence of non-isothermal effects of Reynold viscosity model, using Optimal Homotopy Asymptotic Method (OHAM) and New Iterative Method (NIM). We obtained expressions for velocity profile, temperature distribution, average velocity, volume flux and shear stress. The solutions obtained using these methods are in the form of infinite series; therefore, they can be easily computed. Comparative results of solutions obtained by both methods are given using different tables and graphs.

Keywords: Optimal Homotopy Asymptotic Method, New Iterative Method, Couple Stress Fluid.

1. Introduction

Researchers and scientists have shown their keen interest in non-Newtonian fluids because of its applications in numerous industrial, natural and technological problems. Many authors have cited a wide range of applications of non-Newtonian fluids that cover the flow of polymer solutions, drilling oil, gas wells, foodstuffs, extrusion of molten plastics and synthetic fibers. Recently, researchers have utilized their energy to streamline the connection between the flow state and viscosity of the non-Newtonian fluid, as it is very essential to many mathematical models to describe this kind of fluid [1-5]. Authors in [6-14] investigated some of the remarkable fluid flow problems involving non-Newtonian fluids. In order to enlighten the behaviour of the non-Newtonian fluid, numerous constitutive equations have been proposed. Stokes [15] has introduced couple stress fluid model, which has distinct characteristics among these constitutive equations, such as non-symmetric stress tensor, the presence of couple stress and body couples. Authors in [16] have explored some fundamental steady flows of couple stress fluid, such as Poiseuille, Couette and generalized Couette flows between parallel plates slip on the boundary.

The couple stress theory introduced by Stoke proposes models for those fluids, whose microstructure is mechanically significant. If the problem under consideration has the characteristic geometric dimension of the same order of magnitude as the size of microstructure, then on a liquid, microstructure effect can be felt [17]. The said model represents those fluids which comprise of rigid and randomly oriented particles suspended in a viscous medium. The stress tensor in this fluid is non-symmetric, thus in the classical Newtonian theory, the prediction of exact flow behaviour is not easy. The momentous characteristic of the said model is that it outcomes differential equations are in similar form as that of the Navier-Stokes equations. This model has been extensively used as compared to other models established for fluid under consideration, because of its mathematical simplicity. Researchers in [18, 19] explored various problems of couple stress fluids of flows past axisymmetric bodies. The governing equations of the flow of the said model have the non-linear nature and even higher order than that of the Navier-Stokes equations; hence, it is hard to get the exact solution. The study of heat transfer flow is significant in several engineering applications, for example, the design of thrust bearing, drag reduction, transpiration cooling, radial diffusers and thermal recovery of oil. Heat transfer has a vigorous role in processing and handling of non-Newtonian mixtures [20-22].

Different techniques have been used to explore flow problems in literature. Numerical techniques, iterative techniques, perturbation methods and homotopy based techniques are the main tools for finding the approximate solutions of flow problems. Each technique has its own pro and cons. In numerical methods discretization is used which affect the accuracy. These methods also need a lot of time and computational work. In the case of strong non-linear problems, the results obtained using numerical techniques are not more accurate. Perturbation techniques have also some limitations, such as the assumption of small parameter and strong non-linearity.

Optimal homotopy asymptotic method (OHAM) is one of the power homotopy based techniques for the solution of differential equations. This technique was introduced by Marinca et al. [23] in 2008 for the solution of differential equations. This technique does not require discretization like other numerical techniques and is valid in the absence of small or large parameter as compare

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to other perturbation techniques. This method is also free from initial guess unlike the iterative methods [23-27]. This method has been applied to different problems [28-33]. Authors in [34-38] transformed the highly nonlinear partial differential equations (PDE's) set by heat flux circumstances into structure ordinary differential equations (ODE's) with proper conditions and then evaluate using Optimal Homotopy Analysis Method (OHAM).

In 2006 Daftardar-Gejji and Jafari [39] have been proposed a new technique to solve linear and non-linear equations called new iterative method (NIM). Later, many scientists and mathematicians have been employed NIM to different problems including ordinary, partial differential equations, algebraic equations, the system of non-linear dynamical equations and evolution equations. This method provides results with rapid convergence as compared to other analytical methods [40-45].

In this paper, we have applied two methods, namely, OHAM and NIM to investigate the heat transfer flow of incompressible couple stress fluids having a viscosity, which is dependent on temperature between two parallel plates kept at different temperatures, we have explored plane Poiseuille flow with the said methods. This paper consists of six sections. In section 1 brief introduction is given, section 2 is devoted to the basic governing equations and problem formulation for couple stress fluids. In section 3, the description of the methods is given and section 4 consists of solutions of the problem. In section 5, results and discussions are given and last section 6 consists of conclusion.

2. Basic Equations and Problem Formulation

2.1. Basic Equations

The basic governing equations in [46-51] of the balance of momentum, mass conservation and energy for an incompressible fluid are given as,

$$\nabla.\mathbf{Z}=0, \tag{1}$$

$$\rho\dot{\mathbf{Z}}=\nabla.\tau-\eta\nabla^4\mathbf{Z}+\rho f, \tag{2}$$

$$\rho c_p\dot{\Theta}=\kappa\nabla^2\Theta+\tau L, \tag{3}$$

where $\mathbf{Z}$ is the velocity vector, Θ is the temperature, f is the body force per unit mass, ρ is the constant density, κ is the thermal conductivity, the specific heat is symbolized by c_p, τ is the Cauchy stress tensor, also gradient of $\mathbf{Z}$ is represented by L, here η is used for couple stress parameter and the material derivative is represented by D/Dt and defined as following

$$\frac{\mathrm{D}}{\mathrm{Dt}}(*)=\left(\frac{\partial}{\partial t}+\mathbf{Z}.\nabla\right)(*). \tag{4}$$

The Cauchy stress tensor is represented and defined by

$$\tau=-pI+\mu A_1. \tag{5}$$

where p denotes dynamic pressure, I is the unit tensor, μ is the coefficient of viscosity and the first Rivilin-Ericksen tensor is denoted and defined as

$$A_1=L+L^t, \tag{6}$$

The transpose of L, is denoted by L^t.

