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Research Paper

Natural Magneto-velocity Coordinate System for Satellite Attitude Stabilization: The Concept and Kinematic Analysis

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Abstract. An artificial Earth satellite with an electric charge and an intrinsic magnetic moment is considered. Due to the geomagnetic field, the satellite experiences the influence of the Lorentz and magnetic torques. To set the angular position of the satellite, we introduce natural coordinate system associated with the directions of geomagnetic induction vector and Lorentz force vector which is orthogonal both to the geomagnetic induction and relative velocity of the satellite. It is shown that such a natural magneto-velocity coordinate system is convenient for attitude stabilization of a satellite operating in the mode of scanning the Earth's surface. The properties of the trajectory of the satellite axis on the Earth's surface are analysed. The rotation tensor connecting the natural magneto-velocity and the orbital coordinate systems is obtained. The angular velocity of the natural magneto-velocity trihedron is found. Kinematic differential equations for the unit vectors of the natural magneto-velocity coordinate system are derived.

Keywords: artificial Earth satellite, geomagnetic field, electric charge, intrinsic magnetic moment, attitude control.

1. Introduction

We consider an artificial Earth satellite whose mass centre C moves along a circular Keplerian orbit in near-Earth space. The satellite bears electric charge $Q = \int_V \sigma dV$ distributed over the volume V with density σ and possesses an intrinsic magnetic moment $\vec{I}$. The center of charge is at the point O defined by vector $\overrightarrow{CO} = \vec{\rho}_0 = Q^{-1}\int_V \sigma\vec{\rho}\, dV$. Discarding the gradient of the geomagnetic field in the satellite volume, we assume that the magnetic induction $\vec{B}$ is the same in points C and O. Let $\vec{v}$ be the velocity of the point C in Greenwich coordinate system, rotating with angular velocity ω_E. Denote $\vec{P} = Q\vec{\rho}_0$ and $\vec{T} = \vec{v}\times\vec{B}$. In the process of moving in the geomagnetic field, the satellite experiences the influence of the Lorentz [1] and magnetic [2-4] torques:

$$\vec{M}_L = \vec{P}\times\vec{T}, \quad \vec{M}_M = \vec{I}\times\vec{B}. \tag{1}$$

These torques are in the basis of electrodynamic attitude control system, which has been investigated since 2006 [5]. The hybrid parametric control system, which refers to the simultaneous use of Lorentz and magnetic torques (1) for satellite attitude control, occurred promising in various applications [6-9]. Moreover, the use of magneto-electro-mechanical coupling has shown itself to be successful in other topical problems of mechanics, for example, in smart nanostructures [10], in energy harvesting systems [11], in micro/nano-electro-mechanical systems [12], in induction motor drive control systems [13], in hybrid wind-diesel photovoltaic generating complexes [14], in complex wind power generators [15].

2. Coordinate Systems

In what follows we use the right-hand orthogonal Cartesian coordinate systems. The system $Cxyz$ of principal central axes of inertia with unit vectors $\vec{i},\vec{j},\vec{k}$. Orbital coordinate system [2] $C\xi\eta\zeta$ whose axis $C\xi$ (unit vector $\vec{\xi}_0$) is along the tangent to the orbit in direction of motion, axis $C\eta$ (unit vector $\vec{\eta}_0$) is orthogonal to the orbital plane, axis $C\zeta$ (unit vector $\vec{\zeta}_0$) is along the radius-vector $\vec{R} = \overrightarrow{O_E C} = R\vec{\zeta}_0$ of the satellite mass center with respect to the Earth center O_E. The angular velocity of the orbital coordinate system with respect to the König coordinate system [16] is $\vec{\omega}_0 = \omega_0\vec{\eta}_0$. Introducing the orbital inclination i and the angle $u = \omega_0 t$ (argument of latitude), we have

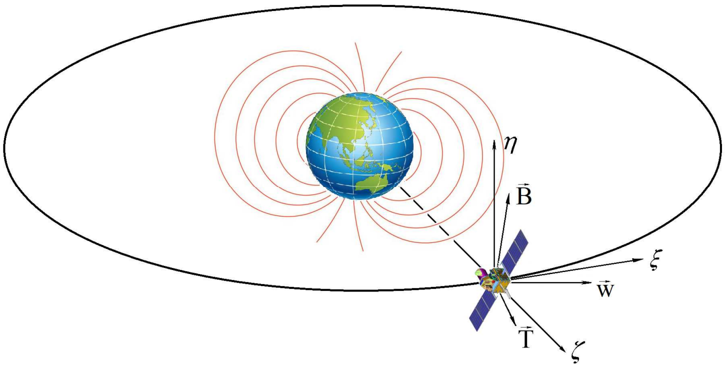

Fig. 1. Magneto-velocity and orbital coordinate systems.

