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Research Paper

Effect of the Gravity and Magnetic Field to Find Regular Precessions of a Satellite-gyrostat with Principal Axes on a Circular Orbit

H.M. Yehia, H.N. El-kenani

Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

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Corresponding author: H.N. El-kenani H.N. (hnabih@yahoo.com)

Abstract. We consider the motion of a magnetized satellite-gyrostat in a circular orbit due to the combined influence of uniform gravity and magnetic fields. Based on the Lagrangian equations, the necessary conditions for the existence of regular precessions are determined in which the axis of precession i\s perpendicular to the orbital plane. All possible regular precessions and permanent rotations are determined and classified. We show the usage of Lagrange equations taking Eulerian angles as generalized coordinates for determining the regular precessions is more effective and accurate than utilization of Euler-Poisson equations.

Keywords: Satellite; Gyrostat; Principal axes; Regular precessions; Permanent rotations; Magnetic field.

1. Introduction

The motion of a rigid artificial satellite in a circular orbit has attracted considerable attention over the centuries. In fact, the scope of work on the motion of the satellite in a circular orbit is a very important issue for further progress in space technology. The study of rigid satellites is considered a microcosm of the study of celestial bodies most of history was devoted to the study of motion under the influence of the gravitational field (see e.g., Beletsky et al. [1]) and likins [2] in addition to in some cases in which a rotor is attached to the satellite implying to a gyrostatic moment. For a detailed description of some of these cases, see, e.g. [3-7].

Since astronomy is one of the most important sciences in the field of classical mechanics and is one of the most important applications in it, especially celestial mechanics, there are many different types of motion that are studied in that field such as (translational motion, oscillatory motion, rotational motion, circular motion, and precession motion): Translational motion means a body shifts from one point to another in space (the motion of a bullet fired from a gun). And another kind of motion that is continually repeated in time with a fixed period is called the oscillatory motion (the motion of the pendulum). As for rotational motion, it has had many important studies, as it means the change in orientation of a body, with respect to other bodies in space for example the spinning top. We must distinguish between this motion and the precession motion where the precession motion means the change in the orientation of the rotational axis of a rotating body.

In addition to the above, there is the circular motion, which shows the motion of a body on a circular path with respect to another body fixed in its center of the path like the motion of the Earth about the Sun. One of the most important studies that have resonated in the history of astronomy was the study of precession motion. The slow change in the orientation of the axis of rotation of the Earth is an important example of precession motion called the precession of the equinoxes. Earth's precession became historically called the precession of the equinoxes because the equinoxes moved westward alongside the ecliptic relative to the constant stars, contrary to every year motion of the Sun alongside the ecliptic. Their mixture changed into named general precession, as opposed to the precession of the equinoxes. The Earth's motions are classified into three types: the first, Spin on-axis in 24 hours makes day and night, the second, revolution around the Sun makes the season, and the third, Wobble of the Earth every 26,000 years.

In reality, Hipparchus, an astronomer from the 2nd century BC, is credited with discovering the precession of the equinoxes in the Western world. Axial precession is a gradual, continual movement in the direction of an astronomical body's rotating axis caused by gravity [8]. This is analogous to the precession of a spinning top, with the axis sketching out a pair of cones linked at their apices. Also, precession was introduced by Grioli found that the rigid body in an asymmetric case that moves about a fixed point can make a regular precession, where the body is rotated about a fixed axis in it. This axis is performed precession with the same uniform angular velocity about a non-vertical axis fixed in space [9]. By adding a rotor to a body moving under the influence of a single gravitational field, Kharlamova expanded Grioli's discovery [10].

Yehia examined the regular precession motion of a rigid body in the case of an asymmetric under the influence of two fields and investigated the identical issue with three fields in [11, 12]. Olshanskii obtained in a linear invariant system, three conditions that are sufficient to allow a regular precession for a non-symmetric mechanical system. He found the velocities of precession

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and rotation and studied the particular case of the permanent rotation of an asymmetric rigid shell [13].

The regular precession with the axis of proper rotation for asymmetrical liquid core-filled rigid mantle structures was studied by Olshanskii in 2020 [14]. He analyzed the case when the shape of the core differs little from the spherical one and investigated a regular precession when the axis of proper rotation coincides with the principal axis. Previous research has shown an overview of an asymmetric heavy rigid body rotating around a fixed point. Leimanis [15] investigated the regular precession of a symmetric rigid body around a fixed point in (Lagrange case). Markeev et al. [16] studied the stability of the conical precession of a symmetric rigid body moving in a circular orbit in a central Newtonian gravitational field. Shevchenko et al. [17] showed small-amplitude periodic movements in the vicinity of regular precessions of a dynamically symmetric satellite on a circular orbit in the hyperboloidal scenario. As with the parameters, analytic formulas for regular forms and generating functions are produced based on the system frequencies.

Hughes [18] distinguished between three forms of regular precessions (conical, cylindrical, and hyperboloid). If the center of mass's orbit is circular, the satellite will experience periodic partial stationary movements known as precessions. Markeev [19] investigated the stability of a dynamically symmetric satellite's rotation around the normal of its orbital plane. In an elliptic orbit, he achieved the stability of the satellite's cylindrical precession. In addition, he solved the stability problem of the steady rotational motion of a satellite around its center of mass which moves in a circular orbit in a central Newtonian gravitational field. The stabilization process may be realized depending on the angular velocity of rotation of the satellite and the ratio between the axial and equatorial moments of inertia and a complete solution for the case of the cylindrical precession of satellite is presented [20].

The rotational motion of a symmetric rigid body moving in a circular orbit in a centripetal field was investigated by Maciejewski [21]. He discovered the regular precessions for this constrained rotational motion issue. Bushenkov et al. [22] presented a study on a satellite's stability challenge. The geomagnetic field in satellite coordinates was the sole measurement used in this investigation. The magnetic moment of current coils placed on the satellite body was used to operate the spacecraft. The stabilizer developed in this study solved difficulties with magnetic and gravitational stability. Makeyev [23] investigated the requirements for the presence and stability of a regular precession of gyrostats with magnetic properties moving in the stationary field of a magnetic dipole, moving approximately in the center of mass.

Beletsky [24] looked at the dynamic symmetric satellite's static and near-static motion. From the Hamilton function, he discovered the equation for the motion of a dynamically symmetric satellite in the presence of a gravitational force field. The three forms of permanent rotation were also identified in this research. Sukhov looked into periodic movements caused by a prior hyperbolic precession of a dynamically symmetric rigid satellite in a circular orbit [25]. Palli and Bucci [3] observed that a Volterra-type gyrostat satellite in a circular orbit under a Newtonian force field experiences regular precessions around the center of mass. They studied the regular precession with respect to the inertial frame by using Euler Poisson equations.