2.2. Problem Formulation

Let us consider a steady flow of an incompressible couple stress fluid between to infinite parallel plates which are separated by distance $2d$, both plates are stationary, where Θ_0 and Θ_1 are the temperatures of the lower and upper plate respectively. Both plates are placed in the (x,y), orthogonal coordinates system at $y=-d$ and $y=d$, the motion of fluid is in the x-axis and y-axis is vertical to the plates as shown in figure 1. The pressure gradient is zero, μ is taken to be a function of $\Theta(y)$, velocity and temperature fields are considered to be of the form.

$$\mathbf{Z}=\mathbf{Z}(u,0,0), u=u(y), \text{and } \Theta=\Theta(y). \tag{7}$$

Equation (1) is identically fulfilled, if there is no body force, equations (2)-(3), that is the equation of momentum and the energy equation becomes

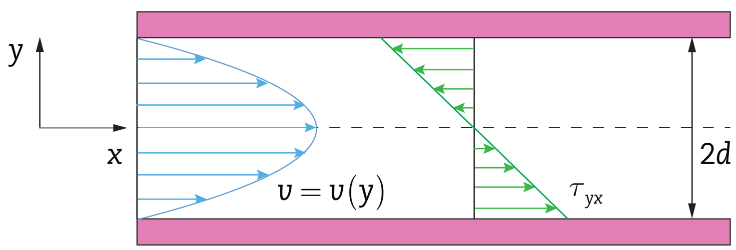

Fig. 1. Geometry of Plane Poiseuille Flow.

$$\mu\frac{d^2u}{dy^2}+\frac{d\mu}{dy}\frac{du}{dy}-\eta\frac{d^4u}{dy^4}-A=0, \tag{8}$$

$$\frac{d^2\Theta}{dy^2}+\frac{\mu}{\kappa}\left(\frac{du}{dy}\right)^2+\frac{\eta}{\kappa}\left(\frac{d^2u}{dy^2}\right)^2=0. \tag{9}$$

The boundary conditions for equations (8)-(9), are

$$u(-d)=0, u(d)=0, \tag{10}$$

$$u''(-d)=0,\ u''(d)=0, \tag{11}$$

$$\Theta(-d)=\Theta_0, \Theta(d)=\Theta_1. \tag{12}$$

Equations (10) are no-slip boundary conditions. Equations (11) show that couple stresses are zero at the plates. Over-all, the dimensionless parameters are:

$$u^*=\frac{u}{U}, y^*=\frac{y}{d}, \Theta^*=\frac{\Theta-\Theta_0}{\Theta_1-\Theta_0}, \mu^*=\frac{\mu}{\mu_0},\ B=d\sqrt{\frac{\mu_0}{\eta}},\ \lambda=\frac{\mu_0 U^2}{\kappa(\Theta_1-\Theta_0)}, A^*=\frac{Ad^4}{V\eta}.$$

Thus equations (8)-(9) having boundary conditions (10)-(12) can be written, by ignoring the asterisks, in the form as

$$\frac{d^4u}{dy^4}-B^2\mu\frac{d^2u}{dy^2}-B^2\frac{d\mu}{dy}\frac{du}{dy}+A=0, \tag{13}$$

$$\frac{d^2\Theta}{dy^2}+\lambda\mu\left(\frac{du}{dy}\right)^2+\frac{\lambda}{B^2}\left(\frac{d^2u}{dy^2}\right)^2=0, \tag{14}$$

$$u(-1)=0,\ u(1)=0,\ u''(-1)=0,\ u''(1)=0, \tag{15}$$

$$\Theta(-1)=0,\ \Theta(1)=1. \tag{16}$$

The Reynolds viscosity in the dimensionless form [52-56] is as

$$\mu=\exp(-M\Theta). \tag{17}$$

Let us assume $M=\in m,$ where $\in$ is a small parameter. Using Taylor series expansion of equation (17) we obtain

$$\mu=1-\in m\Theta, \frac{d\mu}{dy}\cong-\in m\frac{d\Theta}{dy}. \tag{18}$$

3. Description of the Methods

3.1. Basic Idea of OHAM

The background of the Optimal Homotopy Asymptotic Method (OHAM) is given in this sub-section. Let us take the differential equation

$$L(u(x))+f(x)+N(u(x))=0,\quad B(u,\frac{du}{dx})=0. \tag{19}$$

Here L is the linear operator, $f(x)$ is known function, $u(x)$ is an unknown function, $N(u(x))$ is a non-linear term and B is a boundary operator. Using OHAM we have [57]

$$(1-r)[L(u(x,q))+f(x))]=H(r)[L(u(x,q))+f(x))+N(u(x,q))],\qquad B\left(u(x,r),\frac{du(x,r)}{dx}\right)=0. \tag{20}$$

where $r\in[0,1]$ is an embedding parameter, $H(r)$ is an auxiliary function such that it is non-zero for $r\neq 0$ and for $r=0$, $H(0)=0.$ Clearly when $r=0$ and $r=1$ it yields

$$u(x,0)=u_0(x),\ u(x,1)=u(x). \tag{21}$$

The solution $u(x,q)$ varies from $u_0(x),$ to the solution $u(x),$ as r varies from 0 to 1, we have $u_0(x)$ by putting $r=0$ in equation (19),

$$L(u_0(x))+f(x)=0,\ B(u_0\frac{du_0}{dx})=0. \tag{22}$$

Auxiliary function can be written as

$$H(r)=rc_1+r^2c_2+r^3c_3+...,. \tag{23}$$

Here c_1, c_2, c_3... are constants to be evaluated, now consider solution of equation (20) in the form

$$u(x,r,c_i) = u_0(x) + \sum_{j\geq 1} u_j(x,c_i) r^j. \quad i = 1,2,... \tag{24}$$