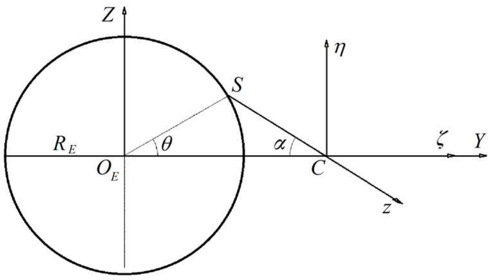

Fig. 2. Finding the trace of the point S on the Earth's surface. The case of equatorial orbit.

$$\vec{\upsilon} = R(\omega_0 - \omega_E \cos i)\vec{\xi}_0 + R\omega_E \sin i \cos u\, \vec{\eta}_0 \,. \tag{2}$$

We introduce the magneto- velocity coordinate system (Fig. 1) as the orthogonal Cartesian coordinate system with unit vectors

$$\vec{w} = \frac{\vec{B}\times\vec{T}}{|\vec{B}|\cdot|\vec{T}|}, \quad \vec{b} = \frac{\vec{B}}{|\vec{B}|}, \quad \vec{l} = \frac{\vec{T}}{|\vec{T}|} \,. \tag{3}$$

Note that in the case of circular equatorial orbit and the simple “straight magnetic dipole” model of the geomagnetic field [2], $\vec{\upsilon} = \upsilon_\xi \vec{\xi}_0$, $\vec{B} = B_\eta \vec{\eta}_0$, $\vec{T} = \upsilon_\xi B_\eta \vec{\zeta}_0$ and trihedron $\vec{w},\vec{b},\vec{l}$ coincides with the orbital trihedron $\vec{\xi}_0,\vec{\eta}_0,\vec{\zeta}_0$. Hence, in this case, the satellite stabilized in the magneto-velocity coordinate system in direct position when the orths $\vec{i},\vec{j},\vec{k}$ coincide with the orths $\vec{w},\vec{b},\vec{l}$, moves in such a way when the axis Cz is along the local vertical and the path of the axis Cz on the Earth’s surface (point S) is the equatorial line.

3. The Path of the Satellite Axis on the Earth’s Surface

In fact, due to the difference of the real geomagnetic field [17-19] from the field of the “straight magnetic dipole”, the trace of the point S (Fig. 2) will differ from the equatorial line. To clarify the trajectory of the point S on the Earth’s surface (sphere of radius R_E in Fig. 2), let us first consider the case of a satellite in an equatorial orbit.

We introduce the auxiliary coordinate system $O_E XYZ$ whose axes are not associated with the Earth. The $O_E Y$ axis is directed along $O_E C = \vec{R}$, and the $O_E Z$ axis is orthogonal to the satellite’s orbit plane. It can be seen from Fig. 2 that the coordinates Y_S and Z_S of the point S satisfy the system of equations

$$X_s^2 + Y_s^2 = R_s^2, \quad Y_s = X_s \tan\theta, \quad Y_s = (R - X_s)\tan\alpha \,. \tag{4}$$

where the angle θ is positive if vector $\vec{T}$ forms an obtuse angle with axis $C\eta$ and negative if vector $\vec{T}$ forms an acute angle with axis $C\eta$. System (4) makes it possible to find the following dependence of $\tan\theta$ on $\tan\alpha$, where $R_* = R / R_E$:

$$\tan\theta = \frac{\tan\alpha}{1 - R_*^2 \tan^2\alpha}\left(-1 + \sqrt{1 - (1 - R_*^2 \tan^2\alpha)(1 - R_*^2)}\right). \tag{5}$$

Since for a satellite with known orbital parameters the vectors $\vec{\upsilon}$ and $\vec{B}$ are known functions of time or, which is the same, a dimensionless variable u, the vector $\vec{T} = \vec{\upsilon}\times\vec{B}$ is also known function $\vec{T}(u)$. Moreover, since

$$\cos\alpha = T_\zeta / |\vec{T}|, \quad \tan^2\alpha = \cos^{-2}\alpha - 1 = (T_\xi^2 + T_\eta^2)/T_\zeta^2 \,, \tag{6}$$

then $\tan\alpha$ is also known function of time. Therefore, the formula (5) allows calculating $\theta(u)$ for the known orbital radius R and the accepted model of the geomagnetic field. Figure 3 shows the dependence of $\theta(u)$ for a satellite stabilized in the magneto-velocity coordinate system and moving in a circular equatorial orbit with a radius of $R = 7000$ km under the conditions of the geomagnetic field calculated taking into account the first five multipole components [18].

