



Journal of Applied and Computational Mechanics



Research Paper

Control of Actuators Torques for Optimal Movement along a Given Trajectory for the DexTAR Robot

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Received August 15 2020; Revised September 25 2020; Accepted for publication September 25 2020.

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Abstract. The control problem of the dynamics of actuators is considered to obtain a given optimal movement of the end-effector for a parallel Dexterous Twin Arms Robot (DexTAR). The trajectory is assumed to be known in advance, and the law of motion along the trajectory is given from some optimality conditions. The equations of dynamics of the robot are written under the condition that the leading rods are driven by torques of a symmetrically arranged pair of engines. The solutions of the direct and inverse kinematic problems are presented as auxiliary material. The resulting nonlinear motion equations are derived. A numerical example shows that the equations can be simplified neglecting the change in the angle between the rods at the end-effector. Numerical examples of calculating the torques are given.

Keywords: Robot dynamics, Dynamics inverse problem, Optimal motion, Robot control, Parallel robot.

1. Introduction

A variety of control problems in robotics is associated with the rapid development of this industry nowadays. Already in the early 90th, control problems began to move from a theoretical field to a practical one, see, for instance, the monograph [1], in which one can find the main bibliography in various areas of control theory and robotics until the 90s. Note that the mentioned monograph does not consider robots of parallel structure, in contrast to the Merle monograph [2], which describes the main types of robots of parallel structure, and also discusses research directions in this area. One of the important problems of robotics is the dynamic problem, which can be either direct or inverse. A bibliography of the works on the principles of constructing dynamic robot models is also given in the monograph [2]. Solutions to these problems aim to control the movements of the robot. Due to the great diversity of the design of robots, a lot of research consider the control of specific types of mechanisms, of which we mention the article [3] related to robots with two degrees of freedom. Here the optimal design of a robot similar to the robot considered in this article is proposed. In [4], along with kinematic problems, the inverse dynamics problem for a parallel robot with two degrees of freedom (2DoF) is considered, based on the equations of dynamics in the Lagrange form. A unified approach to deriving dynamics equations for a parallel manipulator was proposed in [5]. Dynamic models of various types of parallel robots were also considered in [6] - [10]. The most complete results are obtained for robots with two degrees of freedom, for example, [11] provides a dynamic analysis of a planar robot, using the Lagrange equations, and computer simulations are performed. In [12], a new robot of 2DoF was proposed, a complete dynamic and kinematic analysis was carried out, and robot motion control is simulated. In [13], various dynamic characteristics of the robot are analyzed, and a set of indicators is proposed as the main characteristic, including speeds, accelerations, and acting forces. A large pool of papers is devoted to optimizing the trajectory of the end-effector of the robot, see, for example, the review [14]. In [17], on the basis of the inverse dynamic problem for a spatial manipulator, a robot trajectory with minimal energy consumption is constructed. The problem of controlling accelerations for repetitive motions for various forms of trajectories of a parallel planar robot is solved in [18]. In [19], the synthesis of the quadrotor control algorithms in the basic flight modes is considered including position control, cruise speed control and reference path tracking. In [20], kinematic and dynamic analysis is performed for a parallel robot with three degrees of freedom such as 3PRC. Based on the inverse dynamic problem, the forces and powers of the actuators are determined and the problem of controlling the movement of a given robot along a given trajectory is considered. In [21], using the inverse dynamic problem, the nonlinear problem of identifying two electro-mechanical systems for the TX40 robot model is successfully solved. In [22], a general formulation of the direct and inverse problems of dynamics for a parallel robot is formulated on the basis of concepts generally accepted in robotics. Further, it is shown how this general formulation can be written for two examples of parallel robots. In [23], three different methods for controlling a parallel 3RPS robot are discussed and compared to ensure movement along a given trajectory. The author of the article [24] studied the kinematics and dynamics of spatial movements of a Stuart type robot and, based on the solution of the inverse dynamics problem, the forces applied to the moving elements are calculated and their powers are determined. In [25], the inverse dynamics problem for a tripod is solved taking into account the masses of control elements and the forces and accelerations are calculated to provide a movement along a straight trajectory. In [26], the direct and inverse problems of kinematics and dynamics for a parallel robot for translational motion are solved. It is shown that



the mathematical model is in good agreement with the experiment. Thus, the cited articles show, that the inverse dynamics problems are the important problems of robotics.

In this paper, we also solve the inverse dynamic problem using the Lagrange equations for 2DoF Dexterous Twin Arm Robot (DexTAR). In engineering applications this robot is known as a five-bar parallel robot, so-called pantograph, and its structure has been invented in 1934 [27]; actually, this robot has been proposed as a first industrial parallel robot. Nowadays, this robot has different modifications and applications [16, 28-31]. Let us name here its academic models [28, 29], models to perform 3D printing and drawing [29], model to perform some pick-and-place operations [30] as well as micro-models aimed to use in medicine [15]. Nowadays, different types of micro robots are used in surgery [31], moreover, remotely controlled micro robots are applied to design new composed materials [32].

Schematic configurations of various models of DexTAR robots starting from the pantograph [27] are described in the User's Manual [29]. Actually, in industrial versions, the robots on the base of a pantograph can have more than 2 degree of freedom, including its movement along the axis orthogonal to its bars' plane and rotations of that axis. But in the presented study we consider its planar movement with 2DoF only.

We assume that the trajectory of motion and the law of motion of the end-effector along the trajectory are given. In our previous study [33], we have considered the problem of optimizing the law of motion of an end-effector along a parametrically specified flat trajectory. As a cost functional, we considered the sum of kinetic energy and the weighted mean square inertia forces. In other words, we minimized the energy consuming and improved the comfortability of motion, due to minimizing inertia forces. The optimal law of motion was built for the coordinates of the end-effector. The next step is to define the motion of actuators that depends on the type of the robot, i.e. the specific inverse kinematics problem should be solved. Further problem is computing forces and torques that should be exerted by the actuators, via the solution of inverse dynamic problem.

Note that for the purpose of this study, it does not matter what was an optimality criterion of the motion along the trajectory. We only assume that the trajectory is set parametrically and the dependence on time for the parameter is known.

This paper is organized as follows. In section 2, we present the solutions of the inverse and direct kinematics. Based on the solution of the inverse problem, the optimal motion for actuators is constructed. In section 3, the inverse dynamic problem is formulated. Using the obtained mathematical model, the moments on the engines are calculated to provide the optimal movement mode. Finally, section 4 consists of discussions and conclusion.

