Scaling Law of Permeability and Porosity for Fluid Transport Phenomena in Porous PCM Media

Yassine Hariti, Ahmed Hader, Hamza Faraji, Yahia Boughaleb

1 LGBIM, Ecole Normale Supérieure, University Hassan II, Casablanca, Morocco
2 Centre régional des métiers d’éducation et de formation Casablanca-Settat, établissement Settat, Morocco
3 Physics Department, LPMMAT Laboratory, Faculty of Sciences Ain Chock, Hassan II University, Casablanca, Morocco
4 LPMC, University Chouaib Doukkali. El Jadida, Morocco

Abstract. The present paper reports the numerical results of fluid flow in porous phase change materials (PCM) media. This is an important topic in potential scientific, technological and engineering fields especially latent heat storage. In this paper, we are only interested in the correlation between permeability and porosity of the porous media and not in latent storage. Fluid flow is characterized by many parameters mainly permeability and porosity. Many models have been proposed for the study of this phenomenon over the years. However, it can be modeled using the complex model that studies the characteristics of pore microstructure and fluid flow in porous media. This model is more accurate and realistic compared to previous models. It predicts permeability and porosity with a good agreement with experimental data. In this paper, the complex model is used to determine the impact of the tortuosity and the density of capillary distribution on the relation between permeability and porosity and check their scaling laws with universal exponents independently of other parameters. The results show that the permeability-porosity relation is proportional to the standard deviation of capillary distribution and its density. The tortuosity affects porosity proportionally, and permeability inversely. The relation between porosity and permeability follows a power law with universal exponents $\beta = 4.06 \pm 0.12$ for different values of the expectation of distribution, the density of capillaries and the tortuosity; and $\beta = 1.69 \pm 0.01$ for different values of the standard deviation, density of capillaries and tortuosity. The universality of these exponents further validates the complex model with various previous experimental and numerical studies.

Keywords: Scaling law, Permeability, Porosity, Porous media, Complex model, Fluid flow.

1. Introduction

In the last decades, various researches were done to understand the fluid transport process in different media especially through porous media. This process is used in diverse areas and presents a promising opportunity in scientific, industrial and technological fields [1], petroleum and oil engineering [2, 3] and hydrology [4–6]. Porous media were embarked in various fields of crucial necessity such as gas storage, oils manufacturing from reservoirs and contaminant recovery. Owing to their internal construction and their important surface to volume ratio giving rise to crucial characteristics of mass [7] and heat distribution mechanisms [8–9], they were developed in various industry processes such as filtration, manufacturing of fuel cells, wiping of mash paper, etc.

Recent investigation attempts were furnished to study the complexity of fluid flow in porous media structure continually [10–11]. Hence, various approaches have been embarked for the purpose of modeling and simulating fluid transport in porous media. For example, it could predict the effect of earthquakes caused in oil storage [12]. Moreover, this process of fluid transport in a stationary failure repartition was adopted for seismic attenuation. This could be observed if waves run via a fluid damaged pebble. If mechanical waves move across alike this media, the fluid pressure gradients are enhanced due to the heterogeneity of the damaged medium. The pressure gradients are stabilized via repartition of fluid in the damaged medium. This mechanism is dissipated and directed by pressure diffusion over the damages. Hence, studying fluid transport through porous media is not a new research concept. The concept producing mechanical reinforcement of fluid flow in porous media is that when the energy is transmitted into a porous medium, it could induce a sequence of impacts, enhance effective permeability, improve gravity segregation, droplets amalgamation, and change the characteristics of fluid and decrease networking during miscible fluids. Other processes could also produce maladaptation and transient particle rearrangement, which may generate a momentary modification in pore sizes [13]. Various fundamental physics are associated to mechanical excitation mechanisms like emulsification, cavitation, coagulation and streaming [15].