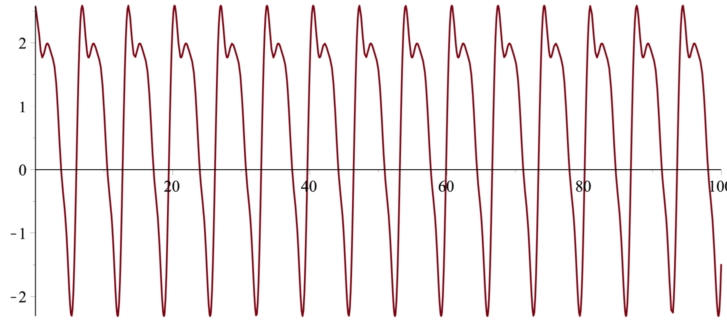

Fig. 3. $\theta(u)$ (degrees). The case of equatorial orbit.

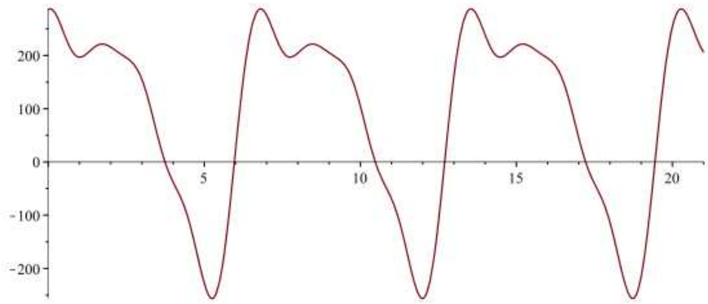

Fig. 4. $Z_S(u)$ (km). The case of equatorial orbit.

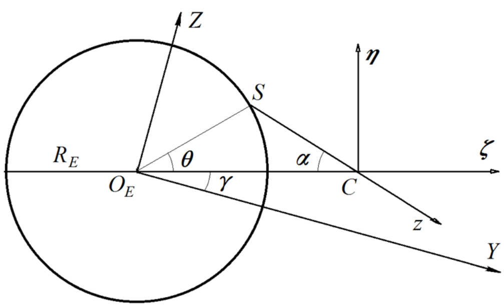

Fig. 5. Angles γ , α and θ .

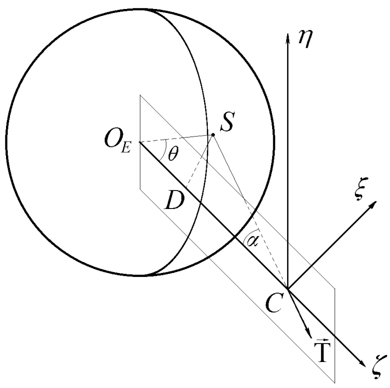

Fig. 6. Planes $C\eta\zeta$ and $O_E CS$.

It can be seen that in the process of satellite motion, the angle $\theta(u)$ changes periodically in the range from -2.5^0 to 2.5^0 . The trajectory of the point S lies in the strip, the width of which is determined by the value of Z_S (Fig. 2).

The system (3) allows us to find the following dependence of Z_S on $\tan\alpha$:

$$Z_S = \cos^2\alpha\left(R - \sqrt{R^2 - (1+\tan^2\alpha)(R^2 - R_E^2)}\right)\tan\alpha \ . \tag{7}$$

Figure 4 shows the values of $Z_S(u)$ calculated on the basis of the formula (7) for the above parameters of the satellite orbit and the geomagnetic field model. It can be seen that the width of the scanned strip is about 500 km. The middle line of the scanned strip coincides with the equator line. However, the trajectory of the point S is not symmetrical about the median line: in the northern hemisphere, the point S stays longer than in the southern.

Let us now proceed to consider the orbits of arbitrary inclination. Let us introduce into consideration the auxiliary coordinate system $O_E XYZ$ with the origin at the center of the Earth, but not associated with the Earth (Fig. 5).

We direct the $O_E X$ axis to the ascending node of the orbit, the $O_E Z$ axis - along the axis of the Earth's daily rotation, and the $O_E Y$ axis - in the equatorial plane so that the $O_E XYZ$ coordinate system is a right-hand Cartesian orthogonal coordinate system. Based on the fact that the latitude argument u is measured from the ascending node of the orbit, the angle γ between the equatorial plane (X, Y) of the Earth and the radius vector $\vec{R} = \overrightarrow{O_E C}$ will be determined by the formula $\gamma = \pi/2 - \arccos(\sin i \sin u)$. For non-equatorial orbits ($i \neq 0$), the angles γ and $\theta(u)$ do not lie in the same plane. An explanatory drawing is shown in Fig. 6.

