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A New Solution for the Classical Problem of a Rigid Body Motion in a Liquid

H.M. Yehia, S.F. Megahid

Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

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Corresponding author: S.F. Megahid (samifikri@mans.edu.eg)

Abstract. We consider the problem of the motion of a rigid body-gyrostat immersed in an incompressible ideal fluid. Based on Yehia's study [1, 2], the equations of the motion of the problem are introduced and they are reduced to the orbital equation. This reduced equation may be used to study the stability of certain motions of the body [3] and to obtain solutions for the classical problems in rigid body dynamics [4]. Using the orbital equation, a single new solution of the considered problem is obtained in which the angle between the body axis of symmetry and the vertical axis is constant.

Keywords: Rigid body dynamics; Motion of a rigid body in a liquid; Particular solutions; The reduced equation.

1. Introduction

The first study of the problem of the motion of a rigid body in a liquid appeared in the work of Tait and Thompson [5], and they considered the simplest form of the problem in which the rigid body is bounded by a simply connected surface in an ideal incompressible fluid. After that, Kirchhoff developed the problem by obtaining the well-known Kirchhoff's equations of motion and deduced a simple integral case in [6]. The equations were reformulated in Hamiltonian form by Clebsch [7] and got two integrable cases in [8]. Steklov, Chaplygin, and Lyapunov constructed additional integrable cases by Kirchhoff's equations. Lamb developed the final form of the equations of motion in Kirchhoff's variables for a perforated multi-connected rigid body in a fluid that rotates through the perforations [8]. Yehia [9] modified the differential equations of motion of a rigid body with a multi-connected surface in an incompressible ideal infinite fluid and obtained the Euler-Poisson vector equations. For this present problem, there exist two conditional and seven general integrable cases. The problem has a few numbers of known particular solutions [10]. Many solutions for such problems of rigid body dynamics with different force cases may be found in Refs ([4], [11]-[20]).

As it is known, the solutions of the equations of motion in the general form are very difficult. Therefore, many researchers tried to simplify the problems of the motion of a rigid body inside a liquid by reducing the order of the corresponding Euler-Poisson equations which give directly the three classical known integrals (The energy integral, The geometric integral, The cyclic integral). These integral expressions allow eliminating three of Euler-Poisson variables to obtain three autonomous equations of the first order in the remaining three variables including the time as an independent variable. Another step can be done by removing the time t from the equations, and a non-autonomous system of two first-order differential equations is obtained considering that the third variable is an independent variable. Finally, one of the two variables can be eliminated, and a single differential equation of the second-order can be given in two variables, one is independent and the other is dependent. Many studies have been published and concerned with arriving at a maximal reduction to a single differential equation of the second-order using algebraic operations for elimination, however, the techniques of solutions resulted in a situation similar to the original equations of motion relying on unresolved constraints or remained incomplete [21]-[25].

Kharlamov used the technique of elimination of Poisson's variables to reduce the equations of motion for a rigid body-gyrostat in a uniform gravitational field to two first-order differential equations [26] and he cannot generalized the reduction neither to cases containing gyroscopic forces nor to cases containing more general potentials. Yahia obtained the final form to the problem of reducing the order of the differential equations of motion of a rigid body by the use of Hamilton's principle in two works: for a body moving under the influence of an axisymmetric general composition of gyroscopic forces and conservative potential [2], and for a gyrostat under the influence of arbitrary potential forces [1]. A single differential equation of the second order in the variables γ_1, γ_3 was obtained for each of the previous two cases this equation is called the reduced equation. This equation, which connects geometric quantities only, has proven to be important in some analytical and qualitative investigations for a rigid body motion, e.g. [3] and [27].

As a result of the rigid body-gyrostat is usually exerted as a model for diverse applications in physics and applications such as the motion of the satellite and underwater vehicles, it enhances a beneficial model for the study from the different aspects. One

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of them is the problem of constructing the explicit solution for the Euler-Poisson equation although it is a very hard task due to the complexity of the equations of the motion. In the present work, we explore the results obtained by Yehia [2] that proves there is an equivalent between the problem of the motion of a rigid body in an incompressible ideal fluid and the problem of the motion of a rigid body about its fixed point under the action of the potential and gyroscopic forces. Hence, a solution for one problem of them implies to a solution for another problem. The explicit solution for the problem manages us to predict the behavior of the motion in an infinite interval of time.

