J. Appl. Comput. Mech., 7(1) (2021) 182-188
DOI: 10.22055/JACM.2020.34895.2500

ISSN: 2383-4536
jacm.scu.ac.ir

Shahid Chamran University of Ahvaz

Journal of Applied and Computational Mechanics

Research Paper

Fractal Variational Theory for Chaplygin-He Gas in a Microgravity Condition

KangLe Wang, ShaoWen Yao

School of Mathematic and Information Science, Henan Polytechnic University, Jiaozuo, Henan 454000, P. R. China

Received September 04 2020; Revised September 25 2020; Accepted for publication September 27 2020.
Corresponding author: S.W. Yao (ysw140917@163.com)

Abstract. On the microgravity condition, gravity affects the motion of objects and the flow of fluids, and the continuum assumption is not valid, therefore, a fractal Chaplygin-He gas model is developed by a new fractal derivative in microgravity space. A fractal variational principle is successfully established via the fractal semi-inverse method.

Keywords: Fractal derivative, Chaplygin-He gas model, Microgravity space, Fractal variational theory, Fractal semi-inverse method.

1. Introduction

Microgravity [1, 2] means that the acceleration caused by gravity or other external forces does not exceed 0.00001~0.0001g, where g is the acceleration of gravity on the Earth's surface. An astronaut (Fig. 1) is subject to a condition of microgravity, the blood flow and the air flow in the respiratory system are different from those in normal conditions. Under the condition of microgravity, the air molecules are segregated each other with a very larger distance than those on the Earth's surface, and the continuum assumption in the fluid mechanics becomes invalid, and the governing equations have to be established according to physical laws in a fractal space.

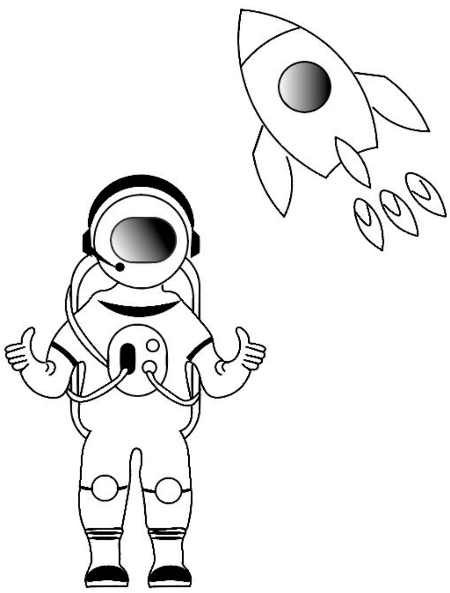

Fig. 1. An astronaut in microgravity ($g_{space} << g_{earth}$)

In a discontinuous space or time, He's fractal derivative [3-9] can perfectly describes the motion of objects and the flow of fluids in a microgravity condition or fractal space. He's fractal derivative is defined as follows

$$\frac{Dv}{{}_H Dx^{\beta}}(x_0)=\Gamma(1+\beta)\lim_{\substack{x-x_0\to\Delta x\\ \Delta x\neq 0}}\frac{v(x)-v(x_0)}{(x-x_0)^{\beta}} \tag{1.1}$$

where $\Delta x \neq 0$ is the average distance among molecules.

In [2], Prof. He studied the variational principle of the one-dimension flow in microgravity space as follows

$$\frac{D\rho}{{}_H Dt^{\alpha}}+\frac{D(\rho v)}{{}_H Dx^{\beta}}=0\ , \tag{1.2}$$

$$\frac{Dv}{{}_H Dt^{\alpha}}+\frac{D}{{}_H Dx^{\beta}}\left(\frac{v^2}{2}+\frac{k\lambda}{\lambda-1}\rho^{\lambda-1}\right)=0\ , \tag{1.3}$$

$$P=k\rho^{\lambda}\ , \tag{1.4}$$

where ρ is the density of flow , v is the velocity of flow, P is the pressure and λ, k are constants. $D/{}_H Dt^{\alpha}$ and $D/{}_H Dx^{\beta}$ are the He's fractal operators. When $\alpha=\beta=1$, the eqs. (1.2)-(1.3) is the classical Euler equation for one dimensional compressible flow.

However, He proposed that if eq. (1.4) is changed, a new fractal model can be obtained and it is called as fractal Chaplygin-He gas model (FCHGM).

