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Research Paper

Theoretical Study of Convective Heat Transfer in Ternary Nanofluid Flowing past a Stretching Sheet

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Abstract. A new theoretical tri-hybrid nanofluid model for enhancing the heat transfer is presented in this article. This model explains the method to obtain a better heat conductor than the hybrid nanofluid. The tri-hybrid nanofluid is formed by suspending three types of nanoparticles with different physical and chemical bonds into a base fluid. In this study, the nanoparticles TiO_2, Al_2O_3 and SiO_2 are suspended into water thus forming the combination TiO_2-SiO_2-Al_2O_3-H_2O. This combination helps in decomposing harmful substances, environmental purification and other appliances that requires cooling. The properties of tri-hybrid nanofluid such as Density, Viscosity, Thermal Conductivity, Electrical Conductivity and Specific Heat capacitance are defined mathematically in this article. The system of equations that governs the flow and temperature of the fluid are converted to ordinary differential equations and are solved using RKF-45 method. The results are discussed through graphs and it is observed that the tri-hybrid nanofluid has a better thermal conductivity than the hybrid nanofluid.

Keywords: Heat Transfer, Tri hybrid Nanofluid, Stretching Sheet, Magnetic Field.

1. Introduction

The boundary layer flow and heat transfer of a fluid finds applications in various fields such as, drawing of polymer sheet, laments extruded from a die, cooling of a metallic plate in a bath etc. The fluids are usually used for cooling purposes due to their heat conducting capacity and its abundant availability. Boundary layer theory gives the information about the shape of the body in order to avoid the boundary layer separation. In this view, Sakiadis [1] introduced the concept of boundary layer flow and it was experimentally supported by Tsou et al. [2]. Rollins and Vajravelu [3] discussed the heat transfer properties of a second order fluid flowing past a stretching sheet. Sharidan et al. [4], Puneeth et al. [5] and Abel et al. [6] studied flow of fluid past a stretching sheet. Elbashbeshy and Aldawody [7-9] discussed mixed convection in the unsteady flow of fluid over a porous sheet. Grubka and Bobba [10] discussed the heat transfer in the flow past a stretching sheet subjected to uniform heat flux.

The development of industries increased the air pollution caused by factory fumes, emissions of oxides of Sulphur and dust particles. Accordingly, the problem related to the environment such as global warming are emerging as global issues. Also, the vehicle movement forms the source of pollution along with the industrial boilers and power generation facilities. These sources produce oxides of Nitrogen that are harmful air pollutants. In recent times, the emission of oxides of Nitrogen has reached the alarming levels in major cities due to the increase in the usage of motor vehicles. This causes respiratory disease, photochemical smog and acid rain. Measures for reducing the emission of NO_x include purification and detoxification through photocatalytic reactions. The photocatalyst used for the above-mentioned purpose should possess high optical activity, high absorbance of visible light, an optimal energy range that is suitable for reactions, chemical inertness, biological inertness, optical stability, low cost etc. All these characteristics are found to be available in TiO_2 and due to these features TiO_2 is used in many appliances.

Titanium dioxide (TiO_2) finds its significance in photocatalytic applications [11-15]. High photo catalysis is obtained by fabricating TiO_2 materials with high specific surface areas. In particular, the nanostructured (nanotubes and nanofibers) TiO_2 materials that are made-up through a template synthesis method introduced by Martin's group [16, 17] attracted researchers in recent years due to its large surface areas and various applications of the materials. Along with these characteristics, the metallic nanoparticles similar to TiO_2 such as Cu, Ag, CuO, Au etc. possess better thermal conductivity when compared to fluids. This

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motivated the researchers to discover nanofluid in which the metallic particles of nano-meter size are suspended. It was observed by through experiments and theoretical studies that the nanofluid possessed better thermal conductivity than the regular fluids.

Choi [18] introduced the nanofluid concept. Xuan and Roetzel [19] had incurred the thermal dispersion in the motion to augment the process of heat transport. They proposed two methods to deduce the heat transfer relation of nanofluid. Later, Xuan and Li [20] proposed a theoretical model that described the heat transport characteristics of a nanofluid flowing in a tubular region. Kelbinski et al. [21, 22] discussed the nanoparticles clustering. Ghalambaz et al. [23, 24] discussed natural convection in the flow of nanofluid containing phase change particles. Further, Ghalambaz [25, 26] continued to discuss the natural convection in an unsteady flow of nanofluid. These studies indicated that the nanofluid finds various applications in the field of industrialized cooling, manufacturing of detergent, biomedical applications, nuclear reactors, microchip technology etc.

The principle behind synthesizing nanofluid composites is to enhance the properties of single nanoparticle that possess either better thermal conductivity or better rheological properties. By framing a composite nanofluid, the thermal conductivity and the rheological property of the resulting nanofluid can be enhanced. This is ensured by preparing a perfect combination of nanoparticles. A nanofluid formed by adding nanoparticles that has better thermal conductivity may not have better rheological property. Hence, by adding nanoparticles with different rheological or thermal properties enhances the overall ability of the nanofluid and makes it more stable and effective. For example, Al_2O_3 exhibits appreciable chemical inertness and stability but will offer low thermal conductivity. Whereas, particles like Aluminum, Silver, Copper etc. possess higher thermal conductivity and are unstable and chemically reactive. Hence, the mixing of these nanoparticles with different physical and chemical bonds forms nanofluid called hybrid nanofluids. They find applications in nuclear safety, pharmaceutical industry, cooling of electronic heaters etc.

