Abstract. This article explores the MHD natural convective viscous and incompressible fluid flow along with radiation and chemical reaction. The flow is confined to a moving tilted plate under slanted magnetic field with variable temperature in a porous medium. Non-dimensional parameter along Laplace transformation and inversion algorithm are used to investigate the solution of system of dimensionless governing equations. Fractional differential operators namely, Caputo (C), Caputo-Fabrizio (CF) and Atangana-Baleanu in Caputo sense (ABC) are used to compare graphical behavior of for velocity, temperature and concentration for emerging parameters. On comparison, it is observed that fractional order model is better in explaining the memory effect as compared to classical model. Velocity showing increasing behavior for fractional parameter \( \alpha \) whereas there is a decline in temperature, and concentration profiles for \( K_s, G_r \) and \( S \). Tabular comparison is made for velocity and Nusselt number and Sherwood number for fractional models.

Keywords: Fractional order derivatives; Inclined magnetic field; Laplace transformation; Inversion algorithm.

1. Introduction

The viscoelastic fluids gain attention in many fields of science and engineering. MHD natural convection flow of a viscous and incompressible fluid involving heat and mass transfer phenomena has been focused by researchers due to its several practical applications like crystal formation, grain storage, high performance insulation for buildings, and geothermal energy systems etc. Some significant efforts have been made by investigators for study of heat and mass transfer in viscous fluid [1] – [4]. Derivatives play a significant role in modeling mathematical problems. Fractional derivatives, as compared to ordinary derivatives, are more applicable in different fields like fluid mechanics, engineering, chemical industry, Chaotic and fractional attractors, and many more [5] – [11].

Fractional calculus plays a key role in describing the behavior of viscoelastic fluids [12] – [13]. Generally, ordinary derivative is replaced by fractional order derivative in governing equations [14]. Fractional order fluid models have been discussed by many researchers [15] – [24]. The natural convection flow of an MHD viscous fluid over a slanted heated surface has many applications in astronomy, cooling towers, geophysics, and boundary layer problems. Umemura and Law [25] discussed the free natural convection flow on a semi-infinite heated plate of different inclination. Analysis of heat and mass transfer of convective flow in a permeable medium over a tilted moving plate discussed by Pattnaik et al. [26]. Nandkeolyar and Das [27] studied free convection flow with radiative heat under slanted magnetic field in the permeable medium. Endalew and Nayak [28] focused on MHD natural convective flow with radiation and chemical reaction with tilted magnetic field on a slanted moving plate. Study regarding free convective flow through permeable medium past an inclined plate can be seen through [29] – [30]. Makinde [31] worked on the mixed convection stagnation point flow with porosity effect radiation. Khalid et al. [32] studies the Casson fluid flow past a moving plate via a porous medium with constant heat and mass transfer phenomenon. Shah et al. [37] investigated heat and mass transfer phenomena of natural convection viscous flow with Dufour effect via Caputo fractional derivative. Heat and mass transfer phenomena in Maxwell fluid via CF and ABC approach has been discussed by Khan et al. [38]. Tassaddiq [39] explores second grade fluid with tilted plate and slanted magnetic field by ABC approach. Free convection MHD viscous fluid flow near a...
vertical plate and constant heat through Caputo time-fractional derivative has been discussed by Shah et al. [40]. Momentum and heat transfer analysis for MHD second grade fluid enclosed in a permeable medium through Caputo-Fabrizio derivative has been investigated by Sheikh [41]. Imran et al. [42] discussed MHD free convection flow of Newtonian fluid on an inclined plate through porous medium with variable temperature and concentration at the boundary via CF and ABC. Ahmad et al. [43] studied heat and mass transfer of fractional Jeffrey flow over infinite vertical plate moving exponentially with variable temperature and mass diffusion using C and CF. In literature, problems based on fractional order derivatives have been focused by many researchers [44] – [48]. In the paper, viscous fluid flow past a moving tilted plate has been taken into account. Furthermore, slanted magnetic field is applied with addition to radiation and chemical reaction in a permeable medium by using definitions of integer order and fractional derivatives.

2. Problem Statement

Let natural convective flow of the viscous fluid past an infinite tilted plate with slanted magnetic field \( B = (B_{0}\cos\phi, B_{0}\sin\phi, 0) \) makes an angle \( \phi \) with vertical. Let the plate is slanted and makes an angle \( \theta \) with the vertical. At the start, fluid and plate exhibit concentration \( C'_{w} \) and temperature \( T'_{c} \). Velocity of plate is given by \( W' = U_{c}(a't') \). Figure 1 presents the geometry of fluid flow [28].

\[
\begin{align*}
\frac{\partial W'(t',t')}{\partial t'} &= v\frac{\partial^2 W'(t',t')}{\partial \psi'^2} + g\beta_{w}(T'(t',t') - T'_{c})\cos\theta + g\beta_{c}(C'(t',t') - C'_{w})\cos\theta - \left(\frac{\sigma B_{0}^{2}\sin^{2}\phi}{\rho} - \frac{v}{K_{T}}\right)W'(t',t') , \\
\rho c_{f}\frac{\partial T'(t',t')}{\partial t'} &= k\frac{\partial^2 T'(t',t')}{\partial \psi'^2} + S(T'(t',t') - T'_{c}) , \\
D\frac{\partial C'(t',t')}{\partial \psi'^2} &= -K'(C'(t',t') - C'_{w}) .
\end{align*}
\]

