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Effects of Non-Linear Thermal Radiation and Chemical Reaction on Time Dependent Flow of Williamson Nanofluid With Combine Electrical MHD and Activation Energy

Gulzar Ahmad Danish[1], M. Imran[1], Madeeha Tahir[2], Hassan Waqas[1], M.I. Asjad[3], Ali Akgül[4], Dumitru Baleanu[5,6,7]

[1] Department of Mathematics, Government College University, Faisalabad, 38000, Pakistan
Emails: gulzardanish@yahoo.com (G.A.D.), drmimranchaudhry@gcuf.edu.pk (M.I.), hassanwaqas22@gcuf.edu.pk (H.W.)
[2] Department of Mathematics, Government College Women University, Faisalabad, 38000, Pakistan, Email: madeehatahir@gcwuf.edu.pk
[3] Department of Mathematics, University of Management and Technology Lahore, 54782, Pakistan, Email; imran.asjad@umt.edu.pk
[4] Art and Science Faculty, Department of Mathematics, Siirt University, Siirt, TR-56100, Turkey, Email; aliakgul@siirt.edu.tr
[5] Department of Mathematics, Cankaya University, 06790, Ankara, Turkey, Email; dumitru@cankaya.edu.tr
[6] Institute of Space Sciences, 077125, Magurele-Bucharest, Romania
[7] Department of Medical Research, China Medical University Hospital, China Medical University, 40447, Taichung, Taiwan

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Corresponding author: A. Akgül (aliakgul@siirt.edu.tr)

Abstract. The current article will present the impact of the heat and mass transfer of combine electrical MHD flow of time dependent Williamson fluid with nanoparticles by the incorporating the influences of non-linear thermal radiation and the chemical reaction through wedge shape geometry. The fluid flows past a porous stretching wedge with convected Nield boundary conditions. The several (geometrical and physical) conditions have been included to provide more practicable results. The effects of activation energy further discussed. Due to relevant similarity transformation, set of partial differential equations which is non-linear and complicated is converted into simplest system of ordinary differential equations. To obtain the desired solution, famous numerical technique (shooting) used with the help of bvp4c MATLAB coding. The variation physical quantities namely velocity, temperature, concentration of nanoparticles, local Sherwood number, coefficient of skin friction and local Nusselt number have been observed under the influence of emerging parameters. The elaborated discussion presented with graphical and tabular illustrations.

Keywords: Williamson Nano fluid; Non-Linear Thermal Radiation; Wedge Geometry; Numerical Technique.

1. Introduction

The study of transportation of non-Newtonioan phenomena and its extensive role in industrial and technological applications lots of experimental and theoretical investigations have been made by researchers experimentally and theoretically. Because of an extensive use in both industrial and engineering fields, non-Newtonian fluids dragged much consideration of the scientist as compared to Newtonian fluids. There is no evidence of any singleton model which wholly predicts about all the features of non-Newtonian fluids because of its broad applications. Non-Newtonian fluids has vast applications like as emulsions, nuclear fuel slurries, condensed milk, molten plastics, extrusion of polymeric fluids, personal care products, soaps, food stuffs, shampoo, pharmaceutical, molten plastics.

A number of researchers are working on analysis of non-Newtonian fluids. In present days non-Newtonian fluids [1, 2] acquire much attention due to their immense role in polymer processing and chemical engineering processing. Thixotropy, viscoelasticity, shear thinning (shear thickening) are some complex attributes of fluids (non-Newtonian) [3] and [4]. A large number of such non-Newtonian models discovered by modifying momentum conservation equations. Oldroyd-B models, Sisko model, differential Reiner–Rivlin models, power-law model and Williamson model are few existing non-Newtonian fluid models. To discuss the valuable relationship between different types of non-Newtonian liquids several attempts made by researchers. Among these, a comparatively simple, predictable model named as Williamson fluid model was the first attempt made by Williamson [5]. Williamson explored the shear thinning property in non-Newtonian fluid and discovered the Williamson fluid model. To solve the system of equation for flow of Williamson nanofluids Nadeem and Akbar [6] used Homotopy Analysis Method (HAM). Williamson model was suggested to replicate the viscoelastic shear thinning property of non-Newtonian fluids. With the increase in shear rate the decreases in apparent viscosity is a significant application of shear thinning materials.

Recently, the non-Newtonian Williamson fluid in the vision of its extensive role in biological phenomenon, heat transfer analysis and peristaltic flow has a conventional copious attention of investigators, like Nadeem and Akram [7], Vasudev et al. [8]. Recently heat transfer of a Williamson nanofluids and stagnation-point flow is examined by Gorla and Gireesha [9] by using linear stretching/shrinking sheet and provide dual solution with the help of Runge-Kutta scheme. Fang et al. [10] was the pioneer who used non-flat stretching sheet and calculated elegant numerical and analytical solution of two-dimensional boundary layer flow. Further by including the energy equation Subhashini et al. [11] extended the problem and found that for the first solution thermal boundary layer thicknesses were thinner as compared to the second solution. Kumar et al. [12] numerically examined the motion of Williamson fluid under the influence of thermal radiation. The numerical analysis of Williamson fluid was carried out by Ramzan et al. [13] by using the non-Fourier heat transfer.

Fluid investigations are important for the chemical, mechanical and geotechnical engineering, atomic energy, thermal as well as hydroelectric industries. The scientific research has revealed the thermal conductivity of such fluids. The heat transformation of these operating fluids, the lower thermal conductivity is the major obstruction in engineering and many technical branches. Thermal conductivity of the materials improved in solids instead of liquids. The specialized technique for improving thermal conductivity includes amalgamation in the process of solid materials. This kind of fluid is called nanofluid. Nanofluids have developed thermophysical properties that make others to conceivably beneficial unexpected applications in liquid fuels, heat transformation, pharmaceutical mechanisms, microelectronics, cooling ducts heat exchangers, refrigerator, hybrid-powered engines, international refrigerator, boiler fuel cylinder pressure decrease as well as heat exchanger. In addition, basic cancer therapy concepts, drug delivery, nanomedicine, dehydration engineering and therapies are indeed extremely dependent on the motion of the particular nanoparticles in the specified systems. Due to the prominence applications in engineering and industries the phenomenon of thermal conductivity of the fluid has fascinated the attention of many researchers. Many theoretical and experimental analysis have been performed in this regard. Recently, researchers studied nanoscale technology and worked on an embryonic field of science. It is confirmed from experimental studies that the domination of multiscale constructions the heat transport phenomenon is crucial. By the recent explorations Garoosi et al. [14] discussed the process of natural convection of heat transformation in heat exchanger which is filled with nanofluids. He found that the heat transfer rate is enhanced with the decrease in nanoparticles size in nanofluids. Soid et al. [15] analyzed the constantly stirring needle in a nanofluid and gave the concept that only in the condition when the free stream and needle has opposite direction of motion the dual solutions exist. Raju et al. [16] analyzed the heat absorption ascertained the numerical and analytical investigation MHD free flow (convection) over an exponentially moving perpendicular plate. He concluded that there is inverse variation between chemical reaction parameter and Schmidt number rate of mass transfer during heat transform process.

The heat transfer and boundary layer flow due to nanofluids are the thrust areas of current research. The heat transfer property is a special feature of boundary layer flow of a nanofluids over a stretching sheet. Choi [17] narrated that in various areas of fluid flows nanofluids comprising with nanoparticles and base fluids are the important source of heat transfer. Literature study signifies that characteristics of fluids like stability and thermal conductivity are changed by the effect of nanoparticles. In this modern era a huge number of studies(experimental and theoretical) are being made to explore the heat transfer phenomena in nanofluids. Due to enrichment of nanofluids thermal conductivity a terrific improvement in coefficient of heat transfer is observed. Over a vertical plate the characteristics of nanofluids on convective boundary layer flow was studied by Nield and Kuznetsov [18]. In the presence of both processes thermal radiation and induced magnetic field a numerical examination on boundary layer flow prompted in a nanofluid has been made by Gbadeyan et al. [19] over a linearly stretching sheet. Magnetic cell separation, drug-delivery, contrast enhancement in magnetic resonance imaging and hyperthermia are some biomedical applications of nanofluids. Besides this, nano liquids have a large number of applications in biomedical industry, solid state lightening, microfluidics, transportation, detergency, microelectronics, power generation in nuclear reactors and more specifically industrial applications in any heat exclusion. Kakac and Pramuanjaroenkij [20] studied numerically nanofluids and modelling of natural convection heat transfer in nanofluids. Nanotechnology is the most rending topic discussion in public health. Mnyusiwalla et al. [21] carried out an analyses that nanoparticles could present possible dangers in environment and health.