Hayat et al. [27] explored that the hybridization of the fluid increased the rate of heat transfer. Chamkha et al. [28, 29, 30] discussed the natural convection in the hybrid nanofluid under magnetic field in a square enclosure. Manjunatha et al. [31] discussed the flow of $Cu-Al_2O_3-H_2O$ hybrid nanofluid under the influence of variable viscosity. Nihara [32] demonstrated the enhancement of mechanical and thermal properties of base fluid due to the addition of nano particles. Bahiraei et al., [33, 34, 35, 36] discussed the advantages of using hybrid nanofluids with graphene as one of the suspensions over pure water in the liquid blocks and also found an improvement in the performance of heat sink while using nanofluid. Han et al. [37] coated TiO_2 with SiO_2 to improve the dispersibility of TiO_2 in water. Furthermore, many researchers have continued studying the heat transfer characteristics of hybrid nanofluid [38-43]. The improvement of balancing the thermal and rheological properties can be done by designing a tri-hybrid nanofluid.

The high demand for a cooling agent with enhanced heat transfer capability in the industries motivated the researchers to develop the existing nanofluid concept for enhancing the heat transfer. As a result, the hybrid nanofluid was discovered and an enhanced heat transfer characteristic was observed. With this motivation further experimental studies are being conducted for tri-hybrid nanofluid anticipating an improved heat transfer rate. In tri-hybrid nanofluid, three classes of nanoparticles with different physical and chemical bonds are suspended. For instance, TiO_2 possesses acid centers of high acid strength when it is treated with sulfuric acid and forms a covalent surface sulfatase such as $TiOSO_4$. Similarly, Al_2O_3 reacts with sulfuric acid to form surface sulfatase similar to aluminum ionic salts such as $Al_2(SO_4)_3$. The acid sites that are weaker in strength are produced due to the reaction between alumina and sulphuric acid. These acid sites produced will make the nanoparticle composition stable and chemically inert. Whereas, SiO_2 does not form sulfatase due to the higher electron negativity of Si than that of Al and Ti. This study is an initiative to develop a coolant that can cool the appliance using TiO_2. Thus, in order to develop such a coolant using TiO_2, the supporting agents SiO_2 and Al_2O_3 are used. In this paper, a theoretical model for Tri-Hybrid nanofluid is introduced in support of the experimental studies.

2. Mathematical Model

Consider a laminar flow of an incompressible tri-hybrid nanofluid formed by suspending TiO_2, Al_2O_3 and SiO_2 in water by considering the hybrid nanofluid $SiO_2-Al_2O_3-H_2O$ as base fluid. It is allowed to flow past a stretching sheet at velocity $\vec{q}$ whose components are (u,v) along (x,y) direction. The physical configuration is shown in Fig. (1) and the Cartesian co-ordinate system is used to describe the configuration. The sheet is assumed to be stretching at a speed of $U_w = ax$. A uniform magnetic field of strength B_0 is applied perpendicular to the sheet. The room is maintained at an ambient temperature T_∞ and the temperature of the tri-hybrid nanofluid is described by T. Based on these assumptions the governing equations take the following form [31]:

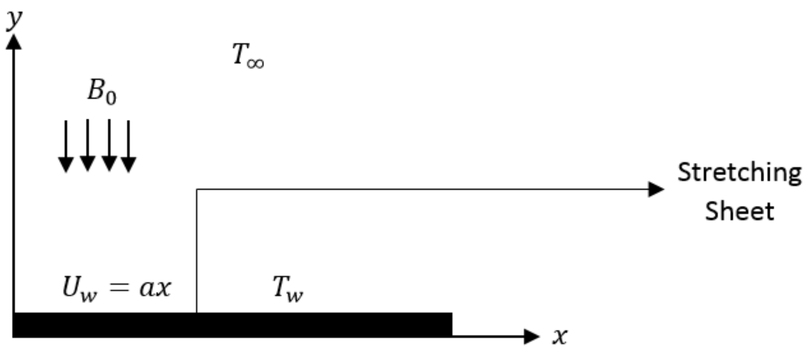

Fig. 1. Physical Configuration

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{thnf}}{\rho_{thnf}}\frac{\partial^2 u}{\partial y^2}+\frac{\sigma_{thnf}}{\rho_{thnf}}B_0^{\,2}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial x}=\kappa_{thnf}\frac{\partial^2 T}{\partial y^2}+Q_c(T-T_\infty) \tag{3}$$

Subjected to the conditions

$$\begin{aligned}&u=U_w,\ v=0,\ \kappa_{thnf}\frac{\partial T}{\partial y}=-h_t(T_w-T)\ \text{at}\ y=0\\&u\to 0,\ T\to T_\infty\ \ \text{as}\ y\to\infty\end{aligned} \tag{4}$$

The physical quantities are defined as:

$$Cf_x=\frac{\mu_{thnf}}{\mu_f(ax)^2}\left[\frac{\partial u}{\partial y}\right]_{y=0}\ \text{and}\ Nu_x=-x\frac{\kappa_{thnf}}{\kappa_f(T_w-T_\infty)}\left[\frac{\partial T}{\partial y}\right]_{y=0} \tag{5}$$

