On the Effect of the End-effector Point Trajectory on the Joint Jerk of the Redundant Manipulators

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Abstract. This paper is focused on investigating the joints jerk of industrial serial redundant manipulators with 6 degrees of freedom (6-DOF) under the variation of the end-effector point (EEP) trajectory in the workspace. The EEP trajectories are initially built in the basic planes because of their simplicity, verification and experimentation are smooth, and most of the actual welded structures are performed on these basic planes. The jerk is determined by solving the inverse kinematics problem of the redundant system. This problem is solved based on the algorithm which is used for adjusting the increments of the generalized coordinate vector (AGV). The efficiency of this algorithm is shown through the error between a given trajectory and the recalculated trajectory through the forward kinematics problem. The result of this study allows us to evaluate the effect of the change of the trajectory on the kinematics characteristics of the robot in general and the jerk of the joints in particular. On the other hand, these results can be used as the basis for planning the EEP trajectory for redundant robots, developing algorithms to reduce joint jerky, increase the life of robot systems, and improve the accuracy of the redundant robot movement.

Keywords: Joint jerk, industrial robots, redundant system, end-effector point trajectory.

1. Introduction

In modern industrial production, automatic systems in general and industrial robots in particular are always oriented to achieve optimum performance. Flexibility, accuracy, load capacity, minimizing joint torque, and high-speed motion are factors that always improve system performance. For industrial robots, dexterity and flexibility are met based on the design of robots which have many degrees of freedom such as redundant manipulators as in [1-5]. Although the redundant system has many links and types of joints which are linked together quite complicatedly making it difficult to modeling, kinematics and dynamics analyzing, and designing the control system. The major challenges are kinematics singularity, joint limitations, and obstacle avoidance in the workspace. The inverse kinematics problem (IK) and the control problem of the redundant robots are complicated because the calculation results of the IK are multi-solutions with kinematics singularities. If the IK algorithms do not handle the kinematics singularities and joint limitations well, the control system design problem will face a lot of difficulties in choosing the possible values, meeting the geometric constraints as well as the physical limitations of the system. Many algorithms are developed to solve the inverse kinematics problems such as Jacobian Transpose [6], Pseudoinverse [7], Damped Least Squares [8], Quasi-Newton, and conjugate gradient [9], [10], Closed-loop inverse kinematics [11-14]. The parallel genetic algorithm is used to solve the IK of the Puma 500 robot in [15]. The neutron network algorithm is used in [16], [17]. The control system is intelligently developed to ensure high precision. Improving the load capacity and acceleration of the robot depends heavily on the working conditions of the transmission system of the joints such as wear, jerk, vibration, joint clearance, etc. A system that has more degrees of freedom the more flexible the movement will be, avoiding singularity positions, avoiding obstacles [18], [19]. However, apart from the complex problem of the control system, the limitation and jerk of the joints also need to be considered. The evaluation of the influence of the varying trajectory on the joint jerk as a basis for designing the trajectory in accordance with the robot’s characteristics has not been clearly considered. Jerk is quite important in the above factors because it is directly related to the wear and load capacity of the powertrain [20], [21]. Jerk is the time derivative of the acceleration and it is therefore related to the driving force of the system. Sudden jerk leads to actuator’s wear, engendering resonant vibrations in the system structure. This makes it difficult for the control system to accurately monitor the motion state of the robot. The jerky of the EEP is investigated through the trajectory planning of the EEP in the workspace and it is considered in [22], [23]. The jerk of the joint is mostly mentioned by trajectory planning of joints in joint space and the derivative is continuous respect to time [20], [21], [24-29]. The problem of given the position, speed, acceleration and jerk of the EEP in the workspace to investigate the jerky of joints in the joint space based on kinematics equations has not been specifically considered and it is just stopped at the level of accelerating joints [13], [30-33].
The trajectory of the EEP directly affects the kinematics and dynamic behaviors of the robot joints. There are trajectories of the EEP that angle joints easily meet, but there are also the EEP trajectories that pose great challenges to accurately calculating the value of the joint angles to ensure movement requirements and fit the configuration physics of the robot. Furthermore, although the joint values assure the above requirements, the values of acceleration and jerk have a great influence on the motion accuracy and the life of the actuators on the joints. Therefore, it is necessary to examine the effect of the EEP trajectory on the kinematics behavior of the robot in general and the jerk of the robot joints in particular. There are many studies on the EEP trajectory optimization with the minimum jerk or the shortest motion time. An algorithm to optimize the jerk in the time domain based on interval analysis is proposed in [24]. The trajectory time of a 7-DOF Kuka robot is optimized in [26] by building a control system with a given joint jerk input. The effects of torque, energy consumption, and jerk on the construction of a robot’s collision avoidance trajectory in complex work environments are analyzed in [28]. The optimal trajectory design to minimize joint jerk contributes to improving the accuracy and increased travel speed for the robot are presented in [34], [35]. The algorithm for adjusting the collision avoidance for the robot and moving in the shortest time to the target is presented in [36]. Speed, acceleration, and jerk are used as input conditions of this adjustment algorithm. The jerk optimization of the robot with 7-DOF based on trajectory planning is described in [37]. The jerk is optimized in [38] based on toolpath design for redundant robots for 3D printing. There are many similar studies on this issue as can be seen in [39], [40], [41].