In all the previous research, the effect of thermal radiation flow and heat transform have not been focused. While heat transformation under the effect of radiation process is significant to all mega manufacturing industries like nuclear plants, space vehicles, satellites, gas turbines, aircraft and missiles etc. Under the influence of thermal radiation Olanrewaju et al. [22] examined the boundary layer flow of nanofluids in a flowing fluid over moving surface. Wang et al. [23] studied the MHD flow of nanofluids over the stagnation point over a stretching sheet with convective boundary condition by considering the impact of thermal radiation. The effect of thermal radiation and heat generation is reported by Gnaneswara Reddy [24] in a micropolar fluid over a stretching sheet. The heat transfer investigation of water-based nanofluids was scrutinized by Akbar et al. [25] over an exponentially stretching sheet. Reddy [26] studied the two dimensional heat and mass flow under the effect of MHD also observed the thermophoresis effects on it over an inclined (radiative isothermal permeable) surface. For the anticipation of chemical reaction a minimal amount of energy is required for reactants, which is the activation energy. Due to concentration variance in mixture type mass transfer process happens. In chemical industry, oil emulsions, geothermal reservoirs and food processing are the fields where, the activation energy plays significant role. Both thermic(exothermic or endothermic)reactions with the backing of activation energy were studied by Maleque [27] on alternative convective flows. Numerical technique for fraction control problems is reported by Zhang et al. [28] by using the technique of Chebyshev polynomials. Shafique et al. [29] proposed numerical technique by using rotating viscoelastic flow with species of chemical reaction and activation energy for Fokker-Planck (linear and nonlinear)equations. Hemeda et al. [30] gave the idea of a numerical approach along with integral iterative scheme. Siddiquea and Akgül [31] investigated the unsteady MHD generalized first problem of Stokes' for an incompressible viscous fluid under isothermal conditions. The developed governing equations for the problem were formulated with the newly introduced fractal fractional operators with power law, exponential decay law and the Mittag-Leffler law kernels. Asjad *et al.* [32] studied the unsteady and incompressible viscous fluid flow with constant proportional Caputo type fractional derivative (hybrid fractional operator) and found analytical solutions of a well-known problem in fluid dynamics known as Stokes' first problem. MHD and porosity were also considered as an additional effects. Hashemi *et al.* [33] constructed a Lie-group integrator based on $GL_4(\mathbb{R})$ and the reproducing kernel functions to investigate the flow characteristics in an electrically conducting second-grade fluid over a stretching sheet. Accurate initial values were achieved when the target equation is matched precisely, and then, they applied the group preserving scheme (GPS) to get a rather accurate results. Sheikholeslami et al. [34] analyzed that the nanofluid in the study of swirl generator and four-lobed pipe minimize the energy consumption and it reduces the heat losses. Sheikholeslami and Farshad [35] studied the nanofluid turbulent flow by installing of helical tape in a solar system and concluded that install of such device generates secondary flow and produce thinner boundary layer.

The colloidal analysis over wedge geometry cannot be neglected owing to its comprehensive application's in various industrial & a number of engineering problems such as engine greasing, thermal vigor storage devices, nuclear reactor cooling, extraction of crude oil, cooling or heating of films, polymer process, plasma studies, cooling of electronic, geothermal enterprises, atomic waste stockpiling and etc. Due to its industrial implementation the flow of fluid in the boundary adjoining to the wedge has wonderful interest to researchers and engineers. The base fluids or nanoliquids flow past wedge geometry open a new avenue for scientists and researchers. Therefore, scientists and researchers concentrated on this way and improved the model day by day with new improvements. Nandi et al. [36] analyzed is carried out into Carreau's nanoliquid flow via the convectively heated stretched wedge through Navier's velocity slip effects together with the magnetic field impact. Ahmed et al. [37] scrutinized the nanoliquid through Buongiorno modal for mixed convective flowing against a vertical porous wedge through the convective boundary constraints. Xiang et al. [38] examined the characteristics of oblique detonation generated by a limited wedge in hydrogen-air formulations with increasing equivalence ratios.

Our inspiration of the current work is to scrutinize and model of the Williamson nanofluid with activation energy over wedge geometry. The behaviors of Brownian and thermophoresis diffusion are analyzed. Nonlinear thermal radiation and activation energy is discussed. The novelty of this analyses is (i) consideration of unsteady Williamson nanoliquid past a wedge. It plays a crucial role in plasma studies, cooling of nuclear reactor, and geothermal heat pips cooling. Resulting dimensionless ODE's are tackled by shooting scheme via bvp4c tool of MATLAB. The careful investigation of literature depicts that the mathematical problem developed in this communication is new and has not been addressed before as per the author's knowledge. The physical behavior of prominent parameters against flow fields are examines through figures and tabular data.

2. Mathematical Model

Let us consider time dependent incompressible flow of a non-Newtonian Williamson fluid in the presence of magnetic field past a wedge shaped geometry as shown in Fig. 1. The activation energy is considered that effects Williamson nano fluid flow along with non-linear thermal radiation and the chemical reaction effects model due to moving edge. Let $u_w(x,t)=\left(bx^m/(1-ct)\right)$ is the stretching edge velocity, in which b and c are denoted as stretching rate and constant having time dimension $(time)^{-1}$ respectively. Since the fluid flow is in the upward direction parallel to axial direction with free stream velocity $u_e(x,t)=\left(ax^m/(1-ct)\right)$, where m constant as is define $0\le m\le 1$, a and c are constant. Let us assume that the angle of wedge is $\Omega=\beta\pi$, in which β is to be related as the pressure gradient and is define as $\beta=\left(2m/(m+1)\right)$. The temperature at the surface of the wedge is to be taken as $T_w(x,t)=T_\infty+T_0u_w(x)/(\nu\sqrt{1-ct})$, where T_0 is the initial temperature at the surface and T_∞ is the temperature at infinity.

A mathematical model for Williamson fluid without viscous dissipation and the external force described as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}+v\frac{\partial^2 u_e}{\partial y^2}\left[\beta^*+\left(1-\beta^*\right)\left(1-\Gamma\frac{\partial u}{\partial x}\right)^{-1}\right]$$
$$+v\Gamma\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial^2 u}{\partial y^2}\right)\left[\left(1-\beta^*\right)\left(1-\Gamma\frac{\partial u}{\partial y}\right)^{-2}\right]-\frac{\sigma B_o^2}{\rho}\left(u-u_e\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha_0+\frac{16\sigma_s T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}\right)\frac{\partial^2 T}{\partial y^2}+\frac{(\rho c)_p}{(\rho c)_f}\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-Kr^2\left(C-C_\infty\right)\left(\frac{T}{T_\infty}\right)^2\exp\left(\frac{-E_a}{kT}\right), \tag{4}$$

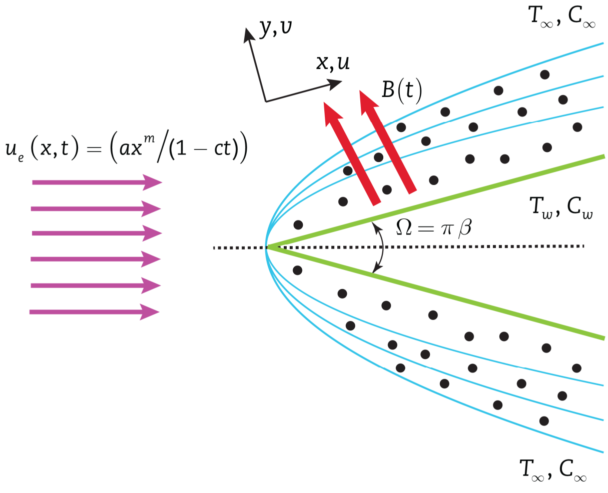

Fig. 1. Geometry of the problem

in above system of equations u and v are respectively the velocity components along x and y directions, μ is dynamic viscosity, ν is kinematic viscosity, ρ_f is base fluid density, T is the temperature of nanofluid, C is the nanoparticle volume friction, α_0 is thermal diffusivity, σ_s is the Stefan-Boltzmann constant, $(\rho c)_f$ heat capacity of liquid, $(\rho c)_p$ effective heat capacity of nanoparticles, ρ_m is the density of the microorganism, k stands for thermal conductivity of the material, ratio of the viscosity is $\beta^* = \mu_0/\mu_\infty$, q_r is radiative heat flux, D_B stands for Brownian diffusivity, D_T for thermophoretic diffusion coefficient, K_r is chemically reaction rate parameter and E_a is activation rate constant.
It is to be noticed that in equation (2) the model for the Williamson fluid converted into the viscous fluid if we put $\beta = \Gamma = 0$.