3. Thermophysical and Rheological Properties

The thermophysical properties of $TiO_2-SiO_2-Al_2O_3-H_2O$ Tri hybrid nanofluid are

1. Density

$$\rho_{thnf}=(1-\phi_1)\{(1-\phi_2)[(1-\phi_3)\rho_f+\phi_3\rho_3]+\phi_2\rho_2\}+\phi_1\rho_1$$

2. Viscosity

$$\mu_{thnf}=\frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}}$$

3. Thermal Conductivity

$$\frac{\kappa_{thnf}}{\kappa_{hnf}}=\frac{\kappa_1+2\kappa_{hnf}-2\phi_1(\kappa_{hnf}-\kappa_1)}{\kappa_1+2\kappa_{hnf}+\phi_1(\kappa_{hnf}-\kappa_1)}$$

where $\frac{\kappa_{hnf}}{\kappa_{nf}}=\frac{\kappa_2+2\kappa_{nf}-2\phi_2(\kappa_{nf}-\kappa_2)}{\kappa_2+2\kappa_{nf}+\phi_2(\kappa_{nf}-\kappa_2)}$ and $\frac{\kappa_{nf}}{\kappa_f}=\frac{\kappa_3+2\kappa_f-2\phi_3(\kappa_f-\kappa_3)}{\kappa_3+2\kappa_f+\phi_3(\kappa_f-\kappa_3)}$

4. Electrical Conductivity

$$\frac{\sigma_{thnf}}{\sigma_{hnf}}=\frac{(1+2\phi_1)\sigma_1+(1-2\phi_1)\sigma_{hnf}}{(1-\phi_1)\sigma_1+(1+\phi_1)\sigma_{hnf}}$$

where $\frac{\sigma_{hnf}}{\sigma_{nf}}=\frac{(1+2\phi_2)\sigma_2+(1-2\phi_2)\sigma_{nf}}{(1-\phi_2)\sigma_2+(1+\phi_2)\sigma_{nf}}$ and $\frac{\sigma_{nf}}{\sigma_f}=\frac{(1+2\phi_3)\sigma_3+(1-2\phi_3)\sigma_f}{(1-\phi_3)\sigma_3+(1+\phi_3)\sigma_f}$

The thermophysical constants with respect to the nanoparticles and base fluid are given in Table 1:

Table 1. Thermophysical properties of the nanoparticles and the base fluid [40]

	ρ $[kg\,m^{-3}]$	σ $[Sm^{-1}]$	κ $[W(mK)^{-1}]$
H_2O	997.1	5.5x10^{-6}	0.6071
TiO_2	4250	2.4x10^{6}	8.953
SiO_2	2270	3.5x10^{6}	1.4013
Al_2O_3	6310	5.96x10^{7}	32.9

4. Similarity Transformation

The partial differential equations (1) - (4) are made dimensionless and are converted to ordinary differential equations by using the following similarity transformation [31]:

$$u = axF'(\eta),\ v = -\sqrt{a\upsilon}F(\eta),\ \Theta = \frac{T - T_\infty}{T_w - T_\infty},\ \eta = \sqrt{\frac{a}{\upsilon}} \tag{6}$$

Eq. (1) is satisfied and the transformed system of equations are:

$$\frac{\mu_{thnf}}{\mu_f}F''' + \frac{\rho_{thnf}}{\rho_f}(FF'' - F'^2) - \frac{\sigma_{thnf}}{\sigma_f}MF' = 0 \tag{7}$$

$$\frac{\kappa_{thnf}}{\kappa_f}\Theta'' + \Pr F\Theta' + Q\Theta = 0 \tag{8}$$

The associated boundary conditions are:

$$F'(0) = 1,\ F(0) = 0,\ \Theta'(0) = -\gamma\frac{\kappa_{thnf}}{\kappa_f}(1 - \Theta(0)),\ F'(\infty) = 0,\ \Theta(\infty) = 0 \tag{9}$$

The expressions corresponding to the physical quantities are defined as:

$$\sqrt{\mathrm{Re}_x}Cf_x = \frac{\mu_{thnf}}{\mu_f}F''(0),\ \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = \frac{\kappa_{thnf}}{\kappa_f}\Theta'(0) \tag{10}$$

The nondimensional parameters are defined as:

$$M = \frac{\sigma_f B_0^{\,2}}{2U\rho_f},\ \Pr = \frac{\upsilon_f}{\alpha_f},\ Q = \frac{Q_c}{T_w - T_\infty},\ \mathrm{Re}_x = \frac{U_w x}{\upsilon_f} \tag{11}$$

5. Solution Methodology

The transformed governing equations (7), (8) and the boundary conditions (9) are converted to initial value problem and are solved using RKF-45 method with the help of shooting technique. The computations are performed by setting $\eta = 10$ for the far field boundary conditions with an accuracy of 10^{-5}. This method determines the proper step size and at every step, two approximations are compared. If these two approximations hold close agreement with each other then it is accepted. Else, the step size is further reduced and the computation is repeated until required accuracy is achieved. The result is verified by comparing it with the existing literature and the comparison is displayed in the below table 2. The comparison is done for the values of $-\Theta'(0)$ by considering $\phi_1 = \phi_2 = \phi_3 = M = 0$.