The fractal Chaplygin-He gas model is given as follows:

(1) The mass conservation law

$$\frac{D\rho}{{}_H Dt^{\alpha}}+\frac{D(\rho v)}{{}_H Dx^{\beta}}=0\ , \tag{1.5}$$

(2) The moment conservation law

$$\frac{Dv}{{}_H Dt^{\alpha}}+\frac{D}{{}_H Dx^{\beta}}\left(\frac{v^2}{2}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}\right)=0\ , \tag{1.6}$$

where A , B , ς are the constants and $0<\varsigma\leq 1$.

(3) The isentropic relationship is as follows

$$P=A\rho-\frac{B}{\rho^{\varsigma}}\ . \tag{1.7}$$

When $\alpha=\beta=1$, eqs. (1.5)-(1.7) is the very famous Chaplygin gas model (CGM). The CGM is used to elaborate on the lift problem of aerodynamics aircraft wings. The model is also often adopted to study dark matter and dark energy. For the physical interpretation of the fractal Chaplygin gas model (FCGM) in the state of microgravity, see [2].

In this paper, we mainly consider the new fractal Chaplygin-He gas model in a microgravity space. We adopt the fractal semi-inverse method [10-13] to establish its variational principle [14-16] and its properties are elaborated via two-scale transform method [17-21].

2. Variational Principle of Fractal Chaplygin-He Gas Model

By eq. (1.5), we construct a function φ which satisfies

$$\frac{D\varpi}{{}_H Dx^{\beta}}=v\ , \tag{2.1}$$

$$\frac{D\varpi}{{}_H Dt^{\alpha}}=-\left(\frac{v^2}{2}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}\right), \tag{2.2}$$

According to the semi-inverse transform method, we establish an energy-like integral operator as follows

$$L(\varpi,\rho)=\iint\{\frac{1}{2}\rho(\frac{D\varpi}{{}_H Dx^{\beta}})^2+K\}Dx^{\beta}Dt^{\alpha} , \tag{2.3}$$

$$\frac{\rho}{2}(\frac{D\varpi}{{}_H Dx^{\beta}})^2=\frac{1}{2}\rho\upsilon^2 , \tag{2.4}$$

where K is the function of ϖ , υ and their fractal derivative. Equation (2.4) is the kinetic of the fluid in the microgravity condition.

Therefore, we obtain the stationary condition as follows

$$-\frac{D}{{}_H Dx^{\beta}}(\rho\frac{D\varpi}{{}_H Dx^{\beta}})+\frac{\delta K}{\delta\varpi}=0 , \tag{2.5}$$

where $\delta K/\delta\varphi$ is the variational derivative operator, which reads

$$\frac{\delta K}{\delta\varpi}=\frac{DK}{{}_H D\varpi}-\frac{D}{{}_H Dt^{\alpha}}(\frac{DK}{{}_H D(DK/{}_H Dt^{\alpha})})-\frac{D}{{}_H Dx^{\beta}}(\frac{DK}{{}_H D(DK/{}_H Dx^{\beta})}) . \tag{2.6}$$

According to eq. (1.5) and eq. (2.1), from eq. (2.5) we obtain

$$\frac{\delta K}{\delta\varpi}=\frac{D}{{}_H Dx^{\beta}}(\rho\frac{D\varpi}{{}_H Dx^{\beta}})=\frac{D(\rho\upsilon)}{{}_H Dx^{\beta}}=-\frac{D\rho}{{}_H Dt^{\beta}} . \tag{2.7}$$

Equation (2.7) can be rewritten into the form

$$\delta K=-\frac{D\rho}{{}_H Dt^{\beta}}\delta\varpi . \tag{2.8}$$

Thus, we can obtain the following result

$$K=\rho\frac{D\varpi}{{}_H Dt^{\alpha}}+K_1 . \tag{2.9}$$

Equation (2.3) can be written into the form

$$L(\varpi,\rho)=\iint\{\frac{1}{2}\rho(\frac{D\varpi}{{}_H Dx^{\beta}})^2+\rho\frac{D\varpi}{{}_H Dt^{\alpha}}+K_1\}Dx^{\beta}Dt^{\alpha} \tag{2.10}$$

The Euler-Lagrange equation is

$$\frac{1}{2}(\frac{D\varpi}{{}_H Dx^{\beta}})^2+\frac{D\varpi}{{}_H Dt^{\alpha}}+\frac{\delta K_1}{\delta\rho}=0 . \tag{2.11}$$

