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Research Paper

# New Exact Traveling Wave Solutions for Fractional Order System Describing the Second Grade Fluid through Medium with Heat Transfer

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**Abstract.** The aim of this paper is to determine the time-dependent MHD fractionalized three-dimensional flow of viscoelastic fluid in porous medium with heat transfer by traveling wave solution. The governing nonlinear partial differential equations are altered by utilizing the wave parameter $\xi = lx + my + nz + \omega\, t^{\beta}/\Gamma(\beta+1)$ into ordinary differential equations. The exact solutions are attained by applying a traveling wave method for two different cases. Here we discuss some precise cases, the solution of MHD Newtonian fluid in porosity can be obtained by substituting $\alpha_1 \to 0$ in general solution. The impact of important governing parameters on the movement of a fluid is examined as well as the comparison of Newtonian and non-viscous fluids have been made by 2D and 3D graphical analysis.

**Keywords**: Magnetohydrodynamic (MHD), Traveling wave method, Second grade fluid, Visco-elastic fluid, 2D and 3D graphs.

## 1. Introduction

The exploration of non-Newtonian fluids has received much concentration due to their practical application. Non-Newtonian properties are demonstrated by various industrial liquids such as clay coatings, greases, polymer melts, extrusion of molten plastic, food and polymer processing, waste liquid, gas and oil well drilling and many emulsions. For example, shampoo, blood, drilling mud, polymer melts, and other emulsions are considered a non-Newtonian liquid. Due to the large variety in the physical structure of non-viscous liquids, it is intricate to purpose a single model that shows all the properties of non-viscous liquids. Therefore, many non-viscous models such as differential type, ratio type and integral type liquids have been proposed in recent years. Among these models, differential-type liquids attracted special attention. In addition, subclass of differential-type liquids, second-class liquids, has been successfully studied by distinct scholars in various types of flows [1 - 4]. Coleman and Noll [5] were the initially to construct the second-class liquid model. Later, this framework was applied to study many problems with a comparatively simple development. Abbas et al. [6] explored the properties of von Kamran stream of non-viscous nanofluid model. They calculated the rotating phenomenon using Oldroyd-B liquid. Gao et al. [7] explored the numerical solution of the disease system of coronavirus by fractional natural decomposition method. Ghadikolaei et al. [8] examined the exact solutions of viscoelastic fluid over a stretching sheet using homotopy perturbation method. Hayat et al. [9] investigated the two-dimensional stream of non-viscous fluid with heat flux by using homotopic convergent solution. Danish et al. [10] studied the Caputo-Fabrizio MHD fractionalized second grade fluid to compute the excellent result between numerical and analytical solutions by Laplace transform and modified Bessel function. Raza et al. [11] inspected the semi analytical solutions of unsteady flow of fractional derivative and twice grade fluid by Laplace transform. Jmail et al. [12] addressed the analytic solutions of fractionalized twice grade fluid because of oscillating flow with the finite Hankel and Laplace transform. Sene [13] studied the approximate solution for stokes problem in the background of fractional derivative by applying the integral balance method. Khan et al. [14] explored the stagnations point flow of casson liquid with MHD on stretching sheet to find the numerical solutions by shooting method. Shen et al. [15] inspected the unsteady second grade nanofluid with heat transfer using the Caputo and Caputo-Fabrizio derivative. Ghadikolaei et al. [16] studied the twice grade fluid of 2-D flow with heat transfer by HPM method to examine the result of solving temperature and velocity equation. Khan et.al [17] analyzed the new analytical solutions of unsteady flow of the type of non-viscous fluid with MHD by Laplace transform to express the solutions of the form in Fox H-function. Some most recent exploration about exact solutions materials can be distinguished in [18 - 24].

The influence of the magnetic field on the flow of a porous medium has gained intensifying thought during the years. Magnetic fields are effectively used to manage convection for electrically leading fluid, for example, incandescent melting and metals, composites and semiconductors. Correspondence between magnetic flux and electrical conductivity has always been investigated

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compared to industrial functions in cooling the pump counters, atomic reactors, and thermal reactor by the research assistant [25 - 26]. Zafar et al. [27] attained the accurate solutions of MHD fractional second grade fluid through porous medium by integral transform. In chemical engineering, liquid flow through the porous media is essential in some areas, such as suction effects and agricultural engineering for the filtration process. Fluid passage through the porous medium, for example aerogels, infusion of sludges, permeable rocks, compounds, slurries or clays, foams, and has several applications for bonding meetings to supplement foamy liquid, and most uses are to squeeze fluids through rocks infusion to reinforce rocks to improve wells as well as oil recovery.

Heat transfer and flow of a liquid through porous channels are very significant together in science and engineering. Their uses have been found in various regions, such as oil exploration, geothermal viability extraction, boundary layer control, discarding polymer fluids, hardening of the liquid precious stone, cooling a metallic plate in the shower, MHD control generators and suspension arrangements. The flow of heat transfer non-viscous liquids is critical in a number of industrial procedures, such as foodstuffs, organic liquids, multiphase mixtures and milk waste, land [28]. Imran et al. [29] investigated the differential type of fluid with mass and heat transfer over the Caputo-fractional derivative by Laplace transform. They showed the obtain solution in terms of G-function, Wright's function, Robotnov-Hartley and Mittage-Leffler function. It is very important to examine the mechanism of viscoelastic fluids. It has many applications in many industries, such as the chemical industry, bioengineering and oil exploitation. Viscoelasticity is the property of fluids that have a major impact on the movement of fluids or the movement of bodies through fluids, even at a micro level. The most important phenomenon corresponding to viscoelasticity is blood circulation, high-speed vehicles and aircraft etc [30].

Fractional calculus (FC) appeared during Newton's time, but recently attracted the concentration of a lot of information. Since the most recently three decades, the most charming rise in engineering and scientific applications have been determined within the FC context. Due to the complexities associated with the heterogeneity phenomenon, the concept of fractional derivatives has been examined by various specialist around the world. The fractional differential equations (FDEs) have been exploited to apprehend the behavior of a versatile medium with diffusion process. There have been basic and necessary tools and various problems are shown more arbitrarily and more precisely with differential equations in arbitrary order. Due to the recent advancement of mathematical producers using computer software, several scientists have begun working on generalized calculations to current their perspectives while examining several complicated cases. Copious leading orientations have been foreseen by many senior researchers for various definitions of FC, which has prepared the foundation in advance [31 - 33]. Differential equations that include differentiation of non-negative integer-order and can be sufficient models for different physical phenomena. The fractional differential equations are depicted in numerous significant phenomena such as material science, viscoelasticity, electromagnetics and electrochemistry. This interest is motivated because it is broadly applied in various fields of petroleum and chemical industries, geophysics and biorheology [34]. Some analyses with the fractional derivatives can be noticed [35 – 38].

The NLPDEs are examined in some areas for example biology, plasma physics, condensed matter physics, solid-state physics, fluid mechanics and applied mathematics [39 – 40]. These nonlinear phenomena must be understood better, then it is essential to determine their accurate solutions. They assist to discuss the solidity of these solutions and analyze the results by making 2D and 3D sketches of the exact solutions. Therefore, finding exact traveling wave solutions from many available other methods is a charming job from a theoretical, applied point of opinion, and also include the properties of traveling wave [41 – 44]. Several numerical and analytical methods have been recommended to determine accurate solutions of NLFPDEs perform a dynamic character in nonlinear science. A kind of numerous reliable methods have been recommended to acquire the accurate solutions of non-linear fractional partial differential equations such as finite element method [45], reduced differential transform method [46], Adomian decomposition method [47], exp-function method [48], (G'/G)–expansion method [49]. There are certain noteworthy studies associated with traveling wave method that come into view in [50 – 54].

In this study, we have discussed the three-dimensional flow of magnetohydrodynamic (MHD) fractionalized second grade fluid in a porous medium with heat transfer by using the traveling wave solution. The mathematical model that studied here is delicately settled in and judge with various standard cases. It is observed that the current model can cover such standard cases and provide novel material on the parameter progression of the dynamic system. The governing equations that we deliberated here are already occurred [56 – 58]. Sajid et al. [56] researched the 2D flow of the type of non-viscous fluid through HAM solution. They found the analytical solution for unsteady non-viscous fluid model with the help of HAM. Our aim is to develop the analysis of [56] to three dimensional flow of MHD fractionalized non-viscous fluid model in a porous medium with heat transfer by traveling wave method for finding the exact solutions. The assortments of new exact traveling wave solutions are accomplished for two different cases. In particular case, the solution of MHD viscous fluid in porosity can be obtained by substituting $\alpha_1 \to 0$ in general solution. Finally, the graphical inferences have been inspected 2D and 3D illustration. Keeping this in view, we acquire the 3D flow of second grade fluid and a hypothetical development and methodology in section 2. We get a set of exact solutions of the system of the equation in section 3. In section 4, we have discussed the results of the graphs. The conclusions are presented in section 5. In the end, the Appendix has also been mentioned in section 6.

## 2. Basic governing equations

We have considered the unsteady three-dimensional flow of MHD fractionalized second grade fluid in a porous medium with heat transfer. The governing equations are the grouping of conservation of mass equation, second grade fluid equation with MHD in porous influence and last equation is conservation of energy equation which are compiled as follows [56 - 58].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\begin{aligned}
&\frac{\partial^{\beta} u}{\partial t^{\beta}}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}=\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\alpha_1}{\rho}\Bigg[\frac{\partial^3 u}{\partial x^2\partial t}+\frac{\partial^3 u}{\partial y^2\partial t}+\frac{\partial^3 u}{\partial z^2\partial t}+u\frac{\partial^3 u}{\partial x^3}\\
&+v\frac{\partial^3 u}{\partial x^2\partial y}+w\frac{\partial^3 u}{\partial x^2\partial z}+v\frac{\partial^3 u}{\partial x^3}+u\frac{\partial^3 u}{\partial x\partial y^2}+w\frac{\partial^3 u}{\partial y^2\partial z}+w\frac{\partial^3 u}{\partial z^3}+u\frac{\partial^3 u}{\partial z^2\partial x}+v\frac{\partial^3 u}{\partial z^2\partial y}+2\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial x}+2\frac{\partial^2 v}{\partial x^2}\frac{\partial v}{\partial x}\\
&+\frac{\partial^2 w}{\partial y\partial x}\frac{\partial w}{\partial y}-\frac{\partial^2 v}{\partial x\partial y}\frac{\partial u}{\partial x}+2\frac{\partial^2 v}{\partial x\partial y}\frac{\partial v}{\partial y}+\frac{\partial^2 u}{\partial x\partial y}\frac{\partial v}{\partial x}-\frac{\partial^2 w}{\partial x\partial z}\frac{\partial u}{\partial x}+2\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z\partial x}+\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z\partial x}+\frac{\partial^2 u}{\partial x\partial z}\frac{\partial w}{\partial x}+\frac{\partial^2 w}{\partial y^2}\frac{\partial w}{\partial x}
\end{aligned} \tag{2}$$

$$+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial z^2}+\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial y^2}+\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial z^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2}-\frac{\partial u}{\partial y}\frac{\partial^2 \upsilon}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial y\partial z}-\frac{\partial u}{\partial z}\frac{\partial^2 \upsilon}{\partial y\partial z}-\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}+\frac{\partial w}{\partial y}\frac{\partial^2 \upsilon}{\partial x\partial z}$$
$$\left.+\frac{\partial u}{\partial y}\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial x^2}+\frac{\partial \upsilon}{\partial z}\frac{\partial^2 w}{\partial x\partial y}+2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}\right]-(\phi+\Phi)u,$$

$$\frac{\partial^\beta \upsilon}{\partial t^\beta}+u\frac{\partial \upsilon}{\partial x}+\upsilon\frac{\partial \upsilon}{\partial y}+w\frac{\partial \upsilon}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial y}=\nu\left(\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial^2 \upsilon}{\partial y^2}+\frac{\partial^2 \upsilon}{\partial z^2}\right)+\frac{\alpha_1}{\rho}\left[\frac{\partial^3 \upsilon}{\partial x^2\partial t}+\frac{\partial^3 \upsilon}{\partial y^2\partial t}+\frac{\partial^3 \upsilon}{\partial z^2\partial t}+u\frac{\partial^3 \upsilon}{\partial x^3}\right.$$
$$+\upsilon\frac{\partial^3 \upsilon}{\partial x^2\partial y}+w\frac{\partial^3 \upsilon}{\partial x^2\partial z}+\upsilon\frac{\partial^3 \upsilon}{\partial y^3}+u\frac{\partial^3 \upsilon}{\partial x\partial y^2}+w\frac{\partial^3 \upsilon}{\partial y^2\partial z}+w\frac{\partial^3 \upsilon}{\partial z^3}+u\frac{\partial^3 \upsilon}{\partial x\partial z^2}+\upsilon\frac{\partial^3 \upsilon}{\partial y\partial z^2}+2\frac{\partial^2 w}{\partial y^2}\frac{\partial w}{\partial y}+2\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y}$$
$$+\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial u}{\partial y}\frac{\partial^2 \upsilon}{\partial x\partial y}+2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial \upsilon}{\partial y}\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial \upsilon}{\partial x}\frac{\partial^2 w}{\partial x\partial y}-\frac{\partial \upsilon}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial x\partial y}-\frac{\partial \upsilon}{\partial y}\frac{\partial^2 w}{\partial y\partial z} \tag{3}$$
$$+\frac{\partial \upsilon}{\partial y}\frac{\partial^2 \upsilon}{\partial x^2}-\frac{\partial \upsilon}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2}+\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial z^2}+\frac{\partial \upsilon}{\partial y}\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial z^2}-\frac{\partial \upsilon}{\partial z}\frac{\partial^2 w}{\partial z^2}+\frac{\partial w}{\partial y}\frac{\partial^2 \upsilon}{\partial y\partial z}+\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial y\partial z}+\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial y\partial z}$$
$$\left.+2\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+\frac{\partial \upsilon}{\partial z}\frac{\partial^2 w}{\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right]-(\phi+\Phi)\upsilon,$$

$$\frac{\partial^\beta w}{\partial t^\beta}+u\frac{\partial w}{\partial x}+\upsilon\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}=\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{\alpha_1}{\rho}\left[\frac{\partial^3 w}{\partial x^2\partial t}+\frac{\partial^3 w}{\partial y^2\partial t}+\frac{\partial^3 w}{\partial z^2\partial t}+u\frac{\partial^3 w}{\partial x^3}\right.$$
$$+\upsilon\frac{\partial^3 w}{\partial x^2\partial y}+w\frac{\partial^3 w}{\partial x^2\partial z}+\upsilon\frac{\partial^3 w}{\partial y^3}+u\frac{\partial^3 w}{\partial x\partial y^2}+w\frac{\partial^3 w}{\partial y^2\partial z}+w\frac{\partial^3 w}{\partial z^3}+u\frac{\partial^3 w}{\partial z^2\partial x}+\upsilon\frac{\partial^3 w}{\partial z^2\partial y}+2\frac{\partial^2 \upsilon}{\partial z^2}\frac{\partial \upsilon}{\partial z}+2\frac{\partial^2 u}{\partial z^2}\frac{\partial u}{\partial z}$$
$$+\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 \upsilon}{\partial x\partial y}\frac{\partial w}{\partial x}-\frac{\partial^2 u}{\partial x\partial y}\frac{\partial w}{\partial y}+2\frac{\partial^2 \upsilon}{\partial y\partial z}\frac{\partial \upsilon}{\partial y}+\frac{\partial^2 w}{\partial x\partial z}\frac{\partial u}{\partial z}+\frac{\partial^2 \upsilon}{\partial x\partial z}\frac{\partial \upsilon}{\partial x}-\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial z}-\frac{\partial w}{\partial z}\frac{\partial^2 \upsilon}{\partial y\partial z} \tag{4}$$
$$+\frac{\partial \upsilon}{\partial z}\frac{\partial^2 \upsilon}{\partial x^2}-\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}+\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial y^2}-\frac{\partial w}{\partial y}\frac{\partial^2 \upsilon}{\partial y^2}+\frac{\partial^2 \upsilon}{\partial y^2}\frac{\partial \upsilon}{\partial z}+\frac{\partial^2 w}{\partial y^2}\frac{\partial u}{\partial z}+\frac{\partial^2 w}{\partial y\partial z}\frac{\partial \upsilon}{\partial z}+\frac{\partial \upsilon}{\partial x}\frac{\partial^2 u}{\partial z\partial y}+\frac{\partial^2 \upsilon}{\partial x\partial z}\frac{\partial u}{\partial y}$$
$$\left.+2\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2}+2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial w}{\partial y}\frac{\partial^2 \upsilon}{\partial z^2}+\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial z^2}\right]-(\phi+\Phi)w,$$