6. Results and Discussion

The dimensionless ordinary differential equations (7)-(9) are solved using RKF-45 method as described in the previous section. The results are validated by comparing it with those existing in the literature [38, 39, 20]. A very close agreement is seen with the results and hence verifies the method used. The outcomes of the study are interpreted through Fig. 2 to Fig. 8 in this section.

Figures 2 and 3 interpret the effect of magnetic field on velocity profile and temperature profile of the tri-hybrid nanofluid respectively. The existence of magnetic field gives rise to a force called Lorentz force that opposes the fluid flow. The magnitude of this force is directly proportional to the magnitude of M. Hence, the increase in M strengthens the Lorentz force. This in turn provides a greater resistivity to the fluid flow and hence the momentum is found to be decreasing with the increasing values of M. This decrement in the velocity of the flow allows nanoparticles to conduct more heat and hence an enhancement in the temperature is observed.

Table 2. Comparison Table

Pr	Manjunatha et al. [37] $-\Theta'(0)$	Khan and Pop [38] $-\Theta'(0)$	Gorla and Sidwai [39] $-\Theta'(0)$	Present Study $-\Theta'(0)$
2	0.9113	0.9113	0.9114	0.9113
6.13	1.7596	1.7597	1.7596	1.7597
7	1.8954	1.8954	1.8954	1.8954
20	1.3539	1.3539	1.3539	1.3539