There are certain boundary conditions which are suggested in the following form:

$$y \to \infty \qquad \begin{aligned} & u = u_w = u_e(x,t),\ v = 0,\ \text{at}\ y = 0, \\ & -k\frac{\partial T}{\partial y} = h_f\left(T_f - T\right),\ \ C = C_w(x,t) = 0\ \text{at}\ y = 0, \\ & u \to u_e = 0,\ v \to 0, T \to T_\infty,\ C \to C_\infty\ \text{as}\ y \to \infty, \end{aligned} \tag{5}$$

where T_∞ and C_∞ depict the fluid temperature and concentration at infinity, respectively.

2.1 Similarity Variables

Now consider the following dimensionless quantities that are used for the transformation of the partial differential equations (PDEs) into a set of ordinary differential equations (ODEs) [39-41]

$$\psi(x,y,t) = \sqrt{\frac{2\nu x u_e}{m+1}} f(\zeta),\ \ \zeta = y\sqrt{\frac{(m+1)u_e}{2\nu x}},\ \ \theta(\zeta) = \frac{T - T_\infty}{T_f - T_\infty},\ \ \varphi(\zeta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{6}$$

Here ψ is the stream function which describe the component of fluid velocity as $u = \partial\psi/\partial y$ whereas $v = -\partial\psi/\partial x$, ζ is the dimensionless coordinate axis along the vertical direction (y-axis) direction. The normalized temperature and concentration are described by θ and φ respectively.

$$\left(\beta^* + (1-\beta^*)(1 - Wef'')^{-2}\right)f''' + ff'' + \beta\left(1 - (f')^2\right) - A(2-\beta)\times\left(f' + \frac{\eta}{2}f'' - 1\right) + ME_1 - M(2-\beta)(f'-1) = 0\ , \tag{7}$$

$$\begin{aligned} & \left[(1+\varepsilon\theta) + \frac{4}{3}Rd\left(1 + (\theta_w - 1)\theta\right)^3\right]\theta'' + [\varepsilon + 4Rd(\theta_w - 1)\left(1 + (\theta_w - 1)\theta\right)^2\theta'^2] \\ & + \Pr\left(f\theta' - 2f'\theta\right) + \Pr\left(Nb\theta'\varphi' + Nt\theta'^2\right) - \Pr\frac{A}{2}(2-\beta)(\eta\theta' + 3\theta) = 0, \end{aligned} \tag{8}$$

$$\phi'' + \left(\frac{N_t}{N_b}\right)\theta'' + \Pr Lef\phi' - \Pr Le\sigma(1+\delta\theta)^n \exp\left(\frac{-E}{1+\delta\theta}\right)\phi = 0. \tag{9}$$

Similarly boundary conditions are transformed as follows

$$f(\zeta) = 0,\ f'(\zeta) = \lambda,\ \varphi(\zeta) = 0,\ \text{at}\ \zeta = 0,\ \ \theta'(\zeta) = -\gamma(2-\beta)^{0.5}(1-\theta),\ \text{at}\ \zeta = 0, \tag{10}$$

$$f' \to 1,\ \theta \to 0,\ \varphi \to 0,\ \text{as}\ \zeta \to \infty, \tag{11}$$

where prime denote the differentiation with respect to ζ, f', θ, φ are the dimensional less velocity, temperature and the nanoparticles concentration profiles respectively.

Where wedge moving parameter is denoted as $\lambda = b/a$, $\lambda > 0$ is stand for stretching wedge, if $\lambda = 0$ is for the static wedge and also if $\lambda < 0$ is for the shrinking wedge. $M = \sigma B_0^2/(\alpha\rho x^{m-1})$ is the magnetic parameter, electric parameter $E_1 = E_0 B_0^2/(\rho a^2 x^{m-1})$, $We = \sqrt{\Gamma^2(m+1)U_e^3}/\nu x$ is the Weissenberg parameter, $\theta_w = T_w/T_\infty$ non-linear thermal radiation parameter, Brownian motion parameter $Nb = (\rho c)_p D_B(C_w - C_\infty)/((\rho c)_f v)$ and thermophoresis parameter, $Nt = (\rho c)_p D_T(T_w - T_\infty)/(\rho c)_f \nu T_\infty$, is the thermophoresis Parameter, $Rn = 16\sigma_1 T_\infty^3/(3k^* k_f)$ is radiation parameter, activation energy parameter $E = E_a/(kT_\infty)$ unsteady parameter $A = c/(ax^{m-1})$, wedge angle parameter $\beta = 2m/(m+1)$. Moreover, local Nusselt number (Nu_x) and local Sherwood number (Sh_x) are defined as $C_f = \tau_w/\rho u_w^2$, $Nu_x = xq_w/k(T_\infty - T_m)$, $Sh_x = xh_m/D_B(C_w - C_\infty)$. Where wall shear stress τ_w, the wall heat flux q_w, and wall mass current h_m are written as

$$\begin{aligned} & \tau_w = \mu_0\frac{\partial u}{\partial y}\left(n + (1-n)\left(1 - \Gamma\frac{\partial u}{\partial y}\right)^{-1}\right), \\ & q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},h_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}. \end{aligned} \tag{12}$$

By using above similarity variables, one can obtain the dimensionless engineering parameters as