Using equation (24) in equation (20) and equating like powers of r we get the following system

$$L(u_1(x)) = c_1 N_0(u_0(x)), \quad B(u_1 \frac{du_1}{dx}) = 0. \tag{25}$$

$$L(u_j(x) - u_{j-1}(x)) = c_j N_0(u_0(x) + \sum_{k=1}^{j-1} c_k [L(u_{j-k}(x)) + N_{j-k}(u_0(x), u_1(x), ..., u_{j-1}(x))], \quad B(u_j \frac{du_j}{dx}) = 0. \; j = 1,2,... \tag{26}$$

In equation (26) the term $N_m(u_0(x), u_1(x), ..., u_m(x))$ is the coefficient of r^m in the expansion

$$N(u(x,r,c_k)) = N_0(u_0(x)) + \sum_{j\geq 1} N_j(u_0(x), u_1(x), ..., u_j(x)) r^j. \quad k = 1,2,... \tag{27}$$

Equations (22), (25) and (26) can easily be computed for $u_j(x), j \geq 0$, the solution of equation (24) i.e., convergence is wholly depending on the constants $c_1, c_2, c_3, ...$ If at $r = 1$ it is convergent, then we have from equation (24)

$$u(x,c_k) = u_0(x) + \sum_{j\geq 1} u_j(x,c_k). \tag{28}$$

Over-all, the solution of equation (19) is approximated by

$$u^n(x,c_k) = u_0(x) + \sum_{j=1}^{n} u_j(x,c_k), \; k = 1,2,...n. \tag{29}$$

After using equation (29) in equation (19) we get the residual

$$R(x,c_k) = L(u^n(x,c_k)) + f(x) + N(u^n(x,c_k)), \; k = 1,2,...,n. \tag{30}$$

when the residual is equal to zero i.e., $R(x,c_k) = 0$ we get the exact solution $u^n(x,c_k)$. However, if residual is not zero, that is $R(x,c_k) \neq 0$, we can minimize as under

$$J(c_l) = \int_a^b R^2(x,c_l) dx. \tag{31}$$

where a and b are constants, depending upon the assumed problem. The unknown constants $c_1, c_2, c_3, ...$ can be achieved from the conditions

$$\frac{\partial J}{\partial c_l} = 0, \quad l = 1,2,...,n. \tag{32}$$

Once we get the values of these constants, we obtain the approximate solution from equation (29).

3.2. Basic Idea of NIM

The background of the New Iterative Method (NIM) is explained in this sub-section; let us choose the differential equation

$$u(x) = L(u(x)) + f(x) + N(u(x)). \tag{33}$$

where L is a linear operator, $f(x)$ is known function, $u(x)$ is unknown function, $N(u(x))$ is the nonlinear term. Assume that the New Iterative Method solution of equation (33) is of the form

$$u(x) = \sum_{i=0}^{\infty} u_i. \tag{34}$$

as L is a linear operator thus

$$L(\sum_{i=0}^{\infty} u_i) = \sum_{i=0}^{\infty} L(u_i). \tag{35}$$

The non-linear operator is given by [58]

$$N(\sum_{i=0}^{\infty} u_i) = N(u_0) + \sum_{i=1}^{\infty} \{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\} = \sum_{i=0}^{\infty} E_i. \tag{36}$$

where $E_0 = N(u_0)$ and

$$E_i = \{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}. \tag{37}$$

Using equations (34), (35) and (36) in equation (33) we get

$$\sum_{i=0}^{\infty} u_i = f(x) + \sum_{j=0}^{i} L(u_j) + \sum_{i=0}^{\infty} E_i. \tag{38}$$

4. Solution of the Problem

OHAM solutions of velocity profile (u_o) and temperature distributions (Θ_o) up to second-order are as under

$$\begin{aligned} u_o = {} & \frac{1}{24}\left(-0.025 + 0.03y^2 - 0.005y^4\right) + \frac{\begin{matrix} 0.00384184 - 6.6586648\times10^{-8}y \\ -0.00472358y^2 + 1.00779792\times10^{-7}y^3 \\ +0.00094471y^4 - 3.77924221\times10^{-8}y^5 \\ -0.00006298y^6 + 3.5992783\times10^{-9}y^7 \end{matrix}}{10080} \\ & + \frac{1}{130767436800}(-8.1883513 + 0.0002199y + 10.13500498y^2 \\ & - 0.00034352y^3 - 2.13727297y^4 + 0.0001443y^5 + 0.20374697y^6 \\ & - 0.00002214y^7 - 0.0131276y^8 + 0.00000145y^9 + 5.04\times10^{-8}y^{10} \\ & + 1.208\times10^{-15}y^{11} - 3.5\times10^{-9}y^{12} - 1.464\times10^{-16}y^{13} - 1.12\times10^{-13}y^{14} \\ & + 6.92722174\times10^{-18}y^{15}). \end{aligned} \tag{39}$$

$$\begin{aligned} \Theta_o = {} & \frac{1+y}{2} + 0.00006123(406.60283342 - 0.00035056y - 546.00329469y^2 \\ & + 173.81186781y^4 + 0.0004914y^5 - 34.21642486y^6 - 0.0001560y^7 \\ & - 0.19498167y^8 + 0.00001516y^9) + 1.38837395\times10^{-13}(-6.0877\times10^9 \\ & + 197366.58y + 8.50261628\times10^9y^2 - 367384.0031y^3 - 3.28638\times10^9y^4 \\ & + 237437.3539y^5 + 9.89643\times10^8y^6 - 74301.09433y^7 - 1.17707\times10^8y^8 \\ & + 6817.91362y^9 - 464976.9188y^{10} + 63.240395y^{11} + 168.64331174y^{12} \\ & + 0.0005914y^{13} - 10.71154811y^{14} - 0.00006938y^{15} - 0.0479270y^{16} \\ & + 0.00000328y^{17}). \end{aligned} \tag{40}$$