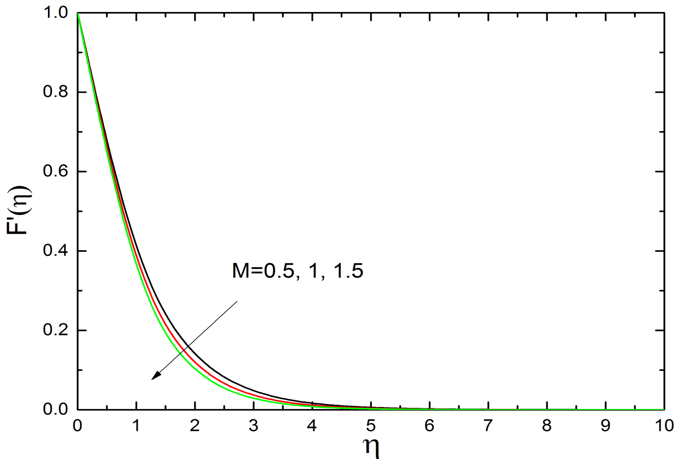

Fig. 2. Effect of Magnetic field on Velocity

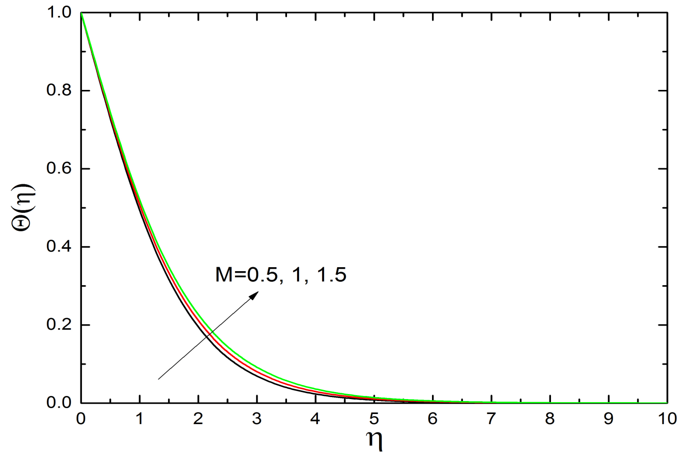

Fig. 3. Effect of Magnetic field on Temperature

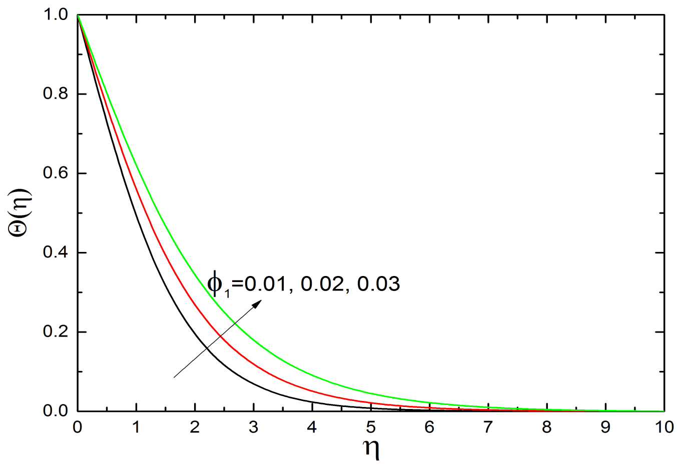

Fig. 4. Effect of Volume Fraction on Temperature

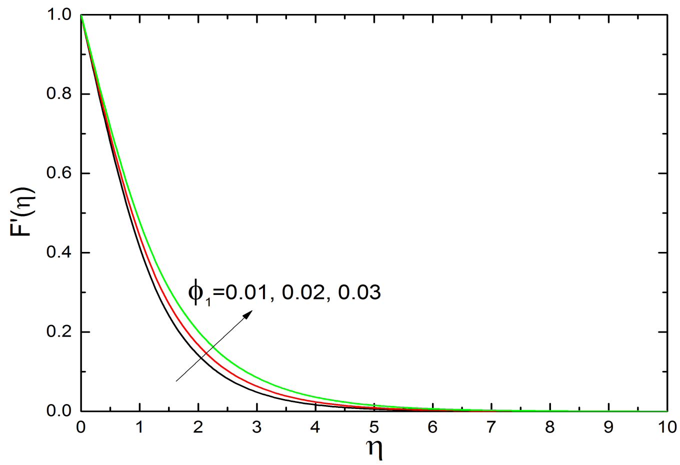

Fig. 5. Effect of Volume fraction on Velocity

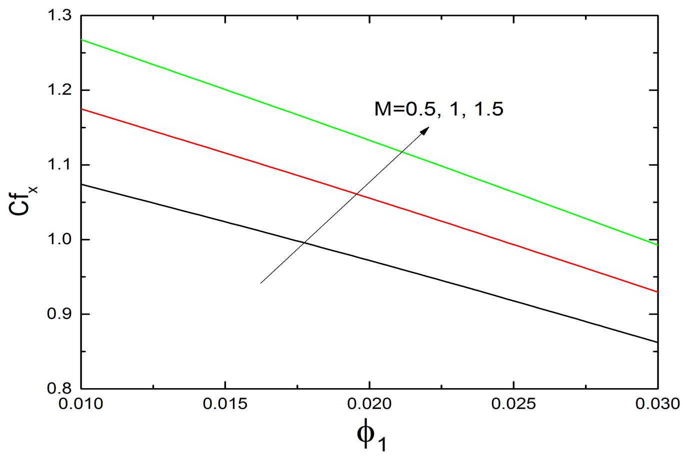

Fig. 6. Effect of Volume fraction and Magnetic field on Skin friction co-efficient

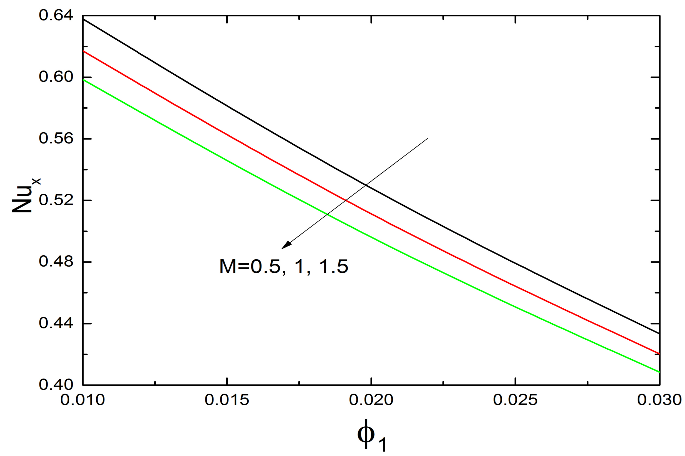

Fig. 7. Effect of Volume fraction and Magnetic field on Nusselt Number

Figures 4 and 5 show the effect of volume fraction over the temperature profile and the velocity profile respectively. With the rise in the volume fraction of TiO_2 nanoparticles, its concentration increases within the nanofluid and provides more room for increased heat conduction. This in turn increases the temperature of the nanofluid as shown in the Fig. 4. This indicates that the absorption of heat from the appliance is enhanced and the appliance is ensured an optimum temperature and long life. The photocatalytic nature of TiO_2 performs better as a heat conductor hence the nanofluid with combination of TiO_2 is used more as coolant. Also, its chemical inertness makes the nanofluid more stable. By using this combination as coolant in the motor vehicles can reduce the air pollution as discussed earlier. Since these nanoparticles conduct more heat it becomes less dense and flows easily in the nanofluid. These less dense nanoparticles pull the fluid along with them in its direction of motion and hence the velocity of the fluid flow rises as shown in the Fig. 5.

The Fig. 6 to Fig. 9 shows the response of Cf_x and Nu_x for variation in non-dimensional parameters. The presence of M gives rise to Lorentz force in the form of frictional force whose magnitude is directly proportional to the magnitude of M. Thus, the increase in the magnetic field parameter increases Cf_x as shown in the Fig. 6. But it decreases with the increase in ϕ_1 due to the fact that the heat conduction is enhanced for higher concentration of nanoparticles. Figure 7 shows that the Nu_x with the increase in M, ϕ_1 and from the Fig. 8 and Fig. 9 it is clear that Nu_x increase with $\Pr$.

Figure 10 displays the comparison of heat transfer performance of fluid, nanofluid, hybrid nanofluid and tri-hybrid nanofluid. It is noted that the tri-hybrid nanofluid has better heat transfer characteristics than that of the other due to the fact that by using different nanoparticles with different chemical bonds helps increase the heat transfer as each nanoparticle with its chemical bond has its own properties to take care of. For example, in this composition, TiO_2 is responsible for more heat conduction due to its photocatalytic feature and high thermal conductivity. SiO_2 is used to enhance the catalytic nature of TiO_2 so that it will be in a position to conduct more heat and the inclusion of Al_2O_3 takes care of chemical inertness and stability of the fluid.