The suitable initial as well as boundary conditions are

\[
t' \leq 0 , \quad W' = 0 , \quad T' = T'_{c} , \quad C' = C'_{w} , \quad \forall \quad \psi' ,
\]

\[
t' > 0 , \quad W' = U_{c}(a't') , \quad T' = T'_{c} + \frac{(T'_{w} - T'_{c})U_{c}^{2}t'}{v} , \quad C' = C'_{w} + \frac{(C'_{w} - C'_{c})U_{c}^{2}t'}{v} , \quad \psi' = 0 , \quad t' > 0 , \quad W' \rightarrow 0 , \quad T' \rightarrow T'_{c} , \quad C' \rightarrow C'_{c} , \quad \psi' \rightarrow \infty ,
\]

where

\[
\frac{\partial q}{\partial \psi'} = -[T'_{w}^{4} - T'_{c}^{4}]4a'\sigma , \quad T''_{w} = -3T'_{w}^{2} + 4T'_{w}T''_{c} .
\]

Equation (1 – 3) can be non-dimensionalized by dimensionless variables as follow:

\[
\psi = \frac{U_{c}}{v} \psi' , \quad W = \frac{W'}{W_{0}} , \quad t = \frac{tU_{c}^{2}}{v} , \quad a = \frac{a'v}{U_{c}^{2}} , \quad T = \frac{T' - T'_{c}}{T'_{w} - T'_{c}} , \quad C = \frac{C' - C'_{c}}{C'_{w} - C'_{c}} , \quad K_{T} = \frac{U_{c}}{v} , \quad \frac{U_{c}}{v} , \quad S = \frac{S_{T}}{\rho c_{f} U_{c}^{2}} , \quad Sc = \frac{v}{D} , \quad Pr = \frac{\mu c_{p}}{k} , \quad \frac{\sigma B_{0}^{2}}{\rho U_{c}^{2}} , \quad M = \frac{\sigma B_{0}^{2}}{\rho U_{c}^{2}} , \quad G_{a} = \frac{g\beta_{a}(T'_{w} - T'_{c})}{U_{c}^{2}} , \quad G_{c} = \frac{g\beta_{c}(C'_{w} - C'_{c})}{U_{c}^{2}} , \quad R = \frac{16a'v^{2}}{\rho U_{c}^{2}} .
\]

![Fig. 1. Geometrical presentation of fluid model](image-url)
Governing equations with corresponding IBC transformed into

\[
\frac{\partial W(\psi,t)}{\partial t} = \frac{\partial^2 W(\psi,t)}{\partial \psi^2} - \lambda W(\psi,t) + G, \text{Cos} T(\psi,t) + G, \text{Cos} C(\psi,t),
\]

(6)

\[
Pr \frac{\partial T(\psi,t)}{\partial t} = \frac{\partial^2 T(\psi,t)}{\partial \psi^2} - dT(\psi,t),
\]

(7)

\[
S \frac{\partial C(\psi,t)}{\partial t} = \frac{\partial^2 C(\psi,t)}{\partial \psi^2} - ScK C(\psi,t),
\]

(8)

where

\[
\lambda = MSin^2 \phi + \frac{1}{K_r}, \quad d = R - SPr,
\]

with initial and boundary conditions:

\[
\begin{align*}
    t & \leq 0, \quad W = 0, \quad T = 0, \quad C = 0, \quad \psi > 0, \\
    t & > 0, \quad W = at, \quad T = t, \quad C = t, \quad \psi = 0, \\
    t & > 0, \quad W \to 0, \quad T \to 0, \quad C \to 0, \quad \psi \to \infty.
\end{align*}
\]

3. Preliminaries

Caputo time derivative [16] with its Laplace transform (LT) with \( \Gamma(.) \) gamma function is given below

\[
{\cal D}^n_{\gamma}(\psi,t) = \frac{1}{\Gamma(n - \psi)} \int_{0}^{t} \frac{\gamma^n(\tau)}{(t - \tau)^{\psi}} d\tau,
\]

(10)

\[
L\left( {\cal D}^n_{\gamma}(\psi,t) \right) = s^n L(\gamma(\psi,t)) - s^{n-1} \gamma(\psi,0).
\]

(11)

The CF fractional derivative and its LT are defined as [44] and [45]

\[
\begin{align*}
    \alpha \frac{\partial}{\partial t} \gamma(\psi,t) &= \frac{1}{1 - \eta} \int_{0}^{t} \frac{\eta(t - \tau)}{1 - \eta} \frac{\partial \gamma(\psi,t)}{\partial \tau} d\tau, \quad 0 < \eta < 1, \\
    \Gamma(\alpha) \frac{\partial}{\partial t} \gamma(\psi,t) &= - s \Gamma(\alpha) \left[ \frac{s^{\alpha - 1}}{(1 - \eta)s^\eta + \eta} \right].
\end{align*}
\]

(12) \quad (13)

The ABC fractional derivative and its LT are defined as [45] and [46] is given by

\[
\begin{align*}
    \frac{\partial \gamma(\psi,t)}{\partial t} &= \frac{1}{1 - \eta} \int_{0}^{t} \left( - \frac{\eta(t - \tau)}{1 - \eta} \frac{\partial \gamma(\psi,t)}{\partial \tau} d\tau, \\
    \Gamma(\alpha) \frac{\partial \gamma(\psi,t)}{\partial t} &= \frac{s \Gamma(\alpha) \gamma(\psi,t) - s^{\alpha - 1} \gamma(\psi,0)}{(1 - \eta)s^\eta + \eta}.
\end{align*}
\]