NIM solutions of velocity profile (u_N) and temperature distributions (Θ_N) up to second order are as follow

$$\begin{aligned} u_N = {} & \frac{1}{24}\left(-0.025 + 0.03y^2 - 0.005y^4\right) + 0.0009(4.68576209\times10^{-17} \\ & (-4.44609\times10^7y^5 + 205.8376y^6 + 2116234.43y^7 - 44.064399y^8 \\ & + 26.46219118y^9 + 6.201818y^{10} + 2.19819\times10^{-7}y^{11} - 0.4153y^{12} \\ & - 2.91353\times10^{-8}y^{13} - 0.00003y^{14} + 1.394\times10^{-9}y^{15} - 3.64\times10^{-10}y^{16} \\ & + 4.28\times10^{-14}y^{17} - 1.26\times10^{-18}y^{18}) - 7.80960\times10^{-14}(-1.3337\times10^9y^4 \\ & + 26676.557y^5 + 8.88729\times10^7y^6 - 3808.0788y^7 + 1428.83305y^8 \\ & - 0.1428958y^9 - 0.0017586y^{10} - 7.816\times10^{-11}y^{11} + 0.00012y^{12} \\ & + 9.237\times10^{-12}y^{13} + 7.93\times10^{-9}y^{14} - 4.595\times10^{-13}y^{15} + 1.36\times10^{-13}y^{16} \\ & - 1.645\times10^{-17}y^{17} + 5.05\times10^{-22}y^{18})). \end{aligned} \tag{41}$$

$$\begin{aligned} \Theta_N = {} & \frac{1+y}{2} + 0.001(0.00023148y^2\left(15 - 5y^2 + y^4\right) - 1.240079\times10^{-9} \\ & \left(\begin{matrix} -419.958y^4 + 0.0252y^5 \\ +111.9888y^6 - 0.008y^7 \\ -9.999y^8 + 0.000777y^9 \end{matrix}\right)) + 0.001(-0.0002y^2\left(15 - 5y^2 + y^4\right) \\ & + 1.24007936\times10^{-9}(-419.958y^4 + 0.0252y^5 + 111.9888y^6 - 0.008y^7 \\ & - 9.999y^8 + 0.0007777777777y^9)) - 0.001(3.47945949\times10^{-10}y^2 \\ & (9979200 - 3324903.269688y^2 - 0.05987519y^3 + 664581.66y^4 \\ & + 0.03420157y^5 + 53.430601y^6 - 0.0033236y^7 + 0.00124715y^8 \\ & - 1.63\times10^{-7}y^9 + 5.44\times10^{-12}y^{10}) + 4.82253086\times10^{-11}(10798.92y^4 \\ & - 0.648y^5 - 2878.41625618704y^6 + 0.20557544y^7 + 256.8163963y^8 \\ & - 0.01996401y^9 + 0.01439102y^{10} - 0.00000196y^{11} + 1.654\times10^{-7}y^{12} \\ & - 4.36108\times10^{-11}y^{13} + 3.65\times10^{-9}y^{14} + 5.1003\times10^{-16}y^{15} + 3.983\times10^{-13}y^{16} \\ & - 2.486\times10^{-17}y^{17} + 1.4905\times10^{-17}y^{18} - 1.882\times10^{-21}y^{19} + 1.903\times10^{-22}y^{20} \\ & - 3.634\times10^{-26}y^{21} + 2.318\times10^{-30}y^{22} - 4.94071146\times10^{-35}y^{23})). \end{aligned} \tag{42}$$

Table 1. OHAM solutions and their residuals of velocity and temperature for $A = 0.005, B = 0.03, M = 0.0002$ and $\lambda = 0.001$.

y	OHAM (u_o)	Residual (u_o)	OHAM (Θ_o)	Residual (Θ_o)
-1.	-1.08259×10^{-19}	1.62863×10^{-18}	-3.02878×10^{-19}	-1.23886×10^{-9}
-0.9	-0.000165794	-1.79627×10^{-16}	0.0535587	7.86566×10^{-8}
-0.8	-0.000326882	-8.81175×10^{-17}	0.107066	1.37772×10^{-7}
-0.7	-0.000479014	6.77432×10^{-17}	0.160463	9.6359×10^{-8}
-0.6	-0.000618442	1.7693×10^{-16}	0.213671	3.68576×10^{-10}
-0.5	-0.000741918	1.98478×10^{-16}	0.266597	-8.09834×10^{-8}
-0.4	-0.000846692	1.37797×10^{-16}	0.319146	-1.04541×10^{-7}
-0.3	-0.000930515	2.66261×10^{-17}	0.371233	-6.6573×10^{-8}
-0.2	-0.000991638	-9.33557×10^{-17}	0.42278	5.50578×10^{-9}
-0.1	-0.00102881	-1.83289×10^{-16}	0.473733	7.14486×10^{-8}
0.	-0.00104129	-2.1623×10^{-16}	0.524055	9.75961×10^{-8}
0.1	-0.00102881	-1.82621×10^{-16}	0.573733	7.10468×10^{-8}
0.2	-0.000991638	-9.20992×10^{-17}	0.62278	4.79287×10^{-9}
0.3	-0.000930515	2.83256×10^{-17}	0.671233	-6.74395×10^{-8}
0.4	-0.000846692	1.39744×10^{-16}	0.719146	-1.0538×10^{-7}
0.5	-0.000741918	2.00459×10^{-16}	0.766597	-8.16411×10^{-8}
0.6	-0.000618442	1.7874×10^{-16}	0.813671	-2.67594×10^{-11}
0.7	-0.000479014	6.92142×10^{-17}	0.860463	9.62104×10^{-8}
0.8	-0.000326882	-8.70955×10^{-17}	0.907066	1.3777×10^{-7}
0.9	-0.000165794	-1.79107×10^{-16}	0.953559	7.86729×10^{-8}
1.	-1.08345×10^{-19}	1.63045×10^{-18}	1.	-1.2556×10^{-9}