Because satellites are of great importance in all areas of life, so we must as much as possible control the movement of the satellite so that it does not get out of control. Precession motion is one of the important motions that affect the stability of the satellite, so in this study, we will search for a gyrostatic satellite in a circular orbit using the Newtonian force and magnetic fields. When the axes of the body are principle, Lagrange equations can be found with respect to the orbital frame. The axis of precession is perpendicular to the orbital plane, which is one of the essential requirements for the existence of regular precessions. All regular precessions and permanent rotations are indicated. The Lagrange technique is said to be more precise, straightforward, and general than other methods.

In the current study, we explain the mathematical formulation which describes the motion, then we drive the Lagrangian equations of motion. We use Euler angles as generalized coordinates. This led to dealing with equations in a clear way and reaching the form of precessional equations in an easy way. After that, the solutions have been calculated under some conditions on the parameters and we classify the types of precessions and permanent rotations. The physical meaning of every type of motion is given blow every solution.

2. Mathematical Formulation

2.1. Motion Description

Consider the motion of magnetized satellite-gyrostat in a circular orbit. We will refer to the center of the Earth as o and o' be the center of mass of the satellite moving in a circular orbit of the radius R with center at o, (e.g., Figure 1). Let XYZ be an inertial frame of reference, $o'x_1y_1z_1$ be the orbital system with x_1 axis along the radius oo', y_1 along the tangent orbit in the direction of motion of the satellite and z_1 in the direction perpendicular to the orbital plane, and let $o'xyz$ be the system of principal axes fixed in the satellite with unit vectors e_1, e_2 and e_3, respectively.

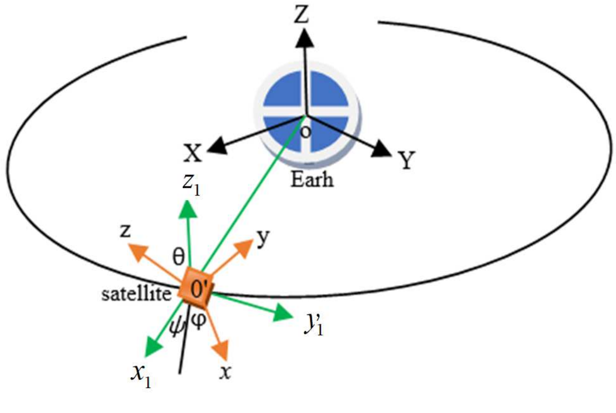

Fig. 1. Euler's angles for describing the orientation of the satellite

Let $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ also be the three-unit vector along x_1, y_1 and z_1, respectively, and $\boldsymbol{\omega}'$ is the satellite angular velocity vector with respect to orbital frame, all components being referred to the body system $o'xyz$. Euler's angles will be used to describe the orientation of the satellite relative to the orbital frame: the precession angle around the z_1 is ψ, the angle between the body axis (z) and z_1 axes is the nutation angle θ and φ the angle of proper rotation of the satellite around its z axis. Assume that the satellite contains magnetized parts which produce a magnetic moment $\boldsymbol{\mu}=(\mu_1,\ \mu_2,\ \mu_3)$ and is affected by an axisymmetric magnetic field, which may be expressed at the current position of the satellite by the vector $\mathbf{H}=h_1\boldsymbol{\alpha}+h_3\boldsymbol{\gamma}$. This magnetic field can be viewed as a uniform and constant field in the orbital frame.

In those variables we can write:

$$\begin{aligned}
&\boldsymbol{\alpha}=(\cos\psi\cos\varphi-\cos\theta\sin\varphi\sin\psi,\ -\cos\psi\sin\varphi-\cos\theta\cos\varphi\sin\psi,\ \sin\theta\sin\psi),\\
&\boldsymbol{\beta}=(\sin\psi\cos\varphi+\cos\theta\sin\varphi\cos\psi,\ -\sin\psi\sin\varphi+\cos\theta\cos\varphi\cos\psi,\ -\sin\theta\cos\psi),\\
&\boldsymbol{\gamma}=(\sin\theta\sin\varphi,\ \sin\theta\cos\varphi,\ \cos\theta),\\
&\boldsymbol{\omega}'=(\dot\psi\sin\theta\sin\varphi+\dot\theta\cos\varphi,\ \dot\psi\sin\theta\cos\varphi-\dot\theta\sin\varphi,\ \dot\psi\cos\theta+\dot\varphi).
\end{aligned}\tag{1}$$

3. Equation of Motion

Following [26], the Lagrangian explaining the satellite motion relative to the orbital frame recognizes the form

$$L=\frac{1}{2}(\boldsymbol{\omega}'+\Omega\boldsymbol{\gamma})\mathbf{I}\cdot(\boldsymbol{\omega}'+\Omega\boldsymbol{\gamma})+(\boldsymbol{\omega}'+\Omega\boldsymbol{\gamma}).\mathbf{k}-V,\tag{2}$$

where $\mathbf{k}$ is the gyrostatic momentum of the satellite, I = diag (A, B, C) is the central matrix of inertia and Ω is the orbital velocity in $\boldsymbol{\gamma}$ direction. The potential function for the gravitational and magnetic fields is given by

$$V=\frac{3}{2}\Omega^2\boldsymbol{\alpha}\mathbf{I}\cdot\boldsymbol{\alpha}+\boldsymbol{\mu}\cdot(h_1\boldsymbol{\alpha}+h_3\boldsymbol{\gamma}).\tag{3}$$

Taking into account equations (1) and (3), the Lagrangian (2) is expressed in the Euler- angles variables as:

$$\begin{aligned}
L=&\frac{1}{2}A[(\dot\psi+\Omega)\sin\theta\sin\varphi+\dot\theta\cos\varphi]^2+\frac{1}{2}B[(\dot\psi+\Omega)\sin\theta\cos\varphi-\dot\theta\sin\varphi]^2+\frac{1}{2}C[\dot\varphi+(\dot\psi+\Omega)\cos\theta]^2\\
&+k_1[(\dot\psi+\Omega)\sin\theta\sin\varphi+\dot\theta\cos\varphi]+k_2[(\dot\psi+\Omega)\sin\theta\cos\varphi-\dot\theta\sin\varphi]+k_3[(\dot\psi+\Omega)\cos\theta+\dot\varphi]\\
&-\frac{3}{2}\Omega^2[A(\cos\psi\cos\varphi-\cos\theta\sin\varphi\sin\psi)^2-B(\cos\psi\sin\varphi+\cos\theta\cos\varphi\sin\psi)^2+C\sin^2\theta\sin^2\psi]\\
&-\mu_1[h_1(\cos\psi\cos\varphi-\cos\theta\sin\varphi\sin\psi)+h_3\sin\theta\sin\varphi]-\mu_2[h_1(\cos\psi\sin\varphi+\cos\theta\cos\varphi\sin\psi)\\
&+h_3\sin\theta\cos\varphi]-\mu_3[h_1\sin\theta\sin\psi+h_3\cos\theta].
\end{aligned}\tag{4}$$