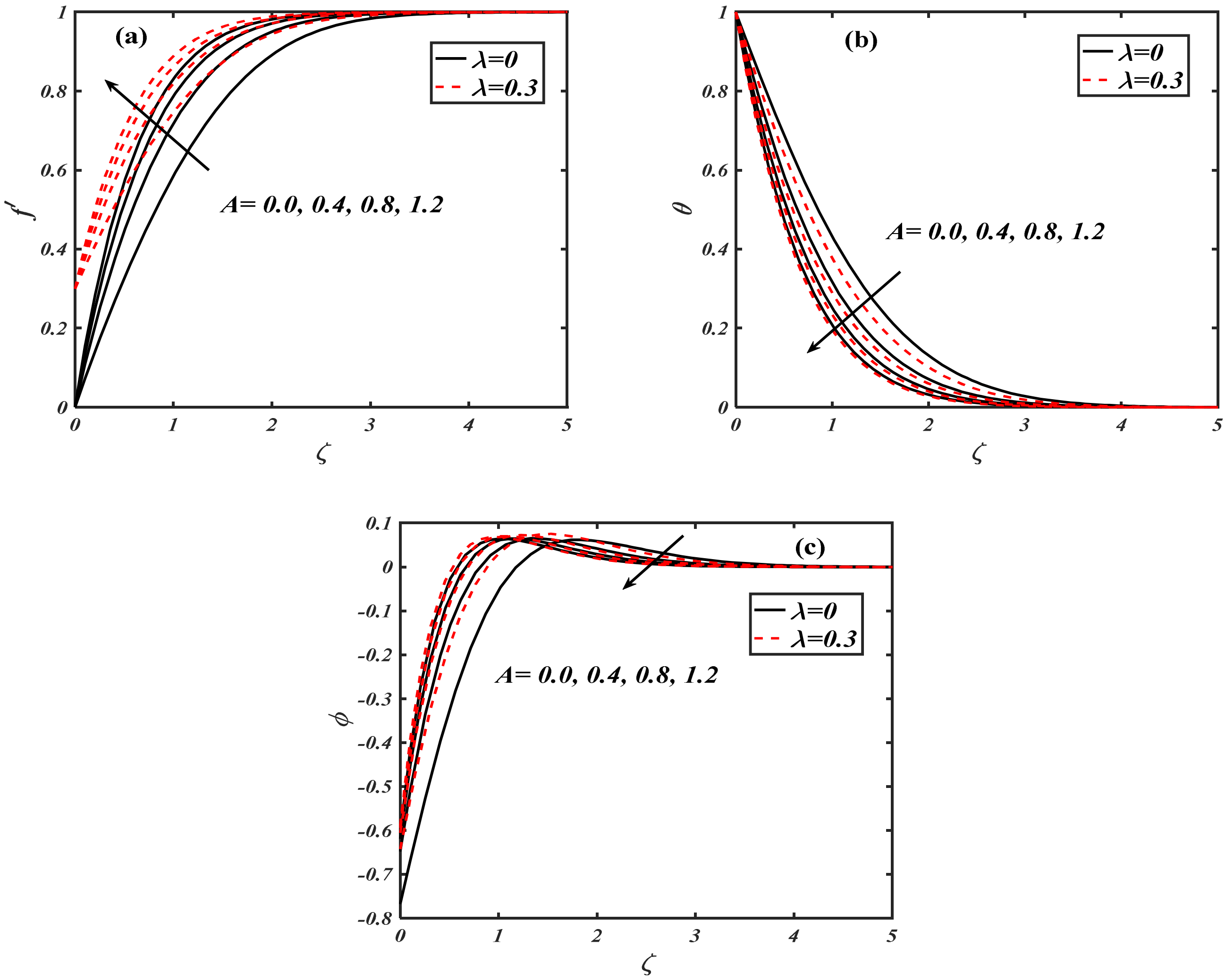

Fig. 2. Illustration of unsteady parameter A on (a) velocity, (b) temperature (c) concentration Profiles.

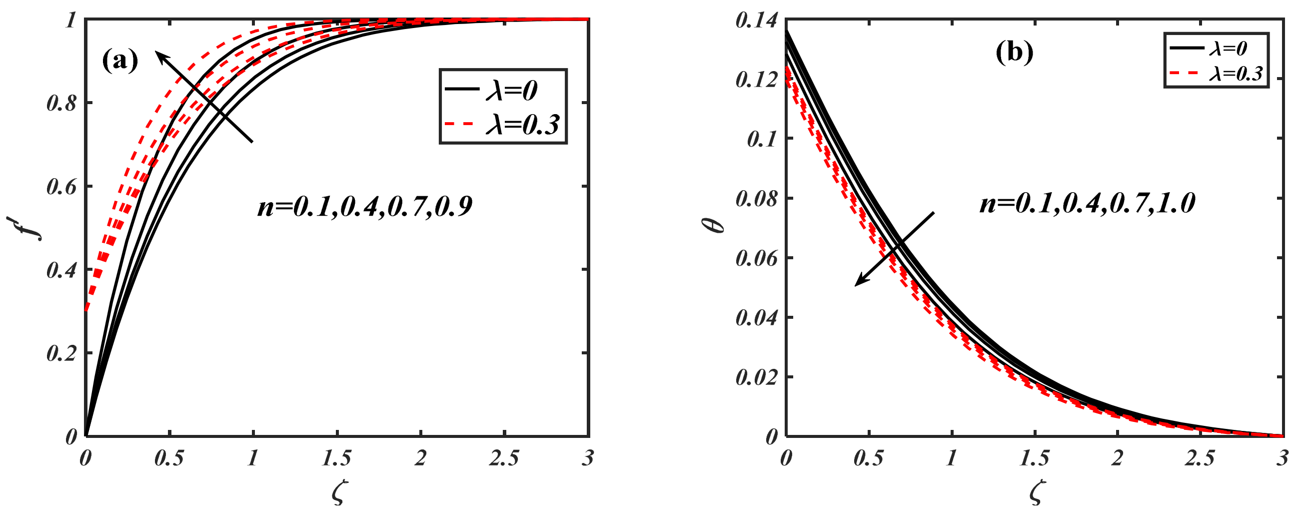

Fig. 3. Illustration of viscosity ratio parameter n on (a) velocity, (b) temperature Profiles.

$$C_f\sqrt{Re_x} = -\frac{1}{\sqrt{2-\beta}} f''(0)\times(\beta^* + (1-\beta^*)(1-Wef''(0))^{-1}), \tag{13}$$

$$\frac{Nu_x}{\sqrt{Re_x}} = -\frac{1}{\sqrt{2-\beta}}\theta'(0), \tag{14}$$

$$\frac{Sh_x}{\sqrt{Re_x}} = -\frac{1}{\sqrt{2-\beta}}\varphi'(0), \tag{15}$$

where $Re = u_w x / \nu$ is named as local Reynolds number.

3. Numerical Solution

By employing well known numerical technique known as shooting technique is used to reduce the highly coupled(non-linear) system of differential equations (7)-(9) by using the boundary conditions (10-11). These set of modified equations are then solved numerically by MATLAB built–in function bvp4c.

$$\begin{aligned}
&y_1' = y_2,\\
&y_2' = y_3,\\
&y_3' = \frac{1}{n+(1-n)+(1-Wey_3)^{-2}}((2-\beta)A(\frac{\zeta}{2}y_3+y_2-1)\\
&\quad +\beta(y_2^2-1)-ME_1+M(2-\beta)(y_2-1)),\\
&y_4' = y_5,\\
&y_5' = \frac{1}{(1+\varepsilon y_4)+0.33333Rd\left(1+(\theta_w-1)y_4\right)^3}(-\varepsilon 0.333Rd(\theta_w-1)\\
&\quad \times(1+(\theta_w-1)y_4)^2 y_5^2-\Pr(y_1y_5-2y_2y_4\\
&\quad -\frac{A}{2}(2-\beta)(\zeta y_5+3y_4)+Nby_5y_7+Nty_5^2,)\\
&y_6' = y_7,\\
&y_7' = \Pr Le\left(\frac{A}{2}(2-\beta)(\zeta y_7+3y_6)-(y_1y_7-2y_2y_6)\right)\\
&\quad -\frac{Nt}{Nb}y_5'+\Pr Le(1+\delta y_4)\exp\left(\frac{-E}{1+\delta y_4}\right)y_6.
\end{aligned}$$

with associate boundary conditions boundary conditions:

$$\begin{aligned}
&y_1(\zeta)=0,\ y_2(\zeta)=\lambda,\ y_5(\zeta)=-\gamma(2-\beta)^{1/2}(1-y_4(\zeta)),\\
&y_6(\zeta)=1,\ \text{at}\ \ \zeta=0,\\
&y_2(\zeta)\to 1,\ y_4(\zeta)\to 0,\ y_6(\zeta)\to 0,\ \text{as}\ \ \zeta\to\infty.
\end{aligned}$$

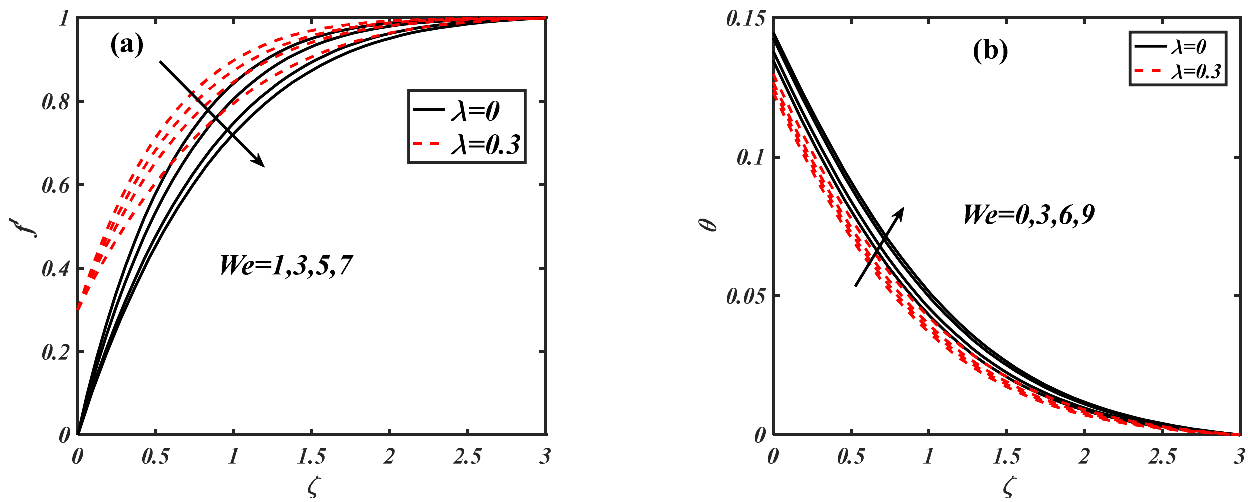

Fig. 4. Illustration of Weissenberg number *We* on (a) velocity, (b) temperature Profiles.