Table 2. NIM solutions and their residuals of velocity and temperature for $A = 0.005, B = 0.03, M = 0.0002$ and $\lambda = 0.001$.

y	NIM (u_N)	Residual (u_N)	NIM (Θ_N)	Residual (Θ_N)
-1.	8.74974×10^{-8}	-7.08702×10^{-14}	-2.54699×10^{-6}	2.17628×10^{-16}
-0.9	-0.000165796	-4.71277×10^{-14}	0.0499978	5.53967×10^{-14}
-0.8	-0.000326963	-2.97731×10^{-14}	0.0999982	6.62283×10^{-14}
-0.7	-0.000479166	-1.7634×10^{-14}	0.149999	5.55478×10^{-14}
-0.6	-0.000618655	-9.60307×10^{-15}	0.199999	3.79546×10^{-14}
-0.5	-0.000742182	-4.66536×10^{-15}	0.249999	2.16078×10^{-14}
-0.4	-0.000846998	-1.92204×10^{-15}	0.299999	9.97216×10^{-15}
-0.3	-0.000930853	-6.10408×10^{-16}	0.35	3.43409×10^{-15}
-0.2	-0.000992	-1.20281×10^{-16}	0.4	7.17974×10^{-16}
-0.1	-0.00102919	-6.71951×10^{-18}	0.45	4.63657×10^{-17}
0.	-0.00104167	8.67362×10^{-19}	0.5	-2.5411×10^{-21}
0.1	-0.00102919	-6.71951×10^{-18}	0.55	4.63666×10^{-17}
0.2	-0.000992	-1.20269×10^{-16}	0.6	7.17938×10^{-16}
0.3	-0.000930853	-6.1032×10^{-16}	0.65	3.43384×10^{-15}
0.4	-0.000846998	-1.92167×10^{-15}	0.699999	9.97121×10^{-15}
0.5	-0.000742182	-4.66424×10^{-15}	0.749999	2.16052×10^{-14}
0.6	-0.000618655	-9.60031×10^{-15}	0.799999	3.79492×10^{-14}
0.7	-0.000479166	-1.76281×10^{-14}	0.849999	5.55385×10^{-14}
0.8	-0.000326963	-2.97617×10^{-14}	0.899998	6.62157×10^{-14}
0.9	-0.000165796	-4.71074×10^{-14}	0.949998	5.53849×10^{-14}
1.	8.74906×10^{-8}	-7.08364×10^{-14}	0.999997	2.17545×10^{-16}

Table 3. The comparison of OHAM and NIM solutions for $M = 0.0002, B = 0.3, A = 0.4$ and $\lambda=0.4$.

y	OHAM (u_o)	NIM (u_N)	Difference	OHAM (Θ)	NIM (Θ)	Difference
-1	1.47193×10^{-17}	7.021×10^{-4}	7.02×10^{-4}	1.06835×10^{-17}	-0.0669758	0.0669758
-0.8182	-0.0230284	-0.0235359	5.07×10^{-4}	0.10737	0.0419694	0.065401
-0.6162	-0.0460977	-0.0476605	1.56×10^{-3}	0.22561	0.161762	0.063848
-0.4141	-0.0643355	-0.0666537	2.32×10^{-3}	0.341111	0.278475	0.062636
-o.2121	-0.076091	-0.0788669	2.77×10^{-3}	0.451867	0.390005	0.061862
-0.0101	-0.0803851	-0.0833231	2.94×10^{-3}	0.556521	0.494941	0.06158
0.2121	-0.076091	-0.0788669	2.77×10^{-3}	0.663967	0.602105	0.061862
0.4141	-0.0643356	-0.0666537	2.32×10^{-3}	0.755211	0.692575	0.062636
0.6162	-0.0460978	-0.0476605	1.56×10^{-3}	0.84181	0.777962	0.063848
0.8182	-0.0230284	-0.0235359	5.07×10^{-4}	0.92557	0.860169	0.065401
1	1.45416×10^{-17}	7.020×10^{-4}	7.02×10^{-4}	1.	0.933024	0.066976

Table 4. The comparison of OHAM and NIM solutions for $M = 0.0015$, $B = 0.4$, $\lambda = 0.1$ and $A = 0.3$.