2. Direct and Inverse Kinematics

Consider the movement of a 2DoF version of DexTAR robot, with a parallel structure, in which the end-effector moves along a flat trajectory. Optimization problems for robots of this type can have different statements. For example, in [15], the size ratio of moving rods to increase the size of the working area was investigated. There, all possible points of assembly of the robot were studied, the number of different positions of the end-effector was determined for the same position of the actuators. In [16], an analysis of the workspace of this robot was carried out and an effective algorithm for constructing the workspace was developed. In this Section, we define the angles of rotation of the driving rods, if the law of motion along the trajectory is specified. The optimal mode of motion along the trajectory was built in advance in [33], for criterion of optimality, which is described below.

The first model of DexTAR robot made previously for academic purposes has been proposed in [28]. Its photo and schematic configuration is presented in Fig.1. In Figure 2, following the notations of [16], we present the kinematic scheme of the robot DexTAR. This scheme is common for all robots on the base of pantograph. Some examples of possible configurations are presented in [15, 16, 28-30].

Let the position of the end-effector at some moment of time be given by the point P (x, y). Consider the case where the lengths of the two driving rods are equal: $CD = AB = l$, the lengths of two rods are $CP = BP = L$, let the distance between the fixed hinges on the base be equal to $AD = d$. We use the coordinate system with the origin in the midpoint of the fixed segment AD. In order to find control angles (φ, ψ) for the known movement along the trajectory, it is necessary to solve the inverse cinematic problem: find the actuator variables (φ, ψ) from the coordinates (x, y).

Although formulas for solving the inverse problem were derived in [15], we give below a shorter way to get it and use other form of the solution for the convenience of further presentation.



Fig. 1. Photo of DexTAR robot and its schematic configuration presented in [28]: real model (left) and virtual prototype: 1 - the base plate; 2 - the first set of rods; 3 - the second set of rods; 4 - end effector; 5 - servo motors,



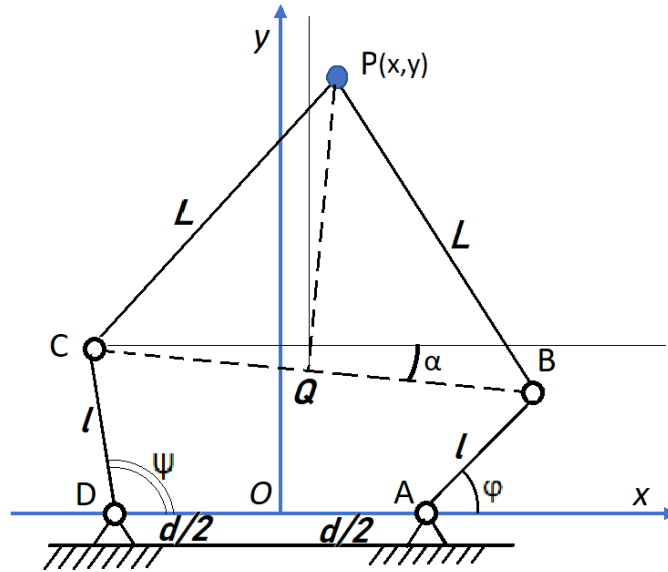


Fig. 2. Kinematic scheme of DexTAR of RRRRR type.

2.1 Inverse kinematics for planar robot DexTAR

Consider the triangle ABP. Temporarily move the origin to point A and write down the conditions that the coordinates of point B satisfy:

$$\begin{cases} x_B^2 + y_B^2 - l^2 = 0 \\ (x - x_B)^2 + (y - y_B)^2 - L^2 = 0 \end{cases} \quad (1)$$

Subtracting the second equation from the first equation of system (1), we obtain:

$$2xx_B + 2yy_B + L^2 - (x^2 + y^2) - l^2 = 0 \quad (2)$$

Using the notations

$$(x^2 + y^2) + l^2 - L^2 = 2d_1, \quad k = \frac{x}{y} \quad (3)$$

we get from the equality (2):

$$y_B = \frac{d_1}{y} - kx_B. \quad (4)$$

Substituting expression (4) into first equation of system (1) we obtain a quadratic equation:

$$(1 + k^2)x_B^2 - 2k\frac{d_1}{y}x_B + \frac{d_1^2}{y^2} - l^2 = 0. \quad (5)$$

Solving it, we have:

$$x_{B1,2} = \frac{1}{(1 + k^2)} \left[\frac{kd_1}{y} \pm \sqrt{(1 + k^2)l^2 - \frac{d_1^2}{y^2}} \right]. \quad (6)$$

Taking into account (4), we get:

$$y_B = -kx_B + \frac{d_1}{y} = -\frac{k^2d_1}{y(1 + k^2)} + \frac{d_1}{y} \mp \frac{k}{(1 + k^2)} \sqrt{(1 + k^2)l^2 - \frac{d_1^2}{y^2}}. \quad (7)$$

Then

$$y_B = \frac{d_1 \mp k\sqrt{y^2l^2(1 + k^2) - d_1^2}}{y(1 + k^2)}. \quad (8)$$

In the coordinate system with the origin at the point O formula (6) is rewritten as follows:

$$x_B = \frac{kd_1 \pm \sqrt{y^2l^2(1 + k^2) - d_1^2}}{y(1 + k^2)} + \frac{d}{2}. \quad (9)$$

Substituting into (8), (9) the value of k, we have:



$$x_B = \frac{xd_1 \pm y\sqrt{l^2(x^2 + y^2) - d_1^2}}{(x^2 + y^2)},$$

$$y_B = \frac{yd_1 \mp x\sqrt{l^2(x^2 + y^2) - d_1^2}}{(x^2 + y^2)}.$$

Returning from the coordinate system with origin at point A back to the initial system, we obtain:

$$x_B = \frac{(x - 0.5d) \pm y\sqrt{l^2((x - 0.5d)^2 + y^2) - d_1^2}}{(x - 0.5d)^2 + y^2} + 0.5d,$$

$$y_B = \frac{yd_1 \mp (x - 0.5d)\sqrt{l^2((x - 0.5d)^2 + y^2) - d_1^2}}{(x - 0.5d)^2 + y^2},$$

$$d_1 = 0.5[(x - 0.5d)^2 + y^2 + l^2 - L^2].$$
(10)

Similarly, the coordinates of point C are calculated by temporarily moving the origin to the point D. Then, moving back to the original coordinate system, we have:

$$x_C = \frac{(x + 0.5d) \pm y\sqrt{l^2((x + 0.5d)^2 + y^2) - d_2^2}}{(x + 0.5d)^2 + y^2} - \frac{d}{2},$$

$$y_C = \frac{yd_2 \pm (x + 0.5d)\sqrt{l^2((x + 0.5d)^2 + y^2) - d_2^2}}{(x + 0.5d)^2 + y^2},$$

$$d_2 = 0.5[(x + 0.5d)^2 + y^2 + l^2 - L^2].$$
(11)

Formulas (10)-(11) define the solution to the kinematic problem, since

$$\varphi(t) = \arcsin \frac{y_B(t)}{l}, \quad \psi(t) = \arcsin \frac{y_C(t)}{l}. \quad (12)$$

Substituting into (10) - (12) the predefined law of motion $x = x(t)$, $y = y(t)$, we obtain the dependence on time for the actuators (leading angles).