A porous medium is an assemblage of a solid matrix holding vacant spaces called pores where fluid flow takes place. This qualification eliminates the unoccupied parts that are surrounded by solid material because they do not participate in fluid flow
The important influence that pore pressure has on the deformation behavior of fluid saturated porous rocks was widely recognized. The pressure of pores produces a much known deformation behavior of fluid in porous media. The temporal and spatial evolution of pore pressure directed not only the fluid transport but also the field of effective applied load in the system mass, where it was jointed, pore pressure repartition may effectively provoke if sliding will happen alongside pre-remaining voids. A coherent theory, which clearly expressed the interaction between the matrix and pore fluid [14]. By analogy to composite materials, fibers are in interaction with a matrix. This interaction improves the material resistance as proved in references [15, 16].

In practice, the failures involve a Darcy-type fluid transport by a supplementary permeability quantity where, in series, and at the extremity of damage, the fluid pressure launches the failure process. These conceptions were protracted concerning failure propagation in various heterogeneous systems and especially for saturated porous media. Moreover, Stokes-type transport, breakdown phase field conception and diffuse interface expressions were integrated in the porous media theory [10]. However, quantitative conceptions for the problem by stimulation were difficult to evaluate, if existent at all. This permits us to identify hydro-mechanical proprieties of fluid transport in porous media, attained via the diffuse interface algorithm, and compared to appropriate numerical, analytical, or experimental results [10].

Although the fundamental concepts that drive the motion of fluid in porous media were well studied by engineers, a few details were not identified about the flow proprieties in low-permeable porous media (shale, clays, etc.). For motion considerations, the principal factors were gradients of fluid potential, permeability and porosity. For shales and clays, these factors were unwell recognized. For example, the hydrocarbons transport coincides with the following petrol motion subsequently ejection from the origin pebble via the basins of water or failures and voids via permeability [14].

The importance of the problem of fluid flow in porous PCM media stems from its various important application in the framework of crucial fields such as the latent energy storage with PCMs. Thus, the capacity to identify and to recognize the fluid transport process allows experts whether the medium is appropriate for utilization. Otherwise, during exploitation, a device is delivered for observing the fluid transport, namely the creation and the propagation of fluid flow. Additionally there are well proved expressions presented which consider the void as pointed interfaces and which may be developed to compare, quantitatively, the approach of phase field [12]. In this paper, the main novelty is the usage of the complex model to determine the impact of the tortuosity and the density of the capillaries distribution on the relation between permeability and porosity, and check their scaling laws for the different parameters: the standard deviation of the pores distribution, and the distribution expectation, for this model and previous ones.

2. Mathematical Modelling

Fluid flow in porous media is still not well understood. Because of the complexity and disordered porous media microstructures, it is complicated to characterize the flow process clearly according to the transport parameters: porosity [17–19], thermal conductivity [20, 21], mechanical proprieties [22, 23] and permeability [18, 19]. The permeability and porosity of such media -crucial parameters in fluid flow- are generally studied numerically, analytically and experimentally. Various models have been established to describe transport phenomena in porous media and to compare numerical results with experiments analysis [24, 25].

Diverse scales may be encountered when studying fluid flow in a porous media. At the pore scale (microscopic scale), the characteristic sizes are in the order of the pores. Hence, variables like disposition of packages or roughness of pore walls are measured via imaging techniques and tools (Scanning Electron Microscopy, tomography, …). The second scale is the macroscopic scale that matches the size of a porous medium core. Characteristics like porosity and permeability are studied with macroscopic scale to be used as tools for simulations to predict fluid flow. Supplementary intermediate or larger scales may be established in hydrology and fluid engineering. Moving from a scale to another is reached via methods of volume averaging or homogenization. The importance of describing a porous medium at the pore scale is to control, recognize and simulate its behavior at the macroscopic level. To be precise, it permits simulating fluid flow and distribution inside the pore [10].

The complex model considers porous media as a set of tortuous capillaries with random diameters distributed randomly in a solid frame, see Fig. 1. The fluid flow is Newtonian. To facilitate the calculations, we saturate the studied porous medium with a fluid in a single phase. The capillaries are assumed to be smooth. For simulation purposes, the isothermal flow obeys Darcy’s law while ignoring the load applied on capillaries and gravity influence.

