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Research Paper

Description of Anomalous Behavior of Aluminum Alloys with Hill48 Yield Criterion by Using Different Experimental Inputs and Weight Coefficients

Bora Sener

Faculty of Mechanical Engineering, Department of Mechanical Engineering, Yildiz Technical University, Besiktas, Istanbul, 34349, Turkey

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Corresponding author: B. Sener (borasen@yildiz.edu.tr)

Abstract. The anomalous behavior of aluminum alloys is modeled with quadratic Hill48 yield criterion in this study. An identification method based on minimization of the error function is applied and the effect of the number of experimental input and weight coefficients used in the identification are investigated. Two highly anisotropic aluminum alloys (AA2090-T3 and AA5182-O) are selected in the study. Firstly, Hill48 parameters are determined with four different experimental data set, then the effect of the weight coefficients for each set is investigated. In-plane variations of plastic properties and yield surfaces of the materials are predicted with determined Hill48 parameters and the most appropriate pair (experimental data set and weight coefficient) are selected by comparison of the predicted results with experiment.

Keywords: Anomalous behavior, aluminum alloys, Hill48 yield criterion, anisotropy, experimental data, weight coefficient.

1. Introduction

Anisotropy is a material property which indicates the variation of mechanical properties with direction. Sheet materials represent anisotropic behavior due to preferred orientation occurs after cold rolling process [1]. Material anisotropy is defined with phenomenological anisotropic yield criteria in plasticity theory. Anisotropic yield criteria involve a certain number of coefficients and these coefficients are calibrated with mechanical tests performed along different directions. The first anisotropic yield criterion was developed by Hill in 1948. Hill included coefficients into isotropic von Mises yield criterion and derived an anisotropic function [2]. Hill48 yield criterion has been widely used in both academy and industry due to simplicity of its coefficient identification procedure. However, this criterion couldn't simultaneously describe planar variations of yield stress and Lankford coefficient. Besides, it couldn't give satisfactory results especially for aluminum alloys. Inconsistent predictions of Hill48 model in aluminum alloys were firstly noticed by Woodthorpe and Pearce and it was called as anomalous behavior [3]. Barlat et al. developed various anisotropic yield functions to define anamolous behavior of aluminum alloys. From Barlat models, Yld89 [4] and Yld91 [5] yield functions could define anomalous behavior, however they couldn't simultaneously capture angular variations of yield strength and Lankford coefficient. Karafillis and Boyce [6] proposed an anisotropic yield criterion based on the combination of two isotropic yield functions and successfully defined the anisotropic behavior of AA2008-T4 aluminium alloy. Then, Barlat et al. extended Yld91 criterion and developed Yld96 [7] criterion. This criterion has seven coefficients for plane stress state (2D) and these coefficients are calibrated with yield stresses and Lankford coefficients in rolling, diagonal and transverse directions (RD, DD and TD) and also one balanced biaxial yield stress. Yld96 criterion could accurately describe angular variations of both mechanical properties and provides satisfactory results for aluminum alloys. However, numerical problems could arise in finite element (FE) simulations performed with this yield criterion due to convexity conditions. Therefore, Barlat et al. developed Yld2000-2d [8] yield criterion to remove the mentioned disadvantage of Yld96. It contains eight coefficients and these coefficients are determined with three yield stresses, three Lankford coefficients, balanced biaxial yield stress and balanced biaxial Lankford coefficient. Yld2000-2d criterion could successfully represent anisotropic behaviors of aluminium alloys, satisfies convexity conditions and give good numerical results [9]. However, this yield criterion has been developed for only 2D stress state and it couldn't use for 3D stress state. Later, Barlat et al. extended Yld2000 for 3D stress state and proposed a new yield criterion called as Yld2004-18p [10]. This model has 18 parameters for 3D stress state and they are defined with yield stresses and Lankford coefficients in seven directions, biaxial yield stress, biaxial Lankford coefficient and two shear yield stresses. Another anisotropic yield criterion was developed by Cazacu and Barlat (CB2001) [11]. Researchers derived CB2001 criterion by extension of isotropic Drucker yield criterion and applied this criterion to model anisotropic behaviors of AA2090-T3 and AA6016-T4 aluminum alloys. Banabic et al. proposed BBC2003 [12] anisotropic yield criterion from isotropic Hershey yield function and then developed BBC2005 criterion [13] by improving this criterion. They applied this criterion in the description of plastic behavior of AA6181-T4 aluminium alloy and could accurately reproduce experimental results.

It is seen from these above-mentioned material models that researchers have continuously developed anisotropic yield criteria

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which have different formulations in order to describe anisotropic behaviors of aluminum alloys. However, accurate description of material behavior depends on not only selected yield function but also the number and type of experimental data used in coefficient identification. Various studies related to this subject have been carried out in the literature. Lazarescu et al. [14] investigated the effect of the number of experimental input on the prediction capability of BBC 2005 yield criterion. Researchers identified the yield criterion with 6, 7 and 8 experimental data and performed FE simulations of bulge test for AA6016-T4 aluminium alloy. After performing simulations, they determined that balanced biaxial yield stress could accurately predicted with 7 and 8 experimental data, whereas it was underestimated with 6 parameter model. A similar study was performed by Paraianu et al. [15]. They used BBC2005 yield criterion to model anisotropic behavior of DC04 sheet and identified the coefficients of the yield function with 4, 6, 7 and 8 mechanical parameters. Then, researchers compared the computed and experimental forming limit diagrams (FLD) and declared that the right side of the FLD could well predicted with identification method which contains biaxial yield stress. Comsa and Banabic [16] developed a yield criteria which is called as BBC2008 for 2D stress state and investigated the effect of the number of parameters on the prediction capability of the model. They identified the model parameters with 8 and 16 experimental input and indicated that the anisotropic behavior of AA2090-T3 aluminium alloy could be correctly defined with 16 coefficients. Chaparro et al. [17] defined the plastic behavior of DC06 steel with four yield criteria (Hill48, Yld91, Karafillis-Boyce93 and CB2001). They identified the material coefficients of yield criteria with different type experimental inputs and observed that average error between experiment and predictions for each yield criterion changed with identification type. Khalfallah et al. [18] applied three different identification procedures to Hill48, Yld89 and CB2001 criteria and investigated the effect of the number and type of the input data on the deep drawing simulation of cross-die. They determined that the prediction capability of CB2001 is influenced from particularly biaxial experimental data. Apart from these studies, yield surfaces could be extract from fine scale models and approximations like deep neural networks could be used [19-20].

It is seen from these studies carried out in the literature that researchers obtained successful results by using different experimental inputs. However, researchers only investigated the effect of the type and number of the experimental data, although the weight coefficients have significant effect on the prediction results. In this study, the effect of both number of experimental data and weight coefficients on the parameter identification are investigated. The developed identification method is applied to Hill48 yield criterion in order to model anisotropic behavior of aluminium alloys. For this purpose, two aluminum alloys (AA2090-T3 and AA5182-O) which are highly anisotropic and exhibit anomalous behavior are selected. The study is conducted in two stages: In the first stage, four different experimental data set (6, 8, 14 and 16 inputs) are used and Hill48 coefficients of the materials are determined for each data set. In the second stage, different weight coefficients are used for each data set and model parameters are determined. Angular variations of both mechanical properties (yield stress and Lankford coefficient) and normalized yield surfaces of the materials are predicted by using determined Hill48 coefficients. Then, the predicted results are compared with experiments and the most appropriate pair (experimental data set and weight coefficients) in the parameter identification are determined. Finally, the results which are predicted from the most appropriate pair are compared with predicted results from analytical identifications of Hill48 criterion and improvement in the prediction capability of the criterion is evaluated.

2. Method

2.1 Hill48 Yield Criterion and Its Identification Procedure

Hill suggested an anisotropic yield criterion based on extension of isotropic von Mises yield function in 1948. Hill48 criterion has four coefficients for 2D stress state and it can be expressed as follows:

$$\sigma_{eq} = [(G+H)\sigma_{xx}{}^2 + (F+H)\sigma_{yy}{}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}{}^2]^{1/2} \tag{1}$$

where F,G,H and N are anisotropy parameters. These parameters could be identified analytically according to stress ratios or Lankford coefficients by using yield function equation and flow rule approach. Stress based and Lankford based identifications are denoted as Hill48_σ and Hill48_r in this study. The definitions of Hill's parameters based on stress ratios and Lankford coefficients are given in eq. (2) and (3),respectively.

$$F_\sigma = \frac{1}{2}\left[\frac{1}{\bar{\sigma}_{90}{}^2} + \frac{1}{\bar{\sigma}_b{}^2} - 1\right], G_\sigma = \frac{1}{2}\left[\frac{1}{\bar{\sigma}_b{}^2} - \frac{1}{\bar{\sigma}_{90}{}^2} + 1\right], H_\sigma = \frac{1}{2}\left[\frac{1}{\bar{\sigma}_{90}{}^2} - \frac{1}{\bar{\sigma}_b{}^2} + 1\right], N_\sigma = \frac{1}{2}\left[\frac{4}{\bar{\sigma}_{45}{}^2} - \frac{1}{\bar{\sigma}_b{}^2}\right] \tag{2}$$

$$F_r = \frac{r_0}{(1+r_0)r_{90}}, G_r = \frac{1}{(1+r_0)}, H_r = \frac{r_0}{(1+r_0)}, N_r = \frac{(r_0+r_{90})(2r_{45}+1)}{2r_{90}(1+r_0)} \tag{3}$$