In this case, Lagrange's equations of motion take the form:

$$\begin{aligned}
&\frac{1}{2}(A-B)\sin2\varphi\sin\theta\ddot\theta+[(A\sin^2\varphi+B\cos^2\varphi)\sin^2\theta+C\cos^2\theta]\ddot\psi+C\cos\theta\ddot\varphi+\frac{1}{2}(A-B)\sin2\varphi\cos\theta\dot\theta^2+\{[(A-B)\cos2\varphi\\
&-C]\sin\theta\dot\varphi+[(A\sin^2\varphi+B\cos^2\varphi-C)\sin2\theta]\dot\psi+[2\Omega\cos\theta(A\sin^2\varphi+B\cos^2\varphi-C)-k_3]\sin\theta+(k_2\cos\varphi+k_1\sin\varphi)\}\dot\theta\\
&+[(A-B)\sin2\varphi\sin^2\theta(\dot\psi+\Omega)+\sin\theta(k_1\cos\varphi-k_2\sin\varphi)]\dot\varphi+\frac{3}{2}\Omega^2C\sin^2\theta\sin2\psi+\mu_3h_1\,\mathrm{os}\,\psi\sin\theta\\
&-\frac{3}{2}\Omega^2(-B\cos^2\theta+A)\sin2\psi\cos^2\varphi+[-3\Omega^2\cos2\psi(A-B)\cos\theta\sin\varphi-h_1(\mu_2\,\mathrm{s}\,\psi\cos\theta+\mu_1\sin\psi)]\cos\varphi\\
&+3\sin\varphi[\frac{\Omega^2}{2}\sin2\psi(A\cos^2\theta-B)\sin\varphi-\frac{1}{3}h_1(\mu_1\cos\psi\cos\theta-\mu_2\sin\psi)]=0,
\end{aligned}\tag{5}$$

$$\begin{aligned}
&\frac{1}{2}(A-B)\sin2\varphi\sin\theta\ddot\psi+(A\cos^2\varphi+B\sin^2\varphi)\ddot\theta+[-\frac{1}{2}(A\sin^2\varphi+B\cos^2\varphi-C)\sin2\theta]\dot\psi^2+\{((A-B)\cos2\varphi+C)\sin\theta\dot\varphi\\
&+\frac{1}{2}(A-B)\sin2\varphi\cos\theta\dot\theta+[-2\Omega\cos\theta(A\sin^2\varphi+B\cos^2\varphi)+2C\Omega\cos\theta+k_3]\sin\theta-\cos\theta(k_2\cos\varphi+k_1\sin\varphi)\}\dot\psi\\
&+[(B-A)\sin2\varphi\dot\theta+((A-B)\cos2\varphi+C)\Omega\sin\theta-(k_2\cos\varphi+k_1\sin\varphi)]\dot\varphi+[\frac{1}{2}(A-B)\sin2\varphi\cos\theta]\Omega\dot\theta\\
&+[-\Omega^2(B+A-C)\cos\theta(3\sin^2\psi+1)+\frac{3}{4}\Omega^2(A-B)\sin2\psi\sin2\varphi+h_1\mu_2\sin\psi\cos\varphi+h_1\mu_1\sin\psi\sin\varphi\\
&+\Omega k_3-h_3\mu_3]\sin\theta-\cos\theta(\Omega k_2-h_3\mu_2)\cos\varphi+(\Omega k_1-h_3\mu_1)\sin\varphi-h_1\mu_3\sin\psi]=0,
\end{aligned}\tag{6}$$

$$\begin{aligned}
&C\cos\theta\ddot\psi+C\ddot\varphi+\frac{1}{2}(A-B)\sin2\varphi\dot\theta^2+\{[((B-A)\cos2\varphi-C)\sin\theta]\dot\psi-((A-B)\cos2\varphi+C)\Omega\sin\theta+(k_1\sin\varphi+k_2\cos\varphi)\}\dot\theta\\
&-[\frac{1}{2}(A-B)\sin2\varphi\sin^2\theta]\dot\psi^2-\Omega\sin\theta\sin2\varphi(A-B)\sin\theta+(k_1\cos\varphi-k_2\sin\varphi)]\dot\psi-\frac{1}{2}\Omega^2(A-B)\sin2\varphi\sin^2\theta\\
&+[-\Omega(k_1\cos\varphi-k_2\,\mathrm{in}\,\varphi)+h_3(\mu_1\cos\varphi-\mu_2\sin\varphi)]\sin\theta-\frac{3}{2}\Omega^2(A-B)[\sin2\psi\cos\theta\cos2\varphi-(\cos^2\psi-\sin^2\psi\cos^2\theta)\sin2\varphi]\\
&-h_1\mu_1\sin\psi\cos\varphi\cos\theta+\cos\psi\sin\varphi)-h_1\mu_2(-\sin\psi\sin\varphi\cos\theta+\cos\psi\cos\varphi)=0.
\end{aligned}\tag{7}$$

4. Precessional Motion

Precession is a type of motion that occurs when the rotational axis of a rotating body is changed its orientation. It may be described as a change in the precession angle in a suitable reference frame. The Lagrange equations (5)-(7) are used in this section to find required conditions of the existence of regular precessions. The study of regular precession requires the rotation velocity of the body to be constant, i.e., $\dot{\varphi} = m$ which performs a precession with the constant angular velocity $\dot{\psi} = n$ about the space axis keeping the nutation angle fixed, i.e., $\theta = \theta_0$. Inserting $\psi = nt + \psi_0$, $\varphi = mt$, $\theta = \theta_0$ into (5), (6), and (7), respectively, we get

$$\begin{aligned}
&\frac{3}{8}\Omega^2(A-B)[(u_1-1)^2\sin(2(m-n)t-2\psi_0(u_1)^2\text{in}(2(m+n)t+2\psi_0)]-\frac{1}{2}h_1\mu_2(u_1-1)\cos((m-n)t-\psi_0))\\
&+(u_1+1)\cos((m+n)t+\psi_0]-\frac{1}{2}h_1\mu_1[(u_1-1)\sin((m-n)t-\psi_0))+(u_1+1)\sin((m+n)t+\psi_0))]\\
&+\frac{3}{4}u_2^2\Omega^2(2C-A-B)\sin(2nt+2\psi_0)+u_2[h_1\mu_3\cos(nt+\psi_0)+mu_2(n+\Omega)(A-B)\sin(2mt)\\
&-mk_1\cos(mt)-mk_2\sin(mt)]=0,
\end{aligned} \tag{8}$$

$$\begin{aligned}
&-\frac{3}{8}\Omega^2u_2(A-B)[(u_1-1)\cos(2(m-n)t-2\psi_0)+(u_1+1)\cos(2(m+n)t+2\psi_0)]+\frac{1}{2}u_2\mu_1h_1[\cos((m-n)t-\psi_0)-\cos((m+n)t+\psi_0)]\\
&-\frac{1}{2}u_2\mu_2h_1[\sin((m-n)t-\psi_0)-\sin((m+n)t+\psi_0)]+\frac{3}{4}\Omega^2u_2u_1(A+B-2C)\cos(2nt+2\psi_0)+u_1\mu_3h_1\sin(nt-\psi_0)\\
&+\frac{5}{4}u_2(A-B)[(\Omega^2+\frac{4}{5}\Omega n+\frac{2}{5}n^2)u_1+\frac{4}{5}m(\Omega+n)]\cos(2mt)+[(\mu_2h_3-(\Omega+n)k_2)u_1+[(\mu_1h_3-(\Omega+n)k_1)u_1-mk_1]\sin(mt)\\
&+u_2(\Omega+n)k_3-\frac{5}{4}u_2[(\Omega^2+\frac{4}{5}\Omega n-mk_2]\cos(mt)+\frac{2}{5}n^2)(A+B-2C)u_1-\frac{4}{5}mC(\Omega+n)+\frac{4}{5}\mu_3h_3]=0,
\end{aligned} \tag{9}$$