$$\left(\frac{\partial^\beta T}{\partial t^\beta}+u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=\frac{k}{\rho c_\rho}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{\nu}{c_\rho}\left[2\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial \upsilon}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right)\right.$$
$$\left.\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial \upsilon}{\partial x}\right)^2+\left(\frac{\partial \upsilon}{\partial z}+\frac{\partial w}{\partial y}\right)^2\right]+\frac{\alpha_1}{\rho c_\rho}\left[\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial t\partial z}+\frac{\partial \upsilon}{\partial z}\frac{\partial^2 \upsilon}{\partial t\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial t\partial z}+w\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}\right.$$
$$+w\frac{\partial \upsilon}{\partial z}\frac{\partial^2 \upsilon}{\partial z^2}+2w\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}+\frac{\partial w}{\partial y}\frac{\partial^2 \upsilon}{\partial t\partial z}+w\frac{\partial w}{\partial y}\frac{\partial^2 \upsilon}{\partial z^2}+\frac{\partial^2 u}{\partial t\partial y}\frac{\partial u}{\partial y}+2\frac{\partial^2 \upsilon}{\partial t\partial y}\frac{\partial \upsilon}{\partial y}+\frac{\partial \upsilon}{\partial z}\frac{\partial^2 w}{\partial t\partial y}+\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial t\partial y}+\upsilon\frac{\partial^2 w}{\partial y^2}\frac{\partial \upsilon}{\partial z}$$
$$+\upsilon\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial y\partial z}+w\frac{\partial^2 u}{\partial y\partial z}\frac{\partial u}{\partial y}+\upsilon\frac{\partial^2 \upsilon}{\partial y\partial z}\frac{\partial \upsilon}{\partial z}+2w\frac{\partial^2 \upsilon}{\partial y\partial z}\frac{\partial \upsilon}{\partial y}+\upsilon\frac{\partial w}{\partial y}\frac{\partial^2 \upsilon}{\partial y\partial z}+w\frac{\partial^2 w}{\partial y\partial z}\frac{\partial \upsilon}{\partial z}+2\upsilon\frac{\partial^2 w}{\partial y\partial z}\frac{\partial w}{\partial z}+w\frac{\partial^2 w}{\partial y\partial z}\frac{\partial w}{\partial y}$$
$$+\upsilon\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+2\upsilon\frac{\partial \upsilon}{\partial y}\frac{\partial^2 \upsilon}{\partial y^2}+\upsilon\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+\frac{\partial \upsilon}{\partial x}\frac{\partial^2 u}{\partial t\partial y}+w\frac{\partial \upsilon}{\partial x}\frac{\partial^2 u}{\partial y\partial z}+\upsilon\frac{\partial \upsilon}{\partial x}\frac{\partial^2 u}{\partial y^2}+w\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial t\partial z}+w\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial z^2}+\upsilon\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial y\partial z} \tag{5}$$
$$+2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial t\partial x}+\frac{\partial u}{\partial y}\frac{\partial^2 \upsilon}{\partial t\partial x}+\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial t\partial x}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial t\partial x}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial t\partial x}+u\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+2w\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial z}+u\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial x\partial z}+u\frac{\partial \upsilon}{\partial z}\frac{\partial^2 \upsilon}{\partial x\partial z}$$
$$+w\frac{\partial u}{\partial y}\frac{\partial^2 \upsilon}{\partial x\partial z}+u\frac{\partial w}{\partial y}\frac{\partial^2 \upsilon}{\partial x\partial z}+w\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial x\partial z}+w\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial x\partial z}+2u\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial x\partial z}+w\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial z}+u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+2\upsilon\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial y}$$
$$+u\frac{\partial \upsilon}{\partial x}\frac{\partial^2 u}{\partial x\partial y}+\upsilon\frac{\partial u}{\partial y}\frac{\partial^2 \upsilon}{\partial x\partial y}+2u\frac{\partial \upsilon}{\partial y}\frac{\partial^2 \upsilon}{\partial x\partial y}+\upsilon\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial x\partial y}+\upsilon\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial x\partial y}+u\frac{\partial \upsilon}{\partial z}\frac{\partial^2 w}{\partial x\partial y}+u\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}+\upsilon\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y}$$
$$\left.+2u\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+u\frac{\partial u}{\partial y}\frac{\partial^2 \upsilon}{\partial x^2}+u\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial x^2}+u\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial x^2}+u\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\right].$$

where the components filed in $x$, $y$ and $z$ directions are $u$, $\upsilon$ and $w$, $p$ and $T$ is pressure and temperature of the fluid respectively, $\alpha_1 \to 0$ is the material parameter, fluid density is $\rho$, $\nu$ is kinematic viscosity, $c_\rho$ is specific heat and heat flux is $k$. The porosity and magnetic constants are $\Phi=\nu\psi/\kappa$, $\phi=\sigma B_0{}^2/\rho$ respectively. The Caputo fractional operator $D_t{}^\beta$ and $\beta$ is fractional parameter is defined by:

$$D_t{}^\beta=\begin{cases}\dfrac{1}{\Gamma(1-\beta)}\displaystyle\int_0^t\frac{f'(\tau)}{(t-\tau)}d\tau\,, & 0<\beta<1\,;\\[2ex] \dfrac{df(t)}{dt} & \beta=1\,,\end{cases}$$

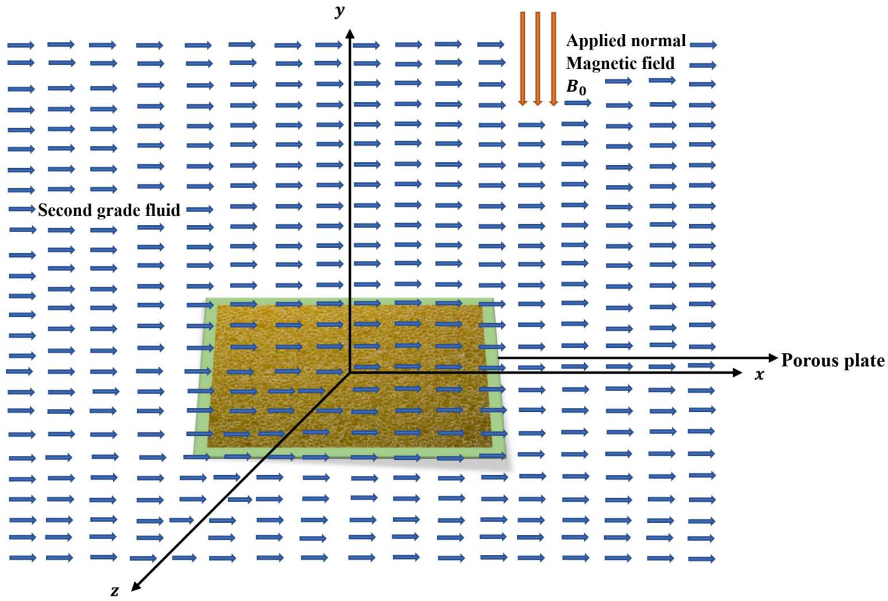


**Fig. 1.** Three-dimensional flow of Second grade fluid in the existence of applied magnetic field on porous plate.

It is clearly observed that to solve Eqs. (1 - 5), Eqs. (1 - 4) should be solved first to get $u$, $v$ and $w$ then Eq. (5) is solved to obtain $T$. In the succeeding sections, the traveling wave solutions of Eqs. (1 - 5), will be achieved and the physical descriptions to Eqs. (1 – 5) are illuminated.

## 2.1 Methodology of traveling wave method

Assume that the NLFPEDs (nonlinear fractional partial differential equations) restrain the four dependent and independent variables are $\Lambda_1$, $\Lambda_2$, $\Lambda_3$, $\Lambda_4$ and $t$, $x$, $y$, $z$ respectively.

$$G_1\left(\Lambda_{1x},\Lambda_{2y},\Lambda_{3z}\right), \tag{6}$$

$$G_2\left(\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_{4x},\Lambda_{1t}{}^{\beta},\Lambda_{1x},\Lambda_{1y},\Lambda_{1z},\Lambda_{1xx},\ldots,\Lambda_{2t}{}^{\beta},\Lambda_{2x},\Lambda_{2y},\Lambda_{2z},\Lambda_{2xx},\ldots,\Lambda_{3t}{}^{\beta},\Lambda_{3x},\Lambda_{3y},\Lambda_{3z},\Lambda_{3xx}\right) \tag{7}$$

$$G_3\left(\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_{4y},\Lambda_{1t}{}^{\beta},\Lambda_{1x},\Lambda_{1y},\Lambda_{1z},\Lambda_{1xx},\ldots,\Lambda_{2t}{}^{\beta},\Lambda_{2x},\Lambda_{2y},\Lambda_{2z},\Lambda_{2xx},\ldots,\Lambda_{3t}{}^{\beta},\Lambda_{3x},\Lambda_{3y},\Lambda_{3z},\Lambda_{3xx}\right) \tag{8}$$

$$G_4\left(\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_{4z},\Lambda_{1t}{}^{\beta},\Lambda_{1x},\Lambda_{1y},\Lambda_{1z},\Lambda_{1xx},\ldots,\Lambda_{2t}{}^{\beta},\Lambda_{2x},\Lambda_{2y},\Lambda_{2z},\Lambda_{2xx},\ldots,\Lambda_{3t}{}^{\beta},\Lambda_{3x},\Lambda_{3y},\Lambda_{3z},\Lambda_{3xx}\right) \tag{9}$$

Here $G_j$ is the polynomial function $\Lambda_1$, $\Lambda_2$ $\Lambda_3$ and $\Lambda_4$ which have higher order and nonlinear terms of differentiation, where $j = 1,2,3,4$. We exhibit the traveling wave solution for the following:

$$\Lambda_1(x,y,z,t) = \Lambda_1(\xi),\ \Lambda_2(x,y,z,t) = \Lambda_2(\xi),\ \Lambda_3(x,y,z,t) = \Lambda_3(\xi) \text{ and } \Lambda_4(x,y,z,t) = \Lambda_4(\xi), \tag{10}$$

where $\xi = lx + my + nz + \omega t^{\beta} / \Gamma(\beta+1)$. The system Eqs. (6 - 9) can be transformed into the system of ordinary differential equations:

$$G_1\left(\Lambda_1',\Lambda_2',\Lambda_3'\right), \tag{11}$$

$$G_2\left(\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4',\Lambda_1',\Lambda_1'',\Lambda_1''',\ldots,\Lambda_2',\Lambda_2'',\Lambda_2''',\ldots,\Lambda_3',\Lambda_3'',\Lambda_3'''\right) = 0, \tag{12}$$

$$G_3\left(\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4',\Lambda_1',\Lambda_1'',\Lambda_1''',\ldots,\Lambda_2',\Lambda_2'',\Lambda_2''',\ldots,\Lambda_3',\Lambda_3'',\Lambda_3'''\right) = 0, \tag{13}$$

$$G_4\left(\Lambda_1,\Lambda_2,\Lambda_3,\Lambda_4',\Lambda_1',\Lambda_1'',\Lambda_1''',\ldots,\Lambda_2',\Lambda_2'',\Lambda_2''',\ldots,\Lambda_3'',\Lambda_3''',\ldots\right) = 0, \tag{14}$$

where $l$, $m$ and $n$ are constants and the prime represent derivative with respect to $\xi$.

## 3. Exact traveling wave solutions and explanations

This section determines the traveling wave solution of the governing Eqs. (1 - 5). Taking the type of traveling wave parameter, the solution is expressed as

$$u = u(\xi)\,,\, v = v(\xi)\,,\, w = w(\xi)\,,\, p = p(\xi)\,,\, T = T(\xi) \text{ and } \xi = lx + my + nz + \frac{\omega\, t^{\beta}}{\Gamma(\beta+1)}$$

Here $\omega$ is constant phase velocity. On using the above traveling wave parameter $\xi$ into Eqs. (1 - 5), we procure representation of the form

$$l\,u' + m\,v' + n\,w' = 0, \tag{15}$$

$$\begin{gathered}(\omega + lu + mv + nw)u' = \frac{-lp'}{\rho} + \nu\left(l^2 + m^2 + n^2\right)u'' + \frac{\alpha_1}{\rho}\Big[\left(l^2 + m^2 + n^2\right)(\omega + lu + mv + nw)u''' \\ +\left(2l^3 + lm^2 + ln^2\right)u'u'' + \left(2l^3 + 3lm^2 + 2ln^2\right)v'v'' + \left(2l^3 + 2lm^2 + 3ln^2\right)w'w'' - \left(m^3 + mn^2\right)u'v'' \\ +l^2m\, v'u'' - \left(m^2n + n^3\right)w''u' + l^2n\, w'v'' + lmn\, v''w' + lmn\, w''v'\Big] - (\phi + \Phi)u,\end{gathered} \tag{16}$$

$$\begin{gathered}(\omega + lu + mv + nw)v' = \frac{-mp'}{\rho} + \nu\left(l^2 + m^2 + n^2\right)v'' + \frac{\alpha_1}{\rho}\Big[\left(l^2 + m^2 + n^2\right)(\omega + lu + mv + nw)v''' \\ +\left(2m^3 + 3lm^2 + 2mn^2\right)u'u'' + \left(2m^3 + l^2m + mn^2\right)v'v'' + \left(2l^2m + 2m^3 + 3mn^2\right)w'w'' - \left(l^3 + ln^2\right)u''v' \\ +lm^2u'\, v'' - \left(l^2n + n^3\right)w''\, v' + m^2n\, w'v'' + lmn\, w'u'' + lmn\, w''u'\Big] - (\phi + \Phi)v,\end{gathered} \tag{17}$$

$$\begin{gathered}(\omega + lu + mv + nw)w' = \frac{-np'}{\rho} + \nu\left(l^2 + m^2 + n^2\right)w'' + \frac{\alpha_1}{\rho}\Big[\left(l^2 + m^2 + n^2\right)(\omega + lu + mv + nw)w''' \\ +\left(2n^3 + 3l^2n + 2m^2n\right)u'u'' + \left(2n^3 + 3m^2n + 2l^2n\right)v'v'' + \left(2n^3 + l^2n + m^2n\right)w'w'' - \left(l^3 + lm^2\right)u''w' \\ +lm^2\, w''u' - \left(l^2m + m^3\right)v''w' + mn^2w''v' + lmn\, v''u' + lmn\, v'u''\Big] - (\phi + \Phi)w,\end{gathered} \tag{18}$$

$$\begin{gathered}(\omega + lu + mv + nw)T' = \frac{k}{\rho c_\rho}\left(l^2 + m^2 + n^2\right)T'' + \frac{\nu}{c_\rho}\Big[\left(2l^2 + m^2 + n^2\right)u'^2 + \left(l^2 + 2m^2 + n^2\right)v'^2 \\ +\left(l^2 + m^2 + 2n^2\right)w'^2 + 2lm\, u'v' + 2ln\, u'w' + 2mn\, v'w'\Big] + \frac{\alpha_1}{\rho c_\rho}(\omega + lu + mv + nw)\Big[\left(2l^2 + m^2 + n^2\right)u'u'' \\ +\left(l^2 + 2m^2 + n^2\right)v'v'' + \left(l^2 + m^2 + 2n^2\right)w'w'' + lm\, v''u' + lm\, v'u'' + nl\, u'w'' + nl\, w'u'' + mn\, v''w' + mn\, v'w''\Big],\end{gathered} \tag{19}$$