4. Results and Discussion

Figs. 2(a-c) exhibit the results under the effect of unsteadiness factor A on concentration, non-dimensional velocity and temperature fields. In Fig. 2(a) shows the behaviour of fluid flow at the huge values of the unsteadiness factor A makes a rushing behaviour in the Nanofluid velocity for both static and stretching wedge on $(\lambda=0, \lambda=0.3)$ respectively. In dual bags the velocity, momentum profile and boundary layer flow is advanced in case of stretching wedge $(\lambda=0.3)$. In Fig. 2(b) mirrors that the unsteadiness factor A shrinkages in cooperation of the related thermal boundary layer depth with energy profile. Really, in the increment of unsteadiness factor A makes opposite effects on the stretching sheet and loss its heat, under the shrank temperature of the nanofluid . Fig. 2(c) shows the behaviour likely as Fig. 2(b) as the influence of A over the concentration profile. A declivity is witnessed in both the boundary layer depth and concentration about the escalating values of the unsteadiness factor A . To elucidate the impact of viscosity ratio parameter.

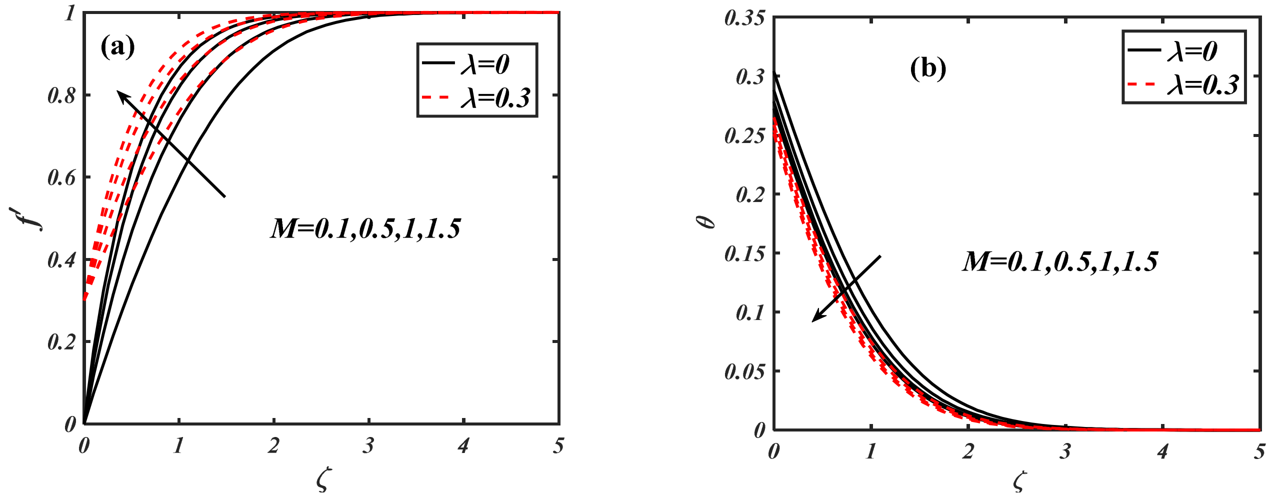

Fig. 5. Illustration of magnetic parameter M on (a) velocity, (b) temperature Profiles.

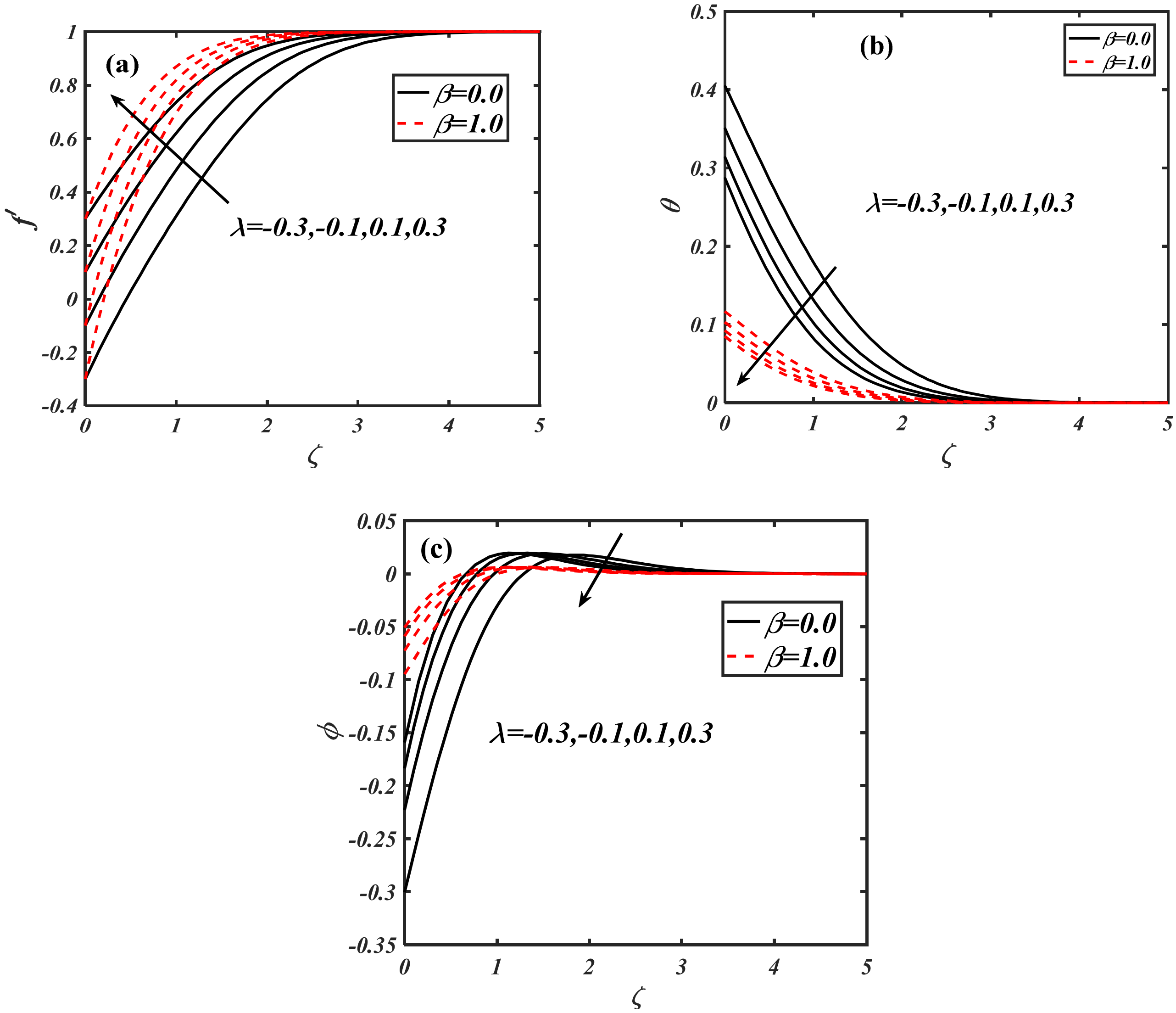

Fig. 6. Illustration of λ on (a) velocity, (b) temperature (c) concentration Profiles.