In these equations, $\bar{\sigma}_{45}, \bar{\sigma}_{90}$ and $\bar{\sigma}_b$ denote yield stress ratios along DD, TD and balanced biaxial loading directions, while r_0, r_{45} and r_{90} indicate Lankford coefficients along three main directions. Yield stress in the RD (σ_0) is accepted as reference in this study and stress ratios in other directions are calculated depend on this parameter. From both two type of identifications, mathematical relationships between the yield stress ratios and Lankford coefficients could be obtained. The first relationship is between balanced biaxial yield stress ratio and Lankford coefficients along RD and TD and it could be written as follows:

$$\left(\frac{\sigma_b}{\sigma_0}\right)^2 = \frac{r_{90}(1+r_0)}{r_0+r_{90}} \tag{4}$$

When planar isotropy (normal anisotropy) state is considered ($r_0 = r_{90}$), Eq.(4) could be expressed as follows:

$$\left(\frac{\sigma_b}{\sigma_0}\right)^2 = \frac{(1+r_n)}{2} \tag{5}$$

where r_n denotes normal anisotropy coefficient. The second relationship is between yield stress ratio along TD and Lankford coefficients along RD and TD and it is given below:

$$\left(\frac{\sigma_{90}}{\sigma_0}\right)^2 = \frac{r_{90}(1+r_0)}{r_0(1+r_{90})} \tag{6}$$

From Eq. (5) and Eq.(6), it is seen that when $r_n > 1$ then $\sigma_b > \sigma_0$ and when $\sigma_{90} > \sigma_0$ then $r_{90} > r_0$. However, some aluminium alloys do not satisfy these conditions and violation of Eq.(5) and Eq.(6) in the literature are called as first and second anomalous behaviors, respectively.

Prediction of the angular variations of yield stress ratio and Lankford coefficient in the sheet plane is an important indicator in the description of the planar anisotropy of the material and evaluation of the prediction capability of the yield criterion. Angular variation of yield stress ratio could be determined by using stress transformations and yield function equation. This parameter is given for Hill48 criterion in Eq. (7).

$$\frac{\sigma_\theta}{\sigma_0} = \bar{\sigma}_\theta = \frac{1}{\sqrt{F\sin^4\theta + G\cos^4\theta + H\left(\sin^2\theta - \cos^2\theta\right)^2 + 2N\cos^2\theta\sin^2\theta}} \tag{7}$$

The angular variation of Lankford coefficient is derived by using associated flow rule and volume constancy principle. For Hill48 yield criterion, it could be expressed as follows:

$$r_\theta = \frac{F\sin^4\theta + G\cos^4\theta + H(\sin^2\theta - \cos^2\theta)^2 + 2N\cos^2\theta\sin^2\theta}{G\cos^2\theta + F\sin^2\theta} - 1 \tag{8}$$

When balanced biaxial stress state $(\sigma_1 = \sigma_2 = \sigma_b)$ is considered, Eq.(7) and Eq.(8) are reduced to the following formulas:

$$\frac{\sigma_b}{\sigma_0} = \frac{1}{\sqrt{F+G}} \tag{9}$$

$$r_b = \frac{F}{G} \tag{10}$$

2.2 Developed Alternative Identification Procedure for Hill48 Yield Criterion

Analytical identification procedure of Hill48 criterion contains limited number of experimental data and this restricts the prediction capability of the criterion. Therefore, a different identification procedure which is based on the minimization of error (objective) function is developed in this study. This function is expressed with the summation of the squares of errors and it indicates the deviation between the theoretical values predicted from criterion and experimental data. The proposed error function in the study is given in Eq.(11).

$$Error\_function(E) = w_1\sum_{i=1}^{n}\left(\frac{(\bar{\sigma}_\theta)_{pred} - (\bar{\sigma}_\theta)_{\exp}}{(\bar{\sigma}_\theta)_{\exp}}\right)_i^2 + w_2\sum_{i=1}^{n}\left(\frac{(r_\theta)_{pred} - (r_\theta)_{\exp}}{(r_\theta)_{\exp}}\right)_i^2 + w_3\left(\frac{(\bar{\sigma}_b)_{pred} - (\bar{\sigma}_b)_{\exp}}{(\bar{\sigma}_b)_{\exp}}\right)^2 + w_4\left(\frac{(r_b)_{pred} - (r_b)_{\exp}}{(r_b)_{\exp}}\right)^2 \tag{11}$$

where $(\bar{\sigma}_\theta)_{\exp}$ and $(r_\theta)_{\exp}$ denote experimental yield stress ratios and Lankford coefficients which are determined from uniaxial tensile tests performed along different directions with regard to the RD. $(\bar{\sigma}_b)_{\exp}$ and $(r_b)_{\exp}$ indicate yield stress ratio and Lankford coefficient in the balanced biaxial stress state which these parameters could be determined from biaxial tensile tests or bulge and disc compression tests. $\bar{\sigma}_{\theta pred}, r_{\theta pred}, \bar{\sigma}_{b pred}, r_{b pred}$ are corresponding theoretical values predicted from yield function. n represents the number of uniaxial tensile test data. w_1, w_2, w_3 and w_4 are weight coefficients which are predetermined by the user and represent the relative importance of each experimental data group. The sum of the weight coefficients is taken as 1 in this study. It is seen from eq. (11) that error function can involve all experimental data and this provides flexibility to parameter identification. Hill's coefficients are design variables in this optimization problem and their values are continuously changed during the minimization in order to decrease deviation between theoretical predictions and experimental data. In this study, sequential quadratic programming method (SQP) from numerical optimization techniques is applied to minimize error function and the method is explained in Section 2.3.

2.3 Sequential Quadratic Programming Method

SQP is a numerical optimization method which is developed for solving of nonlinear constrained optimization problems. Method consists of two stages: search direction is determined in the first stage, then step length along direction is calculated. SQP solves a quadratic programming (QP) subproblem to determine search direction at each iteration. Primarily, QP subproblem is formulated with Lagrange function and subproblem is defined as follows:

$$\begin{aligned} &\min_{d\in R^n} \frac{1}{2} d^T H_k d + \nabla f(x_k)^T d \\ &\nabla g(x_k)^T d + g_i(x_k) = 0, i = 1,....m_e \\ &\nabla g(x_k)^T d + g_i(x_k) \le 0, i = m_e + 1,......m \end{aligned} \tag{12}$$

where d is the search direction in the design space, H_k is Hessian matrix, $g_i(x_k)$ is constraint function and x_k is design variable. Then, critical points of Lagrange function are determined with Newton method and solving of quadratic subproblems is continued until convergence is obtained [21]. The variation of the objective function is considered as convergence criteria of the optimization algorithm. A tolerance value is selected which is a lower bound on the change in the value of objective function during a step. Tolerance value is accepted as 1e-10 in this study and the formulation of the convergence criteria is given as follows:

$$\left|f(x_i)-f(x_{i+1})\right|<1e-10 \tag{13}$$

3. Application

In this study, the developed coefficient identification method is applied to describe anisotropic behavior of AA2090-T3 and AA5182-O aluminium alloys. AA2090-T3 is an Al-Li alloy and it is widely used for aircraft components such as fuselage, lower wing due to its high strength and low density [22]. It is a highly textured material and exhibits high planar anisotropy. The second material AA5182-O is an Al-Mg alloy, it has high strength and good formability. Therefore, this material is very popular in automotive industry and it is used for complex stamping applications such as forming of structural parts and inner panels [23].

Application of the method is conducted in two stages. In the first stage, the effect of the number and type of experimental data are investigated on the prediction capability of Hill48 criterion and the coefficient identification is carried out with different experimental data set. In the second stage, the effect of the weight coefficients is investigated and this investigation is separately conducted for each data set.

3.1 The Effect of Experimental Data Set On the Parameter Identification

Four different experimental data set are used to investigate the effect of the number of experimental input. The first set contains 6 experimental data (uniaxial yield stresses and Lankford coefficients along three main directions), the second set involves 8 experimental data (uniaxial yield stresses and Lankford coefficients in three main directions and biaxial yield stress and biaxial Lankford coefficient), the third set includes 14 experimental inputs (uniaxial yield stresses and Lankford coefficients along seven directions) and the last data set uses 16 experimental data set (uniaxial yield stresses and Lankford coefficients in seven directions and biaxial yield stress and biaxial Lankford coefficient). Experimental data of the materials are taken from the literature and the values for AA2090-T3 [24] and AA5182-O [25] alloys are given in Table 1 and Table 2, respectively.

It is seen from Table 1 and Table 2 that AA2090-T3 exhibits the second anomalous behavior, while AA5182-O represents both the first and second anomalous behavior. Error function is minimized for each experimental data set and Hill48 coefficients are determined. Weight coefficients are taken as 0.5 for yield stress ratio and Lankford coefficient in order to investigate only the effect of the number of experimental data set. The determined Hill48 coefficients from each data set for AA2090-T3 and AA5182-O alloys are given in Table 3 and Table 4, respectively. Four different experimental data set are referred as Hill48_6p, Hill48_8p, Hill48_14p and Hill48_16p in the study.

Planar variations of plastic properties and normalized yield surfaces of the materials are computed by using determined Hill48 coefficients for each identification type and the computed results are compared with the experimental results. Comparisons of the predicted results with experiment for AA2090-T3 alloy are given between Fig. 1 and Fig. 3.