y	OHAM (u_o)	NIM (u_N)	Difference	OHAM (Θ)	NIM (Θ)	Difference
-1	-7.62436×10^{-18}	9.38×10^{-4}	9.38×10^{-4}	2.12314×10^{-18}	-5.41×10^{-3}	5.41×10^{-3}
-0.9192	-0.00757213	-7.37×10^{-3}	1.94×10^{-4}	0.0409638	0.0356557	5.30×10^{-3}
-0.7172	-0.0255932	-0.026973	1.38×10^{-3}	0.143328	0.138258	5.07×10^{-3}
-0.5152	-0.0408483	-0.0434041	2.55×10^{-3}	0.245539	0.240666	4.87×10^{-3}
-0.3131	-0.0519164	-0.0552582	3.34×10^{-3}	0.347512	0.342779	4.73×10^{-3}
-0.1111	-0.0578333	-0.061576	3.74×10^{-3}	0.449022	0.444364	4.65×10^{-3}
0.1111	-0.0578334	-0.061576	3.74×10^{-3}	0.560122	0.555464	4.65×10^{-3}
0.3131	-0.0519166	-0.0552582	3.34×10^{-3}	0.660612	0.655879	4.73×10^{-3}
0.5152	-0.0408486	-0.0434041	2.55×10^{-3}	0.760739	0.755866	4.87×10^{-3}
0.7172	-0.0255934	-0.0269731	1.38×10^{-3}	0.860528	0.855458	5.07×10^{-3}
0.9192	-0.00757221	-7.37×10^{-3}	1.94×10^{-4}	0.960164	0.954856	5.30×10^{-3}
1	-8.1625×10^{-18}	9.37×10^{-4}	9.37×10^{-4}	1.	0.99459	5.41×10^{-3}

Table 5. The comparison of OHAM and NIM solutions for $\lambda = 0.5, M = 0.0009, B = 0.3$ and $A = 0.3$.

Y	OHAM (u_O)	NIM (u_N)	Difference	OHAM (Θ)	NIM (Θ)	Difference
-1	-7.43821×10^{-18}	5.26×10^{-3}	5.26×10^{-4}	1.58247×10^{-17}	-0.0470922	0.0470922
-0.9596	-0.00389674	-3.58×10^{-3}	3.09×10^{-4}	0.0227948	-0.0240456	0.0468404
-0.7576	-0.0227509	-0.0233925	6.42×10^{-4}	0.136543	0.090906	0.045637
-0.5556	-0.0391217	-0.0404867	1.36×10^{-3}	0.249219	0.20461	0.044609
-0.3535	-0.05145	-0.0533143	1.86×10^{-3}	0.359573	0.315729	0.043844
-0.1515	-0.058644	-0.0607849	2.14×10^{-3}	0.466226	0.422825	0.043401
0.1515	-0.058644	-0.0607849	2.14×10^{-3}	0.617726	0.574325	0.043401
0.3535	-0.0514501	-0.0533143	1.86×10^{-3}	0.713073	0.669229	0.043844
0.5556	-0.0391218	-0.0404868	1.36×10^{-3}	0.804819	0.76021	0.044609
0.7576	-0.022751	-0.0233926	6.42×10^{-4}	0.894143	0.848506	0.045637
0.9596	-0.00389676	-3.58×10^{-3}	3.09×10^{-4}	0.982395	0.935555	0.04684
1	-7.44693×10^{-18}	5.26×10^{-4}	5.26×10^{-4}	1.	0.952908	0.047092

Table 6. The comparison of OHAM and NIM solutions for $\lambda = 0.2,\ M = 0.001, B = 0.9$ and $A = 0.2$.

Y	OHAM (u_O)	NIM (u_N)	Difference	OHAM (Θ)	NIM (Θ)	Difference
-1	8.50833×10^{-18}	3.24×10^{-3}	3.24×10^{-3}	5.079×10^{-19}	-1.15×10^{-3}	1.15×10^{-3}
-0.8586	-7.3×10^{-3}	-7.55×10^{-3}	2.57×10^{-4}	0.0707929	0.0698346	9.58×10^{-4}
-0.6566	-0.0169159	-0.0210432	4.13×10^{-3}	0.171911	0.171183	7.28×10^{-4}
-0.4545	-0.0247198	-0.031551	6.83×10^{-3}	0.273055	0.272498	5.57×10^{-4}
-0.2525	-0.030008	-0.0384991	8.49×10^{-3}	0.374118	0.373672	4.46×10^{-4}
-0.0505	-0.032351	-0.0415392	9.18×10^{-3}	0.475146	0.474747	3.99×10^{-4}
0.0505	-0.0323511	-0.0415392	9.18×10^{-3}	0.525646	0.525247	3.99×10^{-4}
0.2525	-0.0300083	-0.0384991	8.49×10^{-3}	0.626618	0.626172	4.46×10^{-4}
0.4545	-0.0247202	-0.031551	6.83×10^{-3}	0.727555	0.726998	5.57×10^{-4}
0.6566	-0.0169163	-0.0210433	4.13×10^{-3}	0.828511	0.827783	7.28×10^{-4}
0.8586	-7.30×10^{-3}	-7.55×10^{-3}	2.57×10^{-4}	0.929393	0.928435	9.58×10^{-4}
1	9.1031×10^{-18}	3.23×10^{-3}	3.23×10^{-3}	1.	0.998848	1.15×10^{-3}