Consider an example. Let the trajectory be a semicircle. Let the parameterization of the trajectory have the form:

$$x = R \cos p, y = R \sin p, p \in [0, \pi]. \quad (13)$$

Here R is the radius, p is the parameter, in our case it is the polar angle of movement around the circle. Let the radius of the trajectory be $R = 120$ mm. Let us consider uniform movement around the circle with time dependency of parameter $p(t) = t$.

In [16] the optimal geometry of the robot maximizing the workspace has been computed. Here we use that geometric parameters:

$$l = 72 \text{ mm}, d = 60 \text{ mm}, L = 87 \text{ mm}.$$

Our calculations were carried out in dimensionless quantities, where the length L is a unity of length, the mass of the end-effector is taken as a unit of mass and the whole duration of the movement T is taken as a unit of time.

Figure 3 shows the solution of the inverse kinematics problem for uniform movement of the end-effector in a semicircle: rotation angles are $\varphi(t)$, $\psi(t)$. It can be seen that despite the strongly nonlinear nature of dependences (12), between the angles of rotation of the leading rods and the angle of rotation p of the end-effector, the dependence $\varphi(t)$, $\psi(t)$ is close to the linear one.

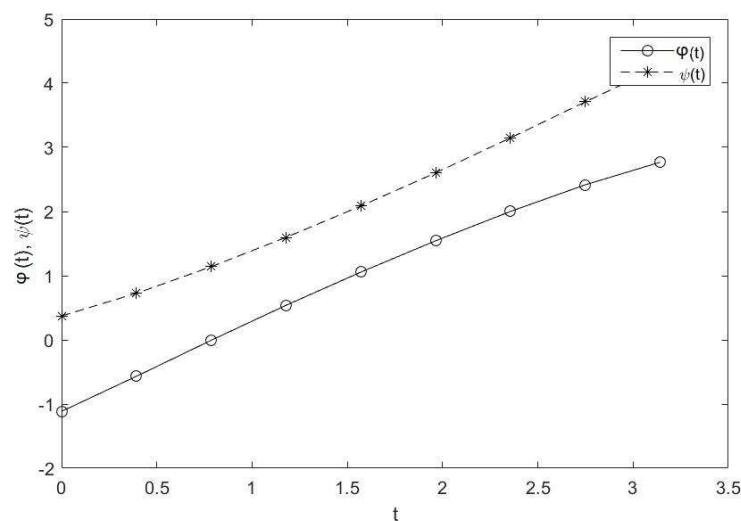


Fig. 3. Solution of the inverse kinematics problem for uniform movement of the end-effector in a semicircle: dependence of the leading angles $\varphi(t)$ and $\psi(t)$ on time.



Now we consider the case when the law of motion along the trajectory is not uniform, but is chosen from some optimality conditions.

2.2 The optimal law of rotation of the driving rods

In our previous study [33], we have constructed a solution to the problem of optimizing the motion of a point mass along a given path:

$$\begin{aligned}x(t) &= x(p(t)), \\ y(t) &= y(p(t)).\end{aligned}$$

Here the function $p(t)$ is taken from the minimality condition of a cost functional of the following form:

$$\Phi(p) = \frac{m}{2} \int_0^1 (\dot{x}^2(t) + \dot{y}^2(t)) dt + \frac{\eta m^2}{2} \int_0^1 (\ddot{x}^2(t) + \ddot{y}^2(t)) dt. \quad (14)$$

The functional (14) represents the sum of the average kinetic energy of motion and the average inertia for the entire period of motion taken with a weight $\eta > 0$.

As it is showed earlier in [33], for motion along an arc of the circle with the minimum of functional (14), the parameter $p(t)$ satisfies a following fourth-order quasilinear ODE:

$$p^{(iv)} - (6\dot{p}^2 + \gamma^2)\ddot{p} = 0, \quad \gamma = (\eta m)^{-1/2}, \quad t \in [0, 1] \quad (15)$$

Here m is the mass of the point, p is the variable that defines the parameter on the arc of a circle (angle of rotation). The start and end positions of the point are stopping points.

Numerical solutions of equation (15) were constructed. Below we take this solution for the unit mass of the point $m = 1$ and the weight in the functional (15) $\eta = 0.1$. As boundary conditions, we take the relations

$$\begin{aligned}p(0) &= 0, \quad \dot{p}(0) = 0, \\ p(1) &= \pi, \quad \dot{p}(1) = 0.\end{aligned} \quad (16)$$

These relations mean that the tool moves along an arc of a semicircle and has zero speeds at the start and end points. Let, as in the previous example, the trajectory of the point P be a semicircle, and be defined by the same parametrization (13).

We substitute the optimal law of motion along the trajectory in the sense of the minimum of the functional (14) into the solution of the inverse kinematics problem (12) and construct the dependence on time of the leading angles. Figure 4 shows successive robot positions for optimal movement along the trajectory taken for moments with time interval $\Delta t = 1/8$. It can be seen that at the initial and final positions, the end-effector slows down, and the changes in the rotation angle are not uniform.

Figure 5 shows the dynamic of the rotation angle of the driving rods at the optimal mode of movement. Thus, using the solution to the problem of optimizing the movement of the end-effector along a given trajectory and solving the inverse kinematics problem, we can construct a time dependence of the leading coordinates $(\varphi(t), \psi(t))$ that ensures this movement.

Now, in order to obtain the optimal coordinates' dynamic of the actuators, we must determine how to control action on actuators. Usually this is the torques on the engines. The solution to this problem depends on the design of the robot. But even in relatively simple cases, solving this problem in explicit formulas can be practically inaccessible. Let us continue the discussion for the robot DexTAR.