Integration of Eq. (15) w. r. t $\xi$ yields

$$l\,u + m\,v + n\,w = b_0. \tag{20}$$

On using Eq. (20) into Eqs. (16 - 19), reduced to

$$\begin{gathered}(\omega + b_0)u' = \frac{-lp'}{\rho} + \nu\left(l^2 + m^2 + n^2\right)u'' + \frac{\alpha_1}{\rho}\Big[\left(l^2 + m^2 + n^2\right)(\omega + b_0)u''' + \frac{u'u''}{m^2}\left(2l^5 + 2lm^4 + 2lm^2n^2\right. \\ \left.+4l^3m^2 + 2n^2l^3\right) + \frac{w'w''}{m^2}\left(2n^2l^3 + 4lm^2n^2 + 2n^4l + 2l^3m^2 + 2lm^4\right) + \frac{u'w''}{m^2}\left(2nl^4 + 2n^3l^2 + 2l^2m^2n\right) \\ + \frac{u''w'}{m^2}\left(2nl^4 + 2n^3l^2 + 2l^2m^2n\right)\Big] - (\phi + \Phi)u,\end{gathered} \tag{21}$$

$$\begin{gathered}(\omega + b_0)v' = \frac{-mp'}{\rho} + \nu\left(l^2 + m^2 + n^2\right)v'' + \frac{\alpha_1}{\rho}\Big[\left(l^2 + m^2 + n^2\right)(\omega + b_0)v''' + \frac{u'u''}{m^2}\left(2m^5 + 2ml^4 + 2l^2mn^2\right. \\ \left.+4l^2m^3 + 2m^3n^2\right) + \frac{w'w''}{m^2}\left(4m^3n^2 + 2l^2mn^2 + 2mn^4 + 2l^2m^3 + 2m^5\right) + \frac{u'w''}{m^2}\left(2lm^3n + 2l^3mn + 2lmn^3\right) \\ + \frac{u''w'}{m^2}\left(2lm^3n + 2l^2mn + 2lmn^3\right)\Big] - (\phi + \Phi)v,\end{gathered} \tag{22}$$

$$(\omega+b_0)w' = \frac{-np'}{\rho} + \nu\left(l^2+m^2+n^2\right)w'' + \frac{\alpha_1}{\rho}\Bigg[\left(l^2+m^2+n^2\right)(\omega+b_0)w''' + \frac{u'u''}{m^2}\left(2m^2n^3+2nl^4+2nm^4 +4l^2m^2n+2l^2m^3\right) + \frac{w'w''}{m^2}\left(2n^5+4m^2n^3+2n^3l^2+2l^2m^2n+2m^4n\right) + \frac{u'w''}{m^2}\left(2lmn^4+2lm^2n^2+2l^3n^2\right) + \frac{u''w'}{m^2}\left(2lmn^4+2lm^2n^2+2l^3n^2\right)\Bigg] - (\phi+\Phi)w, \tag{23}$$

$$(\omega+b_0)T' = \frac{k}{\rho c_\rho}\left(l^2+m^2+n^2\right)T'' + \frac{\nu}{c_\rho}\Big[\left(2l^2+m^2+n^2\right)u'^2 + \left(l^2+2m^2+n^2\right)v'^2 + \left(l^2+m^2+2n^2\right)w'^2 +2lm\,u'v' + 2nl\,u'w' + 2mn\,v'w'\Big] + \frac{\alpha_1}{\rho c_\rho}(\omega+b_0)\Big[\left(2l^2+m^2+n^2\right)u'u'' + \left(l^2+2m^2+n^2\right)v'v'' + lm\,u'v'' + lm\,u''v' + \left(l^2+m^2+2n^2\right)w'w'' + nl\,u'w'' + nl\,u''w' + mn\,v''w' + mn\,v'w''\Big]. \tag{24}$$

In sequence to achieve $u$, $v$, $w$, $p$ and $T$ from beyond four equations, we consider the subsequent two cases separately.

### 3.1 Case I: $\omega + b_0 \neq 0$

Eliminating $p$ from Eqs. (21 - 23), we have

$$(\omega+b_0)\left(mu'-lv'\right) = \nu\left(l^2+m^2+n^2\right)\left(mu''-lv''\right) + \frac{\alpha_1}{\rho}\left[(l+m+n)(\omega+b_0)\left(mu'''-lv'''\right)\right] - (\phi+\Phi)(mu-lv), \tag{25}$$

$$(\omega+b_0)\left(nu'-lw'\right) = \nu\left(l^2+m^2+n^2\right)\left(nu''-lw''\right) + \frac{\alpha_1}{\rho}\left[(l+m+n)(\omega+b_0)\left(nu'''-lw'''\right)\right] - (\phi+\Phi)(nu-lw). \tag{26}$$

Solving Eqs. (25 - 26), we attain

$$u''' + \kappa_1 u'' + \kappa_2 u' + \kappa_3 u = \kappa_4, \tag{27}$$

where $\kappa_1$, $\kappa_2$, $\kappa_3$ and $\kappa_4$ are presented in Appendix A. The solution of Eq. (27) is

$$u(\xi) = \kappa_5 e^{\gamma_{11}\xi} + \kappa_6 e^{\gamma_{12}\xi} + \kappa_7 e^{\gamma_{13}\xi} + \kappa_8, \tag{28}$$

where $\kappa_5$, $\kappa_6$ and $\kappa_7$ are arbitrary constants and $\kappa_8$ presented in Appendix A and $\gamma_{11}$, $\gamma_{12}$ and $\gamma_{13}$ are the roots of the auxiliary equation,

$$m^3 + \kappa_1 m^2 + \kappa_2 m + \kappa_3 = 0.$$

Using Eq. (28) in Eq. (25), we get

$$v''' + \kappa_9 v'' + \kappa_{10} v' + \kappa_{11} v = \kappa_{12} e^{\gamma_{11}\xi} + \kappa_{13} e^{\gamma_{12}\xi} + \kappa_{14} e^{\gamma_{13}\xi} + \kappa_{15}, \tag{29}$$

where $\kappa_9 = \kappa_1$, $\kappa_{10} = \kappa_2$ and $\kappa_{11} = \kappa_3$, $\kappa_{12}$, $\kappa_{13}$, $\kappa_{14}$ and $\kappa_{15}$ are given in Appendix A. The solution of Eq. (29) is

$$v(\xi) = \kappa_{23} e^{\gamma_{11}\xi} + \kappa_{24} e^{\gamma_{12}\xi} + \kappa_{25} e^{\gamma_{13}\xi} + \kappa_{22}. \tag{30}$$

Putting the Eqs. (28), (30) in Eq. (20) we obtain

$$w(\xi) = \kappa_{26} e^{\gamma_{11}\xi} + \kappa_{27} e^{\gamma_{12}\xi} + \kappa_{28} e^{\gamma_{13}\xi} + \kappa_{29}, \tag{31}$$

where $\kappa_{22}$, $\kappa_{23}$, $\kappa_{24}$, $\kappa_{25}$ and $\kappa_{26}$, $\kappa_{27}$, $\kappa_{28}$, $\kappa_{29}$ are presented in Appendix A. Using Eq. (28) in Eq. (21), we obtain the pressure

$$p(\xi) = \kappa_{30} e^{\gamma_{11}\xi} + \kappa_{31} e^{\gamma_{12}\xi} + \kappa_{32} e^{\gamma_{13}\xi} + \kappa_{33} e^{2\gamma_{11}\xi} + \kappa_{34} e^{2\gamma_{12}\xi} + \kappa_{35} e^{2\gamma_{13}\xi} + \kappa_{36} e^{(\gamma_{11}+\gamma_{12})\xi} + \kappa_{37} e^{(\gamma_{12}+\gamma_{13})\xi} + \kappa_{38} e^{(\gamma_{11}+\gamma_{13})\xi} + \kappa_{39}\,\xi + \kappa_{40}, \tag{32}$$

where $\kappa_{30}$, $\kappa_{31}$, $\kappa_{32}$, $\kappa_{33}$ and $\kappa_{34}$, $\kappa_{35}$, $\kappa_{36}$, $\kappa_{37}$ and $\kappa_{38}$, $\kappa_{39}$ are presented in Appendix A and $\kappa_{40}$ is arbitrary constant. Put Eqs. (28), (30) and (31) in Eq. (24) we have

$$T'' - \kappa_{41} T' = \kappa_{42} e^{2\gamma_{11}\xi} + \kappa_{43} e^{2\gamma_{12}\xi} + \kappa_{44} e^{2\gamma_{13}\xi} + \kappa_{45} e^{(\gamma_{11}+\gamma_{12})\xi} + \kappa_{46} e^{(\gamma_{11}+\gamma_{13})\xi} + \kappa_{47} e^{(\gamma_{12}+\gamma_{13})\xi}. \tag{33}$$

The solution of Eq. (33) is

$$T(\xi)=\kappa_{48}+\kappa_{49}\,e^{\kappa_{41}\,\xi}+\kappa_{50}\,e^{2\gamma_{11}\,\xi}+\kappa_{51}\,e^{2\gamma_{12}\,\xi}+\kappa_{52}\,e^{2\gamma_{13}\,\xi}+\kappa_{53}\,e^{(\gamma_{11}+\gamma_{12})\,\xi}+\kappa_{54}\,e^{(\gamma_{11}+\gamma_{13})\,\xi}+\kappa_{55}\,e^{(\gamma_{11}+\gamma_{12})\,\xi}, \tag{34}$$

where $\kappa_{41}$, $\kappa_{42}$, $\kappa_{43}$, $\kappa_{44}$, $\kappa_{45}$ , $\kappa_{46}$, $\kappa_{47}$, $\kappa_{51}$, $\kappa_{52}$, $\kappa_{53}$ and $\kappa_{54}$, $\kappa_{55}$ are presented in Appendix A and $\kappa_{48}$, $\kappa_{49}$ are arbitrary constants. Recurrence the original variables are

$$u(x,y,z,t)=\kappa_5\,e^{\gamma_{11}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_6\,e^{\gamma_{12}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_7\,e^{\gamma_{13}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_8, \tag{35}$$

$$v(x,y,z,t)=\kappa_{23}\,e^{\gamma_{11}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{24}\,e^{\gamma_{12}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{25}\,e^{\gamma_{13}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{22}, \tag{36}$$

$$w(x,y,z,t)=\kappa_{26}\,e^{\gamma_{11}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{27}\,e^{\gamma_{12}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{28}\,e^{\gamma_{13}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{29}, \tag{37}$$

$$\begin{aligned}p(x,y,z,t)&=\kappa_{30}\,e^{\gamma_{11}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{31}\,e^{\gamma_{12}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{32}\,e^{\gamma_{13}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}\\&+\kappa_{33}\,e^{2\gamma_{11}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{34}\,e^{2\gamma_{12}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{35}\,e^{2\gamma_{13}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}\\&+\kappa_{36}\,e^{(\gamma_{11}+\gamma_{12})\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{37}\,e^{(\gamma_{12}+\gamma_{13})\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{38}\,e^{(\gamma_{11}+\gamma_{13})\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{39}\left(lx+my+nz+\frac{\omega\,t^\beta}{\Gamma(\beta+1)}\right)+\kappa_{40},\end{aligned} \tag{38}$$

$$\begin{aligned}T(x,y,z,t)&=\kappa_{48}+\kappa_{49}\,e^{\kappa_{41}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{50}\,e^{2\gamma_{11}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{51}\,e^{2\gamma_{12}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}\\&+\kappa_{52}\,e^{2\gamma_{13}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{53}\,e^{(\gamma_{11}+\gamma_{12})\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{54}\,e^{(\gamma_{12}+\gamma_{13})\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\kappa_{55}\,e^{(\gamma_{11}+\gamma_{13})\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}.\end{aligned} \tag{39}$$

### 3.2 Case II: $\omega+b_0=0$

In this case, Eqs. (21 - 24) can be simplified as

$$\begin{aligned}\frac{lp'}{\rho}&=\nu\left(l^2+m^2+n^2\right)u''+\frac{\alpha_1}{\rho}\Bigg[\left(l^2+m^2+n^2\right)(\omega+b_0)u'''+\frac{u'u''}{m^2}\left(2l^5+2lm^4+2lm^2n^2+4l^3m^2+2n^2l^3\right)\\&+\frac{w'w''}{m^2}\left(2n^2l^3+4lm^2n^2+2n^4l+2l^3m^2+2lm^4\right)+\frac{u'w''}{m^2}\left(2nl^4+2n^3l^2+2l^2m^2n\right)+\frac{u''w'}{m^2}\left(2nl^4+2n^3l^2+2l^2m^2n\right)\Bigg]-(\phi+\Phi)u\end{aligned} \tag{40}$$

$$\begin{aligned}\frac{mp'}{\rho}&=\nu\left(l^2+m^2+n^2\right)v''+\frac{\alpha_1}{\rho}\Bigg[\left(l^2+m^2+n^2\right)(\omega+b_0)v'''+\frac{u'u''}{m^2}\left(2m^5+2ml^4+2l^2mn^2+4l^2m^3+2m^3n^2\right)\\&+\frac{w'w''}{m^2}\left(4m^3n^2+4l^2mn^2+2mn^4+2l^2m^3+2m^5\right)+\frac{u'w''}{m^2}\left(2lm^3n+2l^3mn+2lmn^3\right)+\frac{u''w'}{m^2}\left(2lm^3n+2l^2mn+2lmn^3\right)\Bigg]-(\phi+\Phi)v\end{aligned} \tag{41}$$

$$\begin{aligned}\frac{np'}{\rho}&=\nu\left(l^2+m^2+n^2\right)w''+\frac{\alpha_1}{\rho}\Bigg[\left(l^2+m^2+n^2\right)(\omega+b_0)w'''+\frac{u'u''}{m^2}\left(2m^2n^3+2nl^4+2nm^4+4l^2m^2n+2l^2m^3\right)\\&+\frac{w'w''}{m^2}\left(2n^5+4m^2n^3+2n^3l^2+2l^2m^2n+2m^4n\right)+\frac{u'w''}{m^2}\left(2lmn^4+2lm^2n^2+2l^3n^2\right)+\frac{u''w'}{m^2}\left(2lmn^4+2lm^2n^2+2l^3n^2\right)\Bigg]-(\phi+\Phi)w\end{aligned} \tag{42}$$

$$\frac{k}{\rho c_\rho}\left(l^2+m^2+n^2\right)T''+\frac{\nu}{c_\rho}\Big[\left(2l^2+m^2+n^2\right)u'^2+\left(l^2+2m^2+n^2\right)v'^2+\left(l^2+m^2+2n^2\right)w'^2+2lm\,u'v'+2nl\,u'w'+2mn\,v'w'\Big]=0 \tag{43}$$