(14) \quad (15)

4. Solutions for Temperature Profile

4.1 Integer Order Derivative

Laplace transform of equation (7) is

\[
Pr m T(\psi,m) = \frac{\partial^2 T(\psi,m)}{\partial \psi^2} - dT(\psi,m),
\]

(16)

Laplace solution of equation (16) is

\[
T(\psi,m) = \frac{1}{m^2} e^{-\sqrt{m^2d}},
\]

(17)

4.2 Caputo Fractional Derivative

Applying Caputo definition (10) to equation (7), we get
employing Laplace transformation on (7) with the help of (11), we explored second order partial differential equation,

\[ \text{Pr} \frac{\partial^2 T_c(\psi, t)}{\partial \psi^2} = -\frac{\partial T_c(\psi, t)}{\partial t}, \]  

(18)

Solution of the above equation is

\[ T_c(\psi, m) = \frac{1}{m} e^{-\sqrt{\text{Pr}m \psi}}. \]  

(20)

4.3 Caputo-Fabrizio Fractional Derivative

Temperature field obtained by using (12) to (7)

\[ \text{Pr} \frac{\partial^\alpha T_{cf}(\psi, t)}{\partial \psi^\alpha} = -\frac{\partial T_{cf}(\psi, t)}{\partial t}, \]  

(21)

The Laplace transform of above equation is

\[ \text{Pr} \left[ \frac{m}{(1-\alpha)m + \alpha} \right] T_{cf}(\psi, m) = \frac{\partial T_{cf}(\psi, m)}{\partial \psi^\alpha} - \frac{\partial T_{cf}(\psi, m)}{\partial t}. \]  

(22)

Solution of the above mentioned equation is

\[ T_{cf}(\psi, m) = \frac{1}{m^\alpha} e^{-\frac{\sqrt{\text{Pr}m \psi}}{m^{\alpha}}} \]  

(23)

where \( a_o = 1/(1-\alpha) \) and \( a_1 = \alpha a_o \).

4.4 Atangana-Baleanu Fractional Derivative

ABC version of equation (7) is

\[ \text{Pr} \frac{\partial^\alpha T_{abc}(\psi, t)}{\partial \psi^\alpha} = -\frac{\partial T_{abc}(\psi, t)}{\partial t}, \]  

(24)

Applying definition of ABC-fractional derivative defined as [14] to above equation, we get

\[ \text{Pr} \left[ \frac{m}{(1-\alpha)m + \alpha} \right] T_{abc}(\psi, m) = \frac{\partial T_{abc}(\psi, m)}{\partial \psi^\alpha} - \frac{\partial T_{abc}(\psi, m)}{\partial t}. \]  

(25)

Solution of equation (25) is given as

\[ T_{abc}(\psi, m) = \frac{1}{m^\alpha} e^{-\frac{\sqrt{\text{Pr}m \psi}}{m^{\alpha}}} \]  

(26)

5. Solutions for Concentration Profile

5.1 Integer Order Derivative

Laplace transform of equation (8) is

\[ \text{Scm} \frac{\partial^2 \bar{C}(\psi, m)}{\partial \psi^2} = -K \text{Scm} \bar{C}(\psi, m), \]  

(27)

solution of the above equation is given as

\[ \bar{C}(\psi, m) = \frac{1}{m} e^{-\sqrt{\text{Scm}\psi}}. \]  

(28)

5.2 Caputo Fractional Derivative

Applying Caputo definition (10) to equation (8), we get

\[ \text{Sc} \frac{\partial^\alpha \bar{C}_c(\psi, t)}{\partial \psi^\alpha} = -K \text{Sc} \bar{C}_c(\psi, t), \]  

(29)

employing Laplace transformation on (8) with the help of (11), we explored second order partial differential equation,

\[ \text{Scm} \frac{\partial^2 \bar{C}_c(\psi, m)}{\partial \psi^2} = -K \text{Scm} \bar{C}_c(\psi, m). \]  

(30)
Solution of the above equation is
\[ \mathbb{C}_c(\psi, m) = \frac{1}{m^2} e^{-\sqrt{m(m-\kappa)}}. \] (31)

5.3 Caputo-Fabrizio Fractional Derivative
Concentration field obtained by using (12) to (8)
\[ \text{Sc} \frac{d}{dt} \mathbb{C}_f(\psi, t) = \frac{\partial^\alpha \mathbb{C}_f(\psi, t)}{\partial t^\alpha} - K \mathbb{C}_f(\psi, t). \] (32)
The Laplace transform of above equation is
\[ \text{Sc} \left[ \frac{m}{(1-\alpha)m+\alpha} \right] \mathbb{C}_f(\psi, m) = \frac{\partial^\alpha \mathbb{C}_f(\psi, m)}{\partial t^\alpha} - K \mathbb{C}_f(\psi, m). \] (33)
Solution of the above mentioned equation is
\[ \mathbb{C}_f(\psi, m) = \frac{1}{m^2} e^{-\sqrt{\frac{(m-\kappa)(m+\gamma)}{m-\alpha}}}, \] (34)