In this study, three trajectories in these basic planes are used to determine the joint jerk of a redundant manipulator with 6-DOFs fixed in a vertical plane. There are many different types of trajectories in space depending on the specific task and these are all based on the three basic planes. The position, velocity, acceleration, and jerk of the joints are the results of this problem. The inverse kinematics solving algorithm takes the given error of the joints variables and limits the joints as the conditions for performing the calculation. There are several reasons for choosing 3 basic planes for planning the EEP trajectory in the workspace. Firstly, this study is only at the beginning of the research on the joint’s jerk of robots with simple and basic trajectories. Secondly, parameters of the EEP trajectories on these 3 planes can ensure the EEP of the robot to the positions that is needed to be investigated such as the position outstretched, close to the robot’s body, the position of rising or falling low close to the base. Thirdly, the EEP trajectories are easily built on these basic planes and easily verify reliability in both theoretical and experimental geometry calculations. On the other hand, easily fabricating auxiliary equipment such as jigs for experimenting, measuring, and verifying calculation results. Next, the analysis results of the problem in this paper can be used immediately because in reality most welded structures are mainly machined on these planes. Finally, the generalized EEP trajectory in the workspace can completely be built and investigated, but verifying the reliability and accuracy of the calculations will be a huge challenge, especially experimentally being verified. This paper is structured in 4 sections including Introduction, Materials and Methods, Simulation results and discussions, and finally Conclusion. Section 2 describes kinematics modeling the welding robot 6-DOF based multi-bodies system theory and DH method and analyzes the inverse kinematics problem using AGV algorithm. Section 3 presents the numerical simulation results which are calculated by MATLAB software.

2. Materials and methods

Consider the kinematics model and working range of industrial FD-V8 welding robot with 6-DOF as shown in Fig. 1. The fixed coordinate system is (OXYZ)o and (OXYZ)i, (i = 1, \ldots, 6) are the local coordinate systems attached to link i. Table 1 describes the kinematic parameters according to the D-H rule [31]. Accordingly, with the transformation \( H_i \), (i = 1, \ldots, 6) homogeneous matrices are determined.

![Fig. 1. Kinematic model of the industrial robot FD-V8 and the EEP working range [42]](image-url)
The position of the EEP (point E in Fig. 1) following the fixed coordinate system is determined as follows [31]

\[ \mathbf{D}_e = H_4 H_3 H_2 H_1 H_6 \]  

(1)

The vector \( \mathbf{q} = [q_1, q_2, q_3, q_4, q_5] \) is the generalized coordinate vector. The forward kinematics equations are present as follows

\[ \mathbf{x} = f(\mathbf{q}), \mathbf{x} \in \mathbb{R}^7, \mathbf{q} \in \mathbb{R}^5 \]  

(2)

where \( f(\mathbf{q}) \) is a vector function representing the robot forward kinematics. Derivative eq. (2):

\[ \mathbf{x} = J(\mathbf{q}) \dot{\mathbf{q}} \]  

(3)

where \( J(\mathbf{q}) \) is the Jacobian matrix with size \( 3 \times 6 \). The acceleration of the EEP can be given by derivation eq. (3) [13], [31], [33]

\[ \dot{\mathbf{x}} = J(\mathbf{q}) \ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q}) \dot{\mathbf{q}} \]  

(4)

Derivative eq. (4), the jerk of the EEP is determined as [30], [32]

\[ \ddot{\mathbf{x}} = J(\mathbf{q}) \dddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{J}(\mathbf{q}) \dot{\mathbf{J}}(\mathbf{q}) \dot{\mathbf{q}} \]  

(5)

The inverse kinematics equations are determined as follows

\[ \mathbf{q}(t) = f^{-1}(\mathbf{x}(t)) \]  

(6)