Eliminating $p$ from Eqs. (40) - (42),

$$\nu\left(l^2+m^2+n^2\right)\left(mu''-lv''\right)-(\phi+\Phi)(mu-lv)=0, \tag{44}$$

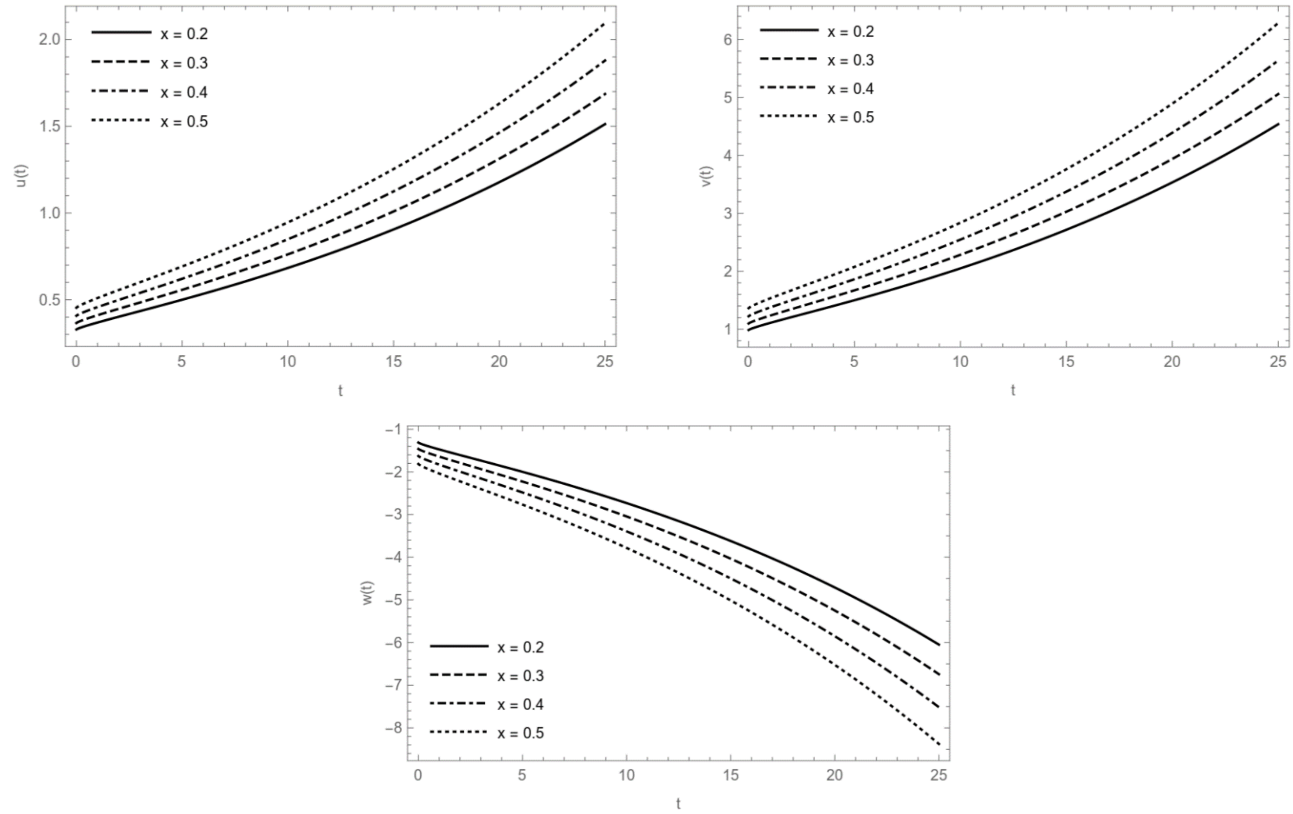


**Fig. 2.** Component field profile $u(x,y), v(x,y)$ and $w(x,y)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37), for $\omega = 0.1$ , $\nu = 0.515$ , $\phi = 2$ , $\Phi = 0.1$ , $\alpha_1 = 0.001$ , $\beta = 0.8$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ and various value of $x$ .

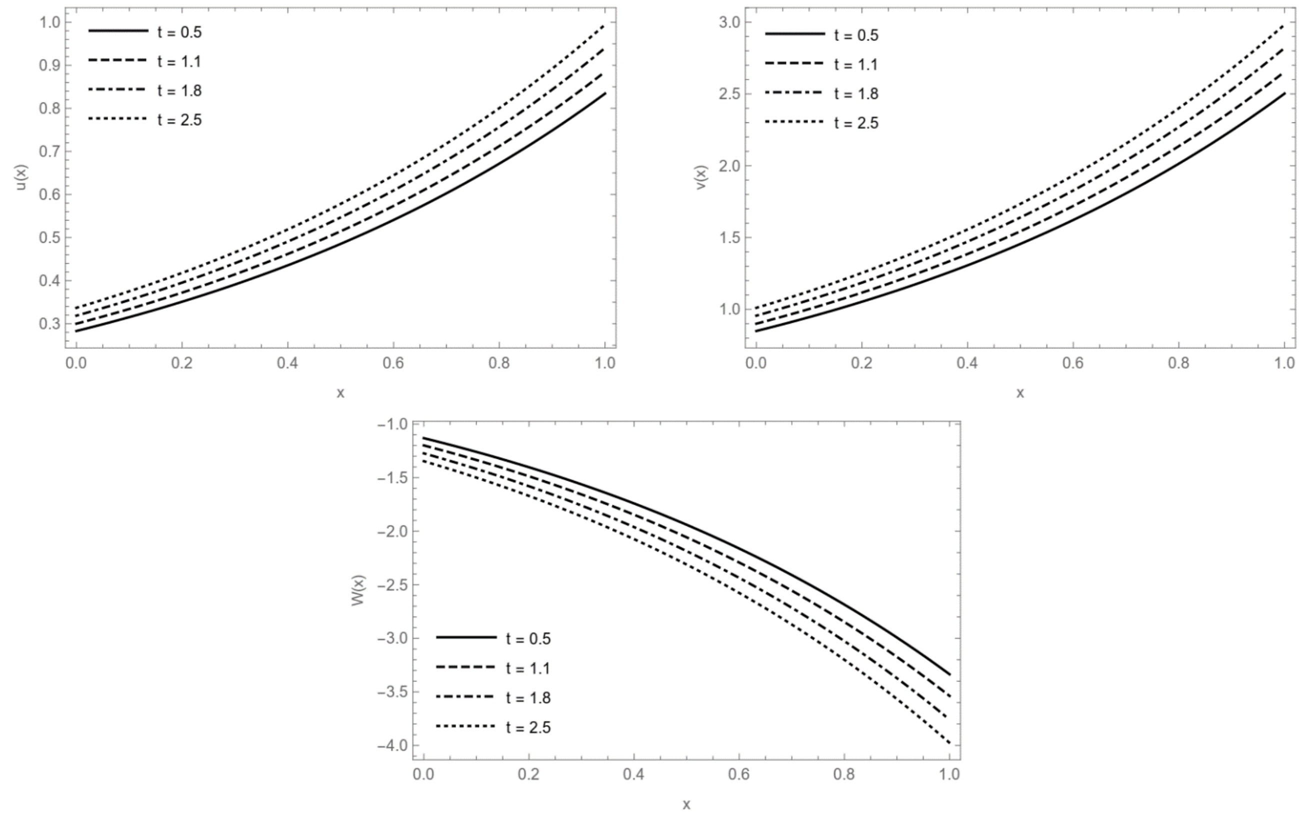


**Fig. 3.** Component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37), for $\omega = 0.1$ , $\nu = 0.515$ , $\phi = 2$ , $\Phi = 0.1$ , $\alpha_1 = 0.001$ , $\beta = 0.8$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ and various value of $t$ .

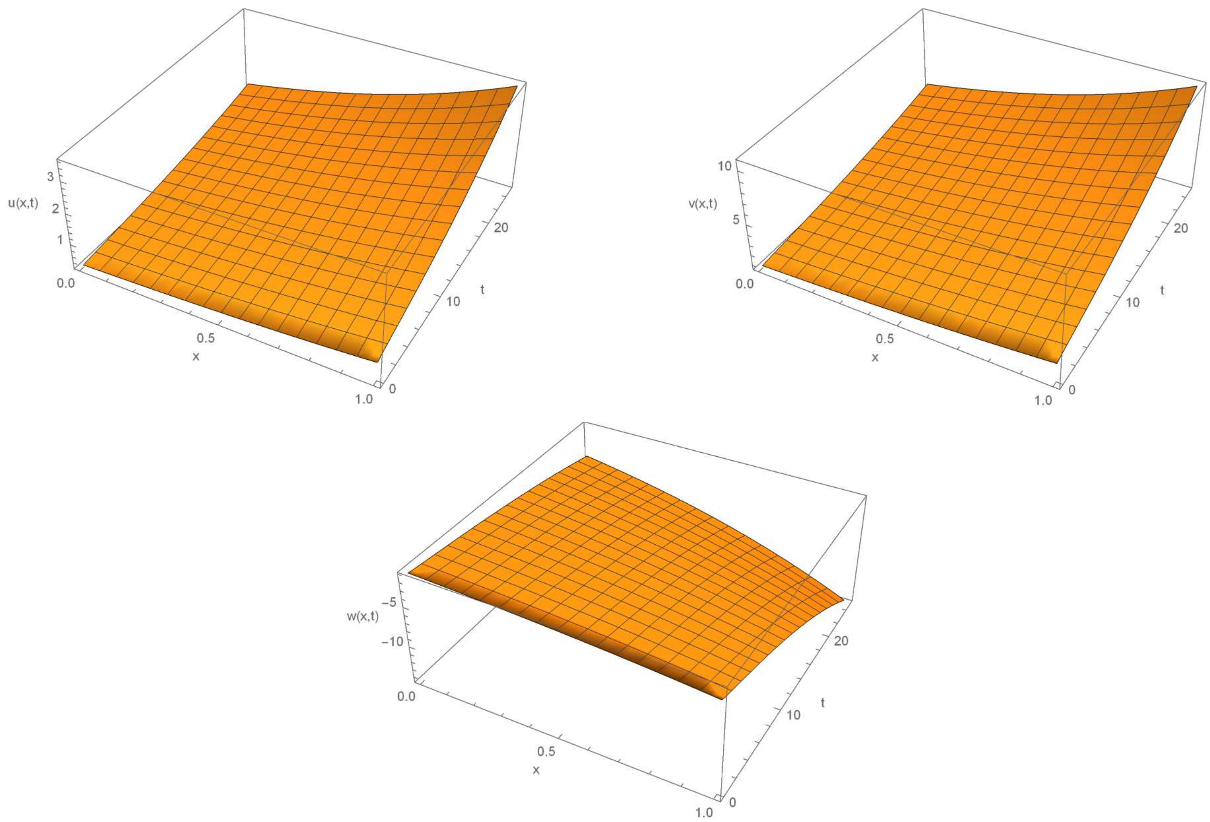


**Fig. 4.** 3D component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37), for $\omega = 0.1$ , $v = 0.515$ , $\phi = 2$ , $\Phi = 0.1$ , $\alpha_1 = 0.001$ , $\beta = 0.8$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ .

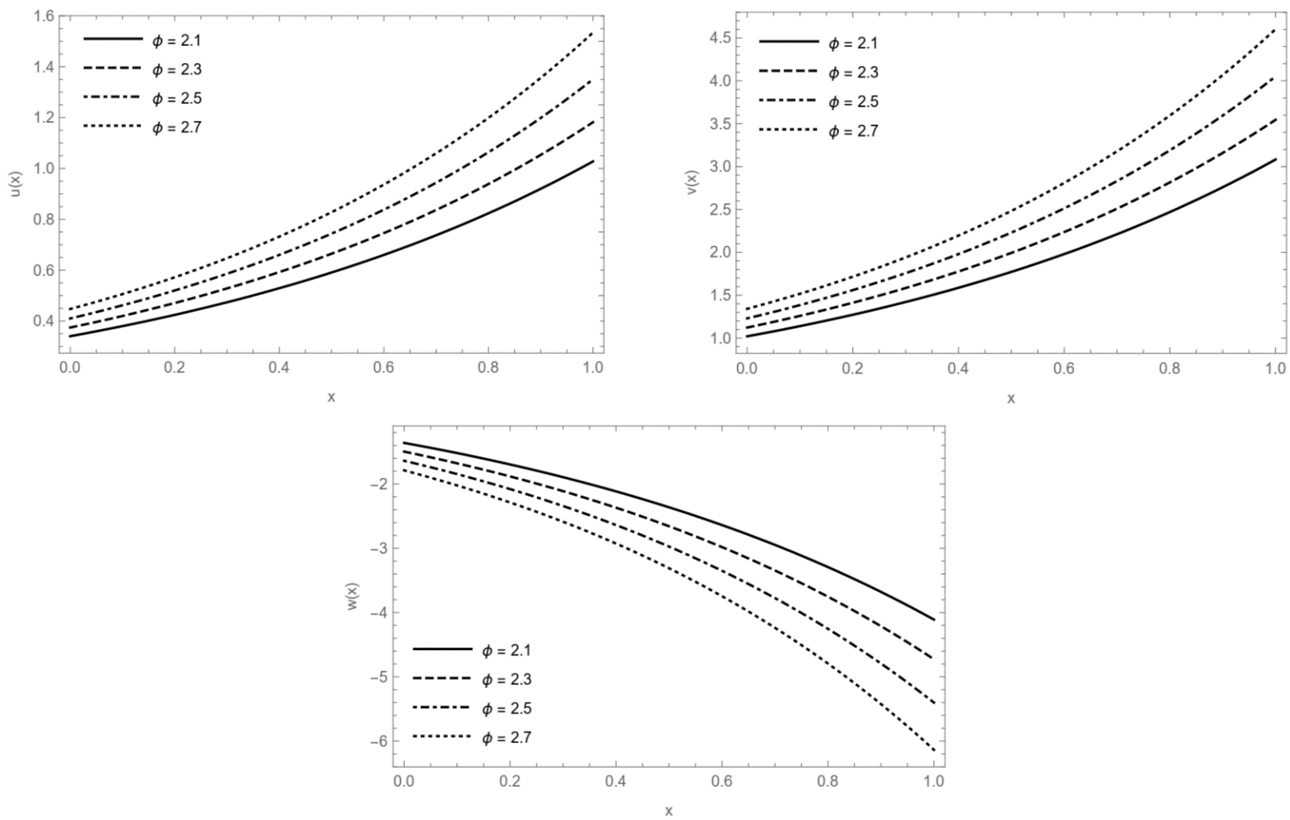


**Fig. 5.** Component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37), for $\omega = 0.1$ , $v = 0.515$ , $\Phi = 0.1$ , $\alpha_1 = 0.001$ , $\beta = 0.8$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ , $t = 2$s and various value of $\phi$ .