5.4 Atangana-Baleanu Fractional Derivative
ABC version of equation (8) is
\[ \text{Sc} \frac{d}{dt} \mathbb{C}_{ab}(\psi, t) = \frac{\partial^\alpha \mathbb{C}_{ab}(\psi, t)}{\partial t^\alpha} - K \mathbb{C}_{ab}(\psi, t). \] (35)
Applying definition of ABC-fractional derivative defined as [14] to above equation, we get
\[ \text{Sc} \left[ \frac{m^\alpha}{(1-\alpha)m+\alpha} \right] \mathbb{C}_{ab}(\psi, m) = \frac{\partial^\alpha \mathbb{C}_{ab}(\psi, m)}{\partial t^\alpha} - K \mathbb{C}_{ab}(\psi, m). \] (36)
Solution of equation (36) is given as
\[ \mathbb{C}_{ab}(\psi, m) = \frac{1}{m^2} e^{-\sqrt{\frac{(m-\kappa)(m+\gamma)}{m-\alpha}}}. \] (37)

6. Solution for Velocity Field
6.1 Integer Order Derivative
By applying Laplace transform to equation (6) and after some simplification, we get
\[ \frac{\partial^2 \mathbb{W}(\psi, m)}{\partial \psi^2} - (m + \lambda) \mathbb{W}(\psi, m) = -G \cos \mathbb{C}_T(\psi, m) - G \cos \mathbb{C}_C(\psi, m), \] (38)
putting the value of \( \mathbb{T}(\psi, m) \) from (17) and \( \mathbb{C}(\psi, m) \) from (28) in (38), we have
\[ \frac{\partial^2 \mathbb{W}(\psi, m)}{\partial \psi^2} - (m + \lambda) \mathbb{W}(\psi, m) = -G \cos \left( \frac{1}{m^2} e^{-\sqrt{m(m-\kappa)}} \right) - G \cos \left( \frac{1}{m^2} e^{-\sqrt{m(m-\kappa)}} \right). \] (39)
The solution of the equation (39) is given by
\[ \mathbb{W}(\psi, m) = \frac{\alpha}{m^2} e^{-\sqrt{m(m-\kappa)}} + \frac{G \cos \left( \frac{1}{m^2} e^{-\sqrt{m(m-\kappa)}} \right)}{d - \lambda} \left( \frac{1}{m(d - \lambda)} + \frac{1}{m^2} + \frac{1}{d - \lambda} \right) \frac{1}{(K \mathbb{S} - \lambda) \mathbb{S} + 1} + \frac{1}{m} \frac{1}{(K \mathbb{S} - \lambda) \mathbb{S} + 1}. \] (40)

6.2 Caputo Fractional Derivative
Velocity equation (6) with reference to Caputo definition (10) is given by
\[ \zeta D^\alpha \mathbb{W}_c(\psi, t) = \frac{\partial^2 \mathbb{W}_c(\psi, t)}{\partial \psi^2} - \lambda \mathbb{W}_c(\psi, t) + G \cos \mathbb{T}_c(\psi, t) + G \cos \mathbb{C}_c(\psi, t), \] (41)
employing Laplace transformation on (6) with the help of (11), we explored second order partial differential equation,
\[ \frac{\partial^2 \mathbb{W}_c(\psi, m)}{\partial \psi^2} - (m + \lambda) \mathbb{W}_c(\psi, m) = -G \cos \mathbb{T}_c(\psi, m) - G \cos \mathbb{C}_c(\psi, m), \] (42)
putting the value of $\mathbf{T}_d(\psi, m)$ from (20) and $\mathbf{\bar{C}}_d(\psi, m)$ from (31) in (42), we have

$$\frac{\partial^2 \mathbf{W}_d(\psi, m)}{\partial \psi^2} - \left(m^2 + \lambda\right) \mathbf{W}_d(\psi, m) = -G\text{Cos}\theta \left(\frac{1}{m^2} e^{-\psi m + \frac{\psi}{m^2}}\right) - G\text{Cos}\theta \left(\frac{1}{m^2} e^{-\psi m - \frac{\psi}{m^2}}\right).$$

(43)

The solution of the equation (43) is given by

$$\mathbf{W}_d(\psi, m) = \frac{a}{m^2} e^{-\psi m + \frac{\psi}{m^2}} + G\text{Cos}\theta \frac{1}{d - \lambda} \left(e^{-\psi m - \frac{\psi}{m^2} - e^{-\psi m + \frac{\psi}{m^2}}\frac{1}{d - \lambda}}\right) \left(1 - \frac{Pr}{m^2 (d - \lambda)} + \frac{1}{m^2 (d - \lambda)(m^2 (d - \lambda) - 1)}\right).$$

(44)

6.3 Caputo–Fabrizio Fractional Derivative

Velocity equation (6) with reference to Caputo definition (12) is given by

$$^CD^\alpha W_d(\psi, t) = \frac{\partial^\alpha W_d(\psi, t)}{\partial \psi^\alpha} - \lambda W_d(\psi, t) + G\text{Cos}\theta T_d(\psi, \psi, t) + G\text{Cos}\theta C_d(\psi, t),$$

(45)

employing Laplace transformation on (6) with the help of (13), we explored second order partial differential equation,

$$\frac{\partial^2 \mathbf{W}_d(\psi, m)}{\partial \psi^2} - \left(m^2 + \lambda\right) \mathbf{W}_d(\psi, m) = -G\text{Cos}\theta T_d(\psi, \psi, m) - G\text{Cos}\theta C_d(\psi, m),$$