Table 1. Experimental data of AA2090-T3 aluminium alloy [24]

Angle from RD (°)	Uniaxial Yield Stress Ratio	r-value	Biaxial Yield Stress Ratio	Biaxial r-value	Normal r value
0	1.0000	0.2115	1.035	0.67	1.0144
15	0.9605	0.3269			
30	0.9102	0.6923			
45	0.8114	1.5769			
60	0.8096	1.0385			
75	0.8815	0.5384			
90	0.9102	0.6923			

Table 2. Experimental data of AA5182-O aluminium alloy [25]

Angle from RD (°)	Uniaxial Yield Stress Ratio	r-value	Biaxial Yield Stress Ratio	Biaxial r-value	Normal r value
0	1.0000	0.6420	1.000	1.126	0.8872
15	0.9911	0.7281			
30	0.9755	0.9111			
45	0.9686	1.0390			
60	0.9740	1.0174			
75	0.9862	0.8968			
90	0.9930	0.8290			

Table 3. The determined Hill48 coefficients from each experimental data set for AA2090-T3 alloy.

Identification Type	F	G	H	N
Hill48_6p	0.4751	1.2907	0.3022	3.5430
Hill48_8p	0.5181	0.9489	0.2439	2.9843
Hill48_14p	0.5829	1.1076	0.2504	2.5497
Hill48_16p	0.5872	0.9931	0.2303	2.4128

Table 4. The determined Hill48 coefficients from each experimental data set for AA5182-O alloy.

Identification Type	F	G	H	N
Hill48_6p	0.5011	0.6319	0.4122	1.7329
Hill48_8p	0.5441	0.5551	0.3955	1.6850
Hill48_14p	0.4963	0.6216	0.4077	1.7131
Hill48_16p	0.5273	0.5740	0.4001	1.6901

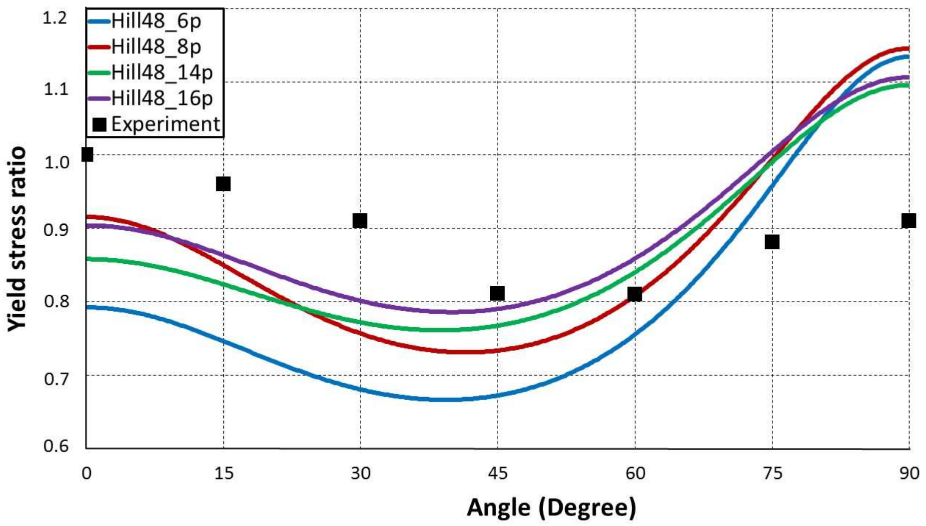

Fig. 1. Comparison of the predicted yield stress ratios from four different identification types with experiment for AA2090-T3 alloy.

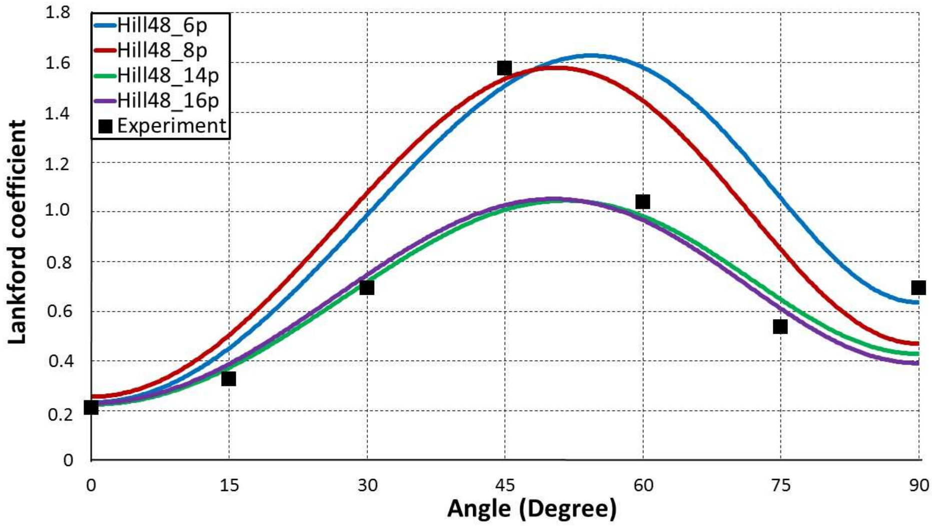

Fig. 2. Comparison of the predicted Lankford coefficients from four different identification types with experiment for AA2090-T3 alloy.

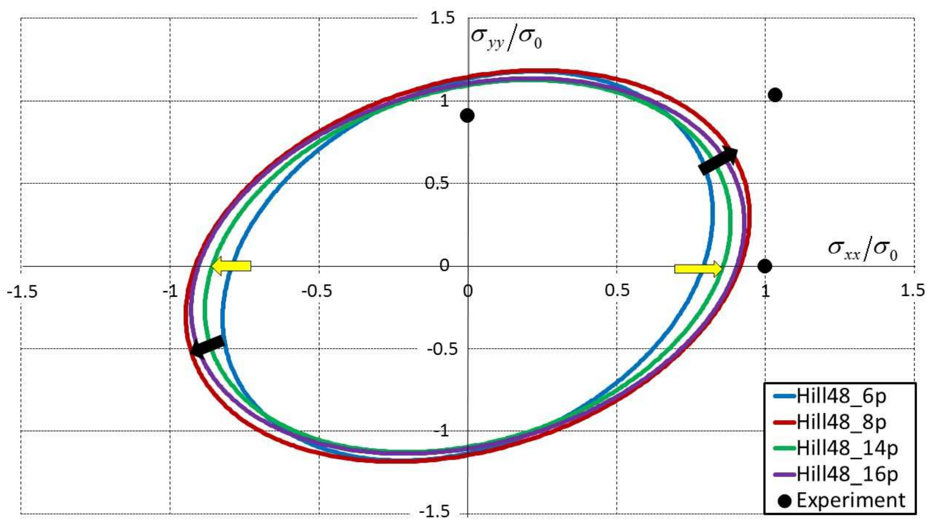

Fig. 3. Comparison of the computed yield surfaces from four different identification types with experimental values for AA2090-T3 alloy.

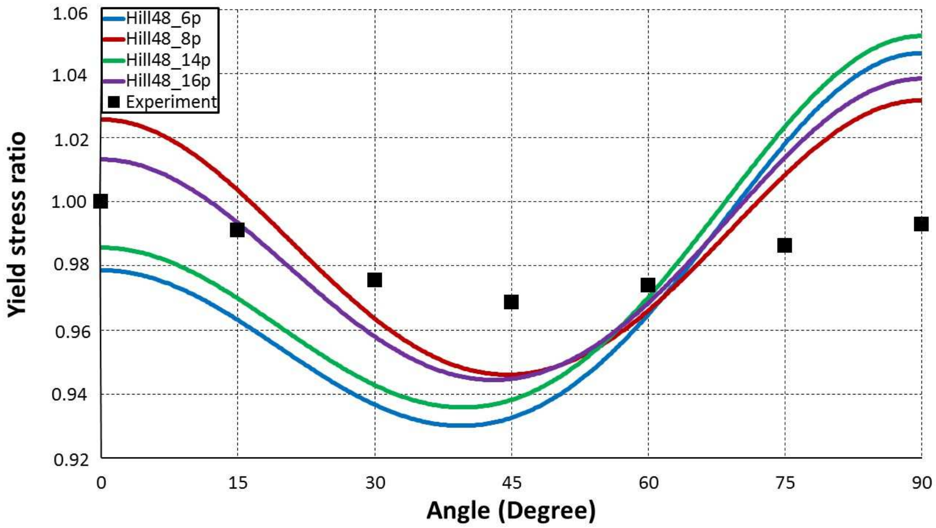

Fig. 4. Comparison of the predicted yield stress ratios from four different identification types with experiment for AA5182-O alloy.