Figs. 3 (a-b) is plotted for testing the behaviour on velocity and temperature profiles for static and stretching wedges. Fig. 3 (a) is depicting the behaviour of viscosity ratio parameter on velocity profile. It has been observed that the velocity profile and the momentum boundary layer thickness get rises in both values i.e. $\lambda = 0$ and $\lambda = 0.3$. While Fig. 3 (b) is taken to illustrate the impact on temperature, it has been observed that both viscosity ratio parameter and temperature profile has inverse variation for both values $\lambda = 0$ and $\lambda = 0.3$ static and stretching wedges, respectively. To investigate the influence of Weissenberg number on temperature and velocity on wedges are graphed. Here we have two cases of wedges, $\lambda = 0$ and $\lambda = 0.3$ presenting static and stretching wedges.

Fig. 4 (a) is depicting the effect of We, on velocity parameter both on static and stretching wedges. It is observed that it shows a declining effect on both the wedges. Physically as we know that We, is the ratio of relaxation time to the processing time. Increasing the number of We, means taking long processing time that causes decrement in velocity. While Fig. 4(b) is displayed to analyze the behaviour on temperature coefficient. It is highly noticed that as we increase We, it results a rise in temperature for both cases $\lambda = 0$ and $\lambda = 0.3$. Physically evaluating We, means a rise in relaxation time that enhances the temperature of the fluid. The Figs. 5(a-b) sketched for magnetic factor M which shows the different influence of the magnetic effects on velocity and temperature profiles. Figs. 5 (a-b) are plotted to exhibit the behaviour of Magnetic parameter on velocity and temperature fields. Here we tested on two cases of wedges $\lambda = 0$ and $\lambda = 0.3$ for static and stretching wedges respectively. It is observed that the velocity parameter get rises when magnetic parameter is enhanced for both static and stretching cases. Fig. 5(b) is sketched to explain the effect of magnetic parameter on temperature field on both static and stretching wedges. It is inspected that temperature profiles show a declining effect on rising the value of magnetic parameter for $\lambda = 0$ and $\lambda = 0.3$ both.

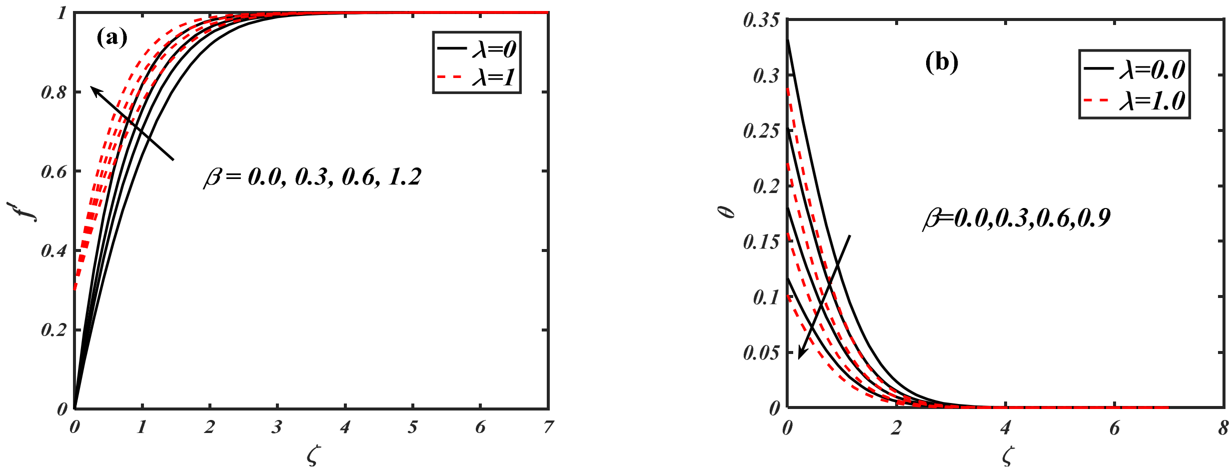

Fig. 7. Illustration of β on (a) velocity, (b) temperature Profiles.

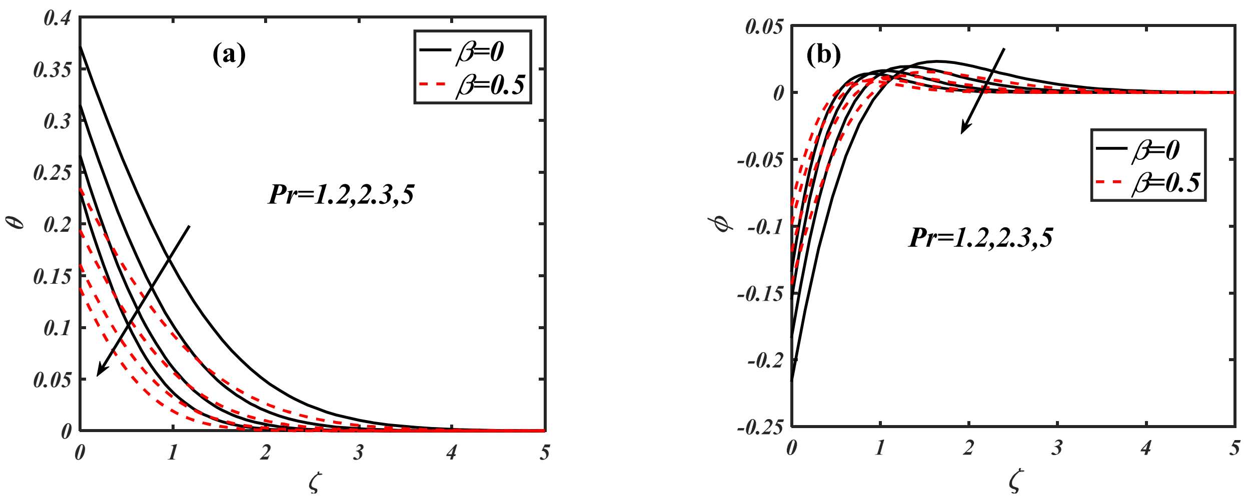

Fig. 8. Illustration of Pr on (a) temperature and (b) concentration Profiles.

Figs. 6 (a-c) illustrate the impact of the moving wedge parameter λ for two different values of the wedge angle parameter as $\beta = 0$ and $\beta = 1$. It can be noticed that from Fig. 6 (a) the velocity distribution profile boosted up as intensify the value of parameter λ against both values of wedge angle parameter as $\beta = 0$ and $\beta = 1$. But opposite behaviour is observed as magnify the value of moving wedge parameter λ, the temperature distribution and the nano particles concentration profile drown swing for both wedge angle parameter as $\beta = 0$ and $\beta = 1$ is illustrate in Figs. 6(b) and 6(c) respectively. Figs. 7(a-b) is sketched for the velocity field and the temperature distribution for the different values of the wedge angel parameter β with $\lambda = 0$ and $\lambda = 0.3$ for static and moving stretching wedges respectively. Similar effect is found for Fig. 7(a) as for unsteady parameter A, against the velocity field. As escalating the value of β, the velocity profile lifted up as depicted in Fig. 7(a), but retard the temperature distribution illustrated in Fig. 7(b). Physically, the wedge angel parameter illustrate the pressure gradient. As enhance the positive valued of the wedge angle parameter, cause the reasonable pressure gradient which escalate the flow. Figs 8(a-b) illustrate the effects of the Prandtl number on temperature distribution and the nanoparticles concentration profile with both values of wedge angle parameter as $\beta = 0$ and $\beta = 1$.