(46)

putting the value of $\mathbf{T}_d(\psi, m)$ from (23) and $\mathbf{\bar{C}}_d(\psi, m)$ from (34) in (46), we have

$$\frac{\partial^2 \mathbf{W}_d(\psi, m)}{\partial \psi^2} - \left(m^2 + \lambda\right) \mathbf{W}_d(\psi, m) = -G\text{Cos}\theta \left(\frac{1}{m^2} e^{-\psi m + \frac{\psi}{m^2}}\right) - G\text{Cos}\theta \left(\frac{1}{m^2} e^{-\psi m - \frac{\psi}{m^2}}\right).$$

(47)

The solution of the equation (47) is given by

$$\mathbf{W}_d(\psi, m) = \frac{a}{m^2} e^{-\psi m + \frac{\psi}{m^2}} \left(\frac{1}{d - \lambda}\left(e^{-\psi m - \frac{\psi}{m^2} - e^{-\psi m + \frac{\psi}{m^2}}\frac{1}{d - \lambda}}\right) \left(1 - \frac{Pr}{m^2 (d - \lambda)} + \frac{1}{m^2 (d - \lambda)(m^2 (d - \lambda) - 1)}\right)\right) \times \left(\lambda + a_0 \frac{a_0 + K_0}{m a_0 (K_0 - \lambda)} + \frac{1}{m^2} + \frac{1}{a_0 (K_0 - \lambda)(m (K_0 - \lambda) - a_0 - \lambda) + a_0 (K_0 - \lambda)}\right).$$

(48)

6.4 Atangana–Baleanu Fractional Derivative

Velocity equation (6) with reference to ABC definition (14) is given by

$$ABC^D^\alpha W_{ABC}(\psi, t) = \frac{\partial^\alpha W_{ABC}(\psi, t)}{\partial \psi^\alpha} - \lambda W_{ABC}(\psi, t) + G\text{Cos}\theta T_{ABC}(\psi, \psi, t) + G\text{Cos}\theta C_{ABC}(\psi, t),$$

(49)

employing Laplace transformation on (6) with the help of (15), we explored second order partial differential equation,

$$\frac{\partial^2 \mathbf{W}_{ABC}(\psi, m)}{\partial \psi^2} - \left(m^2 + \lambda\right) \mathbf{W}_{ABC}(\psi, m) = -G\text{Cos}\theta T_{ABC}(\psi, \psi, m) - G\text{Cos}\theta C_{ABC}(\psi, m),$$

(50)

putting the value of $\mathbf{T}_{ABC}(\psi, m)$ from (26) and $\mathbf{\bar{C}}_{ABC}(\psi, m)$ from (37) in (50), we have

$$\frac{\partial^2 \mathbf{W}_{ABC}(\psi, m)}{\partial \psi^2} - \left(m^2 + \lambda\right) \mathbf{W}_{ABC}(\psi, m) = -G\text{Cos}\theta \left(\frac{1}{m^2} e^{-\psi m + \frac{\psi}{m^2}}\right) - G\text{Cos}\theta \left(\frac{1}{m^2} e^{-\psi m - \frac{\psi}{m^2}}\right).$$

(51)

The required solution is
Fig. 2. Velocity curves for C, CF and ABC with variation in $\alpha$ where

$G_r = 1, Pr = 8, G_r = 1, Sc = 6, K_r = 0.5, K = 3, M = 1, R = 3, S = 0.5, a = 0.01, \theta = \frac{\pi}{3}, \alpha = \frac{\pi}{4}$

$$\Psi_{w_{1u}}(\psi, m) = \frac{a}{m^2} e^{-\frac{\lambda - d - a_1(Pr - 1)}{m^2(d - \lambda)}} + \frac{1}{m^2} \left( a_1(d - \lambda) \left( 1 - \frac{m^2(s_i(Pr - 1) + d - \lambda + a_i(d - \lambda))}{m_i} \right) \right)$$

$$+ \frac{1}{m^2} \left( \frac{\lambda + a_i - Sc(a_i + K)}{m_i a_i(K, Sc - \lambda)} \right) + \frac{1}{m^2} \left( \frac{1}{a_i(K, Sc - \lambda)} \right)$$

(52)
Fig. 3. Velocity curves for C, CF and ABC with variation in $M$ where

$G_r = 1, Pr = 8, G_c = 1, Sc = 6, K_r = 0.5, K_c = 3, \alpha = 0.01, R = 3, S = 0.5, a = 0.01, \theta = \frac{\pi}{3}, \phi = \frac{\pi}{4}$

As $\alpha \to 1$, results are obtained present in Endalew et al. [28].

We apply Laplace transform to our flow model using fractional order derivatives and to approximates inverse Laplace transform, Stehfest’s formula is one of the simplest algorithm that is why in the present work we apply the Stehfest’s algorithm [49],[50] to sort out the inverse Laplace transform.

$$W(r,t) = \frac{e^{\alpha t}}{t^2} \mathcal{W} \left( r, \frac{4.7}{t} \right) + \text{Re} \left[ \sum_{n=1}^{N} (-1)^n \mathcal{W} \left( r, \frac{k_n i + 4.7}{t} \right) \right],$$

where $N$ is a natural number, $\text{Re}(\cdot)$ is a real part, and $i$ is the imaginary unit [50].