However, due to the fact that the robot is a redundant system, solving the system of eq. (6) will give countless answers. Choosing the most suitable answer is a quite difficult problem. Therefore, building an effective algorithm to solve the problem of inverse kinetics is always interested in. Once the values of \( \mathbf{q} \) have been determined from eq. (3), the joint velocities are calculated as

\[ \dot{\mathbf{q}} = J^\dagger(\mathbf{q}) \ddot{\mathbf{x}} \]  

(7)

where, \( J^\dagger(\mathbf{q}) \) is the pseudo-inverse matrix of \( J(\mathbf{q}) \) and defined as [31]

\[ J^\dagger(\mathbf{q}) = (J(\mathbf{q})^T J(\mathbf{q}))^{-1} J(\mathbf{q})^T \]  

(8)

the AGV algorithm [43] is applied to find \( \mathbf{q}(t) \) value with given rules \( \mathbf{x}_i(t), \dot{\mathbf{x}}_i(t), \ddot{\mathbf{x}}_i(t), \dddot{\mathbf{x}}_i(t) \). Assume redundant manipulator works in the period from \( t = 0 \) to \( t = T \). Divide the robot working time into \( N \) equal intervals with \( \Delta t = T / N \) and \( t_{k+1} = t_k + \Delta t \), \( k = 0,1,2,..,N-1 \). Implement the Taylor’s expansion by ignoring infinitely small of order greater than or equal to 2 and use eq. (7) for \( \mathbf{x}_{k+1} \) vector:

\[ \mathbf{q}(t_{k+1}) = \mathbf{q}(t_k) + \dot{\mathbf{q}}(t_k) \Delta t + \frac{1}{2} \ddot{\mathbf{q}}(t_k)(\Delta t)^2 + ... = \mathbf{q}(t_k) + J(\mathbf{q}(t_k)) \dot{\mathbf{x}}(t_k) \Delta t \]  

(9)

For simplicity, it is assumed that \( \mathbf{q}(t_k) = \dot{\mathbf{q}}(t_k) = \ddot{\mathbf{q}}(t_k) = \dddot{\mathbf{q}}(t_k) = \mathbf{q}_s(t_k) = \dot{\mathbf{x}}(t_k) = \mathbf{x}(t_k) = \mathbf{x}_i(t_k) = \mathbf{x}_j(t_k) = \mathbf{x}_k(t_k) = \mathbf{x}_l(t_k) = \mathbf{x}_m(t_k) = \mathbf{x}_n(t_k) \). Also, assume that the approximate value \( \mathbf{q}_s \) is known by the geometric or experimental method. Consider approximate value \( \mathbf{q}_{s+1} \) as follows

\[ \mathbf{q}_{s+1} = \mathbf{q}_s + J(\mathbf{q}_s) \dot{\mathbf{x}} \Delta t \]  

(10)

Determining the exact value of the generalized coordinate vector \( \mathbf{q}_{s+1} \) yields

\[ \Delta \mathbf{q}_{s+1} = 0 \]  

(11)

The increment \( \Delta \mathbf{q}_{s+1} \) needs to be determined. Perform Taylor’s expansion for \( \mathbf{x}_{s+1} \) from the forward kinematics equations (2) as follows

\[ \mathbf{x}_{s+1} = f(\mathbf{q}_{s+1}) = f(\mathbf{q}_s + \Delta \mathbf{q}_{s+1}) = f(\mathbf{q}_s) + \mathbf{J}(\mathbf{q}_s) \Delta \mathbf{q}_{s+1} + ... \]  

(12)

So,

\[ \Delta \mathbf{q}_{s+1} = J^\dagger(\mathbf{q}_s) \Delta \mathbf{x}_{s+1} - f(\mathbf{q}_s) \]  

(13)

Then,

\[ \mathbf{q}_{s+1} = \mathbf{q}_s + \Delta \mathbf{q}_{s+1} \]  

(14)

If \( \|\Delta \mathbf{q}_{s+1}\| \geq \varepsilon \) (where \( \varepsilon \) is the given joint error) then replace eq. (14) into eq. (13) and calculate eq. (13) until \( \|\Delta \mathbf{q}_{s+1}\| < \varepsilon \) and determine the final value of \( \mathbf{q}_{s+1} \) vector as follows

\[ \mathbf{q}_{s+1} = \mathbf{q}_{s+1} \]  

(15)

The velocity vector \( \dot{\mathbf{q}}_{s+1} \) can be achieved from eq. (7). The values of \( \dot{\mathbf{q}}_{s+1} \) vector is tested under the following conditions

\[ \|\dot{\mathbf{q}}_{s+1}\| \leq \mathbf{q}_{\text{max}} \]  

(16)

where \( \mathbf{q}_{\text{max}} \) is the values of the joint position limit of the redundant robot FD-V8.