4.1. Volume flux:

The volume flux Q in the non-dimensional form as is under

$$Q = \int_{-1}^{1} u\,dy. \tag{43}$$

Using equations (39) and (41) in (43) we get

$$\begin{aligned} Q_O = 0.-\frac{4A}{15} &+ 0.10793650AB^2 - 0.04372144AB^4 \\ &- 0.05396825AB^2M + 0.04372144AB^4M \\ &- 0.01150091AB^4M^2 - 0.00397073A^3M\lambda \\ &- 0.00103281A^3B^2M\lambda + 0.0005164A^3B^2M^2\lambda. \end{aligned} \tag{44}$$

$$\begin{aligned} Q_N = -\frac{4A}{15} &+ \frac{AB^2}{126} + \frac{AB^4}{5184} - \frac{1}{252}AB^2M - \frac{AB^4M}{5184} \\ &+ \frac{1093AB^4M^2}{19958400} + \frac{16073A^3M\lambda}{129729600} \\ &+ \frac{1232423A^3B^2M\lambda}{108972864000} + \frac{5474983A^3B^4M\lambda}{14820309504000} \\ &- \frac{1232423A^3B^2M^2\lambda}{217945728000} - \frac{5474983A^3B^4M^2\lambda}{14820309504000} \\ &+ \frac{792750407A^3B^4M^3\lambda}{7602818775552000}. \end{aligned} \tag{45}$$

4.2. Average Velocity

The average velocity of the couple stress is denoted and defined as

$$\bar{u} = \frac{Q}{d}. \tag{46}$$

Equation (46) in the non-dimensional form coincides with the flow rate given in equations (44) and (45).

4.3. Shear Stress on the Plates

The dimensionless shear stress is denoted and defined as follows

$$t_p = -\mu \frac{du}{dy}\Big|_{y=1}. \tag{47}$$

The equations for shear stress are very lengthy; therefore only tables and graphs are given for shear stress of both methods, the minus sign [59] is because of the upper plate facing the negative y-direction of the coordinate system. Where Q_O and Q_N, are the volume fluxes achieved using OHAM and NIM respectively.

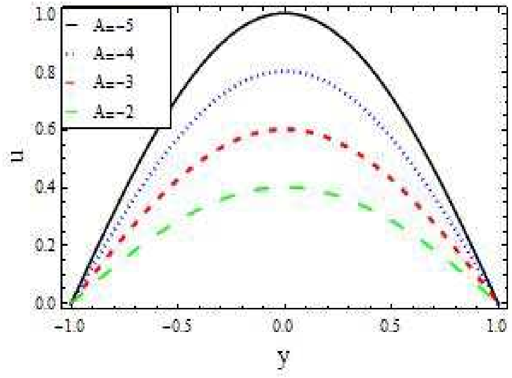

Fig. 2. Velocity profile using OHAM for $M = 0.0002, B = 0.3$ and $\lambda = 2$.

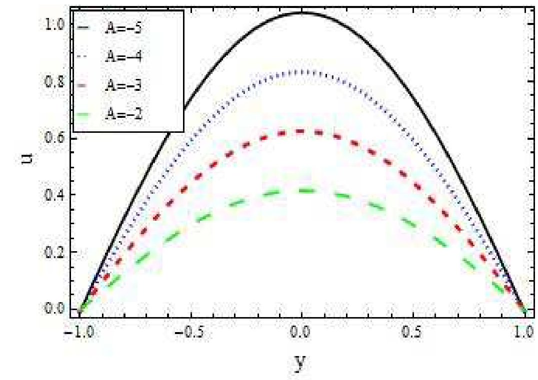

Fig. 3. Velocity profile using NIM for $M = 0.0002, B = 0.3$ and $\lambda = 2$.

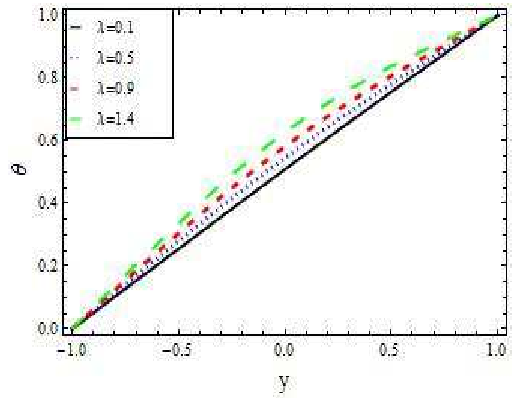

Fig. 4. Temperature profile using OHAM for $M = 0.0002, B = 0.1$ and $A = 0.1$.

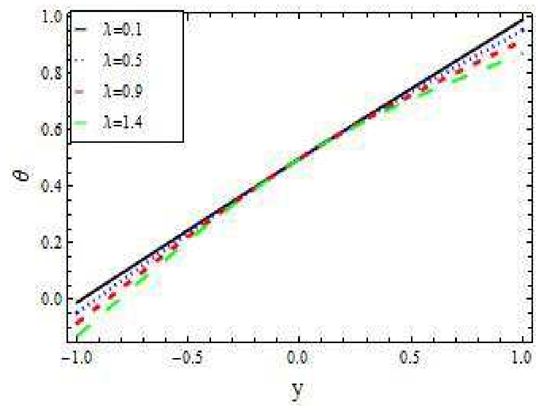

Fig. 5. Temperature profile using NIM for $M = 0.0002, B = 0.1$ and $A = 0.1$.

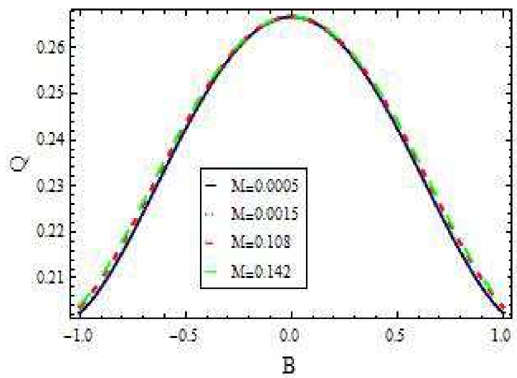

Fig. 6. Volume flux for plane Poiseuille flow varying M and keeping $A = -1, \lambda = 0.1$ using OHAM.