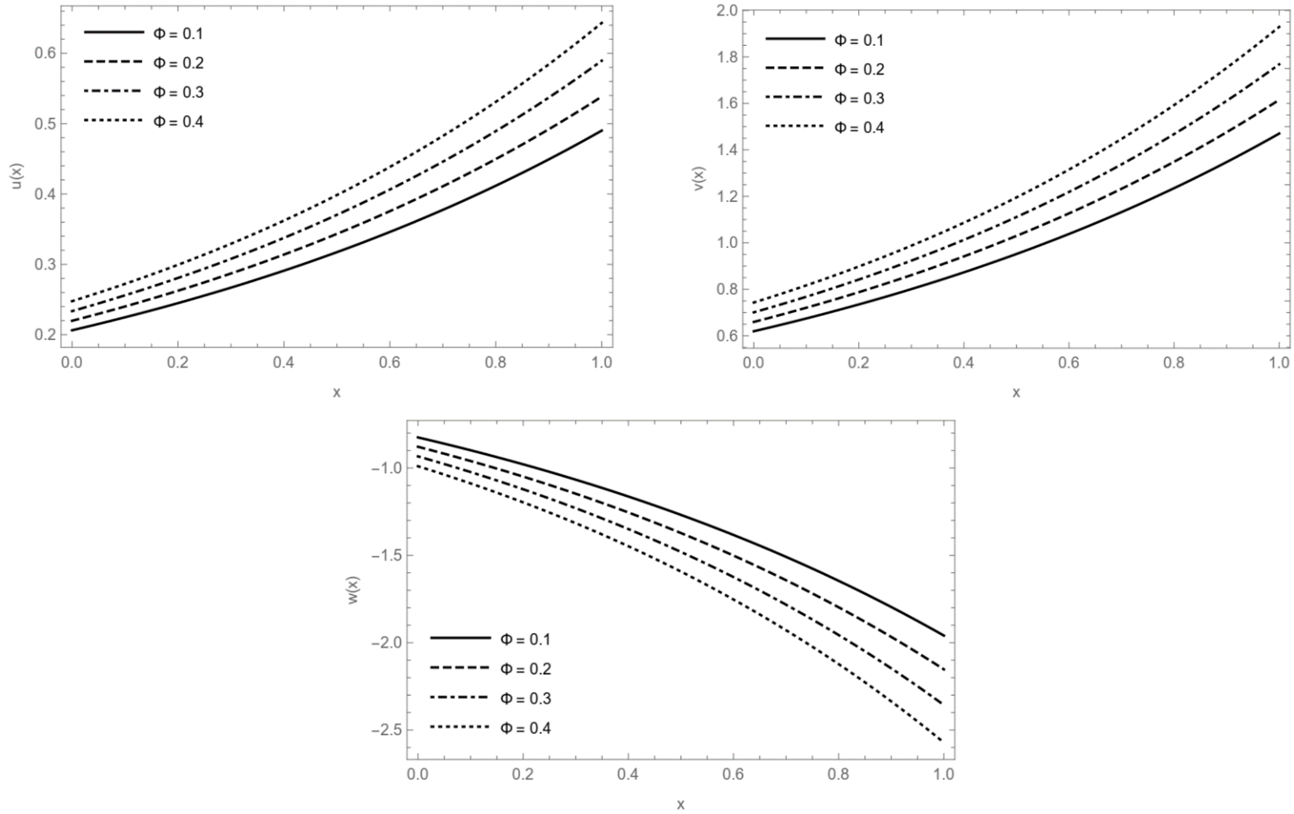


**Fig. 6.** Component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37), for $\omega = 0.1$ , $v = 0.515$ , $\phi = 1.2$ , $\alpha_1 = 0.001$ , $\beta = 0.8$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ , $t = 2s$ and various value of $\Phi$ .

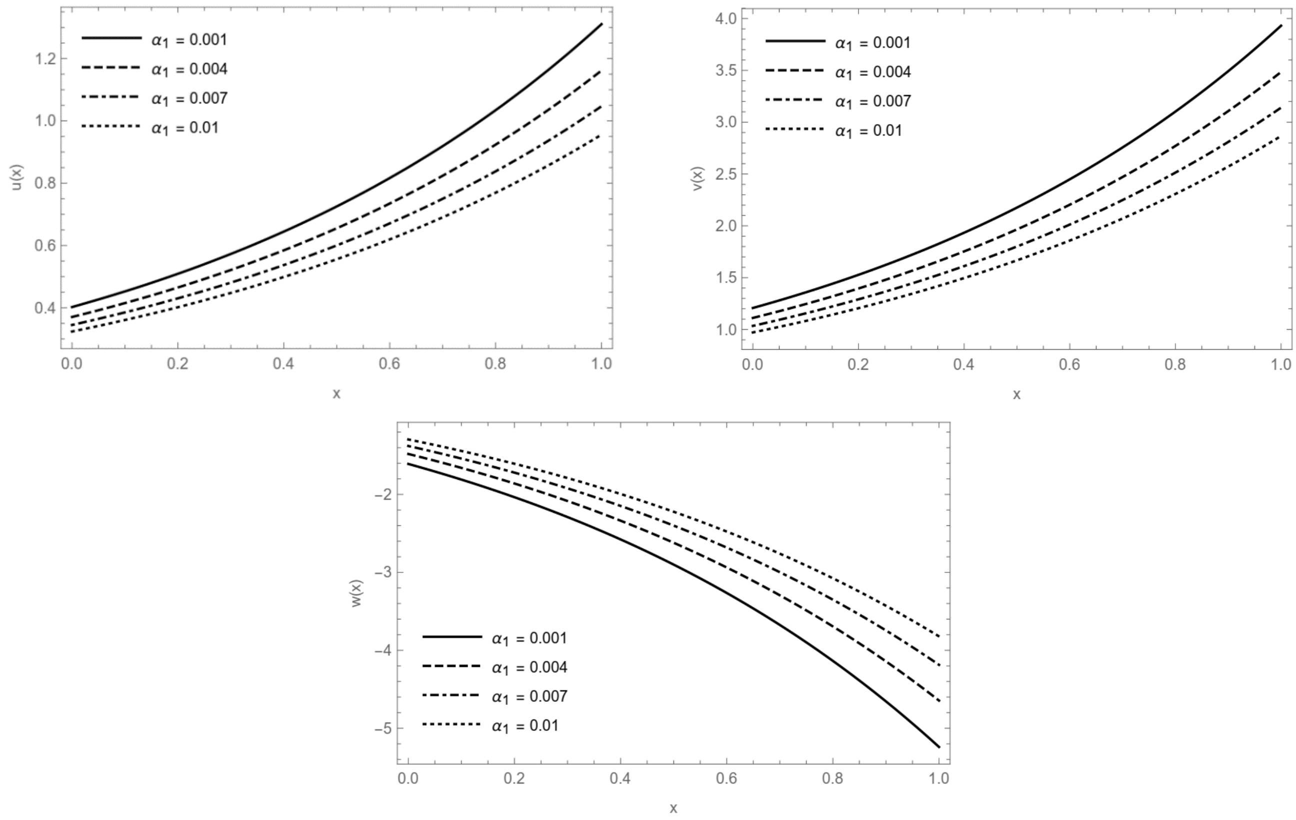


**Fig. 7.** Component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37), for $\omega = 0.1$ , $v = 0.515$ , $\Phi = 0.1$ , $\phi = 2$ , $\beta = 0.8$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ , $t = 2s$ and various value of $\alpha_1$ .

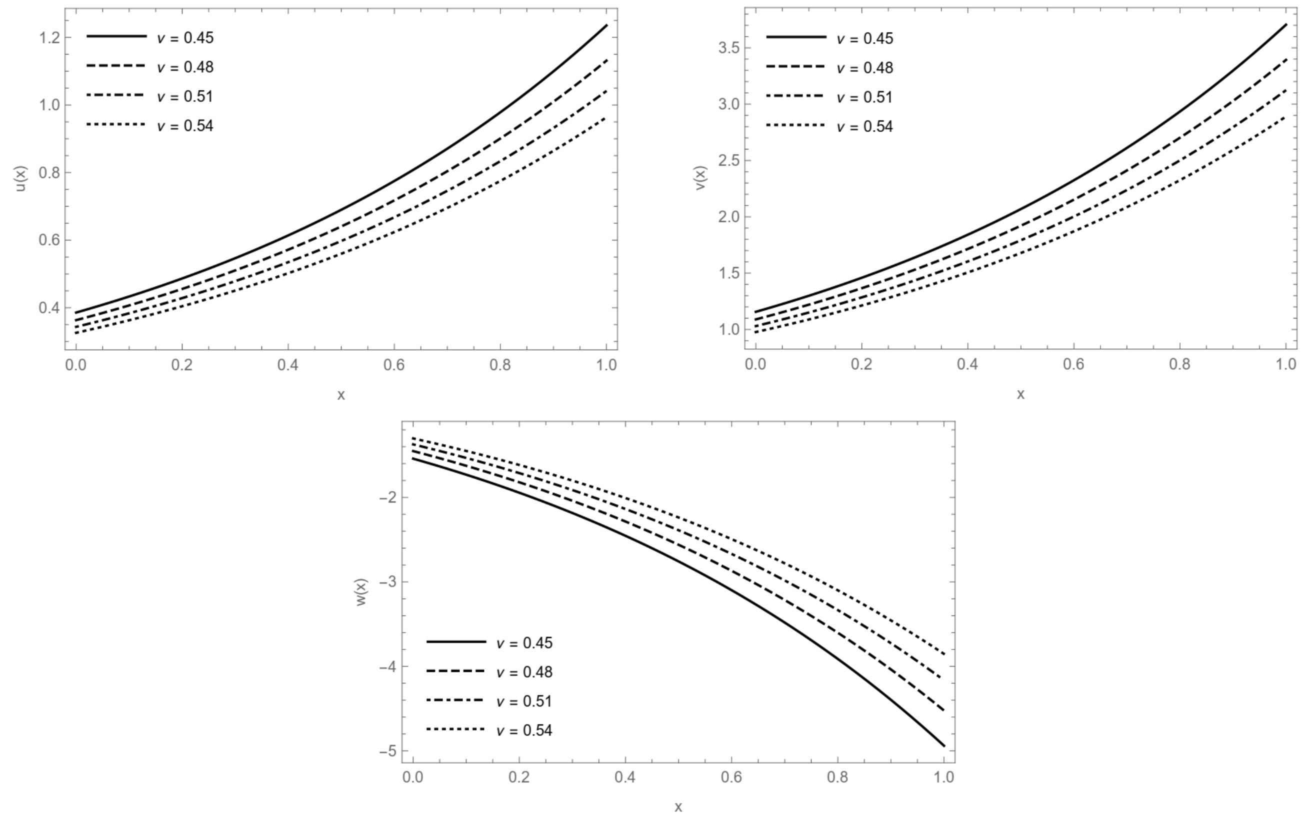


**Fig. 8.** Component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37), for $\omega = 0.1$ , $\beta = 0.8$ , $\phi = 2$ , $\Phi = 0.1$ , $\alpha_1 = 0.001$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ , $t = 2$s and various value of $v$ .

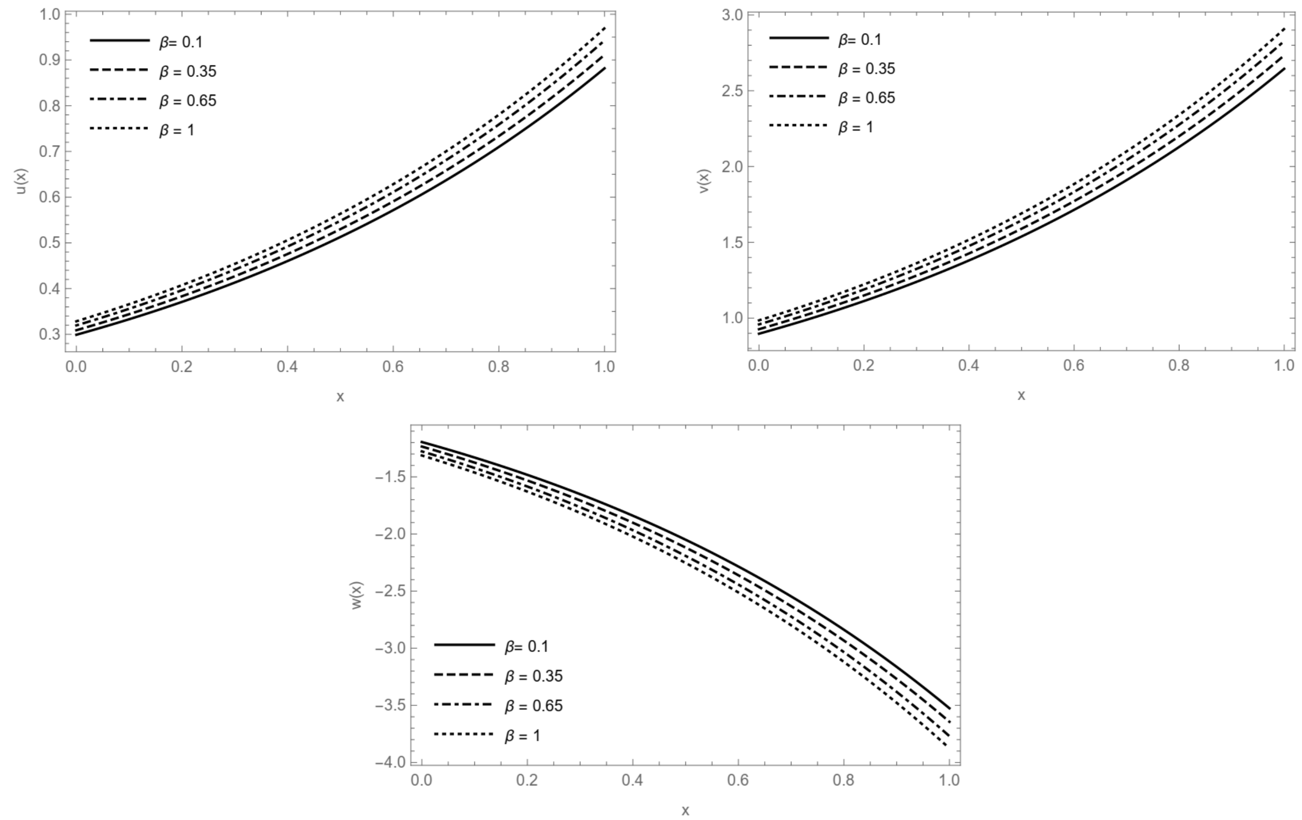


**Fig. 9.** Component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37) for $\omega = 0.1$ , $v = 0.515$ , $\phi = 2$ , $\Phi = 0.1$ , $\alpha_1 = 0.001$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ , $t = 2$s and various value of $\beta$ .

$$\nu\left(l^2+m^2+n^2\right)\left(nu''-lw''\right)-(\phi+\Phi)(nu-lw)=0. \tag{45}$$

Solving Eqs. (44) – (45), we get

$$u''-\tau_1\,u=\tau_2 \tag{46}$$

where $\tau_1$ and $\tau_2$ are presented in Appendix B. The solution of Eq. (46) is

$$u(\xi)=\tau_3\,e^{\alpha_{21}\xi}+\tau_4\,e^{-\alpha_{21}\xi}+\tau_5, \tag{47}$$

where $\tau_3$ , $\tau_4$ are arbitrary constants and $\tau_5$ is presented in Appendix B and $\alpha_{21}$ , $-\alpha_{21}$ are the roots of the auxiliary equation

$$m^2-\tau_1=0.$$

Using the Eq. (47) into Eq. (44), we get

$$v''-\tau_6 v=\tau_7\,e^{\alpha_{21}\xi}+\tau_8\,e^{-\alpha_{21}\xi}+\tau_9, \tag{48}$$

where $\tau_6=\tau_1$ and $\tau_7=\tau_2$ are presented in Appendix B. The solution of Eq. (48) is

$$v(\xi)=\tau_{15}\,e^{\alpha_{21}\xi}+\tau_{16}\,e^{-\alpha_{21}\xi}+\tau_{14}. \tag{49}$$