It is also seen from Fig. 6 that the equalities $\sin\theta = |SD|/R_E$, $\tan\alpha = |SD|/|DC|$, $|DC| = R - R_E\cos\theta$ hold, which allow us to establish the relation $\sin\theta = (R/R_E - \cos\theta)\tan\alpha$, whence, in its turn, introducing the notation $R_* = R/R_E$, we find

$$\cos\theta = R_* \sin^2\alpha + \sqrt{\cos^2\alpha(1 - R_*^2\sin^2\alpha)} \ . \tag{8}$$

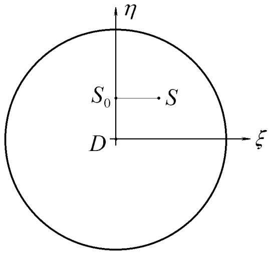

Fig. 7. View from the end of axis $C\zeta$.

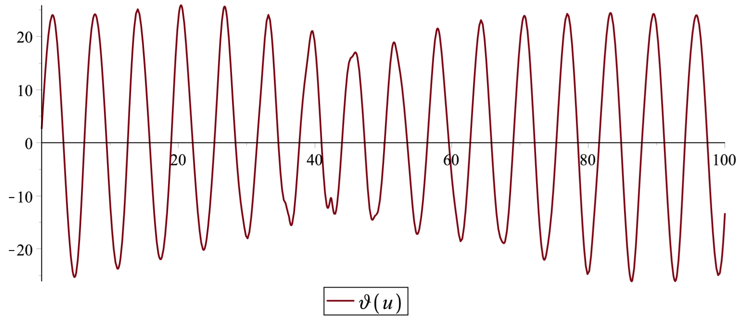

Fig. 8. $\theta(u)$ (degrees) for the orbit with inclination $i = 30^0$.

To find the angular width of the scanning strip, measured along an arc lying in the plane $C\eta\zeta$, one need to go from the found angle θ to the angle θ_0 lying in the plane $C\eta\zeta$. To do this, we pass from the segment $|DS|$ to its projection $|DS_0|$ onto the plane $C\eta\zeta$ (Fig. 7). Since the line (DS) is collinear to the plane $C\xi\eta$, then $|DS_0|$ is equal to the projection of $|CS|$ onto the $C\eta$ axis, i.e. $|DS_0| = |CS|(-\cos(\vec{T},\vec{\eta}_0)) = |CS|T_\eta / |\vec{T}|$.

Considering that $|CS|\sin\alpha = R_E \sin\theta$, we get $|DS_0| = R_E(-T_\eta)\sin\theta / (|\vec{T}|\sin\alpha)$. Then we can find $\sin\theta_0 = |DS_0| / R_E = -T_\eta \sin\theta / (|\vec{T}|\sin\alpha)$, and finally find the angular width of the scan swath by adding the angles γ and θ_0 lying in the same plane $C\eta\zeta$:

$$\vartheta = \gamma + \theta_0 = \pi / 2 - \arccos(\sin i \sin u) - \arcsin\left(\frac{T_\eta \sin\theta}{|\vec{T}|\sin\alpha}\right). \tag{9}$$

For a satellite moving in an orbit with low inclinations i in the regime of rotational motion stabilized in the magneto-velocity coordinate system, the $C\zeta$ axis will describe in the orbital coordinate system a trajectory close to a segment lying in the plane $C\eta\zeta$ and intersecting in its middle with the orbital plane $C\xi\zeta$. On the surface of the Earth, such a trajectory describes a scanning curve lying in a strip, the middle line of which is the trace of the intersection of the $C\zeta$ axis with the rotating surface of the Earth. To construct this strip, one can use the formulas (6), (8), (9).

As an example, Fig. 8 shows the dependence of $\vartheta(u)$ for a satellite stabilized in the magneto-velocity coordinate system and moving in a circular orbit with a radius of $R = 7000$ km and an inclination of $i = 30^0$ in the geomagnetic field, calculated taking into account the first five multipole components. It can be seen that in the process of satellite motion, the angle $\vartheta(u)$ changes almost periodically, and its values lie in the range from -25^0 to 25^0 .

4. Kinematic Differential Equations

Let us determine the orientation of the $Cxyz$ axes relative to the $C\xi\eta\zeta$ axes using the matrix **A** of the direction cosines $\alpha_i, \beta_i, \gamma_i$ (i = 1,2,3) according to equalities

$$\vec{\xi}_0 = \alpha_1 \vec{i} + \alpha_2 \vec{j} + \alpha_3 \vec{k}, \quad \vec{\eta}_0 = \beta_1 \vec{i} + \beta_2 \vec{j} + \beta_3 \vec{k}, \quad \vec{\zeta}_0 = \gamma_1 \vec{i} + \gamma_2 \vec{j} + \gamma_3 \vec{k}.$$