This work is organized as the following: In section 2, we introduce the equation of the motion of a rigid body in an incompressible ideal fluid in an equivalent form to the motion of a rigid body-gyrostat rotating due to the influence of the potential and gyroscopic forces. Section 3, the orbital equation representing the reduced form of the equations of the motion is presented. Section 4 involves the solution of the reduced equation with certain assumptions. Section 5 contains a new explicit solution of the equations of the motion. Section 6 is the conclusion of the obtained solutions.

2. Equations of Motion

The equations of motion of a rigid body with a multi-connected surface in an incompressible ideal infinite fluid are reduced to the Euler-Poisson (for more details see, [9]):

$$\dot{\boldsymbol{\omega}}\mathbf{I} + \boldsymbol{\omega} \times (\boldsymbol{\omega}\mathbf{I} + \mathbf{k} + \boldsymbol{\gamma}\bar{\mathbf{K}}) = \boldsymbol{\gamma} \times (\mathbf{s} + \boldsymbol{\gamma}\mathbf{J}),\ \dot{\boldsymbol{\gamma}} + \boldsymbol{\omega} \times \boldsymbol{\gamma} = 0, \tag{1}$$

where the dots refers to the derivative with respect to the time t, $\mathbf{I} = \mathrm{diag}(A,B,C)$, $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \gamma_3)$, $\boldsymbol{\omega} = (p,q,r)$, A,B,C are the principal moments of inertia; $\gamma_1, \gamma_2, \gamma_3$ are the components of the unit vector $\boldsymbol{\gamma}$ which is the symmetry axis of the force field and p,q,r are the angular velocity's components of the body, all being referred to the principal axes of inertia; $\bar{\mathbf{K}} = (Tr\mathbf{K})\delta / 2 - \mathbf{K}$, δ is the unit matrix; $\mathbf{s}$, $\mathbf{k}$ are constant vectors and $\mathbf{K}$, $\mathbf{J}$ are 3×3 constant matrices. The current system (1) is identical to the equation of motion of a gyrostat about a fixed point (with a gyrostatic moment $\mathbf{k}$ and a position vector of mass center $\mathbf{s}$) under the influence of gyroscopic forces with moment $-\boldsymbol{\omega} \times \boldsymbol{\gamma}\bar{\mathbf{K}}$ and a force field with potential $\mathbf{s}.\boldsymbol{\gamma} + 1/2 \times \boldsymbol{\gamma}\mathbf{J}.\boldsymbol{\gamma}$. The system (1) admits the three first integrals:

The Jacobi integral:

$$I_1 = \frac{1}{2}\boldsymbol{\omega}.\boldsymbol{\omega}\mathbf{I} + \mathbf{s}.\boldsymbol{\gamma} + \frac{1}{2}\boldsymbol{\gamma}\mathbf{J}.\boldsymbol{\gamma} = h \tag{2}$$

The geometric integral:

$$I_2 = \boldsymbol{\gamma}^2 = 1 \tag{3}$$

The cyclic integral:

$$I_3 = \left(\boldsymbol{\omega}\mathbf{I} + \mathbf{k} + \frac{1}{2}\boldsymbol{\gamma}\bar{\mathbf{K}}\right).\boldsymbol{\gamma} = f, \tag{4}$$

where f, h are the areas and energy constants, respectively.

It is worth mentioning that Yehia [2] proved the motion of an immersed rigid body-gyrostat in an incompressible ideal fluid in the absence of external forces and torques, in addition, the center of mass coincides with the center of the volume equivalent to the motion of a rigid body-gyrostat around a fixed point under the action of a different type of forces having an axis of symmetry. Moreover, Elmandouh [28] proved that the equations of the motion describing the motion of a gyrostat in an incompressible ideal fluid under the influence of neutral forces whose centers are not coinciding are mathematically equivalent to the problem of the motion of a rigid body about a fixed point under the action of certain types of potential and variable gyroscopic forces admitting no axis of symmetry.