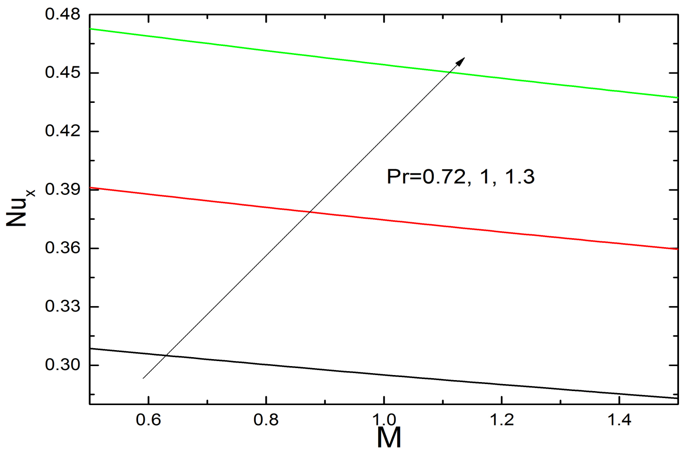

Fig. 8. Effect of Prandtl number and Magnetic field on Nusselt number

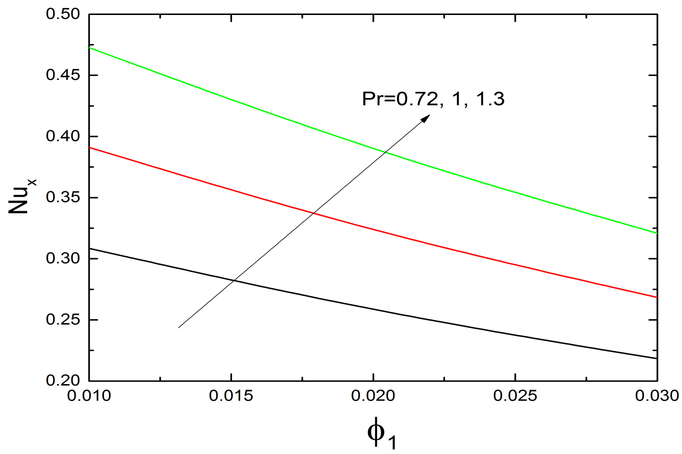

Fig. 9. Effect of Prandtl number and Volume fraction on Nusselt number

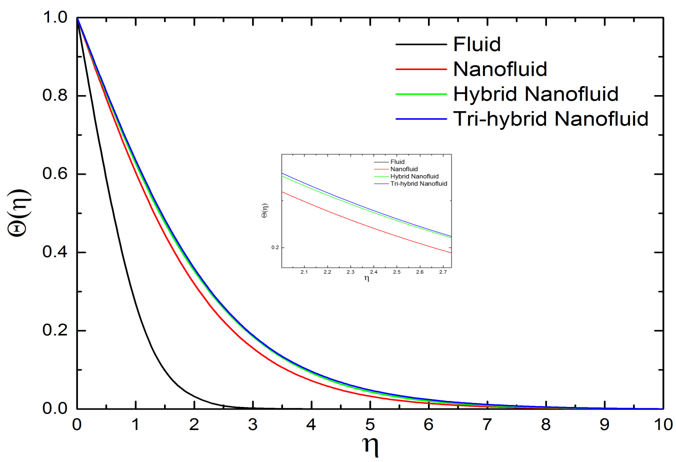

Fig. 10. Comparison.

7. Conclusion

The analysis of heat transfer characteristics and flow behavior for TiO_2-SiO_2-Al_2O_3-H_2O tri-hybrid nanofluid past a linearly stretching sheet is conducted. The governing equations are converted to ordinary differential equations using the suitable similarity transformations. The resulting system of differential equations are solved using RKF 45 method and the results are interpreted through graphs. This work can be further extended to study the behavior of various non Newtonian fluids under different physical situation by considering the suspension of different class of nanoparticles that suits the practical scenario. The major outcomes of the study are:

- Tri-Hybrid nanofluid has better heat transfer property than fluid, nanofluid and hybrid nanofluid.
- The Tri-Hybrid nanofluid conducts more heat for higher volume fraction.
- The momentum of the flow increases for higher volume fraction.
- The increase in the magnetic field resists the flow of Tri-hybrid nanofluid.
- The heat generated due to strong Lorentz force caused the Tri-hybrid nanofluid to conduct more heat.

Author Contributions

S. Manjunatha planned the scheme, initiated the project; V. Puneeth solved the mathematical model and analyzed the empirical results; B.J. Gireesha and Ali J. Chamkha examined the theory validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

(u,v)	Velocity Components $[ms^{-1}]$	Cf_x	Skin friction co-efficient
(x,y)	Coordinates	Q	Heat source or sink parameter
B_0	Strength of magnetic field $[Tesla]$	Re_x	Reynolds number
T	Temperature $[K]$	M	Magnetic field parameter
T_w	Surface temperature $[K]$	Pr	Prandtl number
T_∞	Ambient Temperature $[K]$	κ	Thermal conductivity $[W(mK)^{-1}]$
U_w	Surface velocity $[ms^{-1}]$	σ	Electrical conductivity $[Sm^{-1}]$
a	A positive constant	ρ	Density $[kg\,m^{-3}]$
$\vec{q}$	Velocity vector	μ	Dynamic viscosity $[Pa-s]$
h_t	Heat transfer co-efficient	thnf	Tri hybrid nanofluid
Q_c	Heat source or sink $[J]$	hnf	Hybrid Nanofluid
Nu_x	Nusselt number	nf	Nanofluid