The porous medium flow rate is the sum of all the capillaries flow rates, which may be formulated as follows:

\[
Q = \int_{r_{\text{min}}}^{r_{\text{max}}} q(2r) \, dN
\]

where \(Q\) is the porous medium flow rate, \(N\) is its capillaries total number, and \(2r_{\text{min}}\) and \(2r_{\text{max}}\) are, respectively, the minimal and maximal capillaries radii, and \(q(2r)\) presents the function of the flow rate, via a capillary. Based on statistical principles, \(dN\) can be given as:
\[ dN = 2N_f(2r)dr \]  
\[ \text{where } f(2r) \text{ is the probability density function of capillaries in the porous medium sectional area [26].} \]

Through pivotal experiences in relation to porous media transport, Henry Darcy emphasized in 1856 a linear law existing between the fluid flow rate \( Q \) in a porous medium and the pressure gradient applied amid the entry and withdrawal of medium faces [14]. According to the law of Darcy [11], the porous medium permeability is given by:

\[ K = \frac{QL}{A\Delta P} \]  
\[ \text{where } A \text{ and } L \text{ are the specific area and the length of the porous medium, respectively.} \]

\( L \) is the distance, which the fluid traverses, \( A \) is the porous medium area, \( \mu \) is the fluid viscosity, \( \Delta P \) is the pressure gradient and \( K \) is the permeability of the porous material, which shows the ability of a porous medium to let a fluid flow without changing its interior structure. This material intrinsic characteristic does not depend on the flowing fluid type but merely on the porous medium geometry. The parameter \( K \) is generally named the medium absolute permeability. There is no clear relation between this parameter and porosity. Moreover, various models focus on the pore size importance [26].

Combining equations (1), (2) and (3), the permeability formula of the adopted model is given by:

\[ K = \frac{2LN_f}{\Delta P} \int_{2r}^{2r_{max}} q(2r) f(2r) dr \]  
\[ \text{Moreover, the pore volume is defined by the fractal theory [27] as:} \]

\[ V_p = \int_{2r_{min}}^{2r_{max}} F_p r^2 L_p(2r) dN = 2F_p N \int_{2r_{min}}^{2r_{max}} L_p(2r) f(2r) r^2 dr \]  
\[ \text{where } V_p \text{ is the volume of the capillary, } L_p(2r) \text{ is its length function and } F_p \text{ is its shape factor. It is important to mention that a circular section capillary must be equal to } \pi \text{ [26].} \]

The porous medium porosity is expressed by:

\[ S = \frac{V_p}{V} \]  
\[ \text{where } S \text{ is the medium porosity and } V = L^3 \text{ is the medium volume.} \]

When substituting equation (5) in (6), the porosity formulation of the adopted model is given by:

\[ S = \frac{2F_pN}{V} \int_{2r_{min}}^{2r_{max}} L_p(2r) f(2r) r^2 dr \]  
\[ \text{The flow rate for a fluid flowing through a capillary is established in [34] and expressed by:} \]

\[ q(2r) = \frac{16F_p r^4 \Delta P}{128\mu} \]  
\[ \text{where } q(2r) \text{ is the function of flow rate for a solitary tortuous capillary with a radius } r. \text{ It shows the fluid flow characteristics in a capillary. } q(2r) \text{ contains the main function, which is the permeability factor of a capillary that equals } \pi \text{ for circular capillaries [26].} \]

Tortuosity presents the ratio of the current length of a tortuous capillary and the length of a straight one with the same ends, see Fig. 2. These parameters are related by the equation below:

\[ L_t = \tau L \]  
\[ \text{with } \tau \text{ is the tortuosity of a capillary.} \]

The probability density function of a normal distribution can be written as:

\[ f(2r, \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(2r-\mu)^2}{2\sigma^2}} \]  

\[ \text{Fig. 2. A schematic representing straight and tortuous capillary.} \]
where \( \mu_n \) is the distribution expectation and \( \sigma_n \) represents the standard deviation [26].