3. The Orbital Equation and the Technique of the Solution

In this section, we present the reduced equation of the equations of motion of a rigid body rotating about a fixed point due to the influence of general conservative potential and gyroscopic forces. It is followed by clarifying the procedures used to obtain the complete solution. Following [2], the reduced equation for the equations (1) has been derived and introduced in the form:

$$\begin{aligned}
&ABCD\gamma_2^2\gamma_3'' + ABC^2\left(1-\gamma_3^2\right)\gamma_3 + ABC\gamma_1\left[(A+2C)\gamma_3^2 - A\right]\gamma_3' + ABC\gamma_3\left[C-(C+2A)\gamma_1^2\right]\gamma_3'^2 - A^2BC\gamma_1\left(1-\gamma_1^2\right)\gamma_3'^3 \\
&-\frac{\rho}{D}\{C\gamma_3\left[(A-B)(A+B-C)\gamma_1^2 + B(B-C)\left(1-\gamma_3^2\right)\right] + A\gamma_1[B(A-B)\left(1-\gamma_1^2\right) + (B-C)(B+C-A)\gamma_3^2]\gamma_3'\} \\
&+\frac{\rho}{2(U+h)}\left[(\lambda_2+\lambda_3\gamma_3')\frac{\partial U}{\partial\gamma_1} - (\lambda_1+\lambda_2\gamma_3')\frac{\partial U}{\partial\gamma_3}\right] + \left[\frac{\rho^3}{2D^3(U+h)}\right]^{1/2}\{f[(A-B)(A+B-C)\gamma_1^2 \\
&-B(A-B+C) - (B-C)(B+C-A)\gamma_3^2] + \boldsymbol{\lambda}(\boldsymbol{\gamma})\} = 0,
\end{aligned} \tag{5}$$

where the primes denote derivatives according to γ_1, and

$$\begin{aligned}
&\rho = \lambda_1 + 2\lambda_2\gamma_3' + \lambda_3\gamma_3'^2,\ \lambda_1 = C\left[B\left(1-\gamma_3^2\right) + (A-B)\gamma_1^2\right], \\
&\lambda_2 = AC\gamma_1\gamma_3,\ \lambda_3 = A\left[B\left(1-\gamma_1^2\right) + (C-B)\gamma_3^2\right], \\
&U = -V - \frac{(f - \mathbf{l}.\boldsymbol{\gamma})}{2D},\ \mathbf{l} = \mathbf{k} - \frac{1}{2}\boldsymbol{\gamma}\mathbf{K},\ D = A\gamma_1^2 + B\gamma_2^2 + C\gamma_3^2, \\
&\gamma_2 = \left(1-\gamma_1^2-\gamma_3^2\right)^{1/2},\ \boldsymbol{\lambda}(\boldsymbol{\gamma}) = D^2\frac{\partial}{\partial\boldsymbol{\gamma}}.\left[\frac{1}{D}\boldsymbol{\gamma} \times (\boldsymbol{\gamma}\mathbf{I} \times \mathbf{l})\right].
\end{aligned} \tag{6}$$

In equation (5), V is a potential function depending on the variables γ_1, γ_2, γ_3 only.

Now, the technique of finding solutions to the equations of motion of a rigid body is explained. Let the variable γ_3 be a function in the variable γ_1 in the form

$$\gamma_3 = \gamma_3(\gamma_1) \tag{7}$$

which represents a curve on the Poisson sphere fixed to the rigid body and this curve is described by the end of the vertical unit vector $\boldsymbol{\gamma}$ fixed in space during the motion of the body. Also, This curve is called the orbital of the motion, and equation (5) is called the orbital equation.

Based on the procedures presented in more detail in [2], the reduced equation is assumed to have a solution in the form of equation (7) and consequently, we can now construct the complete solution for the equations of the motion (1). In other words, we aim to express the Euler-Poisson variables as functions of the time t and hence, the complete solution is obtained.