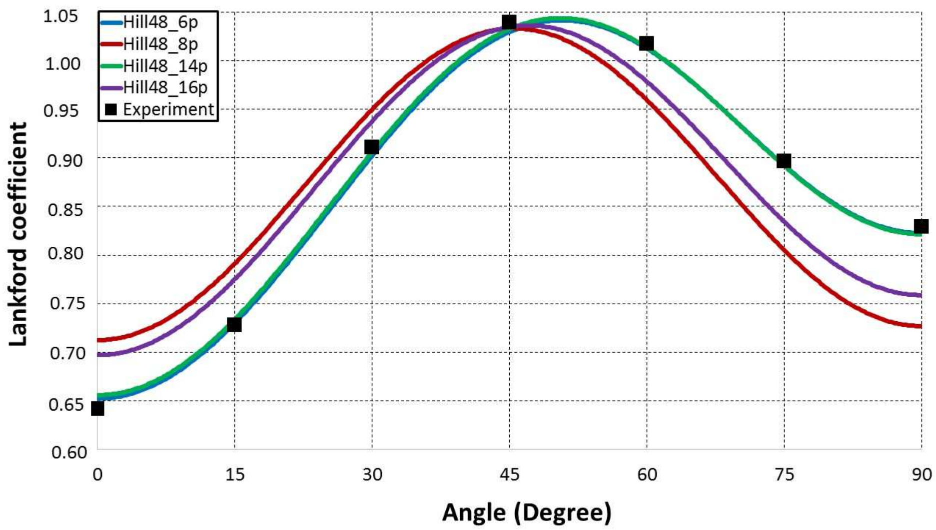

Fig. 5. Comparison of the predicted Lankford coefficients from four different identification types with experiment for AA5182-O alloy.

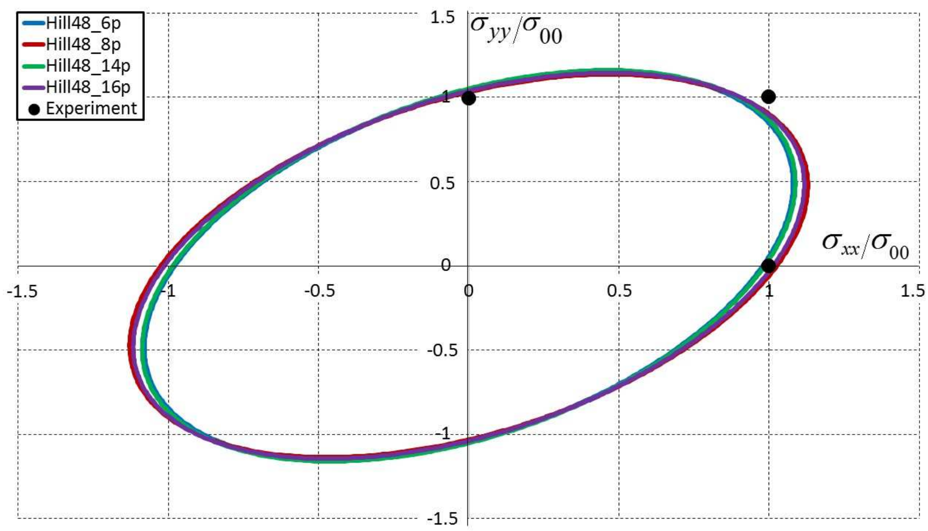

Fig. 6. Comparison of the computed yield surfaces from four different identification types with experimental values for AA5182-O alloy.

It is seen from Fig. 1 and Fig. 2 that the predictions of Hill48-14p and Hill48-16p models are close to each other and they could successfully describe the angular variations of both yield stress ratio and Lankford coefficient when compared to the other two models. It is observed from the computed yield surfaces that yield surface expands in biaxial directions (black arrow direction in Fig.3) when only biaxial tensile test data is used in the identification (Hill48-8p), whereas it expands in rolling direction (yellow arrow direction in Fig. 3) when only uniaxial tensile test data is used (Hill48-14p). In addition to that it is seen from Fig.3 that the biaxial yield stress is underestimated in all models when they are compared with the experimental value.

The predicted results for AA5182-O alloy are compared with experiments and the results are given in Fig. 4, Fig. 5 and Fig. 6. It is seen from the performed comparisons for AA5182-O alloy that the models contain biaxial experimental data (Hill48-8p and Hill48-16p) could correctly predict the planar variation of yield stress ratio, while the models which are identified with only uniaxial tensile test data (Hill48-6p and Hill48-14p) could precisely predict the planar variation of Lankford coefficient. When the contours of yield surface are considered, it is observed that the computed yield surfaces are almost identical and the number of parameters used in the identification has not effect on the yield surface shape for this material. The predicted biaxial data (σ_b, r_b) from models are compared with the experiments and percentage error are calculated for each model. The obtained results for AA2090-T3 and AA5182-O alloys are presented in Fig. 7 and Fig. 8, respectively.

From the results of AA2090-T3 alloy, it is observed that the Hill48-8p model has the lower percentage error than the other models in the prediction of biaxial yield stress ratio, whereas Hill48-16p is the most successful model in the prediction of biaxial Lankford coefficient. For AA5182-O alloy, Hill48-8p model has the minimum error percentage in the prediction of both biaxial yield stress ratio and biaxial Lankford coefficient.

3.2 The Effect of Weight Coefficients on the Parameter Identification

The effect of weight coefficients on the prediction results is investigated in this section. Error function is minimized by using different weight coefficients for each model. The calculated error functions with different weight coefficients are presented from Fig. 9 to Fig. 12 for AA2090-T3.

It is seen from Fig. 9 and Fig. 11 that weight coefficients have contrast effects on Hill48-6p and Hill48-14p models. The value of error function decreases with increase in w_2 coefficient which controls the Lankford parameter in Hill48-6p model, whereas it decreases with increase in w_1 coefficient which controls the yield stress ratio in Hill48-14p model. The weight coefficients are investigated separately in Hill48-8p and Hill48-16p models contain both uniaxial and biaxial experimental data. Firstly, the weight coefficients (w_3 and w_4) which control the biaxial experimental data are kept constant (they are taken as 0.25) and the other weight coefficients (w_1 and w_2) are varied from 0.1 to 0.4 in increments of 0.1. Secondly, the weight coefficients (w_1 and w_2) which control the uniaxial tensile test data are kept constant and the other weight coefficients (w_3 and w_4) are varied within same interval. It is observed from Fig. 10 and Fig. 12 that the variation of w_3 and w_4 coefficients has no significant effect on the value of error function. Besides, the error function decreases when w_1 increases in both Hill-8p and Hill-16p models (except for w_1 = 0.2 in Hill48-8p) and minimum error function is determined with same weight coefficients in both of two models (w_1 = 0.4, w_2 = 0.1, w_3 =w_4=0.25).

Investigation is conducted with same weight coefficients for AA5182-O material and the variation of error function with weight coefficients is given in Fig. 13-16.

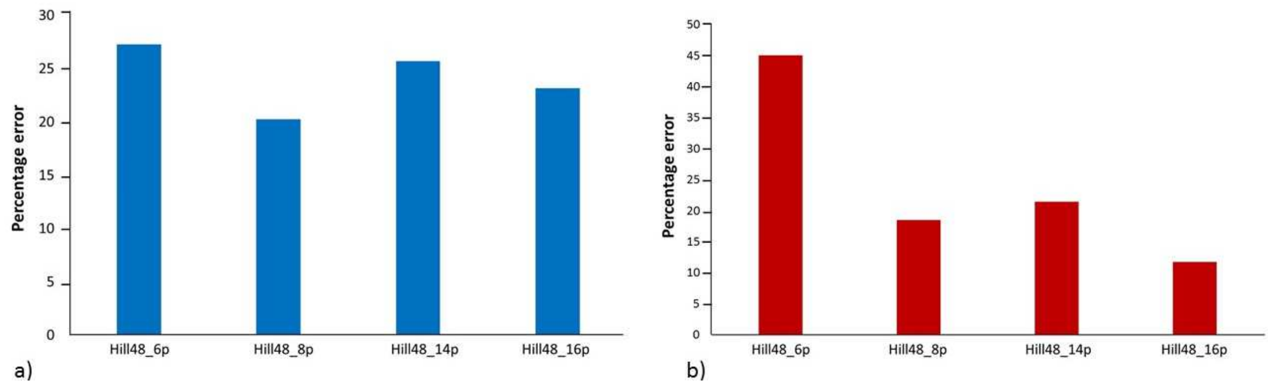

Fig. 7. Percentage error of models for AA2090-T3 alloy a) Biaxial yield stress ratio b) Biaxial Lankford coefficient.

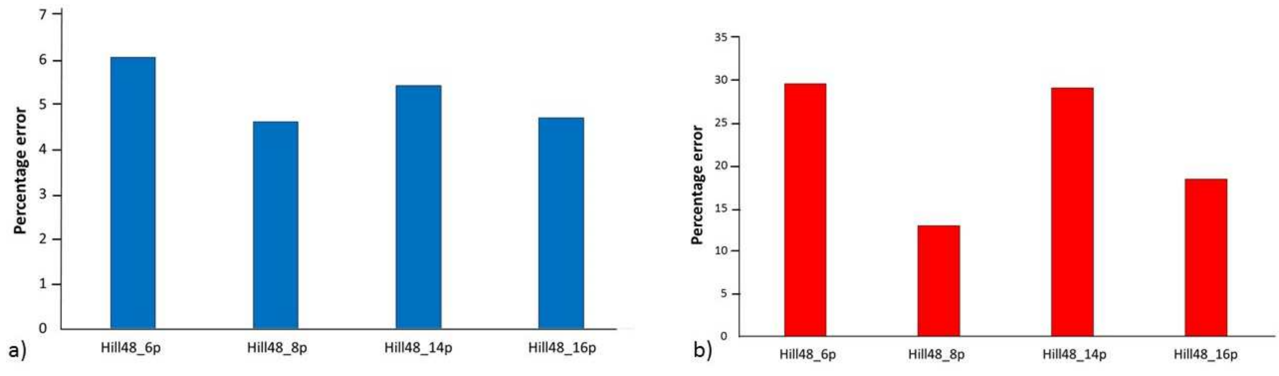

Fig. 8. Percentage error of models for AA5182-O alloy a) Biaxial yield stress ratio b) Biaxial Lankford coefficient.