In addition, we introduce into consideration the orthogonal matrix **B** = (b_{ij}) which determines the orientation of the orbital coordinate system with respect to the magneto-velocity coordinate system so that the equalities

$$\vec{w} = b_{11}\vec{\xi}_0 + b_{12}\vec{\eta}_0 + b_{13}\vec{\zeta}_0, \quad \vec{b} = b_{21}\vec{\xi}_0 + b_{22}\vec{\eta}_0 + b_{23}\vec{\zeta}_0, \quad \vec{l} = b_{31}\vec{\xi}_0 + b_{32}\vec{\eta}_0 + b_{33}\vec{\zeta}_0 .$$

hold true. For a satellite with given orbital parameters, the elements of the matrix **B** can be found explicitly. For this, we use the known expansions of the vectors $\vec{B}$ and $\vec{v}$ in terms of the orbital coordinates. We have the equalities

$$\vec{w} = [(B_\eta T_\zeta - B_\zeta T_\eta)\vec{\xi}_0 + (B_\zeta T_\xi - B_\xi T_\zeta)\vec{\eta}_0 + (B_\xi T_\eta - B_\eta T_\xi)\vec{\zeta}_0] / (|\vec{B}||\vec{T}|),$$

$$\vec{b} = (B_\xi \vec{\xi}_0 + B_\eta \vec{\eta}_0 + B_\zeta \vec{\zeta}_0) / |\vec{B}|, \quad \vec{l} = (T_\xi \vec{\xi}_0 + T_\eta \vec{\eta}_0 + T_\zeta \vec{\zeta}_0) / |\vec{T}|,$$

from which we obtain the elements (b_{ij}) in the form of known functions of time:

$$b_{11} = (B_\eta T_\zeta - B_\zeta T_\eta)/(|\vec{B}||\vec{T}|),\quad b_{12} = (B_\zeta T_\xi - B_\xi T_\zeta)/(|\vec{B}||\vec{T}|),\quad b_{13} = (B_\xi T_\eta - B_\eta T_\xi)/(|\vec{B}||\vec{T}|),$$
$$b_{21} = B_\xi/|\vec{B}|,\quad b_{22} = B_\eta/|\vec{B}|,\quad b_{23} = B_\zeta/|\vec{B}|,$$
$$b_{31} = T_\xi/|\vec{T}|,\quad b_{32} = T_\eta/|\vec{T}|,\quad b_{33} = T_\zeta/|\vec{T}|.$$

To set the orientation of the satellite in the magneto-velocity coordinate system, taken as the base one, we introduce into consideration the orthogonal matrix $\mathbf{N} = (n_{ij}) = \mathbf{BA}$. The satellite program orientation in the basic coordinate system is a direct equilibrium position corresponding to the unit matrix $\mathbf{N} = \mathbf{E}$. In the program orientation of the satellite, the unit vectors $\vec{i},\vec{j},\vec{k}$ of the main central axes of inertia of the satellite coincide, respectively, with the unit vectors $\vec{w},\vec{b},\vec{l}$ of the base coordinate system. Let $\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3$ be the same unit vectors $\vec{w},\vec{b},\vec{l}$ of the base coordinate system, but expressed in basis $\vec{i},\vec{j},\vec{k}$ of the main central axes of inertia of the satellite:

$$\vec{\sigma}_1 = \mathbf{N}^{\mathrm{T}}\vec{w} = (n_{11},n_{12},n_{13})^{\mathrm{T}},\quad \vec{\sigma}_2 = \mathbf{N}^{\mathrm{T}}\vec{b} = (n_{21},n_{22},n_{23})^{\mathrm{T}},\quad \vec{\sigma}_3 = \mathbf{N}^{\mathrm{T}}\vec{t} = (n_{31},n_{32},n_{33})^{\mathrm{T}}.$$

Then the program orientation of the satellite is given by the equalities

$$\vec{\sigma}_1 = \vec{\rho}_1 = (1,0,0)^{\mathrm{T}},\quad \vec{\sigma}_2 = \vec{\rho}_2 = (0,1,0)^{\mathrm{T}},\quad \vec{\sigma}_3 = \vec{\rho}_3 = (0,0,1)^{\mathrm{T}}.$$

The unit vectors of the orbital coordinate system expressed in the basis $\vec{i},\vec{j},\vec{k}$ and denoted below $\vec{s}_1,\vec{s}_2,\vec{s}_3$, take the form

$$\vec{s}_1 = \mathbf{A}^{\mathrm{T}}\vec{\xi}_0 = (\alpha_1,\alpha_2,\alpha_3)^{\mathrm{T}},\quad \vec{s}_2 = \mathbf{A}^{\mathrm{T}}\vec{\eta}_0 = (\beta_1,\beta_2,\beta_3)^{\mathrm{T}},\quad \vec{s}_3 = \mathbf{A}^{\mathrm{T}}\vec{\zeta}_0 = (\gamma_1,\gamma_2,\gamma_3)^{\mathrm{T}}.$$