7. Results & Discussion

Comparative study of flow of natural convective flow over a moving slanted plate by applying inclined magnetic field with porosity is carried out with variable temperature. Momentum, heat and mass profiles are taken into account under the fractional operators C, CF and ABC. Graphical and tabular analysis has been illustrated for temperature, concentration and velocity for several emerging parameters $\alpha, M, G, R, S, Pr$ and $\phi$ using C, CF and ABC fractional operators.

7.1 Velocity profile for $\alpha$

Figure (2) characterizes the control of fractional parameter on fluid flow. The velocity profile gets accelerated with the increment in values of $\alpha$. From the physical perspective, boundary layer thickness grows with increment in time as the result velocity is highest near plate. As $\alpha \to 1$, velocity curves approach to integer order which is the best explanation of fractional derivative for memory effect.
Fig. 5. Velocity curves for C, CF and ABC with variation in $Pr$ where
$$G_t = 1, M = 1, G_c = 1, Sc = 6, K_v = 0.5, K_s = 3, \alpha = 0.01, R = 3, S = 0.5, a = 0.01, \theta = \frac{\pi}{3}, \phi = \frac{\pi}{4}$$

7.2 Velocity profile for $M$

The impact of $M$ can be observed through Figure (3). Velocity declined with the increase in $M$. Physically, the fluid flow reduces due frictional force which is directly proportional to $M$ and hence velocity decreases. The behavior of non-integer order models in velocity for Hartmann number is the same at different times.
7.3 Velocity profile for $C_r$

Figure (4) presents that with the increase in $G_r$, raises velocity. Physically, rise in temperature accelerates buoyancy forces, as a result velocity increases. Clearly, velocity is highest for the ABC model.

7.4 Velocity profile for $Pr$

Figure (5) focuses the significance of $Pr$ in variation of fluid velocity. Fluid flow decays for when $Pr$ accelerates. Physically, thermal conductivity associated with $Pr$ which leads to decline due to which the viscosity enhance and resultanty, velocity decays.
Fig. 7. Velocity curves for C, CF and ABC with variation in $K_r$ where

\[ G_r = 1, Pr = 8, G_i = 1, Sc = 6, M = 1, K_r = 3, \alpha = 0.01, R = 3, S = 0.5, a = 0.01, \theta = \frac{\pi}{3}, \phi = \frac{\pi}{4} \]

7.5 Velocity profile for $Sc$

The consequence of $Sc$ variation on velocity profile is featured in Figure (6). It is espied that enlargement in values of $Sc$ results in deceleration of velocity field. The physical logic of decline in velocity is the reduction molecular diffusivity due to enlargement in $Sc$.

7.6 Velocity profile for $K_r$

Figure (7) analyzes the impact of the porosity $K_r$ on velocity. Velocity profiles increase when we raise porosity. As we compare C, CF and ABC curves we can see that velocity is maximum for ABC.
Figure (8) illustrates impact of radiation parameter on velocity profile. Radiation parameter increases the fluid velocity. Physically, thermal buoyancy force reduces by increasing radiation due to which thermal boundary layer become thin and hence velocity becomes high. While comparing the effect of $R$ on velocity for fractional time derivatives, clearly in case of C velocity is less as compared to CF and ABC.
Fig. 9. Velocity curves for C, CF and ABC with variation in $S$ where

$G = 1, Pr = 8, G_r = 1, Sc = 6, K_r = 0.5, K = 3, \alpha = 0.01, R = 3, M = 1, a = 0.01, \phi = \frac{\pi}{3}, \phi = \frac{\pi}{4}$

### 7.8 Velocity profile for $S$

Velocity curves for heat source parameter $S$ can be observed through Figure (9). Physically, by increasing the heat source more heat is generated due to which temperature increases and hence velocity is enhanced. Velocity raises under the influence of $S$ for all fractional models.
7.9 Velocity profile for $\phi$

Figure (10) shows that rise in angle of inclination causes reduction in velocity. Velocity reduces for $\phi$ for all fractional models.

7.10 Temperature profile for $\alpha$, Pr, R

Figures (11), (12), and (13) highlights the behavior of temperature curves for fractional parameter, Prandtl number and radiation parameter. With the increase in $\alpha$, Pr, R temperature decreases. Temperature is least for Caputo and maximum for ABC.
Fig. 11. Temperature curves for C, CF and ABC with variation in $\alpha$ where
\[ G_\gamma = 1, Pr = 8, G = 1, Sc = 6, K = 0.5, K = 3, M = 1, R = 3, S = 0.5, a = 0.01, \eta = \frac{r}{3}, \phi = \frac{\pi}{4} \]

Figures (14), (15) and (16) represents the impact of $\alpha$, $Sc$, $K$, on concentration field. Clearly, concentration reduces for all models. With the increment in values of $Sc$, molecular diffusivity becomes less so concentration decreases. Graphical behavior of concentration is prominent for ABC as in comparison with other two models.