$$\begin{aligned}
&-\frac{3}{8}\Omega^2(A-B)[(u_1-1)^2\sin(2(m-n)t-2\psi_0)+(u_1+1)^2\sin(2(m+n)t+2\psi_0)]+\frac{1}{2}\mu_2h_1[(u_1-1)\cos((m-n)t-\psi_0)\\
&-(u_1+1)\cos((m+n)t+\psi_0)]+\frac{1}{2}\mu_1h_1[(u_1-1)\sin((m-n)t-\psi_0)+(u_1+1)\sin((m+n)t+\psi_0)]\\
&-\frac{3}{4}u_2^2(A-B)(\frac{5}{2}\Omega^2+\frac{4}{3}\Omega n+\frac{2}{3}n^2)\sin(2mt)+u_2[(-(n+\Omega)k_1+\mu_1h_3)\cos(mt)+((n+\Omega)k_2-\mu_2h_3)\sin(mt)]=0,
\end{aligned} \tag{10}$$

where $\cos\theta_0 = u_1, \sin\theta_0 = u_2$. The conditions for the existence of regular precessions about the Z axis are reduced to solve the three identities (8), (9) and (10) which depend on some parameters besides the numbers n, m. The way of satisfying those identities depends on whether n/m is rational or irrational (see e. g. [27]). We shall treat the two cases separately.

4.1. The case of rational *n/m*

Let the ratio n/m be a rational number. Set $n = i/s_1$, $m = j/s_2$, where i, j, s_1, s_2 are integers. We introduce the variable $\tau = t/s_1s_2$ and $nt = n^*\tau$, $mt = m^*\tau$, where $n^* = is_2$, $m^* = js_1$. By virtue of the adopted notation, we represent them in the form:

$$\begin{aligned}
&\frac{3}{8}\Omega^2(A-B)[(u_1-1)^2\sin(2(m^*-n^*)\tau-2\psi_0)-(u_1+1)^2\sin(2(m^*+n^*)\tau+2\psi_0)]-\frac{1}{2}h_1\mu_2[(u_1-1)\cos((m^*-n^*)\tau-\psi_0))\\
&+(u_1+1)\cos((m^*+n^*)\tau+\psi_0))]-\frac{1}{2}h_1\mu_1[(u_1-1)\sin((m^*-n^*)\tau-\psi_0))+(u_1+1)\sin((m^*+n^*)\tau+\psi_0))]\\
&+\frac{3}{4}\Omega^2(2C-A-B)u_2^2.\sin(2n^*\tau+2\psi_0)+u_2.[h_1\mu_3\cos(n^*\tau+\psi_0)+m^*u_2.(n^*+\Omega)(A-B)\sin(2m^*\tau)\\
&-mk_1\cos(m^*\tau)-m^*k_2\sin(m^*\tau)]=0,
\end{aligned} \tag{11}$$

$$\begin{aligned}
&-\frac{3}{8}\Omega^2u_2(A-B)[(u_1-1)\cos(2(m^*-n^*)\tau-2\psi_0)-(u_1+1)\cos(2(m^*+n^*)\tau+2\psi_0)]+\frac{1}{2}u_2.\mu_1h_1[\cos((m^*-n^*)\tau-\psi_0)\\
&-\cos((m^*+n^*)\tau+\psi_0)]-\frac{1}{2}u_2\mu_2h_1[\sin((m^*-n^*)\tau-\psi_0)-\sin((m^*+n^*)\tau+\psi_0)]+\frac{3}{4}u_2\Omega^2u_1(A+B-2C)\cos(2n^*\tau+2\psi_0)\\
&+u_1\mu_3h_1\sin(n^*\tau-\psi_0)+\frac{5}{4}u_2(A-B)[(\Omega^2+\frac{4}{5}\Omega n^*+\frac{2}{5}n^{*2})u_1+\frac{4}{5}m^*(\Omega+n^*)]\cos(2m^*\tau)+[(\mu_2h_3-(\Omega+n^*)k_2)u_1\\
&-m^*k_2]\cos(m^*\tau)+[(\mu_1h_3-(\Omega+n^*)k_1)u_1-m^*k_1]\sin(m^*\tau)+u_2.(\Omega+n^*)k_3-\frac{5}{4}u_2.[(\Omega^2+\frac{4}{5}\Omega n^*+\frac{2}{5}n^{*2})(A+B-2C)u_1\\
&-\frac{4}{5}m^*C(\Omega+n^*)+\frac{4}{5}\mu_3h_3]=0,
\end{aligned} \tag{12}$$

$$\begin{aligned}
&-\frac{3}{8}\Omega^2(A-B)[(u_1-1)^2\sin(2(m^*-n^*)\tau-2\psi_0)+(u_1+1)^2\sin(2(m^*+n^*)\tau+2\psi_0)]+\frac{1}{2}\mu_2h_1[(u_1-1)\cos((m^*-n^*)\tau-\psi_0)\\
&-(u_1+1)\cos((m^*+n^*)\tau+\psi_0)]+\frac{1}{2}\mu_1h_1[(u_1-1)\sin((m^*-n^*)\tau-\psi_0)+(u_1+1)\sin((m^*+n^*)\tau+\psi_0)]\\
&-\frac{3}{4}u_2^2(A-B)(\frac{5}{2}\Omega^2+\frac{4}{3}\Omega n^*+\frac{2}{3}n^{*2}\sin(2m^*\tau)+u_2[(-(n^*+\Omega)k_1+\mu_1h_3)\cos(m^*\tau)-(-(n^*+\Omega)k_2+\mu_2h_3)\sin(m^*\tau)]=0.
\end{aligned} \tag{13}$$

4.1.1. The special case n=0*

This physically is an important case that deserves special treatment. It is obvious that the precession velocity and the orbital velocity are equal. For the orbital frame, the motion is merely a permanent rotation (without precession) about an axis fixed in that frame. Setting $n^* = 0$ in the equations (11) - (13) and equating the coefficients of the trigonometric functions to zero, we obtain the following possible two sets of values of parameters:

Case I: This case is characterized by $\psi = \psi_0 = 0,\ \ \varphi = mt$ and $A = B$. We study the following two subcases in which $\sin\theta_0$ is either zero or not, individually:

a) If $\sin\theta_0 = 0$, we have the following three possible combinations of the parameters. They are

$$1)\ \mu_1 = \mu_2 = 0,\ k_1 = k_2 = 0,\qquad 2)\ \mu_1 = \mu_2 = 0,\ m^* = -\Omega,\qquad 3)\ h_1 = 0,\ k_1 = \frac{h_3\mu_1}{m^*+\Omega},\ k_2 = \frac{h_3\mu_2}{m^*+\Omega}. \tag{14}$$

The solution is :

$$\boldsymbol{\alpha} = (\cos mt,\ -\sin mt,\ 0),\ \boldsymbol{\beta} = (\sin mt,\ \cos mt,\ 0),\ \boldsymbol{\gamma} = (0,\ 0,\ 1),\ \boldsymbol{\omega}' = (0,\ 0,\ m). \tag{15}$$

For the first choice in (14), the satellite is in the symmetric case, where the axis of the body coincides with the perpendicular to the orbital plane and there is only a gyrostatic moment in the direction of the third axis of the body. The satellite does a cylindrical permanent rotation in the orbital frame with angular velocity m, but it gives in the inertial frame cylindrical regular precession whereas the precession velocity and the orbital velocity are equal. For the second choice in (14), the solution admits the form:

$$\boldsymbol{\alpha} = (\cos\Omega t,\ \sin\Omega t,\ 0),\ \boldsymbol{\beta} = (-\sin\Omega t,\ \cos\Omega t\ ,\ 0),\ \boldsymbol{\gamma} = (0,\ 0,\ 1),\ \boldsymbol{\omega}' = (0,\ 0,\ -\Omega). \tag{16}$$

This solution is a special case from solution (15) when $m = -\Omega$. The body makes cylindrical permanent rotation in the orbital frame and only translation motion in the inertial frame. For the last choice in (14), the magnetic field of the body has a direction that is perpendicular to the orbital plane.

b) If $\sin\theta_0 \neq 0$, there are two possible subcases. They are:

$$\begin{aligned} &1)\ h_1 = 0,\ \mu_1 = \mu_2 = 0,\ k_1 = k_2 = 0,\ k_3 = \frac{\Omega^2(B-C)\cos\theta_0 - m^*\Omega C + \mu_3 h_3}{\Omega},\\ &2)\ \boldsymbol{\mu} = \mathbf{0},\ k_1 = k_2 = 0,\ k_3 = \cos\theta_0(B-C)\Omega - Cm^*, \end{aligned} \tag{17}$$

The solution is :

$$\begin{aligned} &\boldsymbol{\alpha} = (\cos mt,\ -\sin mt,\ 0),\ \boldsymbol{\beta} = (\cos\theta_0 \sin mt,\ \cos\theta_0 \cos mt,\ -\sin\theta_0),\\ &\boldsymbol{\gamma} = (\sin\theta_0 \sin mt,\ \sin\theta_0 \cos mt,\ \cos\theta_0),\ \boldsymbol{\omega}' = (0,\ 0\ ,\ m). \end{aligned} \tag{18}$$

For the first choice in (17), these conditions describe the satellite in a state of symmetry, and the magnetic field is in the direction perpendicular to the orbital plane. The magnetic moment and the gyrostatic moment are in the direction of the third axis of the satellite.

It is clear that the second choice in (17) is a special case of the first one in (17) due to the absence of the magnetic moment. The satellite is symmetrical under the influence of the gravitational field in addition to the influence of the gyrostat. Notice, the gyrostatic moment acts in the direction of the third axis of the body.

Thus, the motion of the satellite in the circular orbit is conical permanent rotations with angular velocity *m*. On the other hand, it forms conical regular precession in the inertial frame. In the case of conical precession, the axis of dynamical symmetry of the satellite forms a fixed angle with the normal to the plane of the orbit, while the satellite rotates uniformly around its axis of dynamic symmetry.

Case II: This case is specified by $\psi = \psi_0 = \pi/2,\ \varphi = m^* t$ in addition to A and B are either equal or not. We will study each case separately:

- The first case which is $A \neq B$ gives: after some manipulations $\cos\theta_0 = 1/2,\ m = -\Omega$. There are the following two sets of complementary conditions

$$\begin{aligned} &1)\ \mu_1 = \mu_2 = 0,\ k_1 = k_2 = 0,\ k_3 = \frac{-\mu_3 h_1\sqrt{3}(\Omega^2(A+B-C) + \mu_3 h_3)}{\sqrt{3}\Omega},\\ &2)\ h_1 = -\sqrt{3}h_3,\ k_1 \frac{2\mu_1 h_3}{\Omega},\ k_2 = \frac{2\mu_2 h_3}{\Omega},\ k_3 = \frac{\Omega^2(A+B-C) + 2\mu_3 h_3}{\Omega}, \end{aligned} \tag{19}$$

which characterize the same precession motion described by

$$\begin{aligned} &\boldsymbol{\alpha} = (\frac{1}{2}\sin\Omega t,\ -\frac{1}{2}\cos\Omega t,\ \frac{\sqrt{3}}{2}),\ \boldsymbol{\beta} = (\cos\Omega t,\ \sin\Omega t,\ 0),\\ &\boldsymbol{\gamma} = (-\frac{\sqrt{3}}{2}\sin\Omega t,\ \frac{\sqrt{3}}{2}\cos\Omega t,\ \frac{1}{2}),\ \boldsymbol{\omega}' = (0,\ 0,\ -\Omega). \end{aligned} \tag{20}$$

The solution (19) describes hyperboloidal permanent rotation in the orbital frame with angular velocity Ω. In the space coordinate system, the motion is seen as the hyperboloidal regular precession.

The case in which $B = C$ represents a special case from (19). Inserting $\mu_3 = 0$ or $h_1 = h_3 = 0$ into (19) implies to the conditions of the regular precessions of gyrostatic satellite in a circular orbit in the case of non-symmetry [3].

- The second case is $A = B$. We consider the two subcases in which $\sin\theta_0$ is zero or not.

1. If $\sin\theta_0 = 0$, the solution has the form

$$\boldsymbol{\alpha} = (\sin(mt),\ \cos(mt),\ 0),\ \boldsymbol{\beta} = (\cos(mt),\ -\sin(mt),\ 0),\ \boldsymbol{\gamma} = (0,\ 0,\ 1),\ \boldsymbol{\omega}' = (0,\ 0,\ m), \tag{21}$$

under the following two sets of complementary conditions:

$$1)\ \boldsymbol{\mu} = \mathbf{0},\ k_1 = k_2 = 0 \tag{22}$$

$$2)\ h_1 = 0,\ k_1 = \frac{\mu_1 h_3}{m+\Omega},\ k_2 = \frac{\mu_2 h_3}{m+\Omega}. \tag{23}$$

whereas $\theta_0 = 0$. This means the axis of the body will coincide with the perpendicular to the orbital plane, and the body is symmetric. The satellite makes up the cylindrical permanent rotation in the orbital frame with angular velocity m and gives cylindrical regular precession with respect to the inertial frame. This case generalizes the one found in [3] and turns into it (22) and $(\mu_1 = \mu_2 = 0$ or $h_3 = 0)$ in (23). It should be noted that the condition $k_3 = -mA - (C-A)(n\gamma_3 + m)$ appearing [3] is not necessary but the usage of the Lagrange equations makes k_3 arbitrary.