Taking into account the equality $\mathbf{A}^{\mathrm{T}} = \mathbf{N}^{\mathrm{T}}\mathbf{B}$, we rewrite $\vec{s}_1,\vec{s}_2,\vec{s}_3$ as

$$\vec{s}_1 = b_{11}\vec{\sigma}_1 + b_{21}\vec{\sigma}_2 + b_{31}\vec{\sigma}_3,\quad \vec{s}_2 = b_{12}\vec{\sigma}_1 + b_{22}\vec{\sigma}_2 + b_{32}\vec{\sigma}_3,\quad \vec{s}_3 = b_{13}\vec{\sigma}_1 + b_{23}\vec{\sigma}_2 + b_{33}\vec{\sigma}_3. \tag{10}$$

Let us introduce the unit vectors

$$\vec{r}_1 = b_{11}\vec{\rho}_1 + b_{21}\vec{\rho}_2 + b_{31}\vec{\rho}_3,\quad \vec{r}_2 = b_{12}\vec{\rho}_1 + b_{22}\vec{\rho}_2 + b_{32}\vec{\rho}_3,\quad \vec{r}_3 = b_{13}\vec{\rho}_1 + b_{23}\vec{\rho}_2 + b_{33}\vec{\rho}_3.$$

Then the program orientation of the satellite can be represented in an equivalent way in the form

$$\vec{s}_1 = \vec{r}_1 = (b_{11},b_{21},b_{31})^{\mathrm{T}},\quad \vec{s}_2 = \vec{r}_2 = (b_{12},b_{22},b_{32})^{\mathrm{T}},\quad \vec{s}_3 = \vec{r}_3 = (b_{13},b_{23},b_{33})^{\mathrm{T}}.$$

Let us calculate the time derivatives of the unit vectors $\vec{s}_i$ ($i = 1,2,3$) in the König coordinate system. On the one hand, by the transition theorem, we have the equalities

$$\dot{\vec{s}}_i = \left(\dot{\vec{s}}_i\right)_{xyz} + \vec{\omega}\times\vec{s}_i. \tag{11}$$

where local derivatives are calculated in the coordinate system $Cxyz$. On the other hand, taking the orbital system as the moving coordinate system, we have the equalities

$$\dot{\vec{s}}_i = \vec{\omega}_0\times\vec{s}_i. \tag{12}$$

Equating the right-hand sides of equations (11) and (12), we obtain the kinematic equations for the unit vectors $\vec{s}_i$ (i=1,2,3)

$$\left(\dot{\vec{s}}_i\right)_{xyz} + \vec{\omega}\times\vec{s}_i = \vec{\omega}_0\times\vec{s}_i. \tag{13}$$

Taking into account the equality $\vec{\omega}_0 = \omega_0\vec{s}_2$, we rewrite the equation (13) as

$$\dot{\vec{s}}_1 + \vec{\omega}\times\vec{s}_1 = -\omega_0\vec{s}_3,\quad \dot{\vec{s}}_2 + \vec{\omega}\times\vec{s}_2 = \vec{0},\quad \dot{\vec{s}}_3 + \vec{\omega}\times\vec{s}_3 = \omega_0\vec{s}_1. \tag{14}$$

The programmed orientation of the satellite in the base coordinate system is at the same time a special mode of rotational motion of the satellite in the König or in the orbital coordinate systems. To establish the relationships between the angular velocities of these motions and obtain kinematic equations, we introduce the necessary notation in addition to those that were introduced earlier. Let $\vec{\omega}$ be the absolute angular velocity of the satellite, $\vec{\omega}'$ is the angular velocity of the satellite relative to the base coordinate system with the unit vectors $\vec{w},\vec{b},\vec{l}$, $\vec{\omega}_0'$ is the angular velocity of the satellite relative to the orbital coordinate system, $\vec{\omega}_b$ - angular velocity of the base coordinate system relative to the König coordinate system, $\vec{\omega}_{b0}$ - angular velocity of the base coordinate system relative to the orbital coordinate system. Since $\vec{\omega} = \vec{\omega}_0 + \vec{\omega}_0'$ and $\vec{\omega}_0' = \vec{\omega}_{b0} + \vec{\omega}'$, we obtain the equality $\vec{\omega} = \vec{\omega}_0 + \vec{\omega}_{b0} + \vec{\omega}'$. It is expedient to express the terms on the right-hand side of this equality in terms of the unit vectors of the same coordinate system and their time derivatives. For the first term we have: $\vec{\omega}_0 = \omega_0\,\vec{\eta}_0$. Consider the expression

$$\vec{w}\times\left(\dot{\vec{w}}\right)_{\xi\eta\zeta} + \vec{b}\times\left(\dot{\vec{b}}\right)_{\xi\eta\zeta} + \vec{l}\times\left(\dot{\vec{l}}\right)_{\xi\eta\zeta}. \tag{15}$$