7.11 Concentration profile for $\alpha$, $Sc$, $K$,
In Table 1, inversion algorithm is used to compute velocity for all fractional models for $\phi$. Velocity showing increasing behavior for all models. Table 2, and 3 interpret numerical calculations for $Nu$ corresponding to $Pr$ are $Sh$ corresponding to $Sc$ respectively via Stehfest’s and Tzou’s algorithms. Heat and mass transfer rate is high for the ABC model as compared to the rest of the two models.
Fig. 13. Temperature curves for C, CF and ABC with variation in $R$ where

$G_r = 1, Pr = 8, G_c = 1, Sc = 6, K_w = 0.5, K = 3, n = 0.01, M = 1, S = 0.5, a = 0.01, \theta = \frac{\pi}{3}, \phi = \frac{\pi}{4}$
Fig. 14. Concentration curves for C, CF and ABC with variation in $\alpha$ where $G_r = 1, Pr = 8, G_r = 1, Sc = 6, K_r = 0.5, K_r = 3, M = 1, R = 3, S = 0.5, a = 0.01, \theta = \frac{\pi}{3}, \phi = \frac{\pi}{4}$
Fig. 15. Concentration curves for C, CF and ABC with variation in $Sc$ where

$G_{r} = 1, \text{Pr} = 8, G_{r} = 1, M = 1, K_{r} = 0.5, K_{p} = 3, n = 0.01, R = 3, S = 0.5, a = 0.01, \theta = \frac{\pi}{3}, \phi = \frac{\pi}{4}$
Fig. 16. Velocity curves for C, CF and ABC with variation in $K_r$ where

$G_i = 1, Pr = 8, G_i = 1, Sc = 6, K_r = 0.5, M = 1, \alpha = 0.01, R = 3, S = 0.5, a = 0.01, \theta = \frac{\pi}{3}, \phi = \frac{\pi}{4}$
Table 1. Calculations for velocity by numerical inversion Laplace transform for Stehfest’s and Tzou’s

<table>
<thead>
<tr>
<th>Pr</th>
<th>C (Stehfest’s)</th>
<th>C (Tzou’s)</th>
<th>CF (Stehfest’s)</th>
<th>CF (Tzou’s)</th>
<th>ABC (Stehfest’s)</th>
<th>ABC (Tzou’s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.218</td>
<td>0.206</td>
<td>0.373</td>
<td>0.319</td>
<td>0.334</td>
<td>0.328</td>
</tr>
<tr>
<td>0.6</td>
<td>0.213</td>
<td>0.199</td>
<td>0.283</td>
<td>0.257</td>
<td>0.326</td>
<td>0.314</td>
</tr>
<tr>
<td>0.9</td>
<td>0.186</td>
<td>0.172</td>
<td>0.209</td>
<td>0.143</td>
<td>0.302</td>
<td>0.294</td>
</tr>
<tr>
<td>1.3</td>
<td>0.152</td>
<td>0.135</td>
<td>0.152</td>
<td>0.141</td>
<td>0.285</td>
<td>0.255</td>
</tr>
<tr>
<td>1.6</td>
<td>0.118</td>
<td>0.103</td>
<td>0.118</td>
<td>0.108</td>
<td>0.238</td>
<td>0.138</td>
</tr>
<tr>
<td>1.9</td>
<td>0.089</td>
<td>0.056</td>
<td>0.076</td>
<td>0.047</td>
<td>0.179</td>
<td>0.163</td>
</tr>
<tr>
<td>2.2</td>
<td>0.065</td>
<td>0.039</td>
<td>0.053</td>
<td>0.062</td>
<td>0.135</td>
<td>0.121</td>
</tr>
<tr>
<td>2.5</td>
<td>0.046</td>
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<td>0.037</td>
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<td>0.115</td>
<td>0.104</td>
</tr>
<tr>
<td>2.8</td>
<td>0.033</td>
<td>0.020</td>
<td>0.026</td>
<td>0.021</td>
<td>0.073</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Table 2. Calculations for Nusselt number by numerical inversion Laplace transform for Stehfest’s and Tzou’s

<table>
<thead>
<tr>
<th>Pr</th>
<th>Nu (Stehfest’s)</th>
<th>Nu (Tzou’s)</th>
<th>Nu (Stehfest’s)</th>
<th>Nu (Tzou’s)</th>
<th>Nu (Stehfest’s)</th>
<th>Nu (Tzou’s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.214</td>
<td>0.210</td>
<td>0.234</td>
<td>0.206</td>
<td>0.282</td>
<td>0.265</td>
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<tr>
<td>2</td>
<td>0.284</td>
<td>0.243</td>
<td>0.308</td>
<td>0.301</td>
<td>0.385</td>
<td>0.351</td>
</tr>
<tr>
<td>3</td>
<td>0.344</td>
<td>0.363</td>
<td>0.371</td>
<td>0.353</td>
<td>0.466</td>
<td>0.401</td>
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<tr>
<td>4</td>
<td>0.387</td>
<td>0.371</td>
<td>0.422</td>
<td>0.414</td>
<td>0.535</td>
<td>0.519</td>
</tr>
<tr>
<td>5</td>
<td>0.433</td>
<td>0.442</td>
<td>0.469</td>
<td>0.436</td>
<td>0.596</td>
<td>0.552</td>
</tr>
<tr>
<td>6</td>
<td>0.468</td>
<td>0.451</td>
<td>0.512</td>
<td>0.503</td>
<td>0.651</td>
<td>0.632</td>
</tr>
<tr>
<td>7</td>
<td>0.504</td>
<td>0.516</td>
<td>0.551</td>
<td>0.547</td>
<td>0.732</td>
<td>0.709</td>
</tr>
<tr>
<td>9</td>
<td>0.537</td>
<td>0.520</td>
<td>0.568</td>
<td>0.564</td>
<td>0.749</td>
<td>0.721</td>
</tr>
<tr>
<td>9</td>
<td>0.569</td>
<td>0.542</td>
<td>0.622</td>
<td>0.619</td>
<td>0.794</td>
<td>0.741</td>
</tr>
<tr>
<td>10</td>
<td>0.576</td>
<td>0.589</td>
<td>0.655</td>
<td>0.649</td>
<td>0.836</td>
<td>0.812</td>
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</table>

