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Research Paper

Buckling of Shell Panels Made of Fiberglass and Reinforced with an Orthogonal Grid of Stiffeners

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Abstract. The paper presents an approach to the stress-strain and buckling analysis in fiberglass cylindrical and conical panels reinforced from the concave side with an orthogonal grid of stiffeners. A mathematical model of the Timoshenko (Mindlin–Reissner) type is used. Transverse shears and geometric nonlinearity are taken into account. The stiffeners are introduced in two ways: using the method of refined discrete introduction and the method of structural anisotropy. We use a computational algorithm based on the Ritz method and the best parameter continuation method. We also provide buckling load values and make a comparison between two types of approaches to account for stiffeners, which shows good convergence.

Keywords: Shells, cylindrical panels, conical panels, buckling, Ritz method.

1. Introduction

Studying the process of nonlinear deformations in structural members under external loading is a highly relevant task since it makes it possible to identify critical operating conditions and, therefore, avoid buckling [1]. Such studies are especially important in the case of structures made of orthotropic and composite materials considering that many patterns of their deformation have not been studied sufficiently.

Structures reinforced with a grid of stiffeners are of particular interest since with such a reinforcement, it is possible to improve the operating life of the structure and increase possible critical loads. For instance, the authors of paper [2] determined the global buckling load for various versions of grid stiffened cylindrical shells with pin support along the contour and fixed support.

McElman, Mikulas, and Stein [3] considered the effect of stiffening on vibration and flutter. Lee and Kim [4], Zhao et al. [5], and Talebitooti et al. [6] also considered vibration in their studies. In paper [7], the authors analyzed the free vibration and buckling response of stiffened panels under general loading.

Reviews on stiffened shells can be found in some prominent papers [8–13].

The purpose of this study was to describe a method of buckling analysis for cylindrical and conical panels reinforced with an orthogonal grid of stiffeners with account for material orthotropy.

2. Theory and Methods

2.1 Mathematical Model

Let us examine thin-walled shells under external mechanical loading. The geometry of these structures is defined by the Lame parameters A, B and radii of principal curvatures R_1, R_2 along the x, y coordinates, respectively. We set $A = 1, B = R_2, R_1 = \infty, R_2 = const$ for cylindrical panels and $A = 1, B = x\sin\theta, R_1 = \infty, R_2 = x\tan\theta$ for conical panels.

Let us use a mathematical model of the Timoshenko (Mindlin–Reissner) type, which takes into account transverse shears, material orthotropy, and geometric nonlinearity. According to this model, in the case of static problems, three functions characterizing displacement of the coordinate surface points $U(x,y), V(x,y), W(x,y)$ and two functions characterizing the normal rotation angles in the planes xOz, yOz ($\Psi_x(x,y), \Psi_y(x,y)$) will be the unknown functions. This model is based on the functional of total potential deformation energy, which can be written in the following form:

$$E_s = E_s^0 + E_p^R,$$

$$E_s^0 = \frac{1}{2}\int_{a_1}^{a}\int_{0}^{b}\Big(N_x^0\varepsilon_x + N_y^0\varepsilon_y + \frac{1}{2}\big(N_{xy}^0 + N_{yx}^0\big)\gamma_{xy} + M_x^0\chi_1 + M_y^0\chi_2 +$$
$$+\big(M_{xy}^0 + M_{yx}^0\big)\chi_{12} + Q_x^0\left(\Psi_x - \theta_1\right) + Q_y^0\left(\Psi_y - \theta_2\right) - 2qW - 2P_xU - 2P_yV\Big)ABdxdy, \tag{1}$$

$$E_p^R = \frac{1}{2}\int_{a_1}^{a}\int_{0}^{b}\Big(N_x^R\varepsilon_x + N_y^R\varepsilon_y + \frac{1}{2}\big(N_{xy}^R + N_{yx}^R\big)\gamma_{xy} + M_x^R\chi_1 + M_y^R\chi_2 +$$
$$+\big(M_{xy}^R + M_{yx}^R\big)\chi_{12} + Q_x^R\left(\Psi_x - \theta_1\right) + Q_y^R\left(\Psi_y - \theta_2\right)\Big)ABdxdy,$$

where q, P_x, P_y denote load components; N_x, N_y denote normal forces in the direction of the x, y coordinates; N_{xy}, N_{yx} denote shear forces in the corresponding plane xOy; M_x, M_y denote bending moments; M_{xy}, M_{yx} denote torque moments; Q_x, Q_y denote transverse forces in the planes xOz and yOz, which are defined by the following relationships for the structural skin (superscript 0):