Using eq. (2.1) and eq. (2.2), we have

$$\begin{aligned}\frac{\delta K_1}{\delta\rho}&=-\frac{1}{2}(\frac{D\varpi}{{}_H Dx^{\beta}})^2-\frac{D\varpi}{{}_H Dt^{\alpha}}\\&=-\frac{1}{2}\upsilon^2+\frac{1}{2}\upsilon^2+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}\\&=A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}\end{aligned} \tag{2.12}$$

Eq. (2.12) can be reduced the following form

$$\delta K_1=(A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)})\delta\rho . \tag{2.13}$$

By calculating eq. (2.13), K_1 can be determined as follows

$$K_1 = A\rho\ln\rho - A\rho + \frac{B\rho^{-\varsigma}}{1+\varsigma}. \tag{2.14}$$

Therefore, we successfully establish the variational principle as follows

$$L(\varpi,\rho) = \iint \{\frac{1}{2}\rho(\frac{D\varpi}{{}_H Dx^{\beta}})^2 + \rho\frac{D\varpi}{{}_H Dt^{\alpha}} + A\rho\ln\rho - A\rho + \frac{B\rho^{-\varsigma}}{1+\varsigma}\} Dx^{\beta}Dt^{\alpha}. \tag{2.15}$$

Finally, the Euler-Lagrange equation of eq. (2.15) can be easily obtained as follows

$$-\frac{D}{{}_H Dx^{\beta}}(\rho\frac{D\varpi}{{}_H Dx^{\beta}}) - \frac{D\rho}{{}_H Dt^{\alpha}} = 0, \tag{2.16}$$

$$\frac{1}{2}(\frac{D\varpi}{{}_H Dx^{\beta}})^2 + \frac{D\varpi}{{}_H Dt^{\alpha}} + A\ln\rho - \frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)} = 0. \tag{2.17}$$

Equation (2.16) and eq. (2.17) are equivalent as eq. (1.5) and eq. (2.2), respectively.

Proof: Since

$$\frac{D\varpi}{{}_H Dx^{\beta}} = \upsilon, \tag{2.18}$$

So, eq. (2.16) and eq. (2.17) can be rewritten in the following form

$$\frac{D}{{}_H Dx^{\beta}}(\rho\upsilon) + \frac{D\rho}{{}_H Dt^{\alpha}} = 0, \tag{2.19}$$

$$\frac{1}{2}(\upsilon)^2 + \frac{D\varpi}{{}_H Dt^{\alpha}} + A\ln\rho - \frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)} = 0. \tag{2.20}$$

Hence, eq. (2.19) and eq. (2.20) are completely same as eq. (1.5) and eq. (2.2), respectively.
Equation (1.3) can be further modified the following form

$$\frac{D\upsilon}{{}_H Dt^{\alpha}} + \frac{D}{{}_H Dx^{\beta}}(\frac{\upsilon^2}{2} + A\ln\rho - \frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)} - C\rho^{-1}) = 0 \tag{2.21}$$

where A, B, C, ς are arbitrary constants. Equation (2.21) is a new Chaplygin-He gas modeling, and which is established by He in 2019.

We will established the variational principle of eq. (2.21) as the similar technique.

In a similar way, we construct a function ϖ, and it satisfy

$$\frac{D\varpi}{{}_H Dx^{\beta}} = \upsilon, \tag{2.22}$$

$$\frac{D\varpi}{{}_H Dt^{\alpha}} = -(\frac{\upsilon^2}{2} + A\ln\rho - \frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)} - C\rho^{-1}). \tag{2.23}$$

Similarly, we can obtain the parameter K_1 as follows

$$K_1 = A\rho\ln\rho - A\rho + \frac{B\rho^{-\varsigma}}{1+\varsigma} - C\ln\rho. \tag{2.24}$$

So, we establish the variational principle as follows

$$L(\varpi,\rho) = \iint \{\frac{1}{2}\rho(\frac{D\varpi}{{}_H Dx^{\beta}})^2 + \rho\frac{D\varpi}{{}_H Dt^{\alpha}} + A\rho\ln\rho - A\rho + \frac{B\rho^{-\varsigma}}{1+\varsigma} - C\ln\rho\} Dx^{\beta}Dt^{\alpha}. \tag{2.25}$$

The Euler-Lagrange equation of eq. (2.25) can be obtained as follows

$$-\frac{D}{{}_H Dx^{\beta}}(\rho\frac{D\varpi}{{}_H Dx^{\beta}})-\frac{D\rho}{{}_H Dt^{\alpha}}=0\ , \tag{2.26}$$

$$\frac{1}{2}(\frac{D\varpi}{{}_H Dx^{\beta}})^2+\frac{D\varpi}{{}_H Dt^{\alpha}}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}-C\ln\rho=0\ . \tag{2.27}$$

Equation (2.26) and eq. (2.27) are same as eq. (1.5) and eq. (2.21), respectively.