\[ \mathbf{q}_{\text{max}} = [q_{1\max}, q_{2\max}, q_{3\max}, q_{4\max}, q_{5\max}, q_{6\max}] \]  

(17)

After condition (16) is assured, the robot’s acceleration and jerk can be determined by eq. (7) with the above-mentioned values of \( \mathbf{q}_{s+1} \) and \( \dot{\mathbf{q}}_{s+1} \) vectors. If (16) is not met with joint \( \{i = 1..6\} \) such as \( \|\dot{\mathbf{q}}_{s+1}\| > \mathbf{q}_{\text{max}} \) then \( \Delta \mathbf{q}_{s+1} = 0 \) [44]. The \( \mathbf{q}_{s+1} \) value is recalculated from eq. (13). The value of joint acceleration can be obtained from derivative eq. (7) with respect to time as follows.
Table 2. Trajectories in the workspace

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>$x_i$ (m)</th>
<th>$y_i$ (m)</th>
<th>$z_i$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.7 + 0.3sin(2$t$)</td>
<td>0.3cos(2$t$)</td>
<td>0.45</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.7</td>
<td>0.3cos(2$t$)</td>
<td>0.65 + 0.3sin(2$t$)</td>
</tr>
<tr>
<td>Case 3</td>
<td>1.075 + 0.3sin(2$t$)</td>
<td>0</td>
<td>0.81 + 0.3cos(2$t$)</td>
</tr>
</tbody>
</table>

Similarly, the joint jerk is also determined from derivative eq. (18) with respect to time as follows

$$q_{i+1} = J^T(q_{i+1}, q_{i+1}) \dot{x}_{i+1} + J^T(q_{i}, \ddot{x}_{i}) \ddot{x}_{i+1} + J^T(q_{i}, \dddot{x}_i) \dddot{x}_{i+1}$$

(18)

The position error $e_z$ of the EEP can be determined as follows

$$e_z = \dot{x}_z - \dot{f}(q)$$

(20)

3. Simulation results and discussion

The geometry parameters of the robot FD-V8 are presented as $[42]$ $d_1 = 0.43(m), a_1 = 0.16(m), a_2 = 0.615(m), a_3 = 0.125(m)$, $a_4 = 0.125(m), d_4 = 0.65(m), d_6 = 0.475(m)$. The limit values of velocity is given as $q_{\text{max}} = [2.97 \ 2.7 \ 2.97 \ 3.14 \ 4.01 \ 6.28]$ (rad). The allowable error of joint variables in the algorithm is $\varepsilon = 10^{-6}$ (rad). Given the trajectories of the EEP in three cases are shown in Tab. 2.

The numerical simulation results of case 1 with the trajectory in the workspace, the value of joint variables and simulation model in MATLAB are described respectively in Fig. 3, Fig. 4, and Fig. 5. Similar, simulation results of case 2 are shown in Fig. 6, Fig. 7, and Fig. 8. Case 3 is determined in Fig. 9, Fig. 10, and Fig. 11, respectively.

Fig. 12, Fig. 13, and Fig. 14 present the position error of the EEP between desired and calculated trajectories on axes OX, OY, and OZ respectively with 3 cases. The calculated positions are determined through solving forward kinematics problems with the value of joint variables which are obtained from solving the IK are input data. Fig. 15 to Fig. 20 show the velocity and acceleration value of joints in three cases, respectively. The jerk values of the joints are shown in Fig. 21 to Fig. 25 for all 3 cases. Table 3 presents the maximum joint values of joints in all three cases.

![Fig. 2. Diagram of calculation steps](image)

**Case 1 (C1):** The trajectory on OXY plane

![Fig. 3. Trajectory in C1](image)

![Fig. 4. Joint position in C1](image)

![Fig. 5. Case 1](image)
Case 2 (C2): The trajectory on OYZ plane

Fig. 6. Trajectory in C2
Fig. 7. Joint position in C2
Fig. 8. Case 2

Case 3 (C3): The trajectory on OXZ plane

Fig. 9. Trajectory in C3
Fig. 10. Joint position in C3
Fig. 11. Case 3

Fig. 12. Error position OX
Fig. 13. Error position OY
Fig. 14. Error position OZ

Fig. 15. Velocity value of joints in C1
Fig. 16. Velocity value of joints in C2
Fig. 17. Velocity value of joints in C3
Table 3. Maximum value of jerk at the joint