2.3. Direct kinematics

Let the positions of the leading rods AB and DC , which are defined by the angles φ and ψ , be known. Find the coordinates (x, y) of the point P . From the vector equality (Fig. 2) we have:

$$\overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{QP}$$

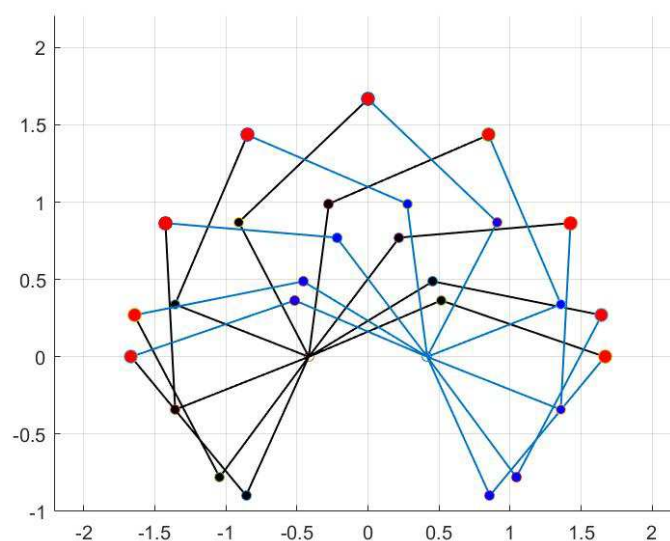


Fig. 4. Consecutive positions of the robot with the optimal choice of the law of motion along a given path.



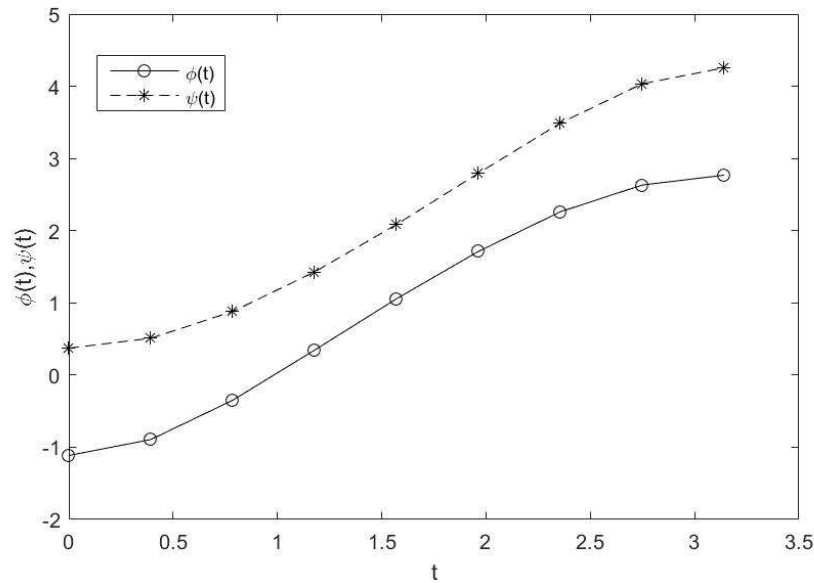


Fig. 5. Rotation angle of the driving rods at the optimal mode of the motion.

where Q is the midpoint of the segment CB, hence:

$$x = x_Q - H \sin \alpha, \quad y = y_Q + H \cos \alpha, \quad (17)$$

where H is the height of the isosceles triangle CPB, and the angle α is the angle between the segment CB and the abscissa Ox. The coordinates of the point Q are easily determined via positions of points B and C:

$$x_Q = 0.5l(\cos \varphi + \cos \psi), \quad y_Q = 0.5l(\sin \varphi + \sin \psi). \quad (18)$$

Height H is also easily expressed via hypotenuse L and leg QB:

$$\begin{aligned} H &= \sqrt{L^2 - 0.25(x_B - x_C)^2 + (y_B - y_C)^2} \\ &= \sqrt{L^2 - \frac{(l \cos \varphi - l \cos \psi + d)^2 + (l \sin \varphi - l \sin \psi)^2}{4}}. \end{aligned} \quad (19)$$

The angle of inclination of the segment CB to the abscissa Ox is determined from the formula

$$\alpha = \arctan \frac{y_B - y_C}{x_B - x_C} = \arctan \frac{\sin \varphi - \sin \psi}{\cos \varphi - \cos \psi + d/l}. \quad (20)$$

Substituting expressions (18) - (20) into (17), we obtain the solution of the DKP (direct kinematics problem):

$$\begin{aligned} x &= 0.5l(\cos \varphi + \cos \psi) - \frac{Hl(\sin \varphi - \sin \psi)}{2\sqrt{L^2 - H^2}}, \\ y &= 0.5l(\sin \varphi + \sin \psi) + \frac{Hl(\cos \varphi - \cos \psi + d)}{2\sqrt{L^2 - H^2}}. \end{aligned} \quad (21)$$

Let β denote the angle at the base of the triangle CBP:

$$\beta = \arctan \frac{H}{\sqrt{L^2 - H^2}}. \quad (22)$$

Then formulas (21) can be written in more compact form:

$$\begin{aligned} x &= 0.5l(\cos \varphi + \cos \psi) - 0.5l \tan \beta (\sin \varphi - \sin \psi), \\ y &= 0.5l(\sin \varphi + \sin \psi) + 0.5l \tan \beta (\cos \varphi - \cos \psi + d). \end{aligned} \quad (23)$$

Suppose that due to the technical implementation of the robot, in extreme left and right positions the segment BC is placed perpendicular to the abscissa axis. Then the following restrictions for the rotation angles of the rods held:

$$\begin{aligned} -\arccos\left(-\frac{d}{2l}\right) &\leq \varphi \leq \arccos\left(-\frac{d}{2l}\right), \\ \arccos\left(-\frac{d}{2l}\right) &\leq \psi \leq \pi + \arccos\left(-\frac{d}{2l}\right) \end{aligned} \quad (24)$$

These restrictions correspond to the extreme positions when the segment BC is placed perpendicular to the abscissa axis. Constraints (24) are obtained from a geometric consideration of the extreme positions of the robot with the vertical position of the segment BC.



3 Inverse dynamics and control torques

Let us now consider the dynamic synthesis problem, i.e. we need to generate a control action that implements a given mode of changing the angle of rotation of the rods (φ, ψ).