By substituting equations (8), (9), and (10) in equation (4), the permeability formula becomes:

\[
K = \frac{NF}{4A} r^{\nu_{\text{max}}} \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} \frac{r^l}{\sigma_n \sqrt{2\pi}} e^{-\frac{(r-\mu_r)^2}{2\sigma_r^2}} dr
\]

(11)

By considering all capillaries in porous media are circular and injecting the equations (9) and (10) in equation (7), the porosity formula becomes:

\[
S = \frac{2\tau LNF}{V} \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} \frac{r^l}{\sigma_n \sqrt{2\pi}} e^{-\frac{(r-\mu_r)^2}{2\sigma_r^2}} dr
\]

(12)

Thus, equation (12) can be rewritten as:

\[
S = \frac{2\tau LNF}{A} \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} \frac{r^l}{\sigma_n \sqrt{2\pi}} e^{-\frac{(r-\mu_r)^2}{2\sigma_r^2}} dr
\]

(13)

In this study, we investigate the relation between porosity and permeability of porous media using the complex model proposed by Xiao-Hua Tan et al. [26]. This model assumes porous media to be composed of capillary bundles with random radii distributed following a normal distribution in a solid frame. They validated the theoretical results with experimental data that proved the accuracy and realism of this complex model. However, those results did not show the scaling behavior of permeability and porosity with respect to the model parameters, which is the aim of this paper. First, we focus on the relation between permeability and porosity versus the tortuosity of the capillaries, their density, the standard deviation of their distribution throughout the solid frame and the expectation of the distribution that gives an idea about the mean diameter of the capillaries. Then, we check the scaling laws for permeability-porosity relation according to the aforementioned quantities. This relation presents a power law with two universal exponents comparable to previously calculated values by different models [28–32].

Fig. 3. The relation between permeability and porosity versus standard deviation \( \sigma_n \), tortuosity \( \tau \) and density of capillaries N/A.
3. Results and Discussions

The objective of this study is to analyze the impact of the tortuosity and the density of the capillaries distribution on the relation between permeability and porosity and check their scaling laws with universal exponents independently of the parameters. Numerical simulations were made using the standard deviation $\sigma_n$ and the expectation of distribution $\mu_n$ as control parameters. The standard deviation $\sigma_n$ varies from $0.2 \times 10^{-6}$ m to $0.7 \times 10^{-6}$ m and the expectation of distribution $\mu_n$ varies from $0.3 \times 10^{-6}$ m to $2.4 \times 10^{-6}$ m. The computational results of this study are presented in terms of permeability variation according to the porosity.

3.1 The effect of tortuosity, standard deviation of the capillaries distribution and density of capillaries on the relation between the permeability and porosity

Figures 3a and 3b represent the effect of the standard deviation $\sigma_n$ on the relation between permeability and porosity in porous media. With the increase of the standard deviation $\sigma_n$ from $0.2 \times 10^{-6}$ m to $0.7 \times 10^{-6}$ m, a slow evolution of the curve is shown upwards. Accordingly, the porosity increases from $0.018–0.029$ to $0.383–0.398$. Furthermore, the permeability increases from $0.034–35.374 \times 10^{-15}$ m$^2$ to $83.651–101.231 \times 10^{-15}$ m$^2$. The curve moves slowly upwards with the increase of the porosity. Accordingly, the porosity increases from $0.005–0.274$ to $0.021–0.294$. The permeability increases from $0.034–35.374 \times 10^{-15}$ m$^2$ to $83.651–101.231 \times 10^{-15}$ m$^2$. The computational results of this study are presented in terms of permeability variation according to the porosity.