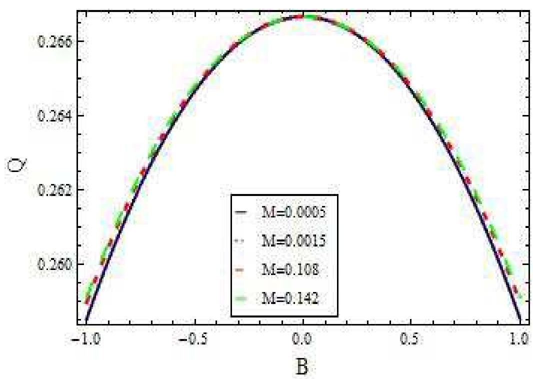

Fig. 7. Volume flux for plane Poiseuille flow varying M and keeping $A = -1, \lambda = 0.1$ using NIM.

5. Results and Discussion

In the present paper, we have studied the variation of the velocity profile and temperature distribution on different parameters such as A, M, B and λ. Tables 1-2 demonstrated solutions of OHAM and NIM for velocity profile, temperature distributions and residual of these methods respectively. Tables 3-6 show differences of the velocity profile, temperature distributions of both methods on different parameters. Figures 2-3 shows the velocity profile of the two methods which are in good agreement, the velocity of fluid increases as one move from lower plate to the upper plate and vanishes at both plates, we observed maximum velocity in the middle of the plates as shown in figures 2-3 for both methods. Figures 4-5 are the temperature distributions for different parameters, the temperature of the fluid increases as fluid moves from the lower plate towards the upper one. In the said methods temperature increases when the parameter λ increases. The volume flux of the fluid is studied in figures 6 and 7 for both methods. In these figures, we can observe the influence of M. Increasing the value of M reduces the volume flux of the fluid, which is inversely related between M and Q. Figures 8 and 9 are sketched for both methods to notice the behaviour of shear stress τ_p in the Plane Poiseuille flow while varying the value of the parameter B. The relationship between B and τ_p can be observed in these figures.

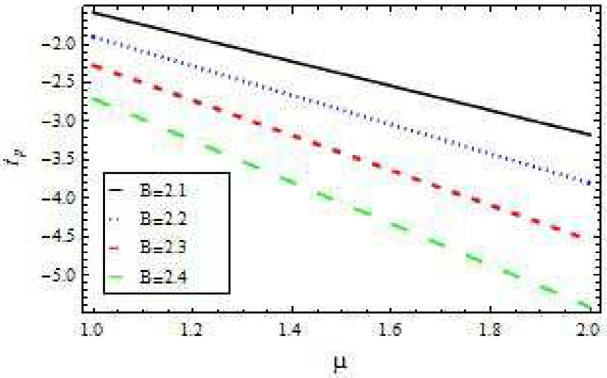

Fig. 8. Shear stress of plane Poiseuille for $M = 0.0005, \lambda = 1$ and $A = 2$ using OHAM.

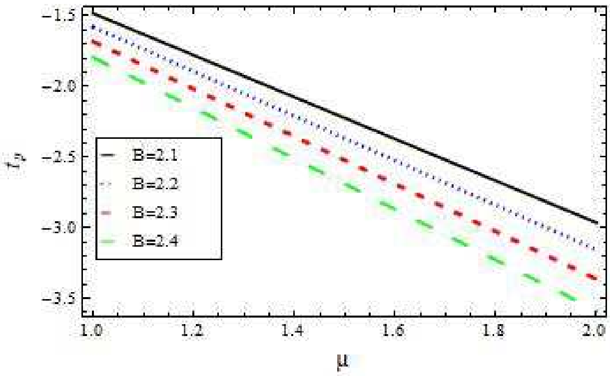

Fig. 9. Shear stress of plane Poiseuille flow for $M = 0.0005, \lambda = 1$ and $A = 2$ using OHAM.

6. Conclusion

In this work, we have investigated the flow of couple stress fluids between two parallel plates, using two methods OHAM and NIM. Applying the said methods, we have explored the approximate solution of the strongly nonlinear differential equations of couple stress fluids for velocity profile, temperature distributions, average velocity, volumetric flow rate and shear stress on the plates. The solutions obtained by these methods are in the form of infinite power series. The velocity profile and temperature distributions obtained using both methods are in excellent agreement. Also, volume flux and shear stress obtained by both these methods are in good agreement. The convergence and effectiveness of both these methods is crystal clear from the tables and figures of this work.

Author Contributions

Muhammad Farooq suggested the basic idea of the problem and developed the mathematical model, Alamgeer Khan explored the solutions using two reliable methods, Rashid Nawaz verified the obtained results, Muhammad Ayaz contributed to the data analysis and Saeed Islam contributed to the physical reasoning of the results. All authors made contribution in writing the manuscript. All authors discussed and reviewed the obtained results and approved the final version of the manuscript.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Nomenclature

$\mathbf{Z}$	Velocity vector	U	Reference velocity
Θ	Dimensional temperature	Θ^*	Dimensionless temperature
ρ	Constant density	κ	Thermal conductivity
c_p	Specific heat	τ	Cauchy stress tensor
L	Gradient of $\mathbf{Z}$	η	Couple stress parameter
Θ_0	Lower plate temperature	Θ_1	Upper plate temperature
D / Dt	Material derivative	τ	Cauchy stress tensor
p	Dynamic pressure	I	Unit tensor
μ	Dimensional coefficient of viscosity	μ^*	Dimensionless coefficient of viscosity
μ_0	Reference viscosity	A_1	First Rivilin-Ericksen tensor
u	Dimensional velocity	u^*	Dimensionless velocity
f	Body force	B	Dimensionless parameter
λ	Brinkmen number	M	Viscosity index
A	Pressure gradient		

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