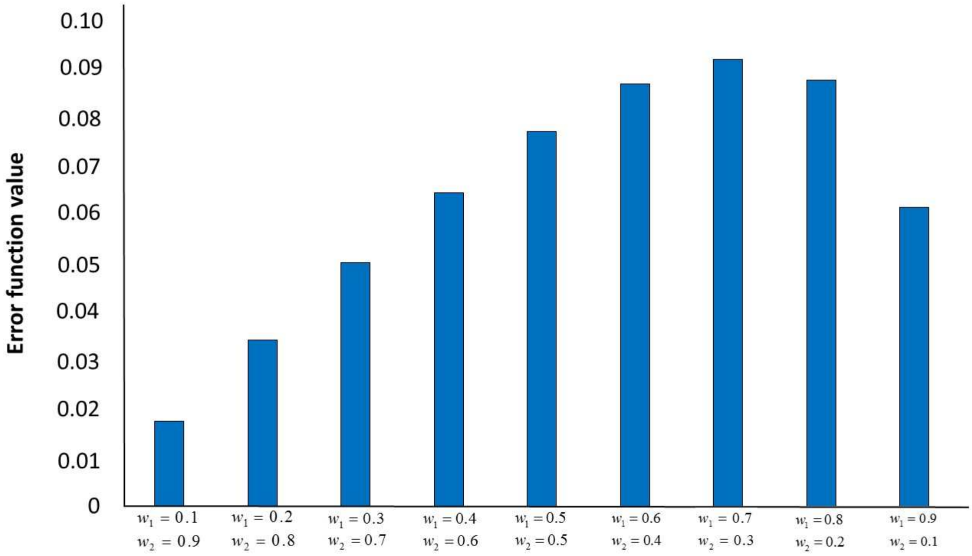

Fig. 9. The values of the error functions obtained from different weight coefficients for Hill48_6p model.

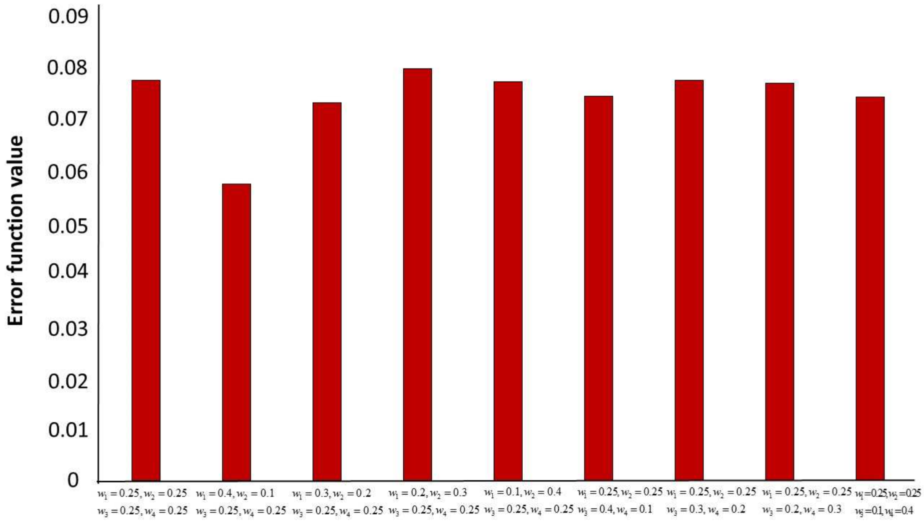

Fig. 10. The values of the error functions obtained from different weight coefficients for Hill48_8p model.

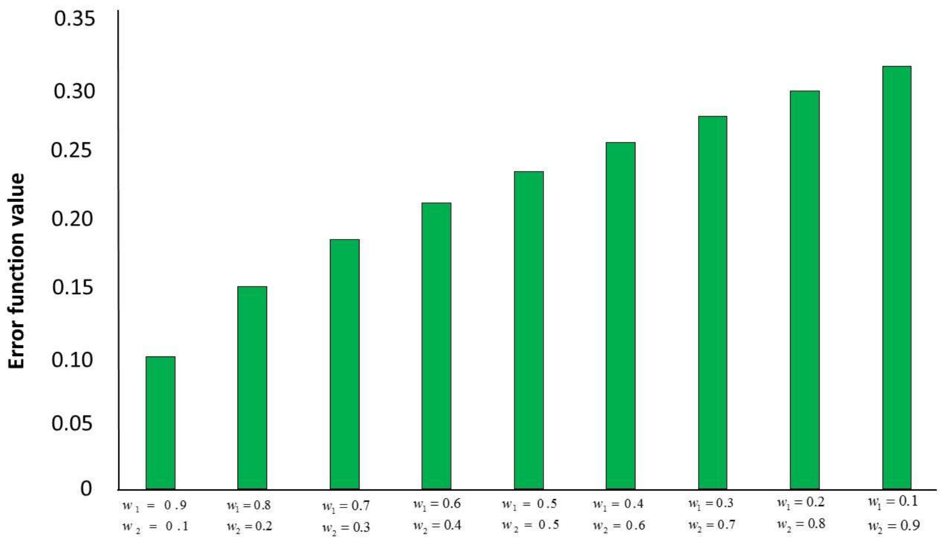

Fig. 11. The values of the error functions obtained from different weight coefficients for Hill48_14p model.

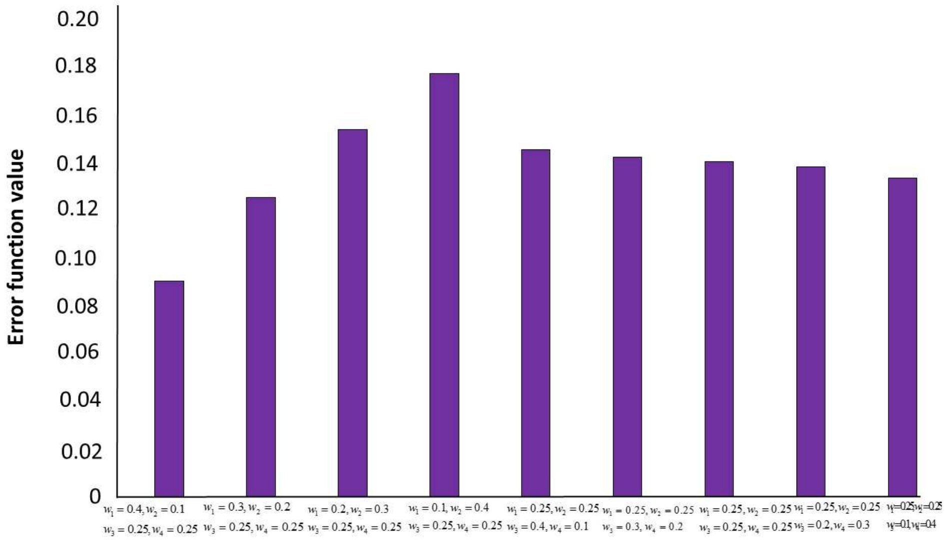

Fig. 12. The values of the error functions obtained from different weight coefficients for Hill48_16p model.

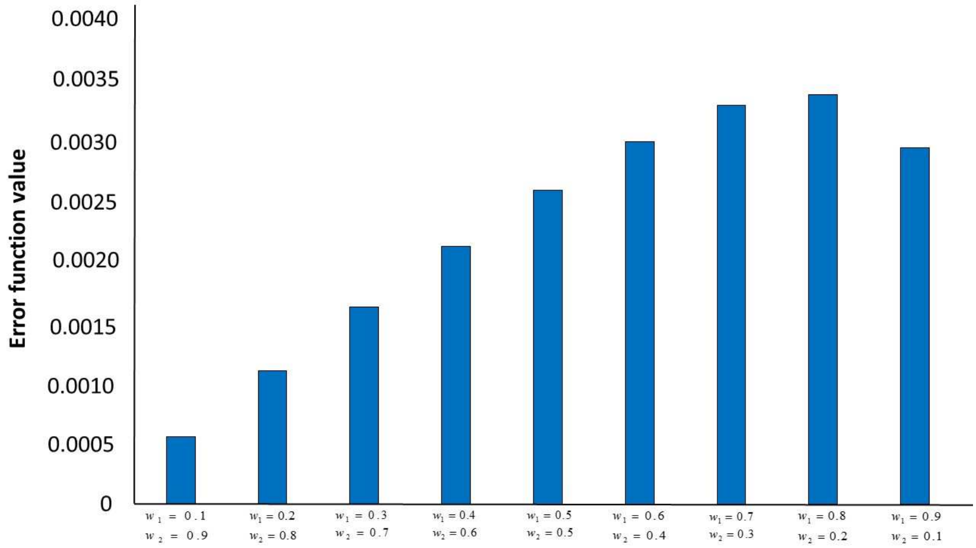

Fig. 13. The values of the error functions obtained from different weight coefficients for Hill48_6p model.