2. If $\sin\theta_0 \neq 0$

$$k_1 = k_2 = 0,\ \mu_1 = \mu_2 = 0,\ k_3 = \frac{\sin\theta_0(4\cos\theta_0(B-C)\Omega^2 - C\Omega m + \mu_3 h_3) - \mu_3 h_1 \cos\theta_0}{\Omega \sin\theta_0}. \tag{24}$$

Again, if we substitute $\mu_3 = 0$, we will get the conditions of regular precessions of a symmetric satellite-gyrostat in a circular orbit [3].

The solution is:

$$\begin{aligned} &\boldsymbol{\alpha} = (-\cos\theta_0 \sin mt,\ -\cos\theta_0 \cos mt,\ \sin\theta_0),\ \boldsymbol{\beta} = (\cos mt,\ -\sin mt,\ 0),\\ &\boldsymbol{\gamma} = (\sin\theta_0 \sin mt,\ \sin\theta_0 \cos mt,\ \cos\theta_0),\ \boldsymbol{\omega}' = (0,\ 0,\ m). \end{aligned} \tag{25}$$

This case allows to getting hyperboloidal permanent rotation in the orbital frame with angular velocity m. The same body gives hyperboloidal regular precession such that precession velocity equals the orbital velocity in the inertial frame.

4.1.2. The case $n^* \neq 0$

Trigonometric functions have a maximum argument equal to the value $2(m^* + n^*)$ [27]. Therefore, equations (11), (12), and (13) are identities under the conditions:

$$A = B, \text{or}\ \ \theta_0 = \pi. \tag{26}$$

If $m^* > 0$, similar conditions will be obtained when $m^* < 0$ taking into account the conditions in (26).

1) First, we will get the conditions when $m^* = n^*$:

$$\begin{aligned} &\cos\theta_0 = -1,\ \psi = nt + \psi_0\ \ \varphi = nt,\ A \neq B\\ &k_1 = \frac{\mu_3 h_1 \cos\psi_0 + \mu_1 h_3}{\Omega},\ k_2 = \frac{\mu_3 h_1 \sin\psi_0 + \mu_2 h_3}{\Omega},\ h_1 = -\frac{\frac{3}{2}\Omega^2(A-B)\sin 2\psi_0}{\mu_1 \sin\psi_0 - \mu_2 \cos\psi_0}. \end{aligned} \tag{27}$$

This case describes the motion of an asymmetric satellite moving on a circular orbit under the influence of symmetrically magnetic field and gyrostatic moment vector its components depends on the values of the magnetic field. We note that if the satellite is in a symmetric state, the component will be equals zero, and the magnetic field will be in the direction of $\boldsymbol{\gamma}$ the solution, in this case, takes the form:

$$\begin{aligned} &\alpha = (\cos\psi_0\ ,\ \sin\psi_0,\ 0),\ \gamma = (0,\ 0,\ \text{-}1)\\ &\beta = (\sin\psi_0,\ \text{-}\cos\psi_0,\ 0),\ \omega' = (0,\ 0,\ 0). \end{aligned} \tag{28}$$

From (28) the body makes cylindrical regular precession in the inertial frame where the angular velocity equals Ω, $\theta = \pi$ at $m = n$. While getting an equilibrium position in the orbital frame.

2) If $A = B$, the satellite is in state of a symmetry and it form regular precession under the following two sets of condition:

$$\begin{aligned} &1)\ \theta_0 = 0,\ \psi = nt + \psi_0,\ \varphi = nt,\ \mu_1 = \mu_2 = 0, k_1 = \frac{\mu_3 h_1 \cos\psi_0}{\Omega + 2n}, k_2 = \frac{\mu_3 h_1 \sin\psi_0}{\Omega + 2n}\\ &2)\ \theta_0 = 0,\ \psi = nt + \psi_0,\ \varphi = nt,\ h_1 = 0,\ k_1 = \frac{\mu_1 h_3}{\Omega + 2n},\ k_2 = \frac{\mu_2 h_3}{\Omega + 2n}, \end{aligned} \tag{29}$$

and the solution is given by:

$$\begin{aligned} &\boldsymbol{\alpha} = (\cos(2nt + \psi_0),\ -\sin(2nt + \psi_0),\ 0),\ \boldsymbol{\gamma} = (0,\ 0,\ 1),\\ &\boldsymbol{\beta} = (\sin(2nt + \psi_0),\ \cos(2nt + \psi_0),\ 0),\ \boldsymbol{\omega}' = (0,\ 0,\ 2n). \end{aligned} \tag{30}$$

The conditions (29) show that the body is in a state of symmetry and in the presence of gyrostatic and a magnetic field makes cylindrical regular precession in the orbital frame where the angular velocity equals $2n$, while getting regular precession in the inertial frame with angular velocity $2n + \Omega$.

3) Under the following condition $\theta_0 = \pi,\ \psi = nt + \psi_0,\ \varphi = 2nt,\ A = B$ and any one of the following two conditions

$$\begin{aligned}&1)\ h_1=0,\ k_1=-\frac{\mu_1 h_3}{(n^*-\Omega)},\ k_2=-\frac{\mu_2 h_3}{(n^*-\Omega)}\\&2)\ \mu_1=\mu_2=\mu_3=0,\ (k_1=k_2=0 \text{ or } n=\Omega).\end{aligned}\tag{31}$$

We note that when $m=2n$ at $\theta_0=\pi$ we get the same case when $m=-2n$ at $\theta_0=0$.
The solution is:

$$\begin{aligned}&\boldsymbol{\alpha}=(\cos(nt-\psi_0),\ \sin(nt-\psi_0),0),\ \boldsymbol{\gamma}=(0,\ 0,\ -1),\\&\boldsymbol{\beta}=(\sin(nt-\psi_0),\ -\cos(nt-\psi_0),\ 0),\ \boldsymbol{\omega}'=(0,\ 0,\ n).\end{aligned}$$

Relations (31) describes the body in a state of symmetry. In this case in the absence of the gyrostatic moments k_1 and k_2 or when the precession velocity equals the orbital velocity then the magnetic moment will be lost. In this case, the body describes cylindrical regular precession motion in the orbital frame with velocity n and the same motion we notice in the inertial frame with a difference of the value of the angular velocity $n+\Omega$. In addition, for any value of $\theta=\theta_0$ except $\theta=0$ or π gives the case of spherical symmetry.