3. Discussion

We consider the Chaplygin-He gas modeling in a microgravity condition or fractal space. We also can establish the variational principle of Chaplygin-He gas modeling in continuous space. We will adopt two-scale transform method to convert fractal space into its continuous space partner.

We assume

$$T=t^{\alpha}\ , \tag{3.1}$$

$$X=x^{\beta}\ . \tag{3.2}$$

Equation (1.5) and eq. (1.6) can be converted into its partners, which reads

$$\frac{D\rho}{{}_H DT}+\frac{D(\rho\upsilon)}{{}_H DX}=0\ , \tag{3.3}$$

$$\frac{D\upsilon}{{}_H DT}+\frac{D}{{}_H DX}(\frac{\upsilon^2}{2}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)})=0\ . \tag{3.4}$$

We can obtain its variational principle as the similar method.
Construct a function φ , and satisfy the following relation

$$\frac{D\varpi}{{}_H DX}=\upsilon\ , \tag{3.5}$$

$$\frac{D\varpi}{{}_H DT}=-(\frac{\upsilon^2}{2}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)})\ . \tag{3.6}$$

Hence, the variational principle is established as follows

$$L(\varpi,\rho)=\iint\{\frac{1}{2}\rho(\frac{D\varpi}{{}_H DX})^2+\rho\frac{D\varpi}{{}_H DT}+A\rho\ln\rho-A\rho+\frac{B\rho^{-\varsigma}}{1+\varsigma}\}DXDT\ . \tag{3.7}$$

The Euler-Lagrange equations of eq. (3.7) can be easily obtained as follows

$$-\frac{D}{{}_H DX}(\rho\frac{D\varpi}{{}_H DX})-\frac{D\rho}{{}_H DT}=0\ , \tag{3.8}$$

$$\frac{1}{2}(\frac{D\varpi}{{}_H DX})^2+\frac{D\varpi}{{}_H DT}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}=0\ . \tag{3.9}$$

In this way, the eq. (2.21) can be written into the form

$$\frac{D\upsilon}{{}_H DT}+\frac{D}{{}_H DX}(\frac{\upsilon^2}{2}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}-C\rho^{-1})=0\ . \tag{3.10}$$

In a similar way, we construct a function ϖ , and it satisfy

$$\frac{D\varpi}{_{H}DX}=\upsilon\,, \tag{3.11}$$

$$\frac{D\varpi}{_{H}DT}=-(\frac{\upsilon^2}{2}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}-C\ln\rho)\,. \tag{3.12}$$

So, we establish the variational principle as follows

$$L(\varpi,\rho)=\iint\{\frac{1}{2}\rho(\frac{D\varpi}{_{H}DX})^2+\rho\frac{D\varpi}{_{H}DT}+A\rho\ln\rho-A\rho+\frac{B\rho^{-\varsigma}}{1+\varsigma}-C\ln\rho\}DXDT\,. \tag{3.13}$$

The Euler-Lagrange equation of eq. (3.13) can be obtained as follows

$$-\frac{D}{_{H}DX}(\rho\frac{D\varpi}{_{H}DX})-\frac{D\rho}{_{H}DT}=0\,, \tag{3.14}$$

$$\frac{1}{2}(\frac{D\varpi}{_{H}DX})^2+\frac{D\varpi}{_{H}DT}+A\ln\rho-\frac{B\varsigma}{1+\varsigma}\rho^{-(1+\varsigma)}-C\ln\rho=0\,. \tag{3.15}$$

In this paper, we mainly study the variational theory of fractal Chaplygin-He gas modeling , whose approximate analytical solutions can be obtained by Homotopy perturbation method [22-26], Homotopy analysis method [27,28], Taylor series method [29-32], Reduced differential transform method [33], Laplace transform method [34-36], Variational iteration method [37-40] and other powerful methods [41].

4. Conclusion

In this work, the fractal Chaplygin-He gas model is established by He's fractal derivative in a microgravity space. Its variational principles are obtained via fractal semi-inverse method. The advantage of variational principle is it can be used to construct conservation laws and suggest solution structures of the model.