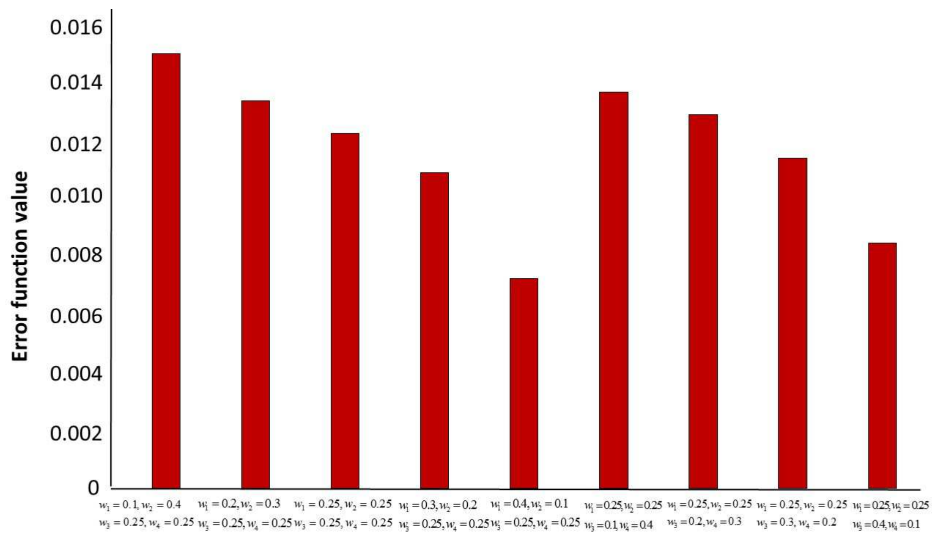

Fig. 14. The values of the error functions obtained from different weight coefficients for Hill48_8p model.

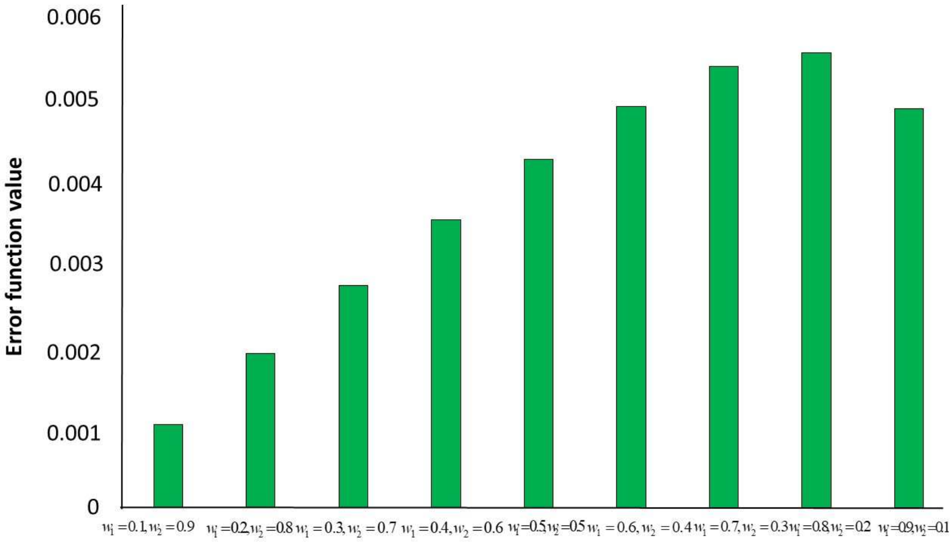

Fig. 15. The values of the error functions obtained from different weight coefficients for Hill48_14p model.

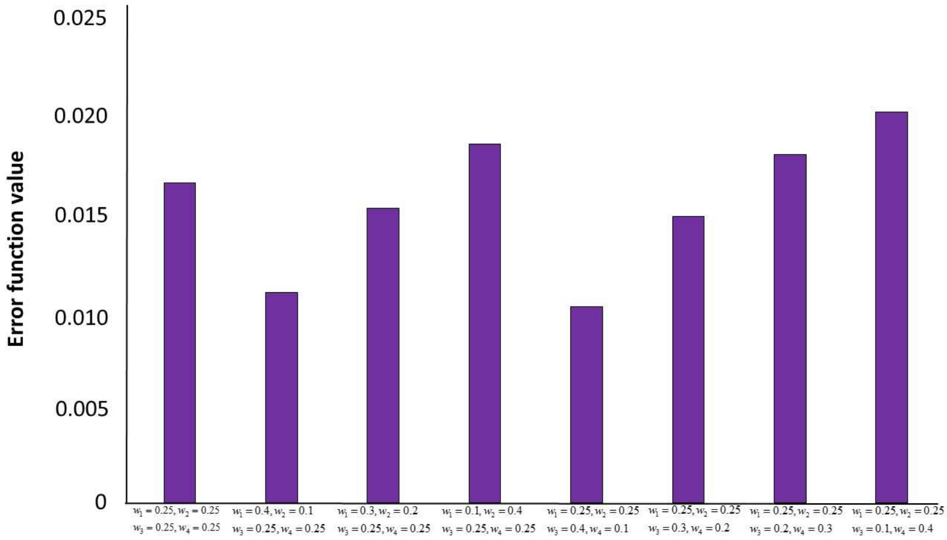

Fig. 16. The values of the error functions obtained from different weight coefficients for Hill48_16p model.

It is observed from Fig. 13 and Fig. 15 that the weight coefficients have same effect on the Hill48-6p and Hill48-14p models as distinct from AA2090-T3 alloy. The value of the error function of both of two models decreases with increase in w_2 coefficient. A similar result is observed between Hill48-8p and Hill48-16p models. The value of error function of both of two models decreases with increase in w_1 coefficient in equal values of w_3 and w_4 coefficients. In equal values of w_1 and w_2 coefficients, it decreases with increase in w_3 coefficient for both Hill48-8p and Hill48-16p models.

3.3 Comparison of Developed Identification Method with Analytical Identification

It is seen from the values of error function determined in section 3.2 that Hill48-6p model with w_1 =0.1 and w_2 = 0.9 coefficients has minimum error function value for both AA2090-T3 and AA5182-O alloy. Therefore, it is selected in description of anisotropic behavior of the materials and the predicted results from this model are compared with analytical identification results of Hill48 in order to evaluate the improvement in the prediction capability of quadratic criterion. Comparisons of the in-plane variation of plastic properties and normalized yield surfaces predicted from Hill48_6p (w_1=0.1, w_2 = 0.9) with classical identification and experimental results are shown in Fig. 17, 18 and Fig.19 for AA2090-T3 alloy.

It is seen from Fig. 17 and Fig. 18 that analytical identifications (Hill48_σ and Hill48_r) can predict either the angular variation of the yield stress ratio or of Lankford coefficient. However, Hill48 model with 6 parameter could simultaneously predict the angular variations of both yield stress ratio and Lankford coefficient. Besides, it is observed from Fig. 18 that angular variations of Lankford coefficient which are predicted from Hill48_6p model and Hill48_r are overlapping each other and this result shows that Hill48-6p model has high prediction capability. From comparisons of the computed yield surfaces, it is seen that only Hill48-σ could accurately predict the biaxial yield stress ratio and elastic region of Hill48-r is larger than Hill48-σ and Hill48-6p model. Comparison results performed for AA5182-O alloy are presented in Fig. 20, Fig. 21 and Fig. 22, respectively.

It is seen from Fig. 20 and Fig. 21 that Hill48-6p model could accurately predict the angular variations of both plastic properties. In addition to that Hill48-6p model could precisely predict the Lankford coefficients along seven directions and the predicted curve is completely the same as that of Hill48-r. However, Hill48-6p model and Hill48-r underestimate the balanced biaxial yield stress ratio, only Hill48-σ could successfully predict this parameter. It is seen that differences between the predicted biaxial yield stress values of AA5182-O alloy are smaller than that of AA2090-T3 alloy.

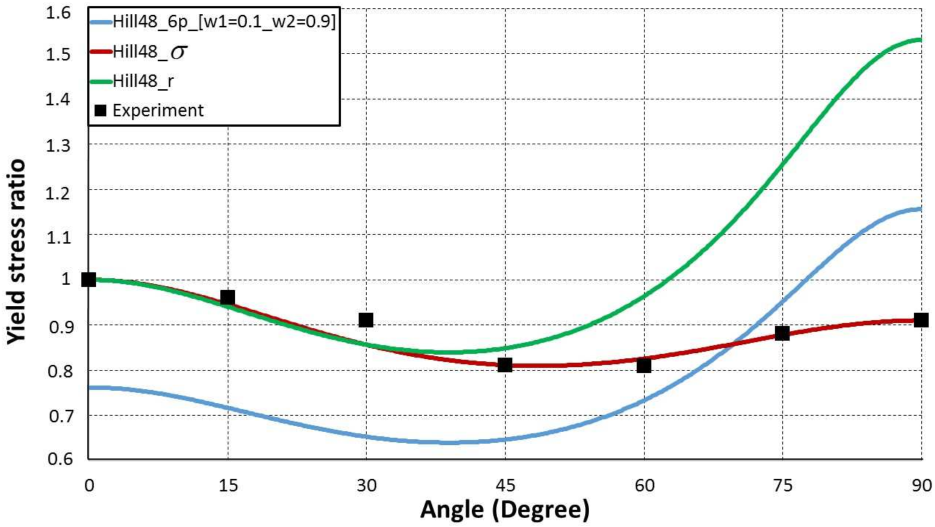

Fig. 17. Comparison of angular variations of yield stress ratios for AA2090-T3.