4.2. The case of irrational *n/m*

In this case, we can obtain after some manipulation on equations (8), (9) and (10) the following relations:

$$\begin{aligned}&\frac{3}{4}\Omega^2(B-A)(u_1^2+1)\sin(2\psi_0+2nt)\cos 2mt+(B-A)[m(n+\Omega)u_2^2+\frac{3}{2}\Omega^2 u_1\cos(2\psi_0+2nt)]\sin 2mt\\&+[-h_1\mu_2u_1\cos(\psi_0+nt)-h_1\mu_1\operatorname{in}(\psi_0+nt)+mk_1u_2]\cos mt+[-h_1\mu_1u_1\cos(\psi_0+nt)+h_1\mu_2\sin(\psi_0+nt)-mu_2k_2]\sin mt\\&+\frac{3}{4}\Omega^2((A+B)u_1^2+2u_2^2A-B)\sin(2\psi_0+2nt)+h_1\mu_3u_2\cos(\psi_0+nt)=0,\end{aligned}\tag{32}$$

$$\begin{aligned}&[-\frac{3}{4}\Omega^2u_1u_2\cos(2\psi_0+2nt)+u_2(\frac{5}{4}u_1\Omega^2+(xn+m)\Omega+n(\frac{1}{2}nu_1+m))](A-B)\cos 2mt+\frac{3}{4}\Omega^2u_2(A-B)\sin(2\psi_0+2nt)\sin 2mt\\&+[h_1\mu_2u_2\sin(\psi_0+nt)-u_1\Omega k_2+u_1(\mu_2h_3-nk_2)-mk_2]\cos mt+[h_1\mu_1u_2\sin(\psi_0+nt)-u_1\Omega k_1+(-nk_1+\mu_1h_3)u_1-mk_1]\sin mt\\&+(\frac{3}{4}\Omega^2u_1u_2(A+B-2C)(\cos(2\psi_0+2nt)+h_1\mu_3u_1\sin(\psi_0+nt)-u_2[(\frac{5}{4}\Omega^2+n\Omega+\frac{1}{2}n^2)(A+B-2C)u_1+(-Cm-k_3)\Omega\\&-nmC+\mu_3h_3-nk_3]=0,\end{aligned}\tag{33}$$

$$\begin{aligned}&-\frac{3}{2}\Omega^2u_1(A-B)\sin(2\psi_0+2nt)\cos 2mt+[(\Omega^2(A-B)(u_1^2+1)(\cos(2\psi_0+2nt)-\frac{1}{3}u_2^2(\Omega^2(A-B)-4n\Omega-2n^2)]\sin 2mt\\&-h_1\mu_1u_1\sin(\psi_0+nt)-h_1\mu_2\cos(\psi_0+nt)-u_2((\Omega+n)k_1-h_3\mu_1)]\cos mt+[h_1\mu_2u_1\sin(\psi_0+nt)-h_1\mu_1\cos(\psi_0+nt)\\&+u_2((\Omega+n)k_2+h_3\mu_2)]\sin mt=0.\end{aligned}\tag{34}$$

Introducing the irrational *n/m* into equations (32), (33) and (34) and taking

$$1)\ t=\frac{\pi}{m}i \qquad i=1,2,3,\ldots\ldots.\tag{35}$$

then we get an infinite system of equations as

$$\begin{aligned}&3\Omega^2[-\frac{1}{2}\sin 2\psi_0(-Bu_1^2-Cu_2^2+A)]\cos\frac{2n\pi i}{m}+3\Omega^2[-\frac{1}{2}\cos 2\psi_0(-Bu_1^2-Cu_2^2+A)]\sin\frac{2n\pi i}{m}\\&+h_1[(-1)^iu_1\mu_2\cos\psi_0+\mu_1\sin\psi_0)+u_2\cos\psi_0\mu_3]\cos\frac{n\pi i}{m}+h_{1[}(-1)^i(u_1\mu_2\sin\psi_0-\mu_1\cos\psi_0)-u_2\sin\psi_0\mu_3]\sin\frac{n\pi i}{m}\\&+u_2m(-1)^ik_1=0,\end{aligned}\tag{36}$$

$$\begin{aligned}&\frac{3}{2}\Omega^2(B-C)u_1u_2\cos 2\psi_0\cos\frac{2n\pi i}{m}-\frac{3}{2}\Omega^2u_1u_2(B-C)\sin 2\psi_0\sin\frac{2n\pi i}{m}+h_1\sin\psi_0[(-1)^iu_2\mu_2+u_1\mu_3]\cos\frac{n\pi i}{m}\\&+h_1\cos\psi_0[(-1)^iu_2\mu_2+u_1\mu_3]\sin\frac{n\pi i}{m}+(-(\Omega+n)u_1k_2+h_3\mu_2u_1-mk_2)(-1)^i+u_2(-\frac{5}{2}(B-C)u_1\Omega^2\\&+(-2n(B-C)u_1+(A-B+C)m+k_3)\Omega-n^2(B-C)u_1+((A-B+C)m+k_3)n-\mu_3h_3)=0,\end{aligned}\tag{37}$$

$$\begin{aligned}&-\frac{3}{2}\Omega^2u_1(A-B)\sin 2\psi_0\cos\frac{2n\pi i}{m}-\frac{3}{2}\Omega^2u_1(A-B)\cos 2\psi_0\sin\frac{2n\pi i}{m}-(-1)^ih_1(\mu_1u_1\sin\psi_0+\mu_2\cos\psi_0)\cos\frac{n\pi i}{m}\\&-h_1(-1)^i(\mu_1u_1\cos\psi_0-\mu_2\sin\psi_0)\sin\frac{n\pi i}{m}+(-1)^iu_2(-(\Omega+n)k_1+h_3\mu_1)=0.\end{aligned}\tag{38}$$

Also, presenting the irrational *n/m* into equations (32) - (34) and assuming that