We use the Poisson kinematic equations for local derivatives of $\vec{w},\vec{b},\vec{t}$, calculated in the orbital coordinate system:

$$\left(\dot{\vec{w}}\right)_{\xi\eta\zeta} = \vec{\omega}_{b0} \times \vec{w}, \quad \left(\dot{\vec{b}}\right)_{\xi\eta\zeta} = \vec{\omega}_{b0} \times \vec{b}, \quad \left(\dot{\vec{l}}\right)_{\xi\eta\zeta} = \vec{\omega}_{b0} \times \vec{l}\,. \tag{16}$$

After substituting (16) into (17) and expanding double vector products, we get equality

$$\vec{w} \times \left(\dot{\vec{w}}\right)_{\xi\eta\zeta} + \vec{b} \times \left(\dot{\vec{b}}\right)_{\xi\eta\zeta} + \vec{l} \times \left(\dot{\vec{l}}\right)_{\xi\eta\zeta} = 2\vec{\omega}_{b0} \tag{17}$$

which, however, is more convenient to represent in terms of vectors specified explicitly in the orbital coordinate system. For this, we represent the unit vectors of the magneto-velocity coordinate system in the basis of the orbital coordinate system and denote $\vec{b}_1, \vec{b}_2, \vec{b}_3$:

$$\vec{b}_1 = \mathbf{B}^{\mathrm{T}}\vec{w} = (b_{11}, b_{12}, b_{13})^{\mathrm{T}}, \quad \vec{b}_2 = \mathbf{B}^{\mathrm{T}}\vec{b} = (b_{21}, b_{22}, b_{23})^{\mathrm{T}}, \quad \vec{b}_3 = \mathbf{B}^{\mathrm{T}}\vec{l} = (b_{31}, b_{32}, b_{33})^{\mathrm{T}}.$$

Taking these notations into account, we rewrite the equality (17) in the form

$$2\vec{\omega}_{b0} = \vec{b}_1 \times \dot{\vec{b}}_1 + \vec{b}_2 \times \dot{\vec{b}}_2 + \vec{b}_3 \times \dot{\vec{b}}_3\,. \tag{18}$$

Here all vectors are specified in the orbital coordinate system and all derivatives are calculated in the same coordinate system: $\vec{b}_i = (b_{1i}, b_{2i}, b_{3i})^{\mathrm{T}}$, $\dot{\vec{b}}_i = (\dot{b}_{1i}, \dot{b}_{2i}, \dot{b}_{3i})^{\mathrm{T}}$ $(i = 1,2,3)$. Based on (10), we represent the expression (18) as the following expansion in terms of vectors $\vec{\sigma}_i$:

$$2\vec{\omega}_{b0} = (\vec{b}_3\,\dot{\vec{b}}_2 - \vec{b}_2\,\dot{\vec{b}}_3)\vec{\sigma}_1 + (\vec{b}_1\,\dot{\vec{b}}_3 - \vec{b}_3\,\dot{\vec{b}}_1)\vec{\sigma}_2 + (\vec{b}_2\,\dot{\vec{b}}_1 - \vec{b}_1\,\dot{\vec{b}}_2)\vec{\sigma}_3\,. \tag{19}$$

The angular velocity $\vec{\omega}_0$ in the same basis has the form $\vec{\omega}_0 = \omega_0(b_{12}\,\vec{\sigma}_1 + b_{22}\,\vec{\sigma}_2 + b_{32}\,\vec{\sigma}_3)$. Therefore, the angular velocity of the magneto-velocity coordinate system $\vec{\omega}_b = \vec{\omega}_0 + \vec{\omega}_{b0}$ can be represented as the following known function of time:

$$\vec{\omega}_b = \frac{1}{2}[(\vec{b}_3\,\dot{\vec{b}}_2 - \vec{b}_2\,\dot{\vec{b}}_3 + 2\omega_0 b_{12})\vec{\sigma}_1 + (\vec{b}_1\,\dot{\vec{b}}_3 - \vec{b}_3\,\dot{\vec{b}}_1 + 2\omega_0 b_{22})\vec{\sigma}_2 + (\vec{b}_2\,\dot{\vec{b}}_1 - \vec{b}_1\,\dot{\vec{b}}_2 + 2\omega_0 b_{32})\vec{\sigma}_3]\,. \tag{20}$$