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint 1</th>
<th>Joint 2</th>
<th>Joint 3</th>
<th>Joint 4</th>
<th>Joint 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>4.95</td>
<td>3.69</td>
<td>4.72</td>
<td>6.4</td>
<td>4.85</td>
</tr>
<tr>
<td>Case 2</td>
<td>4.32</td>
<td>6.73</td>
<td>10.37</td>
<td>6.4</td>
<td>5.45</td>
</tr>
<tr>
<td>Case 3</td>
<td>0</td>
<td>6.73</td>
<td>11.44</td>
<td>0</td>
<td>9.51</td>
</tr>
</tbody>
</table>

On the one hand, case 1 with the trajectory on the OXY plane gives the highest jerk in 3 cases. Jerk of joint 1 in case 1 is the largest with 4.95 (rad/s$^3$) and in case 3 is smallest with zero. This is quite understandable. Joint 1 in case 3 does not need to move because the EEP trajectory is in the XOZ plane. On the other hand, when the EEP point trajectory is close to the center of the robot, the jerk value changes continuously for a short time. The jerk value of joint 4 changed the most (6.4 rad/s$^3$) compared with case 2 (1.94 rad/s$^3$) and case 3 (0 rad/s$^3$). Case 3 also has a short-range of the EEP point trajectory near the robot center and also has the same jerky behavior as in case 1. The sudden change in jerk leads to increasing friction and temperature increase in the joints and it adversely affects the durability and longevity of the joints.

The jerk value of the joints 2, 3, and 5 in case 2 (6.73, 10.37, and 5.45 rad/s$^3$) and case 3 (6.73, 11.44, and 9.51 rad/s$^3$) is much greater than that of case 1 (3.69, 4.72 and 4.85 rad/s$^3$). This occurs when the EEP point trajectory gets too high or too low above the ground. The jerk of these joints in case 3 is the greatest due to the EEP point trajectory having the highest reach and closest to the ground.
The value of joint 1 in case 1 (4.95 rad/s²) and in case 2 (4.32 rad/s²) have a continuous change value and are quite large, except in case 3 (0 rad/s²). Joint 1 usually carries the maximum load due to having to support the entire weight of the robot. Therefore, the designer should limit the motion of joint 1 when designing the trajectory to minimize or avoid sudden changes in jerk value to improve joint life. According to this criterion, the EEP point trajectory is designed not too close to the robot center, preferably the trajectory lying on the XOZ plane (case 3) or the YOZ plane (case 2). Furthermore, this trajectory can be as high as the robot’s maximum reach because the mass of the welding torch on the end-effector link is quite small but should not be too low above the ground. This also depends in part on the construction of the system to be welded.

From the obtained results, it can be seen that the trajectory design should restrain designing on the OXY plane (case 1) with the position of the EEP approaching the center of the robot because the jerk joints will tend to create vibrations on the links. The trajectory should tend to lie on the XOZ and OYZ planes (case 2). The best is the XOZ plane (case 3) because the jerk is quite uniform. This is of great importance in practical welding applications. The results of this study can be performed together with the designing of optimal welding trajectories taking into consideration joint jerk.

4. Conclusion

This paper has effectively applied the AGV algorithm to solve the IK of the redundant robots with 6-DOF. For each different trajectory change in the workspace, the kinematics characteristics of the robot such as the position, velocity, acceleration, and jerk of the joints are calculated to be within geometry limits, and ensure the minimum position error of the EEP. The jerk of the joints is determined to evaluate the effect of the change in their trajectory on them. The results showed that the jerks at joint 1 and joint 2 are large. With industrial robots fixed on the ground vertically, the trajectory in the OXY plane produces the greatest jerk compared to other planes. At the times when the EEP is closer to the center of the robot, joint jerk increases. On the other hand, the calculation results also show that trajectories are designed on the OXZ plane and OYZ plane and directed away from the robot’s center, the jerk of the joint decreases. These will be good references for the design and optimization of the trajectory, optimize the robot’s jerk and serve as the basis for reducing the actuator’s system wear, improving longevity, and working efficiency. On the other hand, the research results show that the AGV algorithm is highly effective when applied to the special redundant robot systems with plenty of DOF combined with the trajectory optimization algorithms, the optimization of the feed rate of the torch.

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Conflict of Interest

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i )</td>
<td>Desired path of the EEP [m]</td>
</tr>
<tr>
<td>( q(t) )</td>
<td>Generalized coordinate vector of Joint variables [rad]</td>
</tr>
<tr>
<td>( J(q) )</td>
<td>The Jacobian matrix</td>
</tr>
<tr>
<td>( H )</td>
<td>Homogeneous matrix in local coordinate frame</td>
</tr>
<tr>
<td>( D )</td>
<td>Homogeneous matrix in fixed coordinate frame</td>
</tr>
</tbody>
</table>

References