Putting the Eqs. (47) and (49) in Eq. (20)

$$w(\xi)=\tau_{17}\,e^{\alpha_{21}\xi}+\tau_{18}\,e^{-\alpha_{21}\xi}+\tau_{19}, \tag{50}$$

where $\tau_{14}$ , $\tau_{15}$ , $\tau_{16}$ , $\tau_{17}$ , $\tau_{18}$ and $\tau_{19}$ are presented in Appendix B. Putting the Eq. (47) and (50) in Eq. (40), we get the pressure

$$p(\xi)=\tau_{20}\,e^{\alpha_{21}\xi}+\tau_{21}\,e^{-\alpha_{21}\xi}+\tau_{22}\,e^{2\alpha_{21}\xi}+\tau_{23}\,e^{-2\alpha_{21}\xi}+\tau_{24}\,\xi+\tau_{25}. \tag{51}$$

Putting the Eqs. (47), (49) and (50) in Eq. (43), we get

$$T(\xi)=\tau_{26}\,e^{2\alpha_{21}\xi}+\tau_{27}\,e^{-2\alpha_{21}\xi}+\tau_{28}\,\xi^2+\tau_{29}\,\xi+\tau_{30}, \tag{52}$$

where $\tau_{20}$ , $\tau_{21}$ , $\tau_{22}$ , $\tau_{23}$ and $\tau_{24}$ , $\tau_{26}$ , $\tau_{27}$ , $\tau_{28}$ are presented in Appendix B and $\tau_{25}$ , $\tau_{29}$ , $\tau_{30}$ are arbitrary constants. Recurrence the original variables are

$$u(x,y,z,t)=\tau_3\,e^{\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_4\,e^{-\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_5, \tag{53}$$

$$v(x,y,z,t)=\tau_{15}\,e^{\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{16}\,e^{-\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{14}. \tag{54}$$

$$w(\xi)=\tau_{17}\,e^{\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{18}\,e^{-\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{19}, \tag{55}$$

$$p(x,y,z,t)=\tau_{20}\,e^{\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{21}\,e^{-\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{22}\,e^{2\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{23}\,e^{-2\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{24}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)+\tau_{25}. \tag{56}$$

$$T(x,y,z,t)=\tau_{26}\,e^{2\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{27}\,e^{-2\alpha_{21}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)}+\tau_{28}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)^2+\tau_{29}\left(lx+my+nz+\frac{\omega t^\beta}{\Gamma(\beta+1)}\right)+\tau_{30}, \tag{57}$$

## 4. Numerical results and discussions

The study of MHD fractionalized viscoelastic second grade fluid in porous medium with heat transfer is considered in this analysis. We have endeavored to attain the exact solutions of velocity components $u$ , $v$ , $w$ pressure and heat transfer. We have studied two different cases which are case I: $\omega+b_0\neq 0$ and case II: $\omega+b_0=0$ for getting the accurate solutions. Further, we can determine some solution of the fractional parameter $\beta$ on the flow by graphical interpretation. It is also be noted when $\alpha_1 \to 0$ the solutions have been recovered from non-Newtonian fluid resulting in the production of a Newtonian fluid. The results are obtained by fractional traveling wave parameter $\xi$ and presented in the exponential form. Presently, to show some graphical consequences of the components field of fluid $u(x,y), v(x,y)$ and $w(x,y)$, it is shown for $x$ changed estimations of $t$ and the appropriate parameters of the liquid.

For comfort, we show the diagram for the case I: $\omega + b_0 \neq 0$ and another case might be talking about similarly. Figs. 2, 3 and 4 displayed the effect of space variable, time and 3D sketches of $u$ and $v$ with respect to $t$ and $x$. It is obvious that first two components $u$ and $v$ are increasing function space variable $x$ and time $t$, whereas the third component $w$ is also intensifying function (in an absolute sense) of these variables in Figs. 2 - 3. The combine influence of these variables is presented in 3D sketches of Figs. 4. It is more obvious from 3D sketches of $u$, $v$ and $w$ that they are becoming reinforce and reinforcing values of $t$ and $x$. In Figs. 5 and 6 are illustrated the impact of magnetic parameter $\phi$ and porosity parameter $\Phi$ on the liquid motion, it is seemed from these diagrams that the components field $u$, $v$ and $w$ (in an absolute sense) are increasing function of these parameter. Figs. 7 and 8 are sketched the effect of $\alpha_1$ material parameter and $\nu$ kinematic viscosity respectively, it is observed from these diagrams that the components field $u$, $v$ and $w$ are decreasing by increasing the values of all two parameters (viscosity and material). It is clear that the effect of $\alpha_1$ and $\nu$ are opposite to magnetic parameter $\phi$. The ultimate important fractional parameter $\beta$ in Figs. 9, clearly show that the increasing values of parameter $\beta$ increase the liquid of motion which is noticeable common experience that when $\beta \to 1$ the fractionalized non-Newtonian fluid decrease to ordinary non-Newtonian fluid. Therefore in Figs. 10, we deduce that the non-Newtonian fluid flow faster than the fractionalized non-Newtonian fluid. We have showed the effect of the parameter $l$ seem in traveling wave parameter $\xi = lx + my + nz + \omega\, t^{\beta}/\Gamma(\beta+1)$. It is noted that for providing time $t$ in every place of the domain of flow the velocity field $u$, $v$ and $w$ varies proportional to the values of $l > 0$.

In the end, to compare the profile of the components field $u$, $v$ and $w$ representing to the stream of the three kinds of liquid (fractionalized MHD non-Newtonian fluid with porous for $\beta = 0.5$, ordinary MHD non-Newtonian fluid with porous for $\beta = 1$ and MHD Newtonian fluid with porous medium fluids) are deliberated and showed in Figs. 11 for alike values of material constants. As expected, and also justified from Figs. 9, that MHD Newtonian fluid with porous medium have greatest velocity in comparison to those of other two liquids. Moreover, it is important to point out that in absolute sense in all figures that the components field $u(x,t), v(x,t)$ and $w(x,t)$ hold the inequality $w(x,t) > v(x,t) > u(x,t)$. All shapes are constructed by Mathematic software with the SI units of the material constants in all diagrams.

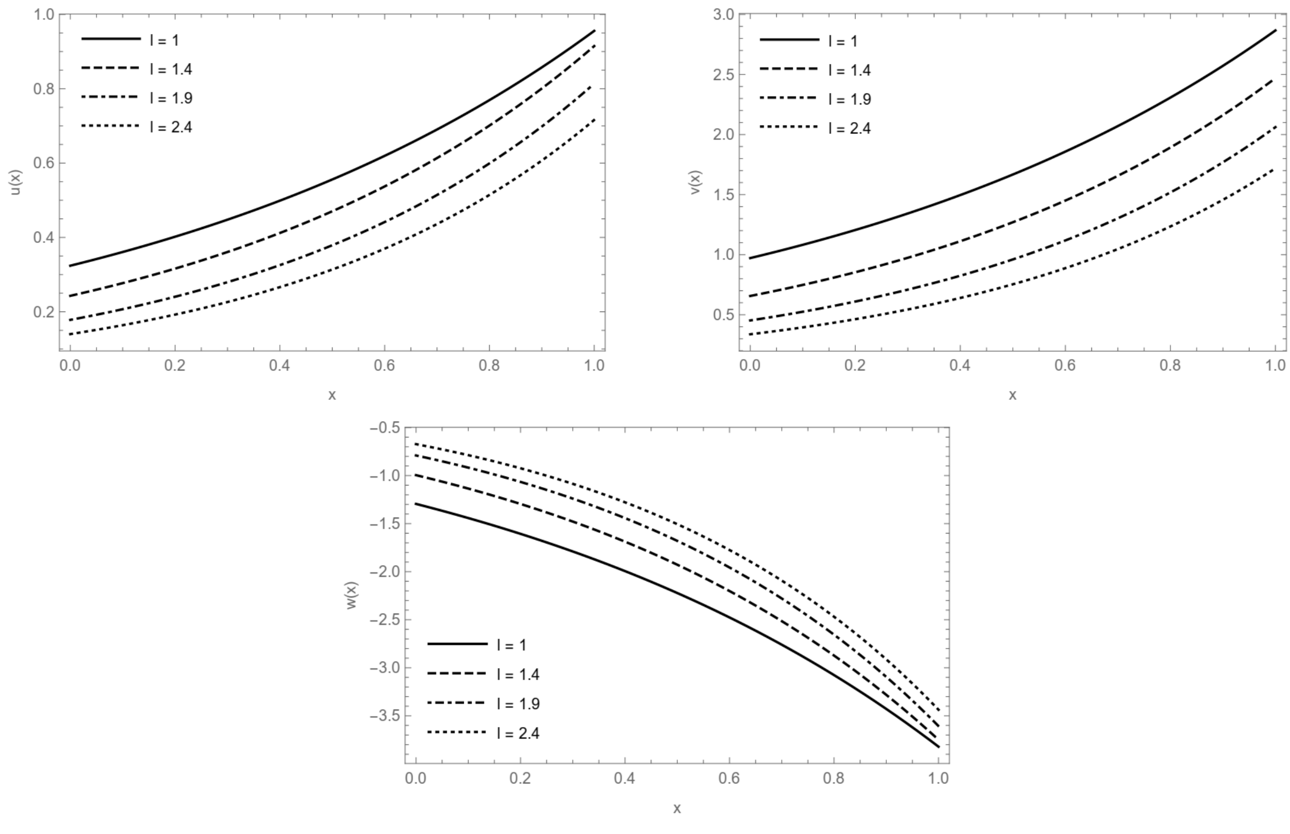


**Fig. 10.** Component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for fractionalized MHD second grade fluid with porous given by Eqs. (35), (36) and (37), for $\omega = 0.1$, $v = 0.515$, $\phi = 2$, $\Phi = 0.1$, $\alpha_1 = 0.001$, $b_0 = 0.001$, $y = 1$, $\rho = 0.001$, $\beta = 0.8$, $m = n = 1$, $t = 2s$ and various value of $l$.

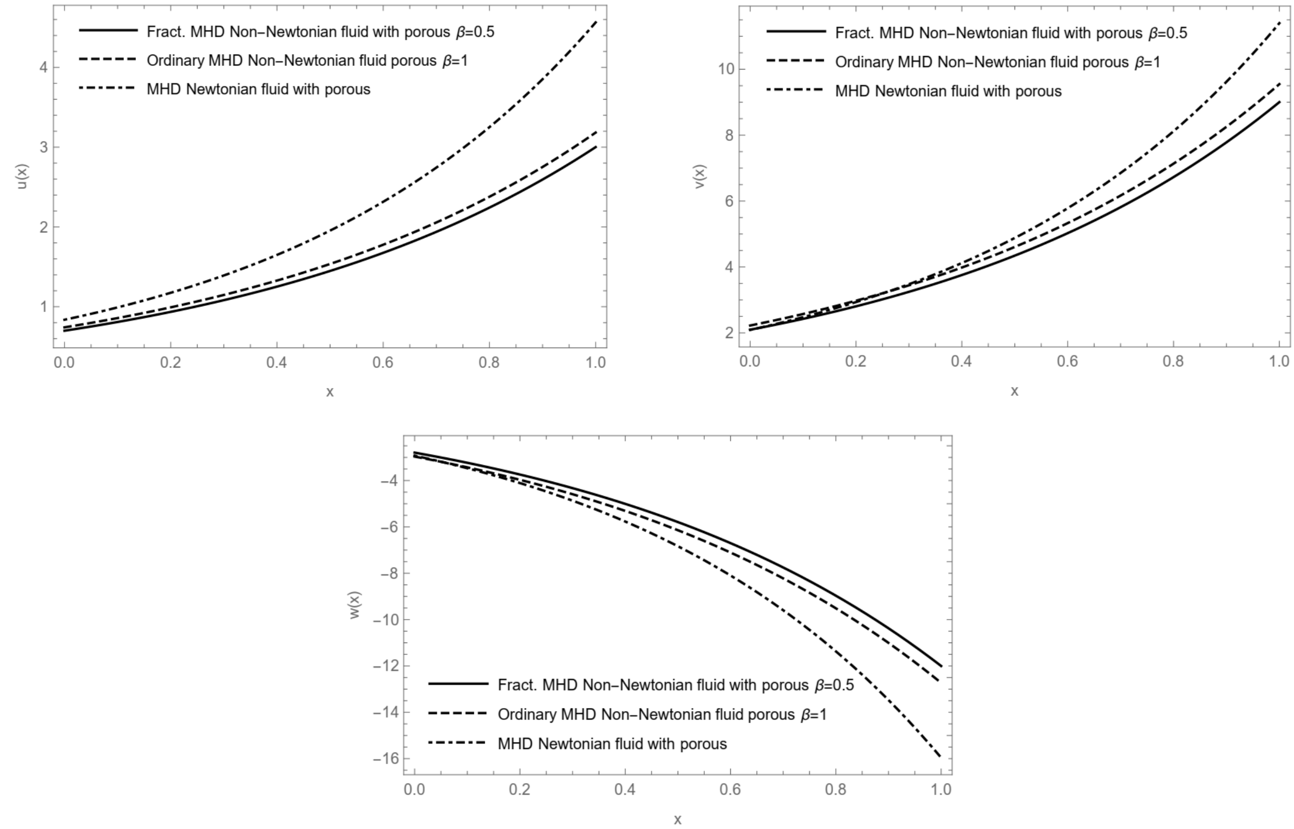


**Fig. 11.** Component field profile $u(x,t), v(x,t)$ and $w(x,t)$ for MHD fractionalized non-Newtonian fluid with porous given by Eqs. (35), (36) and (37), for $\beta = 0.5$ , ordinary MHD non-Newtonian fluid with porous given by Eqs. (35), (36) and (37), for $\beta = 1$ , and MHD Newtonian fluid with porous given by Eqs. (35), (36) and (37) for $\beta = 1$ and $\alpha_1 = 0$ for $\omega = 0.1$ , $v = 0.515$ , $\phi = 2$ , $\Phi = 0.1$ , $b_0 = 0.001$ , $y = 1$ , $\rho = 0.001$ , $l = m = n = 1$ and $t = 2s$ .

## 5. Conclusion

The objective of this paper was to determine the exact solutions of three-dimensional flow of fractionalized second grade fluid MHD in porous medium with heat transfer. The methodology utilized in this study was comfortably for linearizing the stream of equations by contemplating fractional traveling wave parameter $\xi$ . The method was applied directly with no limited assumptions or laborious calculation. It was seen that solutions have been acquired in exponential type. The present results were contrasted with additional group by employing different methods. It was acquired that present method can cover the results and will likewise expand the liquid dynamic model. Exponential kind of solutions have been seen for Newtonian and non- Newtonian fluid [50 – 55]. For further investigation, we will determine three dimensional of non-Newtonian flow model equations by the suggested method. Furthermore, this producer can be implemented to other non-Newtonian liquid to get the accurate solutions for illustration pseudoplastic and rate type fluids.

## Author Contributions

In this article, the foremost idea and geometrical part and their explication fulfilled by Arsalan Ahmed, Poonam K.K, Munam Khalil, Fatima Riaz, and Syed Mohammad Mohsnain executed the mathematical calculation and composed the composition. The paper was composed with the assistance of all authors. We discussed the results, reviewed and approved this article by all authors. All creators examined the outcomes, looked into, and endorsed the last form of the original copy.