Figure 3 shows the effect of the standard deviation and tortuosity on the permeability and porosity, to clarify the relationship between porosity and permeability even more, figure 4 plots the log-log of the permeability and porosity versus standard deviation $\sigma_n$ and tortuosity $\tau$ (Figs. 4a and 4b), and density of capillaries (Figs. 4c and 4d). The plots show that the variation of $\ln(10^{-15}K)$ versus $\ln(S)$ is fitted with a linear function such that: $\ln(10^{-15}K) = \beta \ln(S) + C$ where $\beta$ is the slope and $C$ is the intercept. Therefore, the permeability increases as a power law with porosity as:
where the exponent $\beta = 1.69 \pm 0.01$ for different values of standard deviation, density of capillaries and tortuosity. This scaling law holds true for each value of these parameters, which proves its universal character.

This power law has been established based on the Kozeny-Carman equation (Dastidar et al. [28], Kolodzi [29], Pittman [30] and Hariti et al. [31]). The obtained result for the $\beta$ exponent in this case is close to that found by Kolodzi [29].

3.2 The effect of tortuosity, expectation of distribution and density of capillaries on the relation between the permeability and porosity

Figures 5a and 5b represent the influence of the expectation of distribution $\mu_n$, tortuosity $\tau$ and density of capillaries $N/A$. The relation between permeability and porosity versus the expectation of distribution $\mu_n$, tortuosity $\tau$ and density of capillaries $N/A$.

![Fig. 5. The relation between permeability and porosity versus the expectation of distribution $\mu_n$, tortuosity $\tau$ and density of capillaries $N/A$.](image)

Scaling Law of Permeability and Porosity for Fluid Transport Phenomena in Porous PCM Media

This power law has been established based on the Kozeny-Carman equation (Dastidar et al. [28], Kolodzi [29], Pittman [30] and Hariti et al. [31]). The obtained result for the $\beta$ exponent in this case is close to that found by Kolodzi [29].

$$K \approx 10^{-15} S^\beta$$

3.2 The effect of tortuosity, expectation of distribution and density of capillaries on the relation between the permeability and porosity

Figures 5a and 5b represent the influence of the expectation of distribution $\mu_n$ on the relation between permeability and porosity in porous media. The curve moves slowly upwards with the increase of the expectation of distribution $\mu_n$ from $0.3 \times 10^{-4}$ m to $2.4 \times 10^{-4}$ m, accordingly. The porosity increases from 0.008 – 0.029 to 0.370-0.398, and the permeability increases from 0.066–1.703 $10^{-15}$ m$^2$ to 68.904-101.231 $10^{-15}$ m$^2$ for $\tau = 1$ (straight capillaries in complex model). The porosity increases from 0.037 to 0.039, and the permeability increases from 692.169 $10^{-15}$ m$^2$ to 1016.9 $10^{-15}$ m$^2$ for $\tau = 0.1$ (tortuous capillaries in complex model) and $\mu_n = 2.4 \times 10^{-4}$ m.

Figures 5c and 5d show the impact of the expectation of distribution $\mu_n$ on the relation between permeability and porosity in porous media. The curve moves slowly upwards with the increase of the expectation of distribution $\mu_n$ from $0.3 \times 10^{-4}$ m to $2.4 \times 10^{-4}$ m. The porosity increases from 0.005 – 0.021 to 0.274- 0.294, and the permeability increases from 0.034-0.874 $10^{-15}$ m$^2$ to 35.374-51.970 $10^{-15}$ m$^2$ for $N / A = 0.05 \times 10^{-15}$ m$^2$. The porosity increases from 0.448 to 0.481, and the permeability increases from 34.608 $10^{-15}$ m$^2$ to 50.845 $10^{-15}$ m$^2$ for $N / A = 0.08116.10^{-15}$ m$^2$ and $\mu_n = 2.4 \times 10^{-4}$ m.