To do this, we need to formulate the equations of dynamics. Let us write them in the form of Lagrange. The Lagrangian is equal to the difference between the kinetic and potential energies of the robot. Considering the case of movement in the absence of potential fields, we write down the kinetic energy of the end-effector:

$$T_p = m \frac{\dot{x}^2(\varphi, \psi) + \dot{y}^2(\varphi, \psi)}{2}. \quad (25)$$

To formulate equations of dynamics we need to derive the kinetic energy of the whole mechanism, including rods.

If we use the angles of rotation of the leading rods as generalized coordinates, then the elementary work of the torques on the engine is equal to the product of the torque by the variation of rotation angle. Accordingly, generalized forces are equal to these torques.

If we know the optimal time dependence of rotation angles, then the control actions that implement this movement, i.e. torques at points B and C are defined by the dynamic's equation in Lagrange form:

$$M_1 = \frac{d}{dt} \frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = \frac{d}{dt} \frac{\partial T}{\partial \dot{\varphi}} - \frac{\partial T}{\partial \varphi}, \quad M_2 = \frac{d}{dt} \frac{\partial L}{\partial \dot{\psi}} - \frac{\partial L}{\partial \psi} = \frac{d}{dt} \frac{\partial T}{\partial \dot{\psi}} - \frac{\partial T}{\partial \psi}. \quad (26)$$

This makes it clear that before calculating the torques by (26), we need to know the dependence on the coordinates (x, y) for the angles (φ, ψ).

This means that for this case we need to know the solution to the direct kinematics problem for this robot, which has been built above, and which we rewrite here:

$$\begin{aligned} x &= 0.5l(\cos \varphi + \cos \psi) - 0.5l \tan \beta (\sin \varphi - \sin \psi), \\ y &= 0.5l(\sin \varphi + \sin \psi) + 0.5l \tan \beta (\cos \varphi - \cos \psi + d). \end{aligned} \quad (27)$$

The angle β above depends in a complicated way on the generalized coordinates (φ, ψ) via definition (22), where H is determined by the formula (19). Differentiating (27), we obtain formulas for the velocities of the end-effector:

$$\begin{aligned} \dot{x} &= -0.5l(\dot{\varphi} \sin \varphi + \dot{\psi} \sin \psi) - 0.5l \tan \beta (\dot{\varphi} \cos \varphi - \dot{\psi} \cos \psi) \\ &\quad - 0.5l \cos^{-2} \beta (\sin \varphi - \sin \psi) \dot{\beta} = -\frac{0.5l}{\cos \beta} (\dot{\varphi} \sin(\varphi + \beta) + \dot{\psi} \sin(\psi - \beta)) \\ &\quad - 0.5l \cos^{-2} \beta (\sin \varphi - \sin \psi) \dot{\beta}, \\ \dot{y} &= 0.5l(\dot{\varphi} \cos \varphi + \dot{\psi} \cos \psi) - 0.5l \tan \beta (\dot{\varphi} \sin \varphi - \dot{\psi} \sin \psi) \\ &\quad + 0.5l \cos^{-2} \beta (\cos \varphi - \cos \psi + d) \dot{\beta} = \frac{0.5l}{\cos \beta} (\dot{\varphi} \cos(\varphi + \beta) + \dot{\psi} \cos(\psi - \beta)) \\ &\quad + 0.5l \cos^{-2} \beta (\cos \varphi - \cos \psi + d) \dot{\beta}. \end{aligned} \quad (28)$$

We find the velocities (28) corresponding to the optimal change in the rotation angles $\varphi(t), \psi(t)$. To calculate them numerically, it is convenient to use a formula that defines the trajectory parametrically. In the case of a simple trajectory in the form of (13), the velocities are easily calculated by the formulas:

$$\begin{aligned} \dot{x} &= -R\dot{p} \sin p(t), \\ \dot{y} &= R\dot{p} \cos p(t). \end{aligned} \quad (29)$$

We use formula (29) to test our results only. Actually, we have computed the velocities expressing first derivatives of the angles in (28) approximately via finite differences formulas. The optimal mode for changing the rotation angles was taken from the tabulated numerical solution of the problem of the optimal movement along the semicircle. Squaring expressions (28) and summing, we obtain the following expression for kinetic energy of point P:

$$\begin{aligned} \frac{2T_p}{m} &= \frac{l^2}{4 \cos^2 \beta} [(\dot{\varphi} \sin(\varphi + \beta) + \dot{\psi} \sin(\psi - \beta)) - \cos^{-1} \beta (\sin \varphi - \sin \psi) \dot{\beta}]^2 + \\ &\quad + \frac{l^2}{4 \cos^2 \beta} [(\dot{\varphi} \cos(\varphi + \beta) + \dot{\psi} \cos(\psi - \beta)) + \cos^{-1} \beta (\cos \varphi - \cos \psi + d) \dot{\beta}]^2. \end{aligned} \quad (30)$$

For the special case of motion (29), the kinetic energy T_p is easily calculated from the expression:

$$\frac{2T_p}{m} = \dot{x}^2 + \dot{y}^2 = R^2 \dot{p}^2. \quad (31)$$

We computed the kinetic energy over time for the considered optimal mode. Due to the more general nature of expression (30), we took it as the basis for calculating kinetic energy. As an approximation, neglecting the change in the height of the triangle CPD during movement, we can calculate the speeds of point P depending on the angular velocities of the driving rods. It turned out that if we put in the expressions of the kinetic energy the constant value of the angle β equal to the value β_0 in the middle position, for the moment $t = 0.5$, the time dependence of kinetic energy almost remains the same. In Figure 6 different methods of computing the kinetic energy of the end-effector such as using finite differences for the formula (T_{comp}), and for a constant angle β (T_{approx}) are compared against the exact solution T_{exact} . Note, that to express the results in dimensional form, we must multiply them by the scale of energy equal to mL^2/T^2 , where m, L, T are mass, length of rod CP(=BP), and motion time respectively.