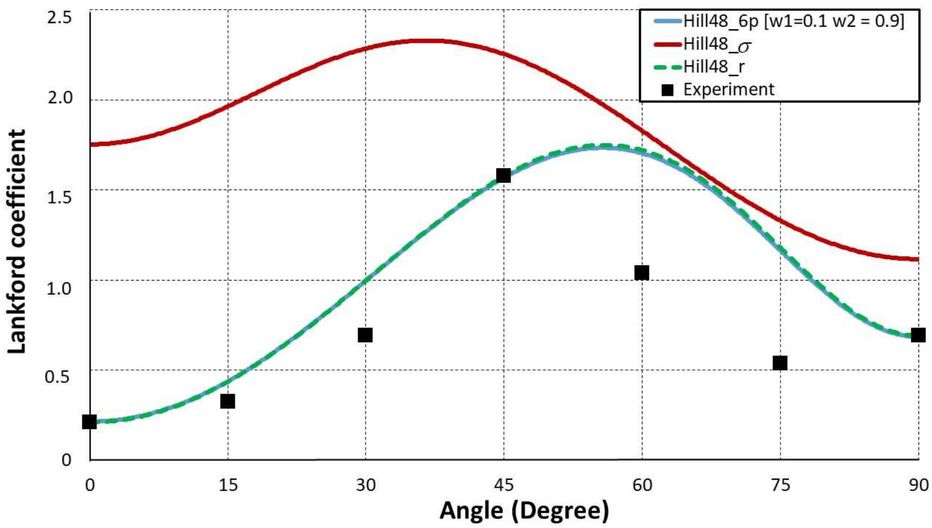

Fig. 18. Comparison of angular variations of Lankford coefficient for AA2090-T3.

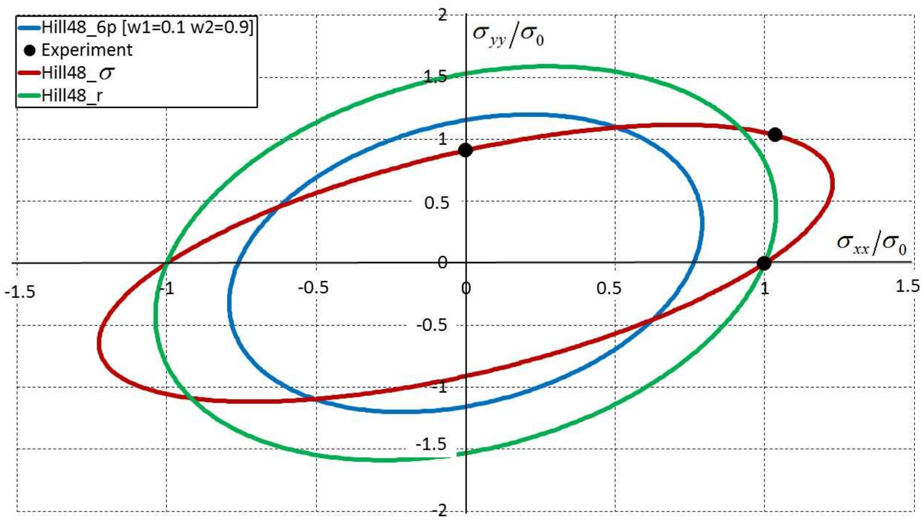

Fig. 19. Comparison of normalized yield surfaces for AA2090-T3.

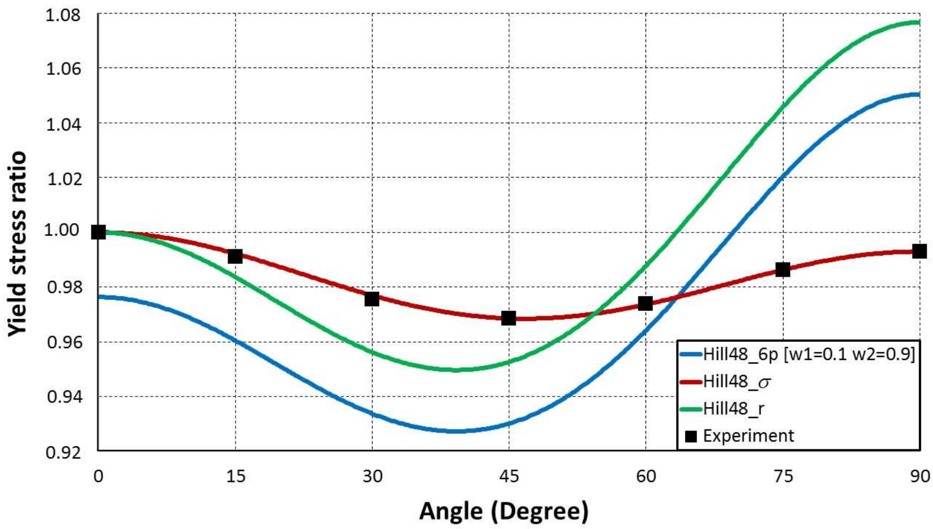

Fig. 20. Comparison of angular variations of yield stress ratios for AA5182-O.

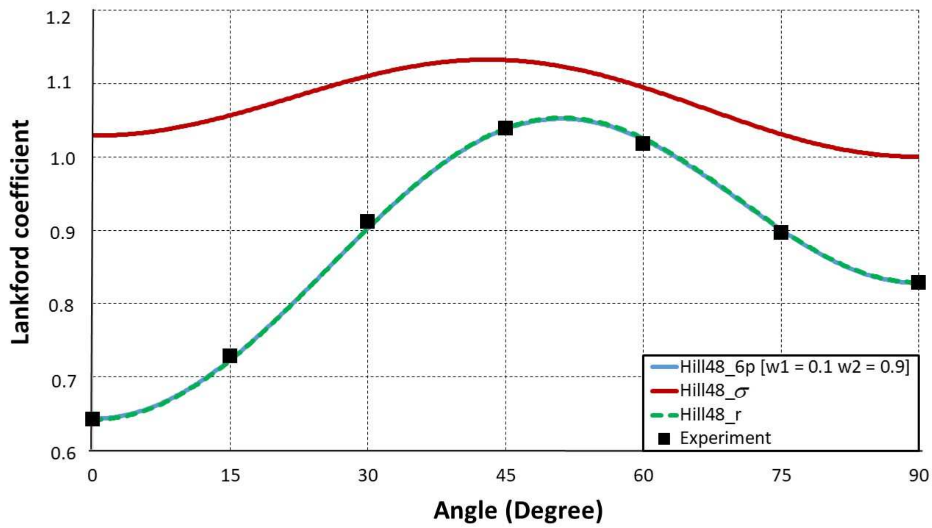

Fig. 21. Comparison of angular variations of Lankford coefficient for AA5182-O.

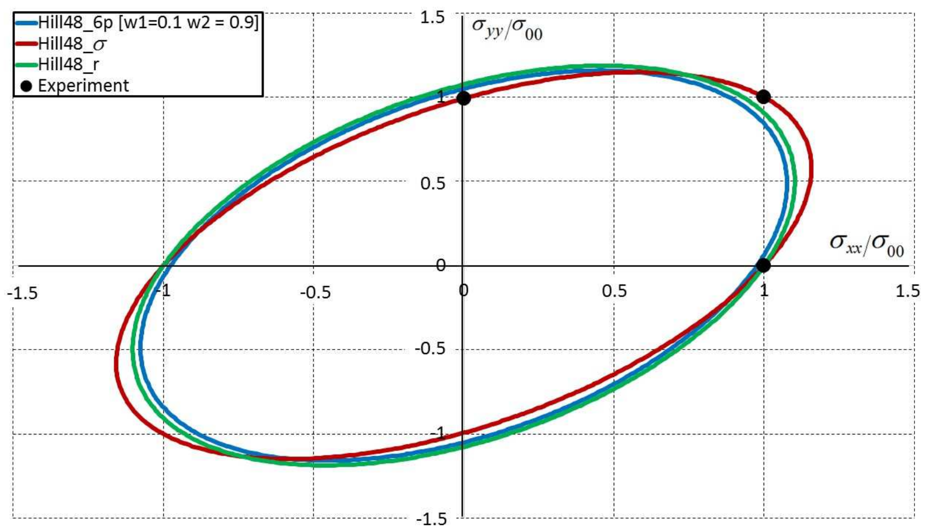

Fig. 22. Comparison of normalized yield surfaces for AA5182-O.