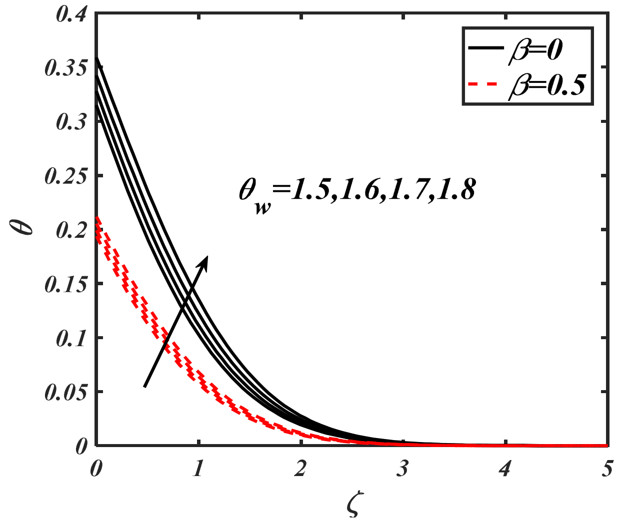

Fig. 9. Illustration of θ_w on temperature.

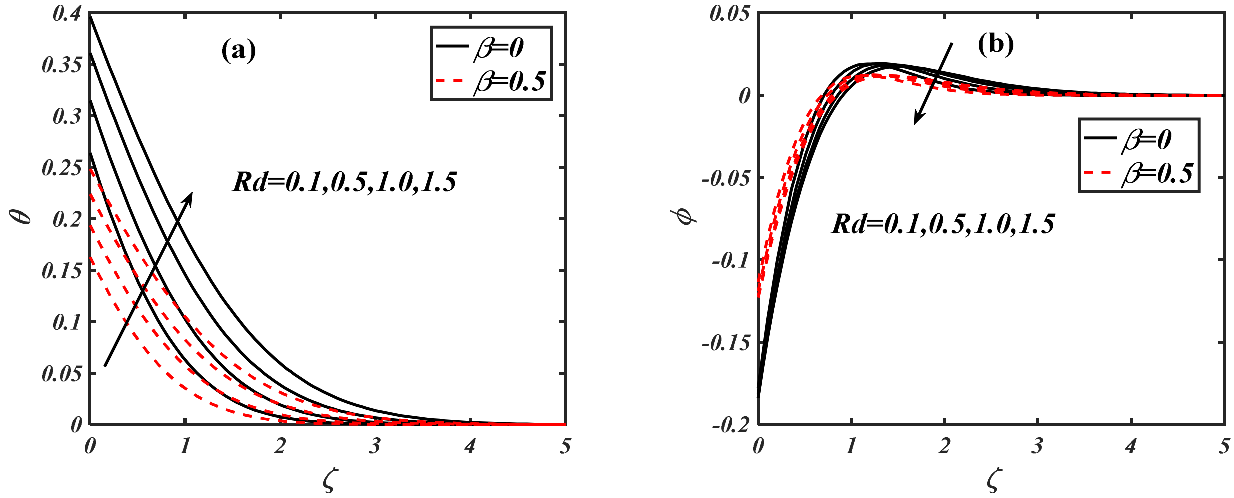

Fig. 10. Illustration of *Rd* on (a) temperature and (b) concentration Profiles.

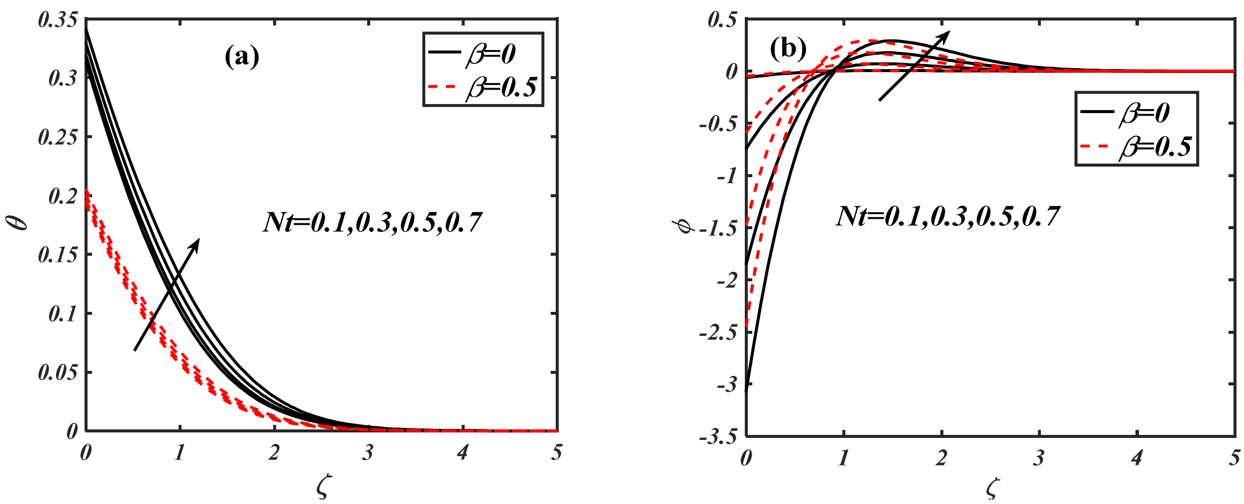

Fig. 11. Illustration of *Nt* on (a) temperature and (b) concentration Profiles.

As supplemented in the values Pr, a depreciation in the temperature and the concentration profile is observed. Fig. 9 demonstrate the impact of the non-linear thermal radiation on the temperature distribution. As enhance in the value of θ_w, the temperature profile become grow. Fig. 10(a-b) depict the impact of the radiation parameter on temperature distribution and the nanoparticles concentration profile with both values of wedge angle parameter as $\beta = 0$ and $\beta = 0.5$. As raised up the values of radiation parameter Rd, the temperature distribution upgrade is sketched in Fig. 10(a), but the concentration field diminished is shown in Fig. 10(b). Figs. 11(a-b) narrates the impact of the thermophoresis parameter Nt on temperature distribution and the nanoparticles concentration profile with both values of wedge angle parameter as $\beta = 0$ and $\beta = 0.5$. As raised up the values of thermophoresis parameter Nt, the temperature distribution and concentration field boosted up.