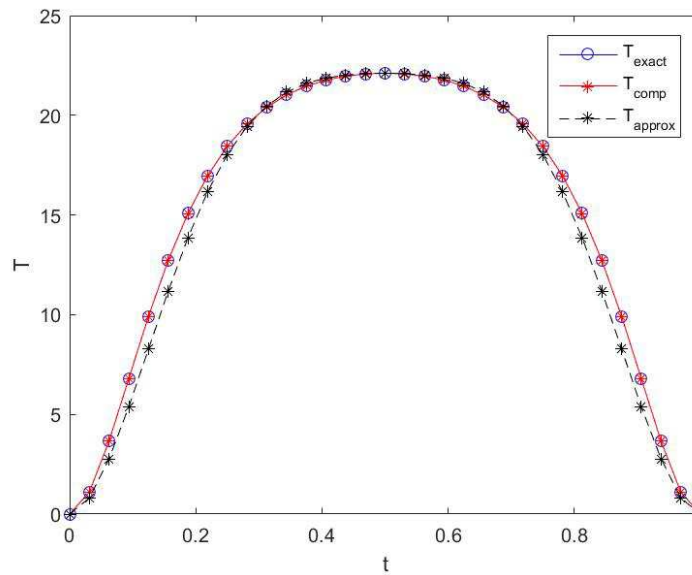


Fig. 6. The kinetic energy of end-effector computed via finite differences approximations (T_{comp}), approximation with a constant angle β (T_{approx}), and the exact solution T_{exact}

The calculation shows that the change in the angle β can be neglected, and then we obtain a simplified expression for the kinetic energy of the mass at point P:

$$\begin{aligned} \frac{2T_p}{m} = (\dot{x}^2 + \dot{y}^2) &= \frac{l^2}{4\cos^2\beta_0} \left[(\dot{\varphi}\sin(\varphi + \beta_0) + \dot{\psi}\sin(\psi - \beta_0))^2 + \right. \\ &\left. + (\dot{\varphi}\cos(\varphi + \beta_0) + \dot{\psi}\cos(\psi - \beta_0))^2 \right] = \frac{l^2}{4\cos^2\beta_0} (\dot{\varphi}^2 + \dot{\psi}^2 + 2\dot{\varphi}\dot{\psi}\cos(\varphi - \psi + 2\beta_0)). \end{aligned} \quad (32)$$

This allows one to simplify the formula for the whole kinetic energy of mechanism. Namely, let us neglect the change of the angle β . Consider the triangle CPB made of two rods CP and BP. Having the predefined trajectory of point P one can compute the middle position of the triangle CPB and use its geometry to compute the kinetic energy of the mechanism. If the angle β is approximately constant then the triangle CPB consisting of two rods moves as a solid body. Then its kinetic energy consists of energy of the inertia center and rotating energy around that center. Position of the inertia center S of the triangle CPB can be computed as follows:

$$\begin{aligned} x_s &= \lambda x + (1 - \lambda)x_Q = \lambda x + 0.5(1 - \lambda)(x_B + x_C), \\ y_s &= \lambda y + (1 - \lambda)y_Q = \lambda y + 0.5(1 - \lambda)(y_B + y_C), \quad \lambda \in (0, 1). \end{aligned} \quad (33)$$

If the density of rod BP is uniformly distributed then one can take the parameter $\lambda = 0.5$ above. Then, we compute the kinetic energy of the translational movement T_s of rods BP and CP, taking into account that the mass of CPB equals $2m_L$:

$$\begin{aligned} T_s = m_L (\dot{x}_s^2 + \dot{y}_s^2) &= m_L \left[\lambda^2 (\dot{x}^2 + \dot{y}^2) + \lambda(1 - \lambda)(\dot{x}\dot{x}_B + \dot{x}\dot{x}_C) + \dot{y}(\dot{y}_B + \dot{y}_C) + \right. \\ &\left. + 0.25(1 - \lambda)^2 (\dot{x}_B + \dot{x}_C)^2 + (\dot{y}_B + \dot{y}_C)^2 \right]. \end{aligned} \quad (34)$$

Substituting into (34) formulas (28) and the following expressions:

$$\begin{aligned} \dot{x}_B &= -l\dot{\varphi}\sin\varphi, \quad \dot{y}_B = l\dot{\varphi}\cos\varphi, \\ \dot{x}_C &= -l\dot{\psi}\sin\psi, \quad \dot{y}_C = l\dot{\psi}\cos\psi, \end{aligned}$$

we have:

$$\begin{aligned} T_s = m_L (\dot{x}_s^2 + \dot{y}_s^2) &= m_L \left[\lambda^2 \frac{2T_p}{m} - \lambda(1 - \lambda)(\dot{x}\dot{\varphi}\sin\varphi + \dot{\psi}\sin\psi) - \dot{y}(\dot{\varphi}\cos\varphi + \dot{\psi}\cos\psi) + \right. \\ &\left. + 0.25(1 - \lambda)^2 l^2 ((\dot{\varphi}\sin\varphi + \dot{\psi}\sin\psi)^2 + (\dot{\varphi}\cos\varphi + \dot{\psi}\cos\psi)^2) \right] \\ &= m_L \left[\lambda^2 \frac{2T_p}{m} + \lambda(1 - \lambda)l^2 \left(0.5(\dot{\varphi}^2 + \dot{\psi}^2) + \dot{\varphi}\dot{\psi} \frac{\cos(\varphi - \psi + \beta_0)}{\cos\beta_0} \right) \right. \\ &\left. + 0.25(1 - \lambda)^2 l^2 (\dot{\varphi}^2 + \dot{\psi}^2 + 2\dot{\varphi}\dot{\psi}\cos(\varphi - \psi)) \right]. \end{aligned} \quad (35)$$

Finally, we get from (35)

$$T_s = \frac{2\lambda^2 m_L}{m} T_p + m_L \frac{(1 - \lambda)^2 l^2 (\dot{\varphi}^2 + \dot{\psi}^2)}{4} + (1 - \lambda)l^2 \dot{\varphi}\dot{\psi} \times \left[\lambda \frac{\cos(\varphi - \psi + \beta_0)}{\cos\beta_0} + (1 - \lambda) \frac{\cos(\varphi - \psi)}{2} \right]. \quad (36)$$



The formula (36) defines the kinetic energy of translational movement of the triangle CPB. Then, to find the rotational energy T_r we should know the angular velocity. In our assumption the angular velocity of the body CPB can be found as a first derivative of the angle α , defined in formula (20). Then we have:

$$\begin{aligned}\dot{\alpha} &= \frac{(\dot{\varphi} \cos \varphi - \dot{\psi} \cos \psi)(\cos \varphi - \cos \psi + d/l) + (\sin \varphi - \sin \psi)(\dot{\varphi} \sin \varphi - \dot{\psi} \sin \psi)}{(\cos \varphi - \cos \psi + d/l)^2 + (\sin \varphi - \sin \psi)^2} = \\ &= \frac{(\dot{\varphi} + \dot{\psi})(1 - \cos(\varphi - \psi)) + d/l(\dot{\varphi} \cos \varphi - \dot{\psi} \cos \psi)}{(\cos \varphi - \cos \psi + d/l)^2 + (\sin \varphi - \sin \psi)^2}.\end{aligned}\quad (37)$$

The whole kinetic energy of the body CPB consisting of rods CP and BP equals

$$T_{CPB} = T_s + T_r = T_s + \frac{J_L \dot{\alpha}^2}{2}, \quad (38)$$

here J_L is an inertia moment of the triangle CPB with respect to its mass center S.