$$2)\quad t=\frac{\pi}{2m}+\frac{\pi j}{m},\quad j=0,1,2,\ldots\ldots\tag{39}$$

$$
\begin{aligned}
&-\frac{3}{4}\Omega^2[\sin 2\psi_0(A-B)(u_1^2+1)(-1)^{2j+1}-\sin 2\psi_0((A+B)u_1^2+2Cu_2^2-A-B)]\cos\frac{(1+2j)n\pi}{m}\\
&+\frac{3}{4}\Omega^2[(u_1^2+1)(A-B)\cos 2\psi_0(-1)^{2j+1}-\cos 2\psi_0((A+B)u_1^2+2u_2^2C-A-B)]\sin\frac{(1+2j)n\pi}{m}\\
&-(-1)^j h_1[\mu_1 u_1\cos\psi_0-\mu_2\sin\psi_0)+\mu_3 h_1 u_2\cos\psi_0]\cos\frac{(1+2j)n\pi}{2m}+h_1[(-1)^j\mu_1\sin\psi_0+(-1)^j\mu_2\cos\psi_0\\
&-\mu_3 u_2\sin\psi_0]\sin\frac{(1+2j)n\pi}{2m}-(-1)^j m u_2 k_2]=0,
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
&-\frac{3}{4}\Omega^2[u_1u_2(-1)^{2j+1}(A-B)\cos 2\psi_0-u_1u_2\cos 2\psi_0(A+B-2C)]\cos\frac{(1+2j)n\pi}{m}+\frac{3}{4}\Omega^2[u_1u_2\sin 2\psi_0(A-B)(-1)^{2j+1}\\
&-u_1u_2\sin 2\psi_0(A+B-2C)]\sin\frac{(1+2j)n\pi}{m}+h_1[(-1)^j\mu_1u_2\sin 2\psi_0+\mu_3u_1\sin 2\psi_0]\cos\frac{(1+2j)n\pi}{2m}+h_1[(-1)^j\mu_1u_2\cos\psi_0\\
&+\mu_3u_1\cos\psi_0]\sin\frac{(1+2j)n\pi}{2m}+\frac{1}{4}[(5\Omega^2+4\Omega n+2n^2)u_1+4m(\Omega+n)](A-B)u_2(-1)^{2j+1}+[(-k_1(n+\Omega)+\mu_1h_3)u_1-mk_1](-1)^j\\
&-u_2[\frac{1}{4}(A+B-2C)(5\Omega^2+4\Omega n+2n^2)u_1-(mC+k_3)(\Omega+n)+\mu_3h_3]=0,
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
&-\frac{3}{2}\Omega^2u_1(A-B)\sin 2\psi_0(-1)^{2j+1}\cos\frac{(1+2j)n\pi}{m}-\frac{3}{2}\Omega^2u_1(A-B)\cos 2\psi_0(-1)^{2j+1}\sin\frac{(1+2j)n\pi}{m}\\
&+(-1)^j h_1[\mu_2u_1\sin\psi_0-\mu_1\cos\psi_0]\cos\frac{(1+2j)n\pi}{2m}+(-1)^j h_1(\mu_2u_1\cos\psi_0+\mu_1\sin\psi_0)\sin\frac{(1+2j)n\pi}{2m}\\
&+(-1)^j u_2[(\Omega+n)k_2+h_3\mu_2]=0,
\end{aligned}
\tag{42}
$$

Considering the first ten equations of the system (36)-(38) the first ten equations of the system (40)-(42) using the assumption that the ratio n/m is irrational and the relation.

$$
\begin{vmatrix}
\cos 2\varphi_1 & \sin 2\varphi_1 & \cos\varphi_1 & \sin\varphi_1 & 1\\
\cos 2\varphi_2 & \sin 2\varphi_2 & \cos\varphi_2 & \sin\varphi_2 & 1\\
\cos 2\varphi_3 & \sin 2\varphi_3 & \cos\varphi_3 & \sin\varphi_3 & 1\\
\cos 2\varphi_4 & \sin 2\varphi_4 & \cos\varphi_4 & \sin\varphi_4 & 1\\
\cos 2\phi_5 & \sin 2\phi_5 & \cos\phi_5 & \sin\phi_5 & 1
\end{vmatrix}
= c_*\sin\frac{1}{2}(\varphi_2-\varphi_1)\sin\frac{1}{2}(\varphi_3-\varphi_1)\sin\frac{1}{2}(\varphi_4-\varphi_1)\sin\frac{1}{2}(\varphi_5-\varphi_1)\times\sin\frac{1}{2}(\varphi_3-\varphi_2)\sin\frac{1}{2}(\varphi_4-\varphi_2)
$$
$$
\times\sin\frac{1}{2}(\varphi_5-\varphi_2)\sin\frac{1}{2}(\varphi_4-\varphi_3)\times\sin\frac{1}{2}(\varphi_5-\varphi_3)\sin\frac{1}{2}(\varphi_5-\varphi_4),
\tag{43}
$$

where c_* is a constant and ϕ_i, $i=1,2,3,4,5$ are arbitrary angles, we come to the conclusion that all the coefficients of the trigonometric functions in (36) - (38) and (40) - (42) are equal to zero.

Now, we will study the solutions for classifying regular precessions and permanent rotations for this case, from (36) - (38), we obtain the following conditions on the parameters:

$$
\begin{aligned}
&\cos\theta=1,\ A=B \text{ in addition to one of the following conditions}\\
&1)\ h_1=0,\ \mu_2=\frac{\Omega+m+n}{h_3}k_2,\ \mu_1=\frac{\Omega+m+n}{h_3}k_1\\
&2)\ \boldsymbol{\mu}=\mathbf{0},\ k_1=k_2=0\\
&3)\ \boldsymbol{\mu}=\mathbf{0},\ n+\Omega+m=0,
\end{aligned}
\tag{44}
$$

It is notable that in the special case $\mu_1=\mu_2=0$ either $k_1=k_2=0$, i.e. only k_3 of the gyrostatic momentum remains and it is in the direction of the third axis of the body or $\Omega+m+n=0$, which means that the body performs only translational motion on the inertial frame and cylindrical regular precession in the orbital frame. In this case, the solution will be in the form

$$
\begin{aligned}
&\boldsymbol{\alpha}=(\cos(nt+\psi_0+mt),\ -\sin(nt+\psi_0+mt),\ 0),\ \boldsymbol{\beta}=(\sin(nt+\psi_0+mt),\ \cos(nt+\psi_0+mt),\ 0),\\
&\boldsymbol{\gamma}=(0,\ 0,\ 1),\quad \boldsymbol{\omega}'=(0,\ 0,\ n+m).
\end{aligned}
\tag{45}
$$

This solution describes cylindrical regular precession in the orbital frame, where the angular velocity equals $n+m$. The solution in the inertial frame describes the same description as in the orbital frame but the angular velocity equals $n+m+\Omega$.

From (40) - (42) we obtain the same conditions on the parameters in the last case of symmetry at $\theta=0,\pi$ and spherical symmetric for any other value of θ.

From the above results we can conclude briefly that using the Lagrange method to find the regular precession and permanent rotation of the moving satellite under the influence of the magnetic field and the Newtonian gravitational field gave a generalization and clarification of what by Pucci and Balli [3] presented. We also note that the Lagrange method added many new solutions that were not shown by the Euler-Poisson method used by Pucci.

5. Conclusion

We have interested in finding certain motions, especially, precession motions and permanent rotation for a magnetized satellite-gyrostat which moves in a circular orbit due to the combined influence of uniform gravity and magnetic fields. Eulerian angles ψ, θ and φ have been utilized as the generalized coordinates to describe this motion. The Lagrange equations of the motion have been presented. The regular precessions of magnetized satellite gyrostats have been categorized into a circular orbit in both the orbital frame and the inertial frame of the satellite axes are assumed to be principle axes. The existence condition of regular precessions for the satellite which is either symmetric or non-symmetric has been introduced in which the axis of precession is perpendicular to the orbital plane. These motions have been physically interpreted. We have been shown that the usage of Lagrange equations to find the precession motion is more effective than the utilization of the Euler-Poisson equations. For instance, Balli and Pucci, who had been used Euler-Poisson equations to find regular precession for asymmetric rigid gyrostatic satellite on a circular orbit, had introduced the condition $k_3 = -mA - (C - A)(n\gamma_3 + m)$ which is not necessary but the usage of the Lagrange equations makes k_3 arbitrary.

Author Contributions

The authors contributed equally to this work. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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