Author Contributions

KangLe Wang proposed the idea of the paper and performed the derivation of conclusion, wrote the paper. Shaowen Yao gave the suggestions and revised the paper. All authors discussed the results, reviewed, and approved the final version of the manuscript.

Acknowledgments

We thank the editor and referees for their careful reading and valuable suggestions to make the article reader-friendly.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

References

[1] Liu,Y., Li, G.H., Kallio, S., Hydrodynamic modeling of dense gas-particle turbulence flows under microgravity space environments, *Microgravity Science and Technology*, 23(1), 2011, 1-11.
[2] He, J.H., A fractal variational theory for one-dimensional compressible flow in a microgravity space, *Fractals,* 28(2), 2020, 2050024.
[3] He, J.H., A Tutorial Review on Fractal Spacetime and Fractional Calculus, *International Journal of Theoretical Physics,* 53(11), 2014, 3698-3718.
[4] He, J.H. *et al.*, A new fractional derivative and its application to explanation of polar bear hairs, *Journal of King Saud University Science,* 28(2), 2016, 190-192.
[5] He, J.H., A new fractal derivation, *Thermal Science*, 15(1), 2011, S145-S147.
[6] Wang, Q. L., Fractal calculus and its application to explanation of biomechanism of polar bear hairs, *Fractals*, 27(5), 2019, 1992001.
[7] Wang, Y., Deng, Q.G., Fractal derivative model for tsunami travelling, *Fractals,* 27(1), 2019, 1950017.
[8] Wang, Y., A fractal derivative model for snow's thermal insulation property, *Thermal Science*, 23(4), 2019, 2351-2354.
[9] He, J.H., A short review on analytical methods for to a fully fourth-order nonlinear integral boundary value problem with fractal derivatives, *International Journal of Numerical Methods for Heat and Fluid Flow,* 2020, doi: 10.1108/HFF-01-2020-0060.
[10] He, J.H., Generalized Variational Principles for Buckling Analysis of Circular Cylinders, *Acta Mechanica*, 231, 2020, 899–906.
[11] He, J.H., Variational Principle for the Generalized KdV-Burgers Equation with Fractal Derivatives for Shallow Water Waves, *Journal of Applied and Computational Mechanics*, 6(4), 2020, 735-740.
[12] He, J.H., A simple approach to one-dimensional convection-diffusion equation and its fractional modification for E reaction arising in rotating disk electrodes, *Journal of Electroanalytical Chemistry,* 854, 2019, 113565.
[13] He, J.H., Lagrange Crisis and Generalized Variational Principle for 3D unsteady flow, *International Journal of Numerical Methods for Heat and Fluid Flow*, 2019, doi: 10.1108/HFF-07-2019-0577.
[14] He, J.H., Sun, C., A variational principle for a thin film equation, *Journal of Mathematical Chemistry*, 57(9), 2019, 2075–2081.
[15] Wang, K.L., He, C.H., A remark on Wang's fractal variational principle, *Fractals*, 27(8), 2019, 1950134.