On the other hand, since $\vec{b}_i = b_{i1}\vec{\xi}_0 + b_{i2}\vec{\eta}_0 + b_{i3}\vec{\zeta}_0$ and $(\dot{\vec{b}}_i)_{\xi\eta\zeta} = \dot{b}_{i1}\,\vec{\xi}_0 + \dot{b}_{i2}\,\vec{\eta}_0 + \dot{b}_{i3}\,\vec{\zeta}_0$ $(i = 1,2,3)$, then the vector $\vec{\omega}_{b0}$ can be represented as

$$\vec{\omega}_{b0} = \omega_{b0\xi}\vec{\xi}_0 + \omega_{b0\eta}\vec{\eta}_0 + \omega_{b0\zeta}\vec{\zeta}_0\,,$$

where $\omega_{b0\xi} = (\vec{r}_2\,\dot{\vec{r}}_3 - \vec{r}_3\,\dot{\vec{r}}_2)/2$, $\omega_{b0\eta} = (\vec{r}_3\,\dot{\vec{r}}_1 - \vec{r}_1\,\dot{\vec{r}}_3)/2$, $\omega_{b0\zeta} = (\vec{r}_1\,\dot{\vec{r}}_2 - \vec{r}_2\,\dot{\vec{r}}_1)/2$, $\vec{r}_i = (b_{1i}, b_{2i}, b_{3i})^{\mathrm{T}}$, $\dot{\vec{r}}_i = (\dot{\vec{b}}_{1i}, \dot{\vec{b}}_{2i}, \dot{\vec{b}}_{3i})^{\mathrm{T}}$ $(i = 1,2,3)$ are known functions of time. As a result, the angular velocity of the magneto-velocity coordinate system $\vec{\omega}_b = \vec{\omega}_0 + \vec{\omega}_{b0}$ can also be represented as the following known function of time:

$$\vec{\omega}_b = \frac{1}{2}[(\vec{r}_2\,\dot{\vec{r}}_3 - \vec{r}_3\,\dot{\vec{r}}_2)\vec{s}_1 + (\vec{r}_3\,\dot{\vec{r}}_1 - \vec{r}_1\,\dot{\vec{r}}_3)\vec{s}_2 + (\vec{r}_1\,\dot{\vec{r}}_2 - \vec{r}_2\,\dot{\vec{r}}_1)\vec{s}_3]\,. \tag{21}$$

In the programmed rotational motion of the satellite

$$\vec{\sigma}_i = \vec{\rho}_i \quad \Leftrightarrow \quad \vec{s}_i = \vec{r}_i = (b_{1i}, b_{2i}, b_{3i})^{\mathrm{T}} \quad (i = 1,2,3), \quad \vec{\omega}' = \vec{0}\,, \tag{22}$$

the angular velocity of the satellite becomes equal to $\vec{\omega}_b$, determined by the formula (20) or (21), the vector $\vec{T}$ takes the value $\vec{T}_0 = |\vec{T}|(b_{31}\vec{r}_1 + b_{32}\vec{r}_2 + b_{33}\vec{r}_3)$, and the vector $\vec{B}$ takes the value $\vec{B}_0 = |\vec{B}|(b_{21}\vec{r}_1 + b_{22}\vec{r}_2 + b_{23}\vec{r}_3)$.

5. Conclusion

An artificial Earth satellite with an electrostatic charge and an intrinsic magnetic moment moving in the geomagnetic field, experiences the influence of the magnetic and Lorentz torques. These torques are in the basis of electrodynamic attitude control system. In the problem of satellite attitude stabilization, the orbital and König coordinate systems are usually used, which are convenient for practical applications, but not related to the specifics of the forces and torques acting on a charged satellite in the geomagnetic field.

In this paper, we introduced into consideration a new - natural magneto-velocity coordinate system (MVCS) associated with the directions of magnetic and Lorentz torques. The main advantage of MVCS in comparison with the orbital and König coordinate systems is that it allows the fully passive variant of attitude stabilization of the satellite with decentralized charge ($\vec{P} = Q\vec{\rho}_0 \neq \vec{0}$) and an intrinsic magnetic moment $\vec{I}$. This results in simplification of attitude control design and seems promising for reduction of on-board energy consumption.

The kinematics of MVCS is analysed. The path of the satellite axis on the surface of the Earth is clarified and built numerically for the geomagnetic field, modelled, taking into account the multipole terms of the 5th order. It is shown that the MVCS is convenient for the implementation of certain modes of scanning the Earth's surface. The kinematic differential equations governing the attitude motion of MVCS trihedron, are derived. These equations are suitable for further use in solving problems of the dynamics of controlled rotational motion of a satellite. Our future goal is the dynamic analysis of a satellite attitude stabilization in MVCS.

Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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