Thus, to write the whole kinetic energy of the robot we need to summarize energy of the end-effector P, of the body CPB, energy of rods AB and CD, and energy of the engines at points A and D:

$$T = T_p + T_{CPB} + (J_l + J_e) \frac{\dot{\varphi}^2 + \dot{\psi}^2}{2}. \quad (39)$$

Here J_e and J_l denote moments of inertia of the engine and the rod AB (CD) with respect to fixed point A(D).

To compare rotating kinetic energy T_r and translational energy T_s in (38) we computed the kinetic energies T_s and T_r for the case when masses of rods are comparable with mass of the end-effector. We have calculated kinetic energy T_{CPB} for mass of BP equal to half of mass of the end-effector: $m_L = 0.5m$. The inertia moment J_L has been taken approximately equal to $m_L l^2/2$ and the parameter $\lambda = 0.5$. In Figure 7 we compare the energy of the end-effector T_p plus translational energy T_s of CPB against the rotational energy T_r of CPB. It turns out that for the considered masses of elements it is possible to neglect the rotational energy of CPB. Then substituting (32) and (36) in (39), the kinetic energy T is represented as follows:

$$\begin{aligned}T &= \left(1 + \frac{2\lambda^2 m_L}{m}\right) \frac{ml^2}{8\cos^2 \beta_0} (\dot{\varphi}^2 + \dot{\psi}^2 + 2\dot{\varphi}\dot{\psi} \cos(\varphi - \psi + 2\beta_0)) \\ &+ \frac{(1-\lambda^2)m_L l^2 (\dot{\varphi}^2 + \dot{\psi}^2)}{4} + (1-\lambda)m_L l^2 \dot{\varphi}\dot{\psi} \left[\lambda \frac{\cos(\varphi - \psi + \beta_0)}{\cos \beta_0} + (1-\lambda) \frac{\cos(\varphi - \psi)}{2} \right] \\ &+ (J_l + J_e) \frac{\dot{\varphi}^2 + \dot{\psi}^2}{2} = C_1 \frac{\dot{\varphi}^2 + \dot{\psi}^2}{2} + \dot{\varphi}\dot{\psi} [C_2 \cos(\varphi - \psi + 2\beta_0) + C_3 \cos(\varphi - \psi + \beta_0) + \\ &+ C_4 \cos(\varphi - \psi)],\end{aligned}\quad (40)$$

Here the following notations are used:

$$\begin{aligned}C_1 &= \left(1 + \frac{2\lambda^2 m_L}{m}\right) \frac{ml^2}{4\cos^2 \beta_0} + \frac{(1-\lambda^2)m_L l^2}{2} + (J_l + J_e), \\ C_2 &= \left(1 + \frac{2\lambda^2 m_L}{m}\right) \frac{ml^2}{4\cos^2 \beta_0}, \quad C_3 = \frac{m_L(1-\lambda)\lambda l^2}{\cos \beta_0}, \quad C_4 = \frac{m_L(1-\lambda)^2 l^2}{2}.\end{aligned}$$

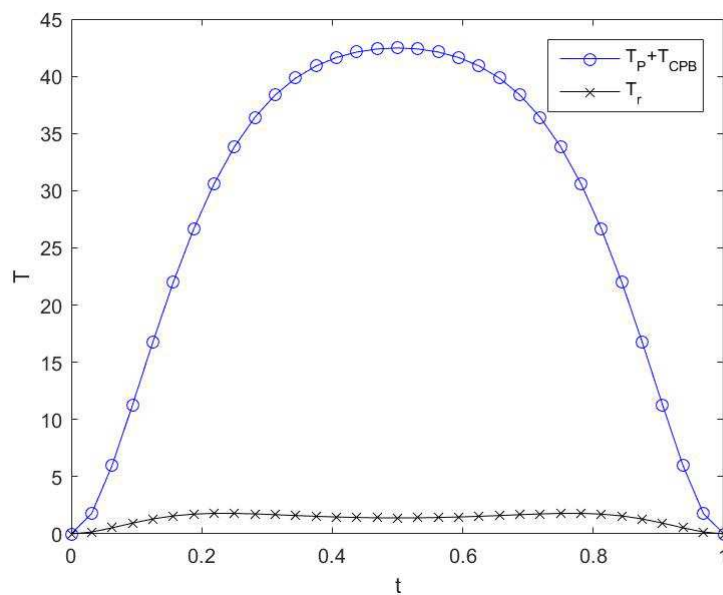


Fig. 7. Comparison of the rotational kinetic energy of CPB and the whole energy of end-effector and rods CB and BP.



Formulas (40) allows one to rewrite the equations of motion (26) in generalized coordinates. Substituting (40) into (26) we have:

$$\begin{aligned} M_1(t) &= C_1 \ddot{\varphi} + \frac{d}{dt} \frac{\partial}{\partial \dot{\varphi}} \dot{\varphi} \dot{\psi} [C_2 \cos(\varphi - \psi + 2\beta_0) + C_3 \cos(\varphi - \psi + \beta_0) + C_4 \cos(\varphi - \psi)] \\ &\quad - \frac{\partial}{\partial \varphi} \dot{\varphi} \dot{\psi} [C_2 \cos(\varphi - \psi + 2\beta_0) + C_3 \cos(\varphi - \psi + \beta_0) + C_4 \cos(\varphi - \psi)], \\ M_2(t) &= C_1 \ddot{\psi} + \frac{d}{dt} \frac{\partial}{\partial \dot{\psi}} \dot{\varphi} \dot{\psi} [C_2 \cos(\varphi - \psi + 2\beta_0) + C_3 \cos(\varphi - \psi + \beta_0) + C_4 \cos(\varphi - \psi)] \\ &\quad - \frac{\partial}{\partial \psi} \dot{\varphi} \dot{\psi} [C_2 \cos(\varphi - \psi + 2\beta_0) + C_3 \cos(\varphi - \psi + \beta_0) + C_4 \cos(\varphi - \psi)]. \end{aligned} \quad (41)$$