These results can be explained by the theory of probability density function. The belief of distribution $\mu_n$ characterizes the average capillary diameter of porous media in the normal distribution function. The more the average capillary diameter becomes higher, the more $\mu_n$ increases. The permeability and porosity both increase by increasing the average capillary diameter [26]. Thus, both quantities are proportional to $\mu_n$, and density of capillaries $N/A$. On the other hand, $\tau$ influences the porosity proportionally and permeability inversely. In addition, the larger the capillary diameter, the less friction there is between the fluid and the inner section of the capillary. This facilitates the flow of fluid through the capillaries and increases the permeability and porosity of the porous medium.
The plots show that the variation in permeability is the slope and \( C \) is the intercept. Therefore, the exponent in this case is close to that found by Dastidar et al. [28].

The tortuosity affected porosity proportionally, and permeability inversely. The relation between porosity and permeability even more, Fig. 6 plots the log-log of the permeability and porosity versus standard deviation, fitted with a linear function such that:

\[
\ln(K_{10 \mu m}) = \beta \ln(S) + C
\]

where the exponent \( \beta = 4.06 \pm 0.12 \) for different values of expectation of distribution, density of capillaries N/A and tortuosity. This scaling law holds true for each value of the standard deviation, the distribution and tortuosity, which proves its universal character.

This power law has been established based on the Kozeny-Carman equation (Dastidar et al. [28], Kolodzie [29], Pittman [30] and Hariti et al. [31]). The obtained result for the \( \beta \) exponent in this case is close to that found by Dastidar et al. [28].

4. Conclusion

This investigation reviewed the previous models for permeability correlations, and adopted the complex model proposed by Xiao-Hua Tan et al. [26]. The results showed that the permeability-porosity relation was proportional to the standard deviation of the capillaries distribution and its density. The tortuosity affected porosity proportionally, and permeability inversely. The relation between porosity and permeability followed a power law with universal exponents \( \beta = 4.06 \pm 0.12 \) (comparable to that found by Dastidar et al [28] for different values of the expectation of distribution, the density of capillaries N/A and the tortuosity); and \( \beta = 1.69 \pm 0.01 \) (comparable to that found by Kolodzie [29] for different values of the standard deviation, density of capillaries and tortuosity). The universality of these exponents validated the complex further. This relation between permeability and porosity follows the Kozeny-Carman equation (Dastidar et al. [28], Kolodzie [29], Pittman [30] and Hariti et al. [31]).

Author Contributions

Yassine Hariti: Conceptualization, methodology, software, conducted analyzed the empirical results, developed the mathematical modeling, and examined the theory validation, writing - original draft, visualization, data curation, investigation;
Ahmed Hader: Supervision, validation, conceptualization, methodology, software, writing - original draft, visualization, data curation, investigation. Hamza Faraji: Conceptualization, Writing - review & editing, investigation. Yahia Boughaleb: Supervision, validation, conceptualization, methodology, software, writing - original draft, visualization, data curation, investigation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

Conflict of Interest
The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

Funding
The authors received no financial support of a significant nature in support of the research in this article.

Nomenclature

\[ Q \] Porous medium flow rate [m\(^3\)/s]  
\[ \mu \] Fluid viscosity [Pa.s]  
\[ f(x) \] Function of the flow rate, through a capillary  
\[ f(x) \] Probability density function of capillaries in the Porous medium sectional area  
\[ A \] Cross-section area of micro-channel [m\(^2\)]  
\[ L \] Distance which the fluid traverses [m]  
\[ \Delta P \] Pressure gradient [Pa]  
\[ K \] Permeability of a porous medium [m\(^2\)]  
\[ \delta \] Distribution expectation [m]  
\[ \sigma \] Standard deviation [m]  

References


ORCID iD
Yassine Hariti https://orcid.org/0000-0002-7308-8467
Ahmed Hader https://orcid.org/ 0000-0002-0597-2576
Hamza Faraji https://orcid.org/0000-0003-2792-4986
Yahia Boughaleb https://orcid.org/0000-0002-1572-0401

© 2020 by the authors. Licensee SCU, Ahvaz, Iran. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC 4.0 license) (http://creativecommons.org/licenses/by-nc/4.0/).