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The authors are indebted to the mysterious referees for the suitable suggestions to improve the quality of paper. The aim of the author Arsalan Ahmed is to dedicate his work to his parents and to his co-author’s parents, and the most respectable person of his life (Late. Ahmed Nawaz, Late. Asadullah Memon, Kewal Mal and Engr. Narain Dass).

## Conflict of Interest

The writers announced no expected irreconcilable circumstances concerning the examination, initiation and distribution of this article.

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## Appendix A

$$\kappa_1=\frac{\nu\ \rho}{\alpha_1(\omega+b_0)},\quad \kappa_2=\frac{-\rho}{\alpha_1(l^2+m^2+n^2)},\quad \kappa_3=\frac{-\rho(\phi+\Phi)}{\alpha_1(\omega+b_0)(l^2+m^2+n^2)} \tag{A.1}$$

$$\kappa_4=\frac{-l\ b_0\ \rho(\phi+\Phi)}{\alpha_1(\omega+b_0)(l^2+m^2+n^2)},\quad \kappa_8=\frac{\kappa_4}{\kappa_3},\quad \kappa_9=\frac{\nu\ \rho}{\alpha_1(\omega+b_0)} \tag{A.2}$$

$$\kappa_{10}=\frac{-\rho}{\alpha_1(l^2+m^2+n^2)},\quad \kappa_{11}=\frac{-\rho(\phi+\Phi)}{\alpha_1(\omega+b_0)(l^2+m^2+n^2)} \tag{A.3}$$

$$\kappa_{12}=\frac{\rho}{l\alpha_1(\omega+b_0)(l^2+m^2+n^2)}\left[\frac{\gamma_{11}{}^3\ \kappa_5\ m\ \alpha_1(\omega+b_0)(l^2+m^2+n^2)}{\rho}+\gamma_{11}{}^2\ \kappa_5\ m\ \nu(\omega+b_0)(l^2+m^2+n^2)-\gamma_{11}\ \kappa_5\ m(\omega+b_0)-\kappa_5\ m(\phi+\Phi)\right] \tag{A.4}$$

$$\kappa_{13}=\frac{\rho}{l\alpha_1(\omega+b_0)(l^2+m^2+n^2)}\left[\frac{\gamma_{12}{}^3\ \kappa_6\ m\ \alpha_1(\omega+b_0)(l^2+m^2+n^2)}{\rho}+\gamma_{12}{}^2\ \kappa_6\ m\ \nu(\omega+b_0)(l^2+m^2+n^2)-\gamma_{12}\ \kappa_6\ m(\omega+b_0)-\kappa_6\ m(\phi+\Phi)\right] \tag{A.5}$$

$$\kappa_{14}=\frac{\rho}{l\alpha_1(\omega+b_0)(l^2+m^2+n^2)}\left[\frac{\gamma_{13}{}^3\ \kappa_7\ m\ \alpha_1(\omega+b_0)(l^2+m^2+n^2)}{\rho}+\gamma_{13}{}^2\ \kappa_7\ m\ \nu(\omega+b_0)(l^2+m^2+n^2)-\gamma_{13}\ \kappa_7\ m(\omega+b_0)-\kappa_7\ m(\phi+\Phi)\right] \tag{A.6}$$

$$\kappa_{15}=\frac{\rho}{l\alpha_1(\omega+b_0)(l^2+m^2+n^2)}\left[-\kappa_8\ m(\phi+\Phi)\right],\quad \kappa_{19}=\frac{\kappa_{12}}{(\gamma_{11})^3+\kappa_9(\gamma_{11})^2+\kappa_{10}\ \gamma_{11}+\kappa_{11}},\quad \kappa_{20}=\frac{\kappa_{13}}{(\gamma_{12})^3+\kappa_9(\gamma_{12})^2+\kappa_{10}\ \gamma_{12}+\kappa_{11}} \tag{A.7}$$

$$\kappa_{21}=\frac{\kappa_{14}}{(\gamma_{13})^3+\kappa_9(\gamma_{13})^2+\kappa_{10}\ \gamma_{13}+\kappa_{11}},\quad \kappa_{22}=\frac{\kappa_{15}}{\kappa_{11}} \tag{A.8}$$

$$\kappa_{23}=\kappa_{16}+\kappa_{19},\quad \kappa_{24}=\kappa_{17}+\kappa_{20},\quad \kappa_{25}=\kappa_{18}+\kappa_{21} \tag{A.9}$$

where $\kappa_{16}$, $\kappa_{17}$ and $\kappa_{18}$ are arbitrary constant.

$$\kappa_{26}=\frac{-1}{n}(l\ \kappa_5+m\ \kappa_{23}),\quad \kappa_{27}=\frac{-1}{n}(l\ \kappa_6+m\ \kappa_{24}),\quad \kappa_{28}=\frac{-1}{n}(l\ \kappa_7+m\ \kappa_{25}) \tag{A.10}$$

$$\kappa_{29}=\frac{1}{n}(b_0-l\ \kappa_8+m\ \kappa_{26}) \tag{A.11}$$

$$\kappa_{30}=\frac{\kappa_5}{l}\left[\rho\ \nu\ \gamma_{11}{}^2(l^2+m^2+n^2)-\rho\ \gamma_{11}(\omega+b_0)+\alpha_1\ \gamma_{11}{}^3(\omega+b_0)(l^2+m^2+n^2)-\rho(\phi+\Phi)\right] \tag{A.12}$$

$$\kappa_{30}=\frac{\kappa_6}{l}\left[\rho\ \nu\ \gamma_{12}{}^2(l^2+m^2+n^2)-\rho\ \gamma_{12}(\omega+b_0)+\alpha_1\ \gamma_{12}{}^3(\omega+b_0)(l^2+m^2+n^2)-\rho(\phi+\Phi)\right] \tag{A.13}$$

$$\kappa_{32}=\frac{\kappa_7}{l}\left[\rho\ \nu\ \gamma_{13}{}^2(l^2+m^2+n^2)-\rho\ \gamma_{13}(\omega+b_0)+\alpha_1\ \gamma_{13}{}^3(\omega+b_0)(l^2+m^2+n^2)-\rho(\phi+\Phi)\right] \tag{A.14}$$

$$\kappa_{33}=\frac{\alpha_1}{m^2}\left[\kappa_5{}^2\ \gamma_{11}{}^3(2l^4+2m^4+2m^2n^2+4l^2m^2+2l^2n^2)+\kappa_{27}{}^2\ \gamma_{11}{}^3(2n^2l^2+4m^2n^2+2n^4+2l^2m^2+2m^4)+2\kappa_5\ \kappa_{27}(2nl^3+2n^3l+2lm^2n)\right] \tag{A.15}$$

$$\kappa_{34}=\frac{\alpha_1}{m^2}\left[\kappa_6{}^2\ \gamma_{12}{}^3(2l^4+2m^4+2m^2n^2+4l^2m^2+2l^2n^2)+\kappa_{28}{}^2\ \gamma_{12}{}^3(2n^2l^2+4m^2n^2+2n^4+2l^2m^2+2m^4)+2\kappa_6\ \kappa_{28}(2nl^3+2n^3l+2lm^2n)\right] \tag{A.16}$$

$$\kappa_{35}=\frac{\alpha_1}{m^2}\left[\kappa_7{}^2\ \gamma_{13}{}^3(2l^4+2m^4+2m^2n^2+4l^2m^2+2l^2n^2)+\kappa_{29}{}^2\ \gamma_{13}{}^3(2n^2l^2+4m^2n^2+2n^4+2l^2m^2+2m^4)+2\kappa_7\ \kappa_{29}(2nl^3+2n^3l+2lm^2n)\right] \tag{A.17}$$

$$\begin{aligned}\kappa_{36}=\frac{\alpha_1}{m^2}\Big[&\kappa_5\ \kappa_6(\gamma_{11}\ \gamma_{12}{}^2+\gamma_{12}\ \gamma_{11}{}^2)(2l^4+2m^4+2m^2n^2+4l^2m^2+2l^2n^2)+\kappa_{27}\ \kappa_{28}(\gamma_{11}\ \gamma_{12}{}^2+\gamma_{12}\ \gamma_{11}{}^2)\times\\&(2n^2l^2+4m^2n^2+2n^4+2l^2m^2+2m^4)+(\kappa_5\kappa_{28}\ \gamma_{11}\gamma_{12}{}^2+\kappa_6\kappa_{29}\ \gamma_{12}\gamma_{11}{}^2)(2nl^3+2n^3l+2lm^2n)+(\kappa_5\kappa_{28}\times\gamma_{12}\ \gamma_{11}{}^2+\kappa_6\kappa_{29}\ \gamma_{11}\gamma_{12}{}^2)(2nl^3+2n^3l+2lm^2n)\Big]\end{aligned} \tag{A.18}$$

$$\begin{aligned}\kappa_{37}=\frac{\alpha_1}{m^2}\Big[&\kappa_5\ \kappa_6(\gamma_{11}\ \gamma_{13}{}^2+\gamma_{13}\ \gamma_{11}{}^2)(2l^4+2m^4+2m^2n^2+4l^2m^2+2l^2n^2)+\kappa_{27}\ \kappa_{29}(\gamma_{11}\ \gamma_{13}{}^2+\gamma_{13}\ \gamma_{11}{}^2)\times\\&(2n^2l^2+4m^2n^2+2n^4+2l^2m^2+2m^4)+(\kappa_5\kappa_{29}\ \gamma_{11}\gamma_{13}{}^2+\kappa_7\kappa_{27}\ \gamma_{13}\gamma_{11}{}^2)(2nl^3+2n^3l+2lm^2n)+(\kappa_5\kappa_{29}\times\gamma_{13}\gamma_{11}{}^2+\kappa_{27}\kappa_{29}\ \gamma_{11}\gamma_{12}{}^2)(2nl^3+2n^3l+2lm^2n)\Big]\end{aligned} \tag{A.19}$$

$$\kappa_{38} = \frac{\alpha_1}{m^2}\Big[\kappa_6\ \kappa_7\left(\gamma_{12}\ \gamma_{13}{}^2 + \gamma_{13}\ \gamma_{12}{}^2\right)\left(2l^4 + 2m^4 + 2m^2n^2 + 4l^2m^2 + 2l^2n^2\right) + \kappa_{28}\ \kappa_{29}\left(\gamma_{12}\ \gamma_{13}{}^2 + \gamma_{13}\ \gamma_{12}{}^2\right)\times$$

$$\left(2n^2l^2 + 4m^2n^2 + 2n^4 + 2l^2m^2 + 2m^4\right) + \left(\kappa_6\kappa_{29}\ \gamma_{12}\gamma_{13}{}^2 + \kappa_7\kappa_{28}\ \gamma_{13}\gamma_{12}{}^2\right)\left(2nl^3 + 2n^3l + 2lm^2n\right) + \left(\kappa_5\kappa_{28}\times\gamma_{13}\gamma_{12}{}^2 + \kappa_7\kappa_{28}\ \gamma_{11}\gamma_{12}{}^2\right)\left(2nl^3 + 2n^3l + 2lm^2n\right)\Big] \tag{A.20}$$

$$\kappa_{39} = \frac{\rho\ \kappa_8\left(\phi + \Phi\right)}{l}\ ,\ \ \kappa_{41} = \frac{\rho\ c_\rho\left(\omega + b_0\right)}{k\left(l^2 + m^2 + n^2\right)} \tag{A.21}$$

$$\kappa_{42} = \frac{1}{\left(l^2 + m^2 + n^2\right)}\Bigg[\frac{-\nu}{\rho\ k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\kappa_5\gamma_{11}\right)^2 + \left(2m^2 + l^2 + n^2\right)\left(\kappa_{23}\gamma_{11}\right)^2 + \left(2n^2 + m^2 + l^2\right)\left(\kappa_{26}\gamma_{11}\right)^2$$

$$+2ml\left(\kappa_5\,\kappa_{23}\right)\gamma_{11}{}^2 + 2nl\left(\kappa_5\,\kappa_{26}\right)\gamma_{11}{}^2 + 2mn\left(\kappa_{23}\,\kappa_{26}\right)\gamma_{11}{}^2\Big] - \frac{\alpha_1\left(\omega + b_0\right)}{k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\kappa_5{}^2\gamma_{11}{}^3\right) + \left(\kappa_{23}{}^2\gamma_{11}{}^3\right)$$

$$\left(2m^2 + l^2 + n^2\right) + \left(2n^2 + m^2 + l^2\right)\left(\kappa_{26}{}^2\gamma_{11}{}^3\right) + 2ml\left(\kappa_5\,\kappa_{23}\right)\gamma_{11}{}^3 + 2nl\left(\kappa_5\,\kappa_{26}\right)\gamma_{11}{}^3 + 2mn\left(\kappa_{23}\,\kappa_{26}\right)\gamma_{11}{}^3\Big]\Bigg] \tag{A.22}$$

$$\kappa_{43} = \frac{1}{\left(l^2 + m^2 + n^2\right)}\Bigg[\frac{-\nu}{\rho\ k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\kappa_6\gamma_{12}\right)^2 + \left(2m^2 + l^2 + n^2\right)\left(\kappa_{24}\gamma_{12}\right)^2 + \left(2n^2 + m^2 + l^2\right)\left(\kappa_{27}\gamma_{12}\right)^2$$

$$+2ml\left(\kappa_6\,\kappa_{24}\right)\gamma_{12}{}^2 + 2nl\left(\kappa_6\,\kappa_{27}\right)\gamma_{12}{}^2 + 2mn\left(\kappa_{24}\,\kappa_{27}\right)\gamma_{12}{}^2\Big] - \frac{\alpha_1\left(\omega + b_0\right)}{k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\kappa_6{}^2\gamma_{12}{}^3\right) + \left(\kappa_{24}{}^2\gamma_{12}{}^3\right)$$

$$\left(2m^2 + l^2 + n^2\right) + \left(2n^2 + m^2 + l^2\right)\left(\kappa_{27}{}^2\gamma_{12}{}^3\right) + 2ml\left(\kappa_6\,\kappa_{24}\right)\gamma_{12}{}^3 + 2nl\left(\kappa_6\,\kappa_{27}\right)\gamma_{12}{}^3 + 2mn\left(\kappa_{24}\,\kappa_{27}\right)\gamma_{12}{}^3\Big]\Bigg] \tag{A.23}$$

$$\kappa_{44} = \frac{1}{\left(l^2 + m^2 + n^2\right)}\Bigg[\frac{-\nu}{\rho\ k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\kappa_7\gamma_{13}\right)^2 + \left(2m^2 + l^2 + n^2\right)\left(\kappa_{25}\gamma_{13}\right)^2 + \left(2n^2 + m^2 + l^2\right)\left(\kappa_{28}\gamma_{13}\right)^2$$

$$+2ml\left(\kappa_7\,\kappa_{25}\right)\gamma_{13}{}^2 + 2nl\left(\kappa_7\,\kappa_{28}\right)\gamma_{13}{}^2 + 2mn\left(\kappa_{25}\,\kappa_{28}\right)\gamma_{13}{}^2\Big] - \frac{\alpha_1\left(\omega + b_0\right)}{k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\kappa_7{}^2\gamma_{13}{}^3\right) + \left(\kappa_{25}{}^2\gamma_{13}{}^3\right)$$