$$N_x^0 = \frac{E_1h}{1-\mu_{12}\mu_{21}}\left(\varepsilon_x + \mu_{21}\varepsilon_y\right),\quad N_y^0 = \frac{E_2h}{1-\mu_{12}\mu_{21}}\left(\varepsilon_y + \mu_{12}\varepsilon_x\right),\quad N_{xy}^0 = N_{yx}^0 = G_{12}h\gamma_{xy},$$
$$M_x^0 = \frac{E_1h^3}{12(1-\mu_{12}\mu_{21})}(\chi_1 + \mu_{21}\chi_2),\quad M_y^0 = \frac{E_2h^3}{12(1-\mu_{12}\mu_{21})}(\chi_2 + \mu_{12}\chi_1), \tag{2}$$
$$M_{xy}^0 = M_{yx}^0 = \frac{G_{12}h^3}{6}\chi_{12},\quad Q_x^0 = G_{13}kh(\Psi_x - \theta_1),\quad Q_y^0 = G_{23}kh\left(\Psi_y - \theta_2\right).$$

Here E_1, E_2 denote elasticity moduli in the directions x, y; $k = 5/6$; G_{12}, G_{13}, G_{23} denote shear moduli in the planes xOy, xOz, yOz, respectively; μ_{12}, μ_{21} denote Poisson's ratios; $\varepsilon_x, \varepsilon_y$ denote tensile strains; γ_{xy} denotes shear strains in the plane xOy; $\chi_1, \chi_2, \chi_{12}$ denote functions of change in curvature and torsion as follows:

$$\varepsilon_x = \frac{1}{A}\frac{\partial U}{\partial x} + \frac{1}{AB}V\frac{\partial A}{\partial y} - k_xW + \frac{1}{2}\theta_1^2,\quad \varepsilon_y = \frac{1}{B}\frac{\partial V}{\partial y} + \frac{1}{AB}U\frac{\partial B}{\partial x} - k_yW + \frac{1}{2}\theta_2^2,$$
$$\gamma_{xy} = \frac{1}{A}\frac{\partial V}{\partial x} + \frac{1}{B}\frac{\partial U}{\partial y} - \frac{1}{AB}U\frac{\partial A}{\partial y} - \frac{1}{AB}V\frac{\partial B}{\partial x} + \theta_1\theta_2,$$
$$\theta_1 = -\left(\frac{1}{A}\frac{\partial W}{\partial x} + k_xU\right),\quad \theta_2 = -\left(\frac{1}{B}\frac{\partial W}{\partial y} + k_yV\right),\quad k_x = \frac{1}{R_1},\quad k_y = \frac{1}{R_2}, \tag{3}$$
$$\chi_1 = \frac{1}{A}\frac{\partial \Psi_x}{\partial x} + \frac{1}{AB}\frac{\partial A}{\partial y}\Psi_y,\quad \chi_2 = \frac{1}{B}\frac{\partial \Psi_y}{\partial y} + \frac{1}{AB}\frac{\partial B}{\partial x}\Psi_x,$$
$$\chi_{12} = \frac{1}{2}\left(\frac{1}{A}\frac{\partial \Psi_y}{\partial x} + \frac{1}{B}\frac{\partial \Psi_x}{\partial y} - \frac{1}{AB}\frac{\partial B}{\partial x}\Psi_y - \frac{1}{AB}\frac{\partial A}{\partial y}\Psi_x\right).$$

Let us use two methods to account for stiffeners: introduction of stiffeners when "smearing" the stiffness using the method of structural anisotropy (Karpov V. V.), and refined discrete introduction of stiffeners in contact with the skin along a strip (Semenov A.A.). We will not present the corresponding relationships due to their cumbersomeness. They are provided in full in paper [14].

2.2 Algorithm for the Solution of Buckling Problems

To solve a buckling analysis problem in a shell structure, we need to find the minimum of the functional (1). For that purpose, let us apply the Ritz method to reduce the variational problem of finding the functional minimum to solving a system of nonlinear algebraic equations. To solve it, we use the best parameter continuation method, which makes it possible to reduce the solution of a nonlinear system to the solution of the initial problem for a system of ordinary differential equations. It is proposed to take the length of the arc of the solution set curve λ as the best parameter [15]. The resulting initial problem can be solved using various methods, e.g. the Euler method.

In this case, the Maple analytical software package is the best option for software implementation, since fairly intensive symbolic computations are required.