References

[1] Sakiadis B.C., Boundary layer behaviour on continuous solid surface; I Boundary layer equations for two dimensional and axisymmetric flow, *AIChE Journal*, 7(1), 1961, 26-28.
[2] Tsou, F.K., Sparrow E.M., Glodstein R.J., Flow and heat transfer in the boundary layer on a continuous moving surface, *International Journal of Heat and Mass Transfer,* 10(2), 1967, 219-237.
[3] Rollins, D., Vajravelu, K., Heat transfer in a second order fluid over a continuous stretching surface, *Acta Mechanica*, 89(1), 1991, 167-178.
[4] Sharidan, S., Mahmood, T., Pop, I., Similarity solution for the unsteady boundary layer flow and heat transfer due to a stretching sheet, *International Journal of Applied Mechanical Engineering*, 11, 2006, 647-65.
[5] Puneeth, V., Manjunatha, S., Gireesha, B.J., Gorla, R.S.R., Magneto convective flow of casson nanofluid due to Stefan blowing in the presence of bio-active mixers, *Proceedings of the Institution of the Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems*, 2021, DOI: 23977914211016692
[6] Abel, M.S., Khan, S.K., Prasad, K.V., Study of Visco-elastic fluid and heat transfer over a stretching sheet with variable viscosity, *International Journal of Non-linear Mechanics*, 37(1), 2002, 81-88.
[7] Elbashbeshy, E.M.A., Heat transfer over a stretching surface with variable surface heat flux, *Journal of Physics D: Applied Physics*, 31(16), 1998, 1951-