$$\left(2m^2 + l^2 + n^2\right) + \left(2n^2 + m^2 + l^2\right)\left(\kappa_{28}{}^2\gamma_{13}{}^3\right) + 2ml\left(\kappa_7\,\kappa_{25}\right)\gamma_{13}{}^3 + 2nl\left(\kappa_7\,\kappa_{28}\right)\gamma_{13}{}^3 + 2mn\left(\kappa_{25}\,\kappa_{28}\right)\gamma_{13}{}^3\Big]\Bigg] \tag{A.24}$$

$$\kappa_{45} = \frac{1}{\left(l^2 + m^2 + n^2\right)}\Bigg[\frac{-\nu}{\rho\ k}\big[2\kappa_5\ \kappa_6\ \gamma_{11}\ \gamma_{12}\left(2l^2 + m^2 + n^2\right) + 2\kappa_{23}\ \kappa_{24}\ \gamma_{11}\ \gamma_{12}\left(2m^2 + l^2 + n^2\right) + 2\kappa_{27}\ \kappa_{28}\ \gamma_{11}\ \gamma_{12}$$

$$\left(2n^2 + m^2 + l^2\right) + 2lm\ \gamma_{11}\gamma_{12}\left(\kappa_5\ \kappa_{24} + \kappa_6\ \kappa_{23}\right) + 2nl\ \gamma_{11}\gamma_{12}\left(\kappa_5\ \kappa_{28} + \kappa_6\ \kappa_{27}\right) + 2mn\ \gamma_{11}\gamma_{12}\left(\kappa_{23}\ \kappa_{28} + \kappa_{24}\ \kappa_{27}\right)\big]$$

$$-\frac{\alpha_1\left(\omega + b_0\right)}{k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\gamma_{11}\gamma_{12}{}^2 + \gamma_{12}\ \gamma_{11}{}^2\right)\kappa_5\kappa_6 + \left(2m^2 + l^2 + n^2\right)\left(\gamma_{11}\gamma_{12}{}^2 + \gamma_{12}\ \gamma_{11}{}^2\right)\kappa_{23}\kappa_{24} + \kappa_{27}\kappa_{28}$$

$$\left(2n^2 + m^2 + l^2\right)\left(\gamma_{11}\gamma_{12}{}^2 + \gamma_{12}\ \gamma_{11}{}^2\right) + lm\left(\kappa_5\kappa_{24}\gamma_{11}\gamma_{12}{}^2 + \kappa_6\kappa_{23}\gamma_{12}\gamma_{11}{}^2\right) + lm\left(\kappa_5\kappa_{24}\gamma_{12}\gamma_{11}{}^2 + \kappa_6\kappa_{23}\gamma_{12}\gamma_{11}{}^2\right) + nl\big(\kappa_5\kappa_{28}$$

$$\gamma_{11}\gamma_{12}{}^2 + \kappa_6\kappa_{27}\gamma_{12}\gamma_{11}{}^2\big) + nl\left(\kappa_5\kappa_{28}\gamma_{12}\gamma_{11}{}^2 + \kappa_6\kappa_{27}\gamma_{11}\gamma_{12}{}^2\right) + mn\left(\kappa_{23}\kappa_{28}\gamma_{11}\gamma_{12}{}^2 + \kappa_{24}\kappa_{27}\gamma_{12}\gamma_{11}{}^2\right) + mn\left(\kappa_{23}\kappa_{28}\gamma_{12}\gamma_{11}{}^2 + \kappa_{24}\kappa_{27}\gamma_{11}\gamma_{12}{}^2\right)\Big]\Bigg] \tag{A.25}$$

$$\kappa_{46} = \frac{1}{\left(l^2 + m^2 + n^2\right)}\Bigg[\frac{-\nu}{\rho\ k}\big[2\kappa_5\ \kappa_7\ \gamma_{11}\ \gamma_{13}\left(2l^2 + m^2 + n^2\right) + 2\kappa_{23}\ \kappa_{25}\ \gamma_{11}\ \gamma_{13}\left(2m^2 + l^2 + n^2\right) + 2\kappa_{27}\ \kappa_{29}\ \gamma_{11}\ \gamma_{13}$$

$$\left(2n^2 + m^2 + l^2\right) + 2lm\ \gamma_{11}\gamma_{13}\left(\kappa_5\ \kappa_{25} + \kappa_7\ \kappa_{23}\right) + 2nl\ \gamma_{11}\gamma_{13}\left(\kappa_5\ \kappa_{29} + \kappa_7\ \kappa_{27}\right) + 2mn\ \gamma_{11}\gamma_{13}\left(\kappa_{23}\ \kappa_{29} + \kappa_{25}\ \kappa_{27}\right)\big]$$

$$-\frac{\alpha_1\left(\omega + b_0\right)}{k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\gamma_{11}\gamma_{13}{}^2 + \gamma_{13}\ \gamma_{11}{}^2\right)\kappa_5\kappa_7 + \left(2m^2 + l^2 + n^2\right)\left(\gamma_{11}\gamma_{13}{}^2 + \gamma_{13}\ \gamma_{11}{}^2\right)\kappa_{23}\kappa_{25} + \kappa_{27}\kappa_{29}$$

$$\left(2n^2 + m^2 + l^2\right)\left(\gamma_{11}\gamma_{13}{}^2 + \gamma_{13}\ \gamma_{11}{}^2\right) + lm\left(\kappa_5\kappa_{25}\gamma_{11}\gamma_{13}{}^2 + \kappa_7\kappa_{23}\gamma_{13}\gamma_{11}{}^2\right) + lm\left(\kappa_5\kappa_{25}\gamma_{13}\gamma_{11}{}^2 + \kappa_7\kappa_{23}\gamma_{11}\gamma_{13}{}^2\right) + nl\big(\kappa_5\kappa_{29}$$

$$\gamma_{11}\gamma_{13}{}^2 + \kappa_7\kappa_{27}\gamma_{13}\gamma_{11}{}^2\big) + nl\left(\kappa_5\kappa_{29}\gamma_{13}\gamma_{11}{}^2 + \kappa_7\kappa_{27}\gamma_{11}\gamma_{13}{}^2\right) + mn\left(\kappa_{23}\kappa_{29}\gamma_{11}\gamma_{13}{}^2 + \kappa_{25}\kappa_{27}\gamma_{13}\gamma_{11}{}^2\right) + mn\left(\kappa_{23}\kappa_{29}\gamma_{13}\gamma_{11}{}^2 + \kappa_{25}\kappa_{27}\gamma_{11}\gamma_{13}{}^2\right)\Big]\Bigg] \tag{A.26}$$

$$\kappa_{47} = \frac{1}{\left(l^2 + m^2 + n^2\right)}\Bigg[\frac{-\nu}{\rho\ k}\big[2\kappa_6\ \kappa_7\ \gamma_{12}\ \gamma_{13}\left(2l^2 + m^2 + n^2\right) + 2\kappa_{24}\ \kappa_{25}\ \gamma_{12}\ \gamma_{13}\left(2m^2 + l^2 + n^2\right) + 2\kappa_{28}\ \kappa_{29}\ \gamma_{12}\ \gamma_{13}$$

$$\left(2n^2 + m^2 + l^2\right) + 2lm\ \gamma_{12}\gamma_{13}\left(\kappa_6\ \kappa_{25} + \kappa_7\ \kappa_{24}\right) + 2nl\ \gamma_{12}\gamma_{13}\left(\kappa_6\ \kappa_{29} + \kappa_7\ \kappa_{28}\right) + 2mn\ \gamma_{12}\gamma_{13}\left(\kappa_{24}\ \kappa_{29} + \kappa_{25}\ \kappa_{28}\right)\big]$$

$$-\frac{\alpha_1\left(\omega + b_0\right)}{k}\Big[\left(2l^2 + m^2 + n^2\right)\left(\gamma_{12}\gamma_{13}{}^2 + \gamma_{13}\ \gamma_{12}{}^2\right)\kappa_6\kappa_7 + \left(2m^2 + l^2 + n^2\right)\left(\gamma_{12}\gamma_{13}{}^2 + \gamma_{12}\ \gamma_{13}{}^2\right)\kappa_{24}\kappa_{25} + \kappa_{28}\kappa_{29}$$

$$\left(2n^2 + m^2 + l^2\right)\left(\gamma_{12}\gamma_{13}{}^2 + \gamma_{13}\ \gamma_{12}{}^2\right) + lm\left(\kappa_6\kappa_{25}\gamma_{12}\gamma_{13}{}^2 + \kappa_7\kappa_{24}\gamma_{12}\gamma_{13}{}^2\right) + lm\left(\kappa_6\kappa_{25}\gamma_{13}\gamma_{12}{}^2 + \kappa_7\kappa_{24}\gamma_{12}\gamma_{13}{}^2\right) + nl\big(\kappa_6\kappa_{29}$$

$$\gamma_{12}\gamma_{13}{}^2 + \kappa_7\kappa_{28}\gamma_{13}\gamma_{12}{}^2\big) + nl\left(\kappa_6\kappa_{29}\gamma_{13}\gamma_{12}{}^2 + \kappa_7\kappa_{28}\gamma_{12}\gamma_{13}{}^2\right) + mn\left(\kappa_{24}\kappa_{29}\gamma_{12}\gamma_{13}{}^2 + \kappa_{25}\kappa_{28}\gamma_{13}\gamma_{12}{}^2\right) + mn\left(\kappa_{24}\kappa_{29}\gamma_{13}\gamma_{12}{}^2 + \kappa_{25}\kappa_{28}\gamma_{13}\gamma_{12}{}^2\right)\Big]\Bigg] \tag{A.27}$$

$$\kappa_{50} = \frac{\kappa_{42}}{\left(2\gamma_{11}\right)^2 - 2\ \kappa_{41}\gamma_{11}}\ ,\ \ \kappa_{51} = \frac{\kappa_{42}}{\left(2\gamma_{12}\right)^2 - 2\ \kappa_{41}\gamma_{12}} \tag{A.28}$$

$$\kappa_{52} = \frac{\kappa_{42}}{\left(2\gamma_{13}\right)^2 - 2\ \kappa_{41}\gamma_{13}}\ ,\ \ \kappa_{53} = \frac{\kappa_{42}}{\left(\gamma_{11} + \gamma_{12}\right)^2 - 2\ \kappa_{41}\left(\gamma_{11} + \gamma_{12}\right)} \tag{A.29}$$

$$\kappa_{54} = \frac{\kappa_{42}}{\left(\gamma_{11} + \gamma_{13}\right)^2 - 2\ \kappa_{41}\left(\gamma_{11} + \gamma_{12}\right)}\ ,\ \kappa_{55} = \frac{\kappa_{42}}{\left(\gamma_{12} + \gamma_{13}\right)^2 - 2\ \kappa_{41}\left(\gamma_{12} + \gamma_{13}\right)} \tag{A.30}$$

## Appendix B

$$\tau_1 = \frac{(\phi+\Phi)}{\nu\left(l^2+m^2+n^2\right)}, \quad \tau_2 = \frac{-l(\phi+\Phi)\,b_0}{\nu\left(l^2+m^2+n^2\right)^2}, \quad \tau_5 = \frac{\tau_2}{\tau_1} \tag{B.1}$$

$$\tau_6 = \frac{(\phi+\Phi)}{\nu\left(l^2+m^2+n^2\right)}, \quad \tau_7 = \left[\frac{m\,\tau_3\,\alpha_{21}{}^2}{l} - \frac{m\,\tau_3(\phi+\Phi)}{l\,\nu\left(l^2+m^2+n^2\right)}\right] \tag{B.2}$$

$$\tau_8 = \left[\frac{m\,\tau_4\,\alpha_{21}{}^2}{l} - \frac{m\,\tau_3(\phi+\Phi)}{l\,\nu\left(l^2+m^2+n^2\right)}\right] \quad \tau_9 = \left[-\frac{m\,\tau_5(\phi+\Phi)}{l\,\nu\left(l^2+m^2+n^2\right)}\right] \tag{B.3}$$

$$\tau_{15} = \tau_{10} + \tau_{12}, \quad \tau_{16} = \tau_{11} + \tau_{13} \tag{B.4}$$

where $\tau_{10}$ and $\tau_{11}$ are arbitrary constants.

$$\tau_{17} = \frac{-1}{n}(l\,\tau_3 + m\,\tau_{15}), \quad \tau_{18} = \frac{-1}{n}(l\,\tau_4 + m\,\tau_{16}), \quad \tau_{19} = \frac{-1}{n}(b_0 - l\,\tau_5 + m\,\tau_{14}) \tag{B.5}$$

$$\tau_{20} = \frac{\rho\,\tau_3}{l\,\alpha_{21}}\left[\nu\ \alpha_{21}{}^2\left(l^2+m^2+n^2\right) - (\phi+\Phi)\right], \quad \tau_{21} = \frac{-\rho\,\tau_4}{l\,\alpha_{21}}\left[\nu\ \alpha_{21}{}^2\left(l^2+m^2+n^2\right) - (\phi+\Phi)\right] \tag{B.6}$$

$$\tau_{22} = \frac{\alpha_1\ \alpha_{21}{}^2}{2m^2}\left[\tau_3{}^2\left(2l^4+2m^4+2m^2n^2+4l^2m^2+2l^2n^2\right) + \tau_{17}{}^2\left(2n^2l^2+4m^2n^2+2n^4+2l^2m^2+2m^4\right) + 2\ \tau_3\tau_5\left(2nl^3+2n^3l+2m^2nl\right)\right] \tag{B.7}$$

$$\tau_{23} = \frac{\alpha_1\ \alpha_{21}{}^2}{2m^2}\left[\tau_4{}^2\left(2l^4+2m^4+2m^2n^2+4l^2m^2+2l^2n^2\right) + \tau_{18}{}^2\left(2n^2l^2+4m^2n^2+2n^4+2l^2m^2+2m^4\right) + 2\ \tau_4\tau_{18}\left(2nl^3+2n^3l+2m^2nl\right)\right] \tag{B.8}$$

$$\tau_{24} = \frac{-\rho\ \tau_5(\phi+\Phi)}{l}, \quad \tau_{26} = \frac{-\nu}{4k\rho}\left[\tau_3{}^2\left(2l^2+m^2+n^2\right) + \tau_{15}{}^2\left(2m^2+l^2+n^2\right) + \tau_{17}{}^2\left(2n^2+m^2+l^2\right) + 2lm\ \tau_3\tau_{15} + 2nl\ \tau_3\tau_{17} + 2mn\ \tau_{15}\tau_{17}\right] \tag{B.9}$$

$$\tau_{27} = \frac{-\nu}{4k\rho}\left[\tau_4{}^2\left(2l^2+m^2+n^2\right) + \tau_{16}{}^2\left(2m^2+l^2+n^2\right) + \tau_{18}{}^2\left(2n^2+m^2+l^2\right) + 2lm\ \tau_4\tau_{16} + 2nl\ \tau_4\tau_{18} + 2mn\ \tau_{16}\tau_{18}\right] \tag{B.10}$$

$$\begin{aligned}\tau_{28} = \frac{-\nu\alpha_{21}{}^2}{4k\rho}\Big[&2\tau_3\tau_4\left(2l^2+m^2+n^2\right) + 2\tau_{15}\tau_{16}\left(2m^2+l^2+n^2\right) + 2\tau_{17}\tau_{18}\left(2n^2+m^2+l^2\right) + 2lm\ \left(\tau_3\tau_{16}+\tau_4\tau_{15}\right) \\ &+2nl\ \left(\tau_3\tau_{18}+\tau_4\tau_{17}\right) + 2mn\ \left(\tau_{15}\tau_{18}+\tau_{16}\tau_{17}\right)\Big]\end{aligned} \tag{B.11}$$


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