[16] Wang, K.L., Yao, S.W., A fractal variational principle for the telegraph equation with fractal derivatives, *Fractals*, 28(4), 2020, 2050058.
[17] Ain, Q.T., He, J.H., On two-scale dimension and its applications, *Thermal Science*, 23(3B), 2019, 1707-1712.
[18] Wang, Y., et al, A short review on analytical methods for fractional equations with He's fractional derivative, *Thermal Science,* 21(4), 2017, 1567-1574.
[19] He, J.H., Ain, Q.T., New promises and future challenges of fractal calculus: from two-scale Thermodynamics to fractal variational principle, *Thermal Science*, 24(2), 2020, 659-681.
[20] He, J.H., Ji, F.Y., Two-scale mathematics and fractional calculus for thermodynamics, *Thermal Science,* 23(4), 2019, 2131-2133.
[21] Li, X.J., Liu, Z., He, J.H., A fractal two-phase flow model for the fiber motion in a polymer filling process, *Fractals,* 2020, doi:10.1142/S0218348X20500930.
[22] Wang, K.L., Wang, K.J., He, C.H., Physical insight of local fractional calculus and its application to fractional Kdv-Burgers-Kuramoto equation, *Fractals*, 27(7), 2019, 1950122.
[23] He, J.H., Homotopy perturbation technique, *Computer Methods in Applied Mechanics and Engineering*, 178, 1999, 257-262.
[24] Yu, D.N., He, J.H, Garcia, A.G., Homotopy perturbation method with an auxiliary parameter for nonlinear oscillators, *Journal of Low Frequency Noise, Vibration and Active Control*, 38, 2019, 3-4.
[25] Sedighi, H.M., Daneshmand, F., Nonlinear transversely vibrating beams by the homotopy perturbation method with an auxiliary term, *Journal of Applied and Computational Mechanics,* 1(1), 2015, 1-9.
[26] Wang, K.L., Yao, S.W., Numerical method for fractional Zakharov-Kuznetsov equation with He's fractional derivative, *Thermal Science,* 23(4), 2019, 2163-2170.
[27] Sedighi, H.M., Shirazi, K.H., Using homotopy analysis method to determine profile for disk cam by means of optimization of dissipated energy, *International Review of Mechanical Engineering,* 5(5), 2011, 941-946.
[28] Ravichandran, C., Valliammal, N., Nieto, J.J., New results on exact controllability of a class of fractional neutral integro-differential systems with state-dependent delay in Banach spaces, *Journal of the Franklin Institute*, 356(3), 2019, 1535-1565.
[29] He, C.H., *et al.*, Taylor series solution for fractal Bratu-type equation arising in electrospinning process, *Fractals,* 2019 doi: 10.1142/S0218348X20500115.
[30] He, J.H., Ji, F.Y., Taylor series solution for Lane-Emden equation, *Journal of Mathematical Chemistry*, 57(8), 2019, 1932–1934.
[31] He, J.H., Taylor series solution for a third order boundary value problem arising in architectural engineering, *Ain Shams Engineering Journal*, 2020, doi: 10.1016/j.asej.2020.01.016.
[32] He, C.H., Shen, Y., Ji, F.Y., He, J.H., Taylor series solution for fractal Bratu-type equation arising in electrospinning process, *Fractals,* 28(1), 2020, 2050011.
[33] Wang, K.L., Wang, K.J., A modification of the reduced differential transform method for fractional calculus, *Thermal Science*, 22(4), 2018, 1871-1875.
[34] Nadeem,M., Li, F.Q., He-Laplace method for nonlinear vibration systems and nonlinear wave equation, *Journal of Low Frequency Noise, Vibration and Active Control*, 38, 2019, 1060-1074.
[35] Nadeem, M., Li, F.Q. , Ahmad, H., Modified Laplace variational iteration method for solving fourth-order parabolic partial differential equation with variable coefficients, *Computer and Mathematics with applications,*78(6), 2019, 2052-2062.
[36] Kumar, S., A new fractional modeling arising in engineering sciences and its analytical approximate solution, *Alexandria Engineering Journal*, 52, 2013, 813-819.
[37] He, J.H., Latifizadeh, H., A general numerical algorithm for nonlinear differential equations by the variational iteration method, *International Journal of Numerical Methods for Heat and Fluid Flow*, 2020, doi:10.1108/HFF-01-2020-0029.
[38] Ahmad, H., Khan, T.A., Variational iteration algorithm-I with an auxiliary parameter for wave-like vibration equations, *Journal of Low Frequency Noise, Vibration and Active Control*, 38(3-4), 2019, 1113-1124.
[39] Ahmad, H., Seadawy, A.R., Khan, T.A., Study on numerical solution of dispersive water wave phenomena by using a reliable modification of variational iteration algorithm, *Mathematics and Computers in Simulation,* 17, 2020, 13-23
[40] Nadeem, M., Li, F.Q., Ahmad, H. Modified Laplace variational iteration method for solving fourth-order parabolic partial differential equation with variable coefficients, *Computers and Mathematics with Applications*, 78, 2019, 2052-2062.
[41] Sedighi, H.M., Shirazi, K.H., Bifurcation analysis in hunting dynamical behavior in a railway bogie: Using novel exact equivalent functions for discontinuous nonlinearities, *Scientia Iranica,* 19(6), 2012, 1493-1501.

ORCID iD

KangLe Wang https://orcid.org/0000-0002-6637-5687
ShaoWen Yao https://orcid.org/0000-0002-0273-4505

How to cite this article: Wang, K.L., Yao, S.W. Fractal Variational Theory for Chaplygin-He Gas in a Microgravity Condition, *J. Appl. Comput. Mech.*, 7(1), 2021, 182-188. https://doi.org/10.22055/JACM.2020.34895.2500