4. Conclusions

In this study, anisotropic behavior of AA2090-T3 and AA5182-O aluminium alloys is described with Hill48 yield criterion by applying an identification method which is based on the minimization of error function. The effect of the number of experimental data and weight coefficients on the prediction accuracy of the criterion is investigated. The coefficients of Hill48 criterion are calibrated with 6, 8, 14 and 16 experimental inputs and then different weight coefficients are used for each identification type. Firstly, weight coefficients are kept constant and the effect of the number of experimental data set is investigated. Hill48-14p and Hill48-16p models accurately predict the in-plane variation of both yield stress ratio and Lankford coefficient for AA2090-T3 alloy. However, it is seen that the results are different for AA5182-O alloy. Hill48-8p and Hill16p models predict closer results to experimental data in the planar variation of yield stress ratio, while Hill-6p and Hill-14p models are more successful than the other models in the planar variation of Lankford coefficient. Positive regions of the yield surfaces couldn't accurately reproduced by these models for both AA2090-T3 and AA5182-O. However, it is observed that differences between the contours of the normalized yield surface for AA5182-O alloy are smaller than that of AA2090-T3. It is seen from the percentage error results calculated for biaxial yield stress ratio and biaxial Lankford coefficient that Hill48-8p and Hill48-16p models have lower percentage error in the prediction of these parameters. However, the predicted results from two models are underestimated. This result shows that addition of parameter improves the prediction of planar anisotropy, whereas it doesn't make a significant contribution to the prediction of biaxial experimental data. Secondly, the effect of the weight coefficients on the prediction results is investigated. Error function is minimized with different weight coefficients and this process is applied for each identification type. It is observed from the obtained error function values that w_2 coefficient has effect on the decreasing of error function in Hill48-6p model both AA2090-T3 and AA5182-O alloys and it also shows same effect on Hill48-14p model for AA5182-O alloy. Weight coefficients are divided into two groups in the models (Hill48-8p and Hill48-16p) which have four weight coefficients. Primarily, the coefficients (w_3 and w_4) which control biaxial experimental data are kept constant and the other two coefficients are varied. From this investigation it is determined that w_1 coefficient has effect on the decreasing of error function of both Hill48-8p and Hill48-16p models and this effect is observed in both of two materials. Then, the coefficients (w_1 and w_2) are kept constant and the other coefficients are changed. It is observed from the obtained results that w_3 and w_4 coefficients have no significant effect on the error function in Hill48-8p and 16 models for AA2090-T3 material, whereas the increasing of w_3 coefficient decrease the value of the error function in both of the models for AA5182-O material.

The values of the error functions obtained from different identification types and different weight coefficients are compared and it is determined that Hill48-6p model with w_1 =0.1 and w_2 = 0.9 coefficients give the minimum error function within the other models and weight coefficients. The results predicted from this model are compared with analytical identification and experimental results in order to evaluate the improvement in the prediction capability of Hill48 yield criterion. It is determined that the model could accurately predict the planar variation of both yield stress ratio and Lankford coefficient for both of materials. Besides the predicted Lankford coefficients from the model is the same as Hill48-r identification and they are close to the experimental values. Obtained results from this study show that the anisotropic behaviors of aluminium alloys which exhibit anomalous behavior could be successfully described by using Hill48-6p model with w_1 =0.1 and w_2 = 0.9 coefficients.

Conflict of Interest

The author declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Nomenclature

F, G, H, N	Hill48 anisotropy coefficients	r_0, r_{45}, r_{90}	Lankford coefficients along three directions
$\bar{\sigma}_0$, $\bar{\sigma}_{45}$, $\bar{\sigma}_{90}$	Yield stress ratios along three directions	r_b	Balanced biaxial Lankford coefficient
$\bar{\sigma}_b$	Balanced biaxial yield stress ratio	r_n	Normal anisotropy coefficient
w_1, w_2, w_3, w_4	Weight coefficients		

References

[1] Mielnik, E.M., *Metalworking Science and Engineering*, McGraw-Hill, New York, 1991.

[2] Hill, R., A Theory of the Yielding and Plastic Flow of Anisotropic Metals, *Proceedings of the Royal Society of London Series A. Mathematical and Physical Sciences*, 193(A), 1948, 281-297.

[3] Woodthorpe, J., Pearce, R., The Anomalous Behavior of Aluminium Sheet Under Balanced Biaxial Tension, *International Journal of Mechanical Sciences,* 12, 1970, 341-347.

[4] Barlat, F., Lian, J., Plastic Behavior and Stretchability of Sheet Metals. Part I: A Yield Function for Orthotropic Sheets Under Plane Stress Conditions, *International Journal of Plasticity,* 5, 1989, 51-66.

[5] Barlat, F., Lege, D.J., Brem, J.C., A Six-Component Yield Function for Anisotropic Materials, *International Journal of Plasticity*, 7, 1991, 693-712.

[6] Karafillis, A.P., Boyce, M.C., A General Anisotropic Yield Criterion Using Bounds and a Transformation Weighting Tensor, *Journal of the Mechanics and Physics of Solids*, 41, 1993, 1859-1886.

[7] Barlat, F., Maeda, Y., Chung, K., Yanagawa, M., Brem, J.C., Hayashida, Y., Lege, D.J., Matsui, K., Murtha, S.J., Hattori, S., Becker, R.C., Makosey, S., Yield Function Development for Aluminum Alloy Sheets, *Journal of the Mechanics and Physics of Solids,* 45, 1997, 1727-1763.

[8] Barlat, F., Brem, J.C., Yoon, J.W., Chung, K., Dick, R.E., Lege, D.J., Pourboghrat, F., Choi, S.-H., Chu, E., Plane Stress Yield Function for Aluminum Alloy Sheets-Part 1: Theory, *International Journal of Plasticity,* 19, 2003, 1297-1319.

[9] Yoon, J.W., Barlat, F., Dick, R.E., Chung, K., Kang, T.J., Plane Stress Yield Function for Aluminum Alloy Sheets-Part II: FE Formulation and Its Implementation, *International Journal of Plasticity*, 20, 2004, 495-522.

[10] Barlat, F., Aretz, H., Yoon, J.W., Karabin, M.E., Brem, J.C., Dick, R.E., Linear Transformation-Based Anisotropic Yield Functions, *International Journal of Plasticity*, 21, 2005, 1009-1039.

[11] Cazacu, O., Barlat, F., Generalization of Drucker's Yield Criterion to Orthotropy, *Mathematics and Mechanics of Solids*, 6, 2001, 613-630.

[12] Banabic, D., Balan, T., Comsa, D.S., A New Yield Criterion for Orthotropic Sheet Metals Under Plane-Stress Conditions, *Proceedings of the 7th Conference TPR2000*, 2000, 217-224.

[13] Banabic, D., Aretz, H., Comsa, D.S., Paraianu, L., An Improved Analytical Description of Orthotropy in Metallic Sheets, *International Journal of Plasticity*, 21, 2005, 493-512.

[14] Lăzărescu, L., Ciobanu, I., Nicodim, I.P., Comsa, D-S., Banabic, D., Effect of the Mechanical Parameters Used as Input Data in the Yield Criteria on the Accuracy of the Finite Element Simulation of Sheet Metal Forming Processes, *Key Engineering Materials*, 554-557, 2013, 204-209.

[15] Paraianu, L., Comsa, D.S, Nicodim, I., Ciobanu, I., Banabic, D., Effect of the Constitutive Law on the Accuracy of Prediction of the Forming Limit Curves, *Key Engineering Materials*, 2012, 77-83.
[16] Comsa, D-S., Banabic, D., Plane-Stress Yield Criterion for Highly-Anisotropic Sheet Metals, *Proceedings of the 7th International Conference and Workshop on Numerical Simulation of 3D Sheet Metal Forming Processes*, 2008, 43-48.
[17] Chaparro, B.M., Alves, J.L., Menezes, L.F., Fernandes, J.V., Optimization of the Phenomenological Constitutive Models Parameters Using Genetic Algorithms, *Advanced Methods in Material Forming*, Springer, Berlin, 2007, 35-54.
[18] Khalfallah, A., Alves, J.L, Oliveira, M.C., Menezes, L.F., Influence of the Characteristics of the Experimental Data Set Used to Identify Anisotropy Parameters, *Simulation Modelling Practice and Theory*, 53, 2015, 15-44.
[19] Vu-Bac, N., Bessa, M.A., Rabczuk, T., Liu, W.K., A Multiscale Model for the Quasi-Static Thermo-Plastic Behavior of Highly Cross-Linked Glassy Polymers, *Macromolecules,* 48, 2015, A-K.
[20] Samaniego, E., Anitescu, C., Goswami, S., Nguyen-Thanh, V.M., Guo, H., Hamdia, K., Zhuang, X., Rabczuk, T., An Energy Approach to the Solution of Partial Differential Equations in Computational Mechanics via Machine Learning: Concepts, Implementation and Applications, *Computer Methods in Applied Mechanics and Engineering*, 362, 2020, 1-29.
[21] Nocedal, J., Wright, S.J., *Numerical Optimization*, Springer, New York, 2006.
[22] El-Aty, A.A., Xu, Y., Guo, X., Zhang, S., Ma, Y., Chen, D. Strengthening Mechanisms, Deformation Behavior, and Anisotropic Mechanical Properties of Al-Li Alloys: A Review, *Journal of Advanced Research*, 10, 2018, 49-67.
[23] Hirsch, J. Recent Development in Aluminium for Automotive Applications, *Transactions of Nonferrous Metals Society of China*, 24, 2014, 1995-2002.
[24] Barlat, F., Lege, D.J., Brem, J.C., Warren, C.J., Constitutive Behavior for Anisotropic Materials and Application to a 2090-T3 Al-Li Alloy, *Modeling the Deformation of Crystalline Solids*, Warrendale, 1991b, 189-203.
[25] Soare, S., On The Use of Homogeneous Polynomials to Develop Anisotropic Yield Functions with Applications to Sheet Forming, *Doctoral Thesis*, University of Florida, USA, 2007.

ORCID iD

Bora Sener https://orcid.org/0000-0002-8237-1950

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