Table 3. Calculations for Sherwood number by numerical inversion Laplace transform for Stehfest’s and Tzou’s

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<tr>
<th>Sc</th>
<th>Sh (Stehfest’s)</th>
<th>Sh (Tzou’s)</th>
<th>Sh (Stehfest’s)</th>
<th>Sh (Tzou’s)</th>
<th>Sh (Stehfest’s)</th>
<th>Sh (Tzou’s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.117</td>
<td>0.113</td>
<td>0.123</td>
<td>0.120</td>
<td>0.128</td>
<td>0.122</td>
</tr>
<tr>
<td>0.6</td>
<td>0.166</td>
<td>0.144</td>
<td>0.171</td>
<td>0.167</td>
<td>0.181</td>
<td>0.154</td>
</tr>
<tr>
<td>0.9</td>
<td>0.213</td>
<td>0.179</td>
<td>0.203</td>
<td>0.185</td>
<td>0.222</td>
<td>0.206</td>
</tr>
<tr>
<td>1.3</td>
<td>0.234</td>
<td>0.215</td>
<td>0.242</td>
<td>0.218</td>
<td>0.256</td>
<td>0.232</td>
</tr>
<tr>
<td>1.6</td>
<td>0.262</td>
<td>0.244</td>
<td>0.271</td>
<td>0.239</td>
<td>0.286</td>
<td>0.258</td>
</tr>
<tr>
<td>1.9</td>
<td>0.287</td>
<td>0.258</td>
<td>0.296</td>
<td>0.268</td>
<td>0.314</td>
<td>0.289</td>
</tr>
<tr>
<td>2.2</td>
<td>0.311</td>
<td>0.279</td>
<td>0.322</td>
<td>0.288</td>
<td>0.339</td>
<td>0.319</td>
</tr>
<tr>
<td>2.5</td>
<td>0.332</td>
<td>0.301</td>
<td>0.342</td>
<td>0.307</td>
<td>0.362</td>
<td>0.325</td>
</tr>
<tr>
<td>2.8</td>
<td>0.352</td>
<td>0.319</td>
<td>0.363</td>
<td>0.325</td>
<td>0.374</td>
<td>0.385</td>
</tr>
</tbody>
</table>

8. Conclusion

The study of natural convective flow with chemical reaction and thermal radiation past a moving slanted infinite plate and slanted magnetic field has been discussed via integer and non-integer models. The solutions of model equations are developed through Laplace transform and inversion algorithm. The main focus is the comparison for different fractional models. From our general results, some results can be deduced from literature. Velocity curves are showing increasing behavior for fractional parameter $\alpha$ whereas temperature and concentration curves are showing decreasing behavior for $\alpha$. There is decline in velocity as $\Gamma$, $\varphi$ increase for integer order models. Velocity enhances for $K_p$, $G_r$ and $S$ for all fractional models. Temperature and velocity have the same behavior for $Pr$ while both have opposite behavior for $R$ in all cases. Concentration declines while we raise the value of $Sc$ and $K_r$. Nusselt number for $Pr$ and Sherwood number for $Sc$ increase for all non-integer order models.

Author Contributions

N. Ifikhar solved the proposed mathematical model and verify the required results; D. Baleanu plotted the graphs using software and verify the results present in literature; M.B. Riaz proposed the problem, initiated the conceptualization, developed the mathematical modeling and examined the theory validation; S.M. Husnine edit the paper. Final version of the article is approved by all authors.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Nomenclature

\( v \) Kinematic viscosity  \( \alpha \) Fractional parameter
\( g \) Acceleration due to gravity  \( t \) Time
\( \rho \) Fluid density  \( \Pr \) Prandtl number
\( C_p \) Specific heat  \( Sc \) Schmidt number
\( \beta \) Volumetric coefficient of expansion for mass transfer  \( \lambda \) Relaxation time
\( \beta_r \) Volumetric coefficient of thermal expansion  \( \sigma \) Electrical conductivity of the fluid
\( \mu \) Dynamic viscosity  \( B_0 \) Magnetic field
\( a' \) Absorption constant  \( M \) Hartmann number
\( a \) Dimensionless acceleration parameter  \( D \) Molecular diffusivity of chemical
\( G_r \) Thermal Grashof number  \( K \) Thermal conductivity
\( G \) Mass Grashof number  \( q \) Relative heat flux
\( R \) Radiation parameter  \( K_e \) Porosity parameter
\( W \) Velocity of the fluid  \( K_e \) Chemical reaction parameter
\( T \) Fluid temperature  \( S \) Heat source parameter
\( C \) Fluid concentration  \( C_w \) Concentration on the plate
\( m \) Laplace transforms parameter  \( C_w \) Concentration of the fluid far away from the plate
\( T_a \) Temperature of the plate  \( \beta \) Angle of inclination of the plate
\( T \) Temperature of fluid far away from the plate  \( \phi \) Angle of inclination of applied magnetic field

References


ORCID iD

Nazish Ifikhar: https://orcid.org/0000-0003-1840-2262
Dumitru Baleanu: https://orcid.org/0000-0002-0286-7244
Muhammad Bilal Riaz: https://orcid.org/0000-0001-5153-297x
Syed Muhammad Husnine: https://orcid.org/0000-0002-3237-6681

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