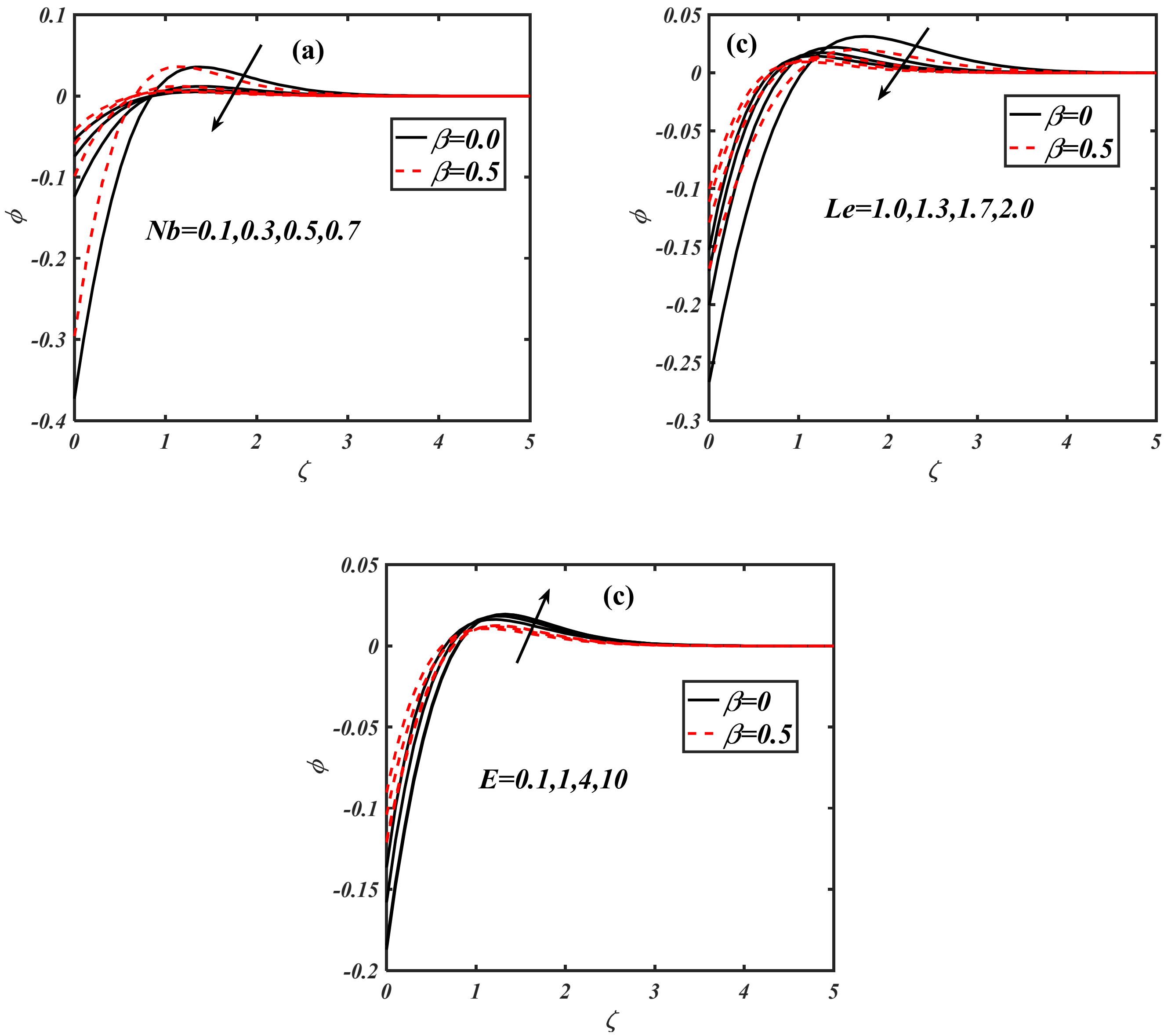

Fig. 12. Illustration of Nb, Le, E on concentration Profile.

Figs. 12(a-c) illustrate the impact of the Brownian motion parameter Nb, Lewis number Le and the activation energy parameter E on the nanoparticles concentration profile with both values of wedge angle parameter as $\beta = 0$ and $\beta = 0.5$. As enhance in the value of the Brownian motion parameter Nb, Lewis number Le, the concentration profile decline as shown in the Fig.12 (a) and Fig. 12 (b) respectively, but lift up as augment in the value of the activation energy depict in Fig. 12(c). The negative value of concentration doesn't means that the nanoparticles concentration is negative but the magnitued of nanoparticles concentration is alwyes positive. Since we have used convected Nield boundary conditions therefore the curve of concentration profile depicts negative behavior but overall, we achieve asymptotic behavior. Moreover, one should keep in mind that magnitude of nano-particles concentration always remains positive. Please see ref. [45].

Tables 1-3 illustrate the effects of the Weissenberg number We, wedge angle parameter β, viscosity ratio parameter n, unsteady parameter M, on the local skin friction coefficient $-f''(0)$, the local Nusselt number $-\theta'(0)$ Sherwood number $\varphi'(0)$ additionally with thermophoresis parameter Sherwood number Nt, Brownian motion parameter Nb and the radiation parameter Sherwood number Rd on the local Nusselt number $-\theta'(0)$ Sherwood number $\varphi'(0)$. It is noticed that increase in value of the unsteady parameter, and the magnetic parameter, the local skin-friction also enhance, but decrease as increase in the value of the Weissenberg number We, viscosity ratio parameter n. The values of local Nusselt number $-\theta'(0)$ rises directly with Pr but the value decline for increasing Rn as shown in Table 2. Table 3 depict the variation of Sherwood number $\varphi'(0)$ under the influence of various parameters. As increase in the value of the parameter p, Sherwood number $\varphi'(0)$ slowdown.

The outcomes are compared with those of others author's Rajagopal [40], Kuo [42], Ishaq et al. [43], and Khan et al. [44] and found to be in excellent agrement as dipect in Table 4.

Table 1. For local skin friction coefficient $-f''(0)$ under the influence of prominent parameters.

We	n	β	A	M	$f''(0)$
1	0.2	0.3	0.1	0.1	1.0461
2					1.0460
3					0.9628
4					0.9266
	0.3				1.1018
	0.4				1.0561
	0.5				1.0265
		0.2			1.1409
		0.5			1.2527
		0.7			1.3167
			0.2		1.3973
			0.3		1.5424
			0.4		1.6494
				0.3	1.3130
				0.5	1.4260
				0.7	1.5287

Table 2. Local Nusselt number under the influence of various parameters.

We	n	β	A	Nt	Nb	δ	Pr	R	$-\theta'(0)$
1	0.2	0.3	0.1	0.5	0.2	0.5	1	0.5	1.0460
2									0.8601
3									0.8465
4					$-\theta'(0)$				0.8389
	0.3								0.8574
	0.4								0.8522
	0.5								0.8486
		0.2							0.8623
		0.5							0.8708
		0.7							0.8736
			0.2						0.9631
			0.3						1.0462
			0.4						1.1202
				0.3					0.8769
				0.4					0.714
				0.6					0.8604
					0.3				0.8659
					0.4				0.8659
					0.5				0.8659
						0.2			0.8661
						0.4			0.8660
						0.6			0.8658
							3		1.2450
							5		1.4228
							7		1.5233
								0.3	0.9136
								0.7	0.8260
								0.9	0.7919

Table 3. Local Sherwood number $\varphi'(0)$ under the influence of prominent parameters.

We	n	β	A	Nt	Nb	δ	Pr	R	$-\phi'(0)$
1	0.2	0.3	0.1	0.5	0.2	0.5	1	0.5	1.0460
2									2.1502
3									2.1162
4									2.0972
	0.3								2.1435
	0.4								2.1305
	0.5								2.1213
		0.2							2.1559
		0.5							2.1769
		0.7							2.1841
			0.2						2.4077
			0.3						2.6155
			0.4						2.8004
				0.3					1.3153
				0.4					1.7427
				0.6					2.5813
					0.3				1.4431
					0.4				1.0823
					0.5				0.8659
						0.2			2.1653
						0.4			2.1649
						0.6			2.1645
							3		3.1125
							5		3.5571
							7		3.8083
								0.1	2.1652
								0.3	2.1650
								0.6	2.1646

Table 4. Comparison of present outcomes of with Rajagopal [42], Kuo [43], Ishaq et al. [44], and Khan et al. [45] as, $\beta^{*} = A = M = We = E = 0$

β	Rajagopal et al. [40]	Kuo [42]	Ishaq et al. [43]	Khan et al. [44]	Current result
0.0	-	0.469600	0.4696	0.469600	0.469605
0.1	0.587035	0.587080	0.5870	0.587035	0.587124
0.3	0.774755	0.774724	0.7748	0.774755	0.774758
0.5	0.927680	0.927905	0.9277	0.927680	0.927682
1.0	1.232585	1.232589	1.2326	1.232588	1.232689

5. Conclusion

The current numerical model is introduced to analyze the features of activation energy with nonlinear thermal radiation on magnetized Williamson nanofluid over a wedge. The Buongiorno model is used to characterize the Brownian and thermophoresis properties of nanofluid. The main outcomes are mentioned bellows

- The velocity profile increase with Weissenberg number while decline for magnetic parameters
- Temperature filed is enhanced by growing the value of temperature ratio parameter.
- The Larger wedge angle parameter reduces the temperature filed.
- Concentration of nanoparticles is boosted up by varying the amount of activation energy while it diminishes for Lewis number and Prandtl number.

Author Contributions

(G.A.D.); (M.I.); (M.T.)(H.W.); (M.I.A.); (A.A.) and (D.B.) modeled the problem, numerically computed results, discussed the results physically, computed the tabulated results, wrote the manuscript and proof read it.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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ORCID iD

Gulzar Ahmad Danish https://orcid.org/0000-0002-9003-0689
M. Imran https://orcid.org/0000-0002-2363-5039
Madeeha Tahir https://orcid.org/0000-0002-6634-2877
Hassan Waqas https://orcid.org/0000-0002-0388-8506
Muhammad Imran Asjad https://orcid.org/0000-0002-1484-5114
Ali Akgül https://orcid.org/0000-0001-9832-1424
Dumitru Baleanu https://orcid.org/0000-0002-0286-7244

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