Calculating the derivatives, we obtain:

$$\begin{aligned} M_1(t) &= C_1 \ddot{\varphi} + \ddot{\psi} [C_2 \cos(\varphi - \psi + 2\beta_0) + C_3 \cos(\varphi - \psi + \beta_0) + C_4 \cos(\varphi - \psi)] \\ &\quad + \dot{\psi}^2 [C_2 \sin(\varphi - \psi + 2\beta_0) + C_3 \sin(\varphi - \psi + \beta_0) + C_4 \sin(\varphi - \psi)], \\ M_2(t) &= C_1 \ddot{\psi} + \ddot{\varphi} [C_2 \cos(\varphi - \psi + 2\beta_0) + C_3 \cos(\varphi - \psi + \beta_0) + C_4 \cos(\varphi - \psi)] \\ &\quad - \dot{\varphi}^2 [C_2 \sin(\varphi - \psi + 2\beta_0) + C_3 \sin(\varphi - \psi + \beta_0) + C_4 \sin(\varphi - \psi)]. \end{aligned} \quad (42)$$

Expressions (42) have a relatively simple form and, using the functions $\varphi(t)$ and $\psi(t)$ make it possible to approximately calculate control torques on the engines. That torques theoretically should provide the optimal mode of movement along a given trajectory. Recall here that by optimal mode we mean such a movement that for a given period of time, the motion is made with a minimum of kinetic energy and weighted inertial forces of the end-effector.

As an example, in Fig. 8 we show the angular velocities of the rods depending on time computed via tabulated values of optimal $\varphi(t)$, $\psi(t)$ using finite differences. In Fig. 9 the corresponding torques of engines, calculated according to formulas (42) are plotted.

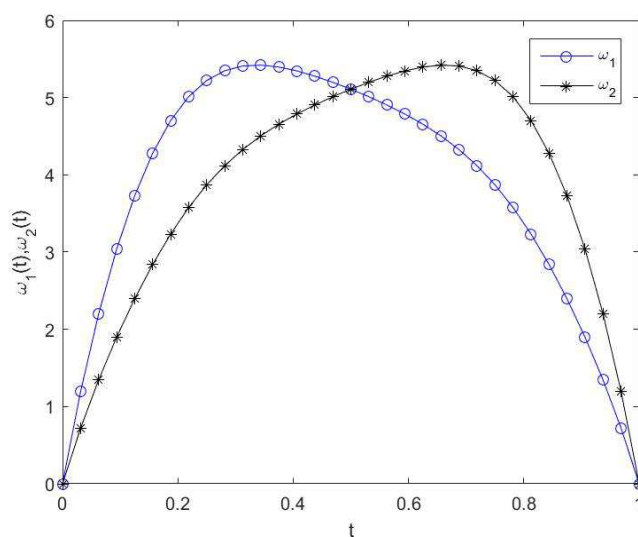


Fig. 8. Angular velocities of the rods at uniform grid of time for an optimal motion along the trajectory.

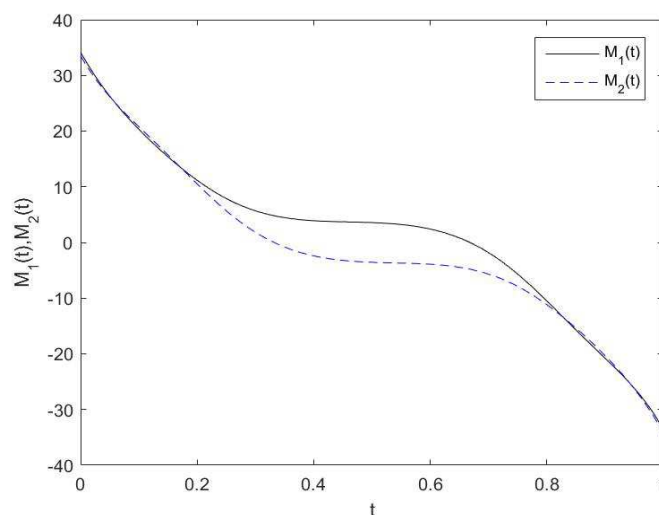


Fig. 9. Rotating moments on the engine implementing a given optimal driving mode for masses of rods $m_L=0.1m$, $m_I=0.08m$.



4. Conclusion

The results of this work are based on knowledge of the trajectory and the optimal mode of motion along it for the DexTAR robot of 2DoF. Using the solution of inverse kinematics problem, the motion of the actuators corresponding to that optimal mode are constructed. Further, the solution to the direct kinematics allows us to derive the equations of motion of the end-effector in the Lagrange form. For the considered trajectory the numerical solution of the direct kinematics has been analyzed. If the angle between legs at the top of the robot does not change significantly, as it is the case of semi-circle, then this change has little effect on the kinetic energy of the robot. This, in turn, helps to simplify the equations of dynamics. On the basis of the obtained equations and the given motions of the actuators, the control actions - torques on the engines have been computed. The further step in this direction is the construction of a program that implements an optimal movement of the robot. In its turn, there will arise other problems related to accurately positioning of the robot and taking into account masses and inertia moments of real robot components.

Author Contributions

The author is the only person who made the presented study, derived formulas, conducted the numerical experiments, wrote the manuscript, as well as formulated the statement to the problem.

Acknowledgments

The author acknowledges Prof. M.N. Kalimoldayev, General Director of Institute of Information and Computational Technologies of MES of Republic of Kazakhstan and M. Akhmetzhanov, PhD, who initiated the project. As well I am very thankful to anonymous reviewers for valuable remarks that helped to improve the presentation of the work.

Conflict of Interest

The author declares no potential conflicts of interest with respect to the research, authorship and publication of this article.

Funding

The work is supported by Ministry of Education and Science of Republic of Kazakhstan, the agreement number AP05133190.

Nomenclature

α	Rotating angle of the segment CB	M_i	Control torques, $i=1,2$
θ	Angle between PC and PB	P	Position of the end-effector
φ, ψ	Rotating angles of leading rods	t	Time
d	Distance between fixed points A and D	T_i	Kinetic energy of different motions with corresponding indice
J_i	Inertia moment of different bodies with corresponding indice	x, y	Cartesian coordinates of different points with corresponding indices
l	Length of rods DC and AB		
L	Length of rods CP and BP		

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How to cite this article: Mukanova B. Control of Actuators Torques for Optimal Movement along a Given Trajectory for the DexTAR Robot, *J. Appl. Comput. Mech.*, 7(1), 2021, 165–176. <https://doi.org/10.22055/JACM.2020.34650.2449>