3. Numerical Results

Let us perform calculations for two options of panels – cylindrical and conical (Table 1, Fig. 1) – based on the proposed model and algorithm. Type of support: pin support; loading: uniformly distributed static loading directed along the normal to the surface. Material: T-10/UPE22-27 fiberglass [16] with $E_1 = 0.294\cdot10^5$MPa, $E_2 = 1.78\cdot10^4$MPa, $G_{12} = G_{13} = G_{23} = 0.301\cdot10^4$MPa, $\mu_{12} = 0.123$. Fiber orientation 1 in the orthotropic material coincides with the direction of the coordinate y (i.e., fiber angle $\phi = 90°$ relative to the initial position). Let us perform calculations at $N = 16$ summands under the approximation of the Ritz method.

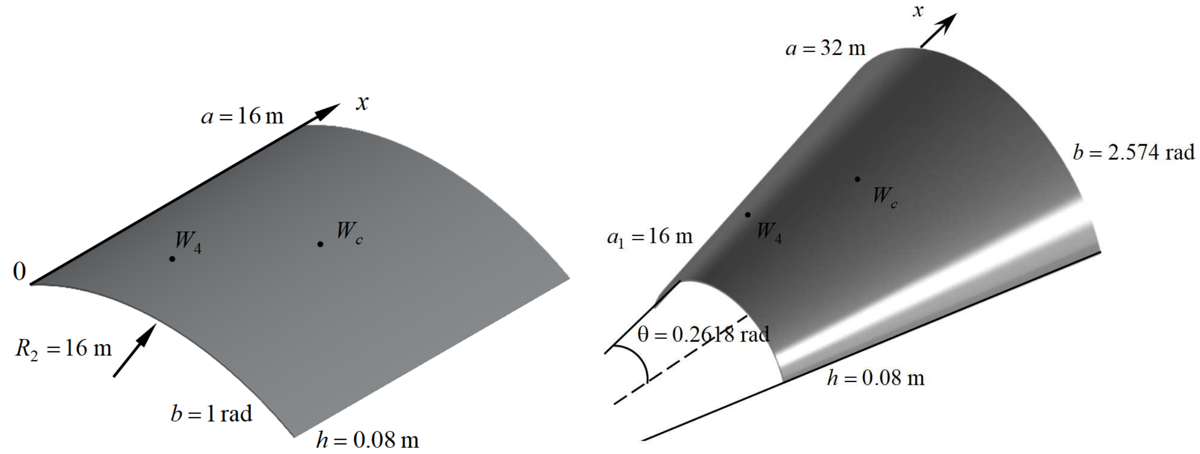

Fig. 1. Skin of structures under consideration.

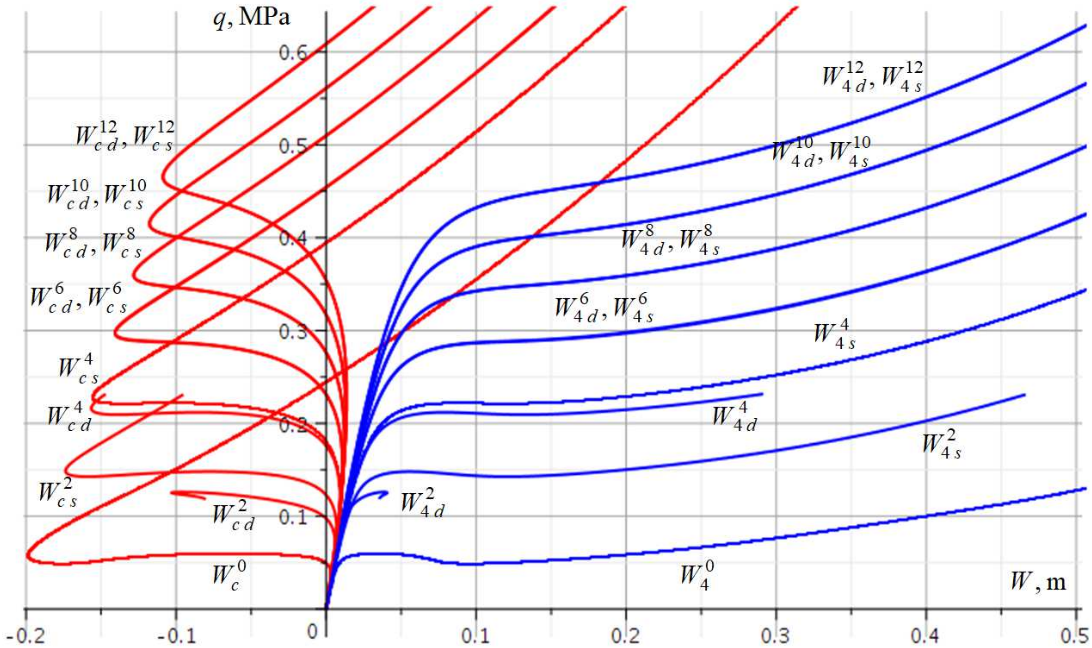

Fig. 2. Load – deflection dependence for a cylindrical panel (Option 1).