1954.
[8] Elbashbeshy, E.M.A., Bazid, M.A.A., Heat transfer over an unsteady stretching surface, *International Journal of Heat and Mass Transfer*, 41(1), 2004, 1-4.
[9] Elbashbeshy, E.M.A., Aldawody, D.A., Effect of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over a porous stretching surface, *International Journal of Non-Linear Sciences*, 9(4), 2010, 448-454.
[10] Grubka, L.J., Bobba, K.M., Heat transfer characteristics of a continuous stretching surface with variable temperature, *International Journal of Heat and Mass Transfer*, 107(1), 1985, 248-250.
[11] Fujishima, A., Honda, K., Electrochemical photolysis of water at a semiconductor electrode, *Nature*, 238(5358), 1972, 37-38.
[12] Bischoff, B.L., Anderson, M.A., Peptization process in the sol-gel preparation of porous anatase (TiO2), *Chemistry of Materials*, 7(10), 1995, 1772-1778.
[13] Hulteen, J., A general template based method for the preparation of nanomaterials, *Journal of Materials Chemistry*, 7(7), 1997, 1075-1087.
[14] Zhang, M., Bando, Y., Wada, K., Sol-gel template preparation of TiO2 nanotubes and nanorods, *Journal of Materials Science Letters*, 20(2), 2001, 167-170.
[15] Seo, D.S., Lee, J.K., Kim, H., Preparation of nanotube shaped TiO2 powder, *Journal of Crystal Growth*, 229(1-4), 2001, 428-432.
[16] Martin, C.R., Nanomaterials: a membrane based synthetic approach, *Science*, 266(5193), 1994, 1961-1966.
[17] Martin, C.R., Parthasarathy, R., Menon, V., Template synthesis of electronically conductive polymers preparation of thin films, *Electrochemica Acta*, 39(8-9), 1994, 1309-1313.
[18] Choi, S.U.S., Eastman, J.A., Enhancing the thermal conductivity of fluids with nanoparticles, *Argonne National Lab 43 (ANL/MSD/CP-84938 CONF-951135-29)*, 1995, 99–105.
[19] Xuan, Y., Roetzel, W., Conceptions for heat transfer correlation of nanofluids, *International Journal of Heat and Mass Transfer*, 43(19), 2000, 3701–3707.
[20] Xuan, Y., Li, Q., Heat transfer enhancement of nanofluids, *International Journal of Heat and Fluid Flow*, 21(1), 2004, 58–64.
[21] Keblinskia, P., Phillpot, S., Choi, S.U.S., Eastman, J.A., Mechanisms of heat flow in suspensions of nano-sized particles (nanofluids), *International Journal of Heat and Mass Transfer*, 45(4), 2002, 36–44.
[22] Keblinskia, P., Eastman, J.A., Cahill, D.G., Nanofluids for thermal transport, *Materials Today*, 8(6), 2005, 36–44.
[23] Mehryan, S.A.M., Ghalambaz, M., Gargari, L.S., Hajjar, A., Sheremet, M., Natural convection flow of a suspension containing nano-encapsulated phase change particles in an eccentric annulus, *Journal of Energy Storage*, 28, 2020, 101236.
[24] Ghalambaz, M., Groşan, T., Pop, I., Mixed convection boundary layer flow and heat transfer over a vertical plate embedded in a porous medium filled with a suspension of nano-encapsulated phase change materials, *Journal of Molecular Liquids*, 293, 2019, 111432.
[25] Ghalambaz, M., Mehryan, S.A.M., Hajjar, A., Veismoradi, A., Unsteady natural convection flow of a suspension comprising Nano-Encapsulated Phase Change Materials (NEPCMs) in a porous medium, *Advanced Powder Technology*, 31(3), 2020, 954-966.
[26] Hajjar, A., Mehryan, S.A.M., Ghalambaz, M., Time periodic natural convection heat transfer in a nano-encapsulated phase-change suspension, *International Journal of Mechanical Sciences*, 166, 2020, 105243.
[27] Hayat, T., Nadeem, S., Heat transfer enhancement with Ag-CuO/water hybrid nanofluid, *Results in Physics*, 7, 2017, 2317-2324.
[28] Tayebi, T., Chamkha, A.J., Entropy generation analysis due to MHD natural convection flow in a cavity occupied with hybrid nanofluid and equipped with a conducting hollow cylinder, *Journal of Thermal analysis and Calorimetry*, 139(3), 2020, 2165-2179.
[29] Ghalambaz, M., Doostani, A., Izadpahani, E., Chamkha, A.J., Conjugate natural convection flow of Ag-MgO/water hybrid nanofluid in a square cavity, *Journal of Thermal Analysis and Calorimetry*, 139(3), 2020, 2321-2336.
[30] Dogonchi, A.S., Nayak, M.K., Karimi, N., Chamkha, A.J., Ganji, D.D., Numerical simulation of hydrothermal features of Cu-H2O nanofluid natural convection within a porous annulus considering diverse configurations of heater, *Journal of Thermal Analysis and Calorimetry*, 141, 2020, 2109-2125.
[31] Manjunatha, S., Kuttan, B.A., Jayanthi, S., Chamkha A.J., Gireesha, B.J., Heat transfer enhancement in the boundary layer flow of hybrid nanofluids due to variable viscosity and natural convection, *Heliyon*, 5(4), 2019, e01469.
[32] Nihara, K., New design concept of structural ceramics-ceramic nano composites, *Journal of Ceramics Society Japan*, 99(1154), 1991, 974-982.
[33] Baheraei, M., Jamshidmofid M., Goodarz, M., Efficacy of a hybrid nanofluid in a new microchannel heat sink equipped with both secondary channels and ribs, *Journal of Molecular Liquids*, 273, 2019, 88-98.
[34] Baheraei, M., Mazaheri, N., Hossein, S.M., Zamani, H., Efficacy of a new graphene platinum nanofluid in tubes fitted with single and twin twisted tapes regarding counter and co-swirling flows for efficient use of energy, *International Journal of Mechanical Sciences*, 150, 2019, 290-303.
[35] Bahiraei, M., Heshmatian, S., Thermal performance and second law characteristics of two new microchannel heat sinks operated with hybrid nanofluid containing graphene-silver nanoparticles, *Energy Conversations and Management*, 168, 2018, 357-370.
[36] Bahiraei, M., Mazaheri, N., Application of a novel hybrid nanofluid containing graphene-platinum nanoparticles in a chaotic twisted geometry for utilization in miniature devices: thermal and energy efficacy considerations, *International Journal of Mechanical Sciences*, 138, 2018, 337-349.
[37] Han, B., Li, Z., Zhang, L., Zeng, S., Yu, X., Ou, J., Reactive powder concrete reinforced with nano SiO2-coated TiO2, *Construction and Building Materials*, 148, 2017, 104-112.
[38] Kuttan, B.A., Manjunatha, S., Jayanthy, S., Gireesha B.J., Archana, M., Effect of variable viscosity on Marangoni convective boundary layer flow of nanofluid in the presence of mixed convection, *Journal of Nanofluids*, 8(4), 2019.
[39] Manjunatha, S., Gireesha, B.J., Effects of variable viscosity and thermal conductivity on MHD flow and heat transfer of a dusty fluid, *Ain Shams Engineering Journal*, 7(1), 2016, 505-515.
[40] Puneeth, V., Manjunatha, S., Makinde, O. D., Gireesha, B. J., Bioconvection of a radiating hybrid nanofluid past a thin needle in the presence of heterogeneous–homogeneous chemical reaction, *Journal of Heat Transfer*, 2021, 143(4), 042502.
[41] Hamza, M.H., Sidik, N.A.C., Ken, T.L., Mamat, R., Najafi, G., Factors affecting the performance of hybrid nanofluids: a comprehensive review, *International Journal of Heat and Mass Transfer*, 115, 2017, 630-646.
[42] Khan, W.A., Pop, I., Boundary layer flow of a nanofluid past a stretching sheet, *International Journal of Heat and Mass transfer*, 53(11-12), 2010, 2477-2483.
[43] Ghalambaz, M., Sheremet, M.A., Mehryan, S.A.M., Kashkooli, F.M., Pop, I., Local thermal non-equilibrium analysis of conjugate free convection within a porous enclosure occupied with Ag–MgO hybrid nanofluid, *Journal of Thermal Analysis and Calorimetry*, 135(2), 2019, 1381-1398.
[44] Gorla, R.S.R., Sidwai, I., Free convection on a vertical surface with suction and blowing, *Applied Scientific Research*, 52(3), 1994, 247-257.

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