Table 1. Options of panels under consideration

No.	h, m	a, m	b, rad	R_1, m	R_2, m	a_1, m	θ, rad
Cylindrical panels							
1	0.08	16	1	∞	16	–	–
Conical panels							
2	0.08	32	2.574	–	–	16	0.2618

Table 2. Buckling loads q_{cr} for fiberglass panels, MPa

No.	Method	0x0	2x2	4x4	6x6	8x8	10x10	12x12
1	Refined discrete method	0,0598	0,1260	0,2118	–	–	–	–
	Structural anisotropy method		0,1480	0,2219	–	–	–	–
2	Refined discrete method	0,2627	0,7732	1,0648	1,2440	1,4099	1,5545	1,6870
	Structural anisotropy method		0,7999	1,0608	1,2478	1,4095	1,5546	1,6872

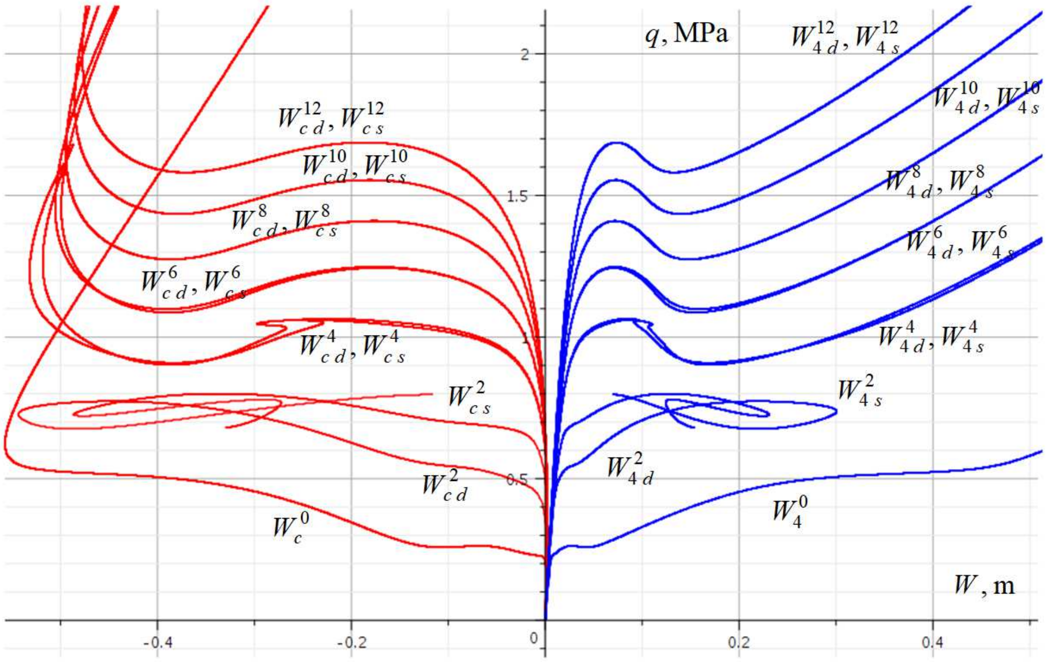

Fig. 3. Load – deflection dependence for a conical panel (Option 2).

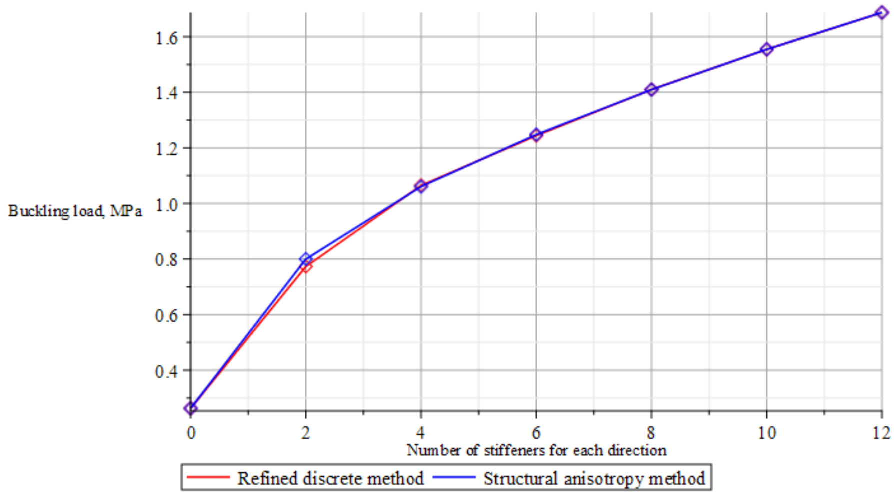

Fig. 4. Buckling load q_{cr} values at different numbers of stiffeners (Option 2).

The orthogonal grid of stiffeners is placed on the inside of the skin. The height and width of stiffeners are $h^i = h^j = 3h,\ r_i = r_j = 2h$, respectively. The number of stiffeners is equal in both directions, and for each new grid option, it is increased by 2 or 4. Stresses will be calculated with regard to the outer surface of the skin.

Table 2 shows buckling load values for different reinforcement options, obtained using different methods to account for stiffeners. The data presented show that in the case of structure option 1, no buckling is observed starting from a particular number of stiffeners. Figs. 2, 3 and 4 present the obtained data in graphic form. Here and further, the red curve W_c in the diagrams depicts the deflection in the center of a structure $(x = (a_1 + a)/2, y = b/2)$, and the blue curve W_4 depicts the deflection in the quadrant of a structure $(x = a_1 + (a - a_1)/4, y = b/4)$.

The obtained values show that, for these structures, the structural anisotropy method converges to the solution resulting from the discrete introduction of stiffeners, starting from an 8×8 grid (the difference in values is less than 1%).

4. Conclusion

In the current study, we performed the strength analysis in thin-walled cylindrical shells reinforced with stiffeners. We also analyzed approaches to introducing stiffeners. The analysis showed that the proposed refined method to account for stiffness properties provides a result close to that obtained with the structural anisotropy method. Besides, we demonstrated that the structural anisotropy method converges as the number of stiffeners increases. Its application instead of discrete approaches significantly reduces computation time.

Author Contributions

The manuscript was written through the contribution of one author. Author discussed the results, reviewed, and approved the final version of the manuscript.

Conflict of Interest

The author declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Nomenclature

A, B	Lame parameters describing the shell geometry;	N	Number of terms in the expansion;
E_1, E_2	Elastic moduli;	q, P_x, P_y	Load components;
E_s	Functional of full potential deformation energy of the shell structure;	q_{cr}	Critical buckling load;
E_s^0	Functional of full potential deformation energy of the skin;	Q_x, Q_y	Transverse forces in the planes xOz and yOz
$f(z)$	Function describing the distribution of stresses τ_{xz} and τ_{yz} through the shell thickness;	U, V, W	Displacement functions;
G_{12}, G_{13}, G_{23}	Shear modules;	W_c	Deflection in the center of a structure;
k_x, k_y	Main curvatures of the shell along the x and y axes;	W_4	Deflection in the quadrant of a structure;
M_x, M_y, M_{xy}, M_{yx}	Moments, occurring in the structure;	γ_{xy}	Shear deformation in the xOy plane;
$M_x^0, M_y^0, M_{xy}^0, M_{yx}^0$	Moments, occurring in the skin;	$\bar{\delta}(x - x_j)$, $\bar{\delta}(y - y_i)$	Unit column functions equal to 1 at points where the stiffeners are located or 0 outside those locations;
$M_x^R, M_y^R, M_{xy}^R, M_{yx}^R$	Forces and moments, occurring in the stiffeners;	$\chi_1, \chi_2, \chi_{12}$	Functions of curvature and torsional change;
N_x, N_y, N_{xy}, N_{yx}	Forces, occurring in the structure;	Ψ_x, Ψ_y	Functions of the normal rotation angles in the xOz and yOz planes, respectively;
$N_x^0, N_y^0, N_{xy}^0, N_{yx}^0$	Forces, occurring in the skin;	$\varepsilon_x, \varepsilon_y$	Deformations of elongation along the x, y coordinates of the middle surface;
$N_x^R, N_y^R, N_{xy}^R, N_{yx}^R$	Forces and moments, occurring in the stiffeners;	μ_{12}, μ_{21}	Poisson's ratios.

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