The relation between γ_1 and the time t is given by

$$t = \int \sqrt{\frac{\lambda_1 + 2\lambda_2\gamma_3' + \lambda_3\gamma_3'^2}{2(U+h)\left(1-\gamma_1^2-\gamma_3^2\right)D}}d\gamma_1. \tag{8}$$

Substituting equation (7) into (8) and hence the integrated function becomes a function in γ_1. Therefore, we can get the variable γ_1 in terms of the time t. Using relation (7) and the geometric integral (3), the variables γ_2 and γ_3 can be expressed as functions of the time t. The vector of the angular velocity $\boldsymbol{\omega}$ of the rigid body can be determined by the relation

$$\boldsymbol{\omega} = \dot{\psi}\boldsymbol{\gamma} + \mathbf{N}, \tag{9}$$

where the body precessional angular velocity $\dot{\psi}$ and the vector $\mathbf{N}$ are given by

$$\dot{\psi} = \frac{1}{D}\left(f - \mathbf{l}.\boldsymbol{\gamma} - \boldsymbol{\gamma}\mathbf{I}\cdot\mathbf{N}\right),$$

$$\mathbf{N} = \frac{\left(-\gamma_2\dot{\gamma}_3,\ \gamma_1\dot{\gamma}_3,\ \gamma_2\dot{\gamma}_1 - \gamma_1\dot{\gamma}_2\right)}{1-\gamma_3^2}.$$

Thus, all Euler-Poisson's variables $\boldsymbol{\gamma}$ and $\boldsymbol{\omega}$ are determined in terms of the time t, which can be considered as a complete solution of the equation of the motion (1).

4. A New Solution for the Orbital Equation

We present a new solution to the reduced equation for the potential of the problem of the motion of a rigid body with a multi-connected surface moving in an incompressible ideal infinite fluid with a certain assumption formula to γ_3. Inserting the potential and the assumed form γ_3 into the reduced equation (5), we get a polynomial in γ_1 with all its coefficients equal to zero. By solving the obtained set of equations, we get the parameter values that satisfy the reduced equation representing a new solution to the reduced equation (5). We will consider the problem of the motion of a rigid body with a multi-connected surface moving in an ideal incompressible infinite fluid and has a potential given by the following relationship

$$V = \mathbf{s}.\boldsymbol{\gamma} + \frac{1}{2}\boldsymbol{\gamma}\mathbf{J}.\boldsymbol{\gamma}. \tag{10}$$

Now for the sake of simplicity, we assume a solution to reduced equation (5) in the form

$$\gamma_3 = c, \tag{11}$$

where c is a constant. This means the body moves under the condition that the angle between the body axis and the vertical axis fixed in space (nutation angle) is constant. Inserting the two expressions (10) and (11) in the equation (5), we obtain a polynomial in γ_1 with all of its coefficients should vanish which imply to a system of the equations containing about 24 the parameters: $c, h, f, s_1, s_2, s_3, k_1, k_2, k_3, A, B, C, J_{11}, J_{22}, J_{33}, J_{12}, J_{13}, J_{23}, K_{11}, K_{22}, K_{33}, K_{12}, K_{13}, K_{23}$. We solve these equations and get the consistent following values for the parameters:

- The principal moments of inertia satisfy

$$B = A \tag{12}$$

- The mass centre of the rigid body is located in the xz principal plane of the inertia, so that

$$s_2 = 0, \tag{13}$$

and s_1 is determined by

$$s_1 = \{(B_2+2C)^2[-2cB_2C(k_3+cK_{11}) + (c^2[B_2(A+2C)+4C^2]-B_3)K_{33}] - 2B_1B_3C[c^2(2B_2+3C-A)+A]a_0\}K_{13}\,/\,(2cB_1B_2B_3C), \tag{14}$$

where

$$B_1 = Ac^2 - Cc^2 + A,$$
$$B_2 = Ac^2 - Cc^2 - A - 2C,$$
$$B_3 = A^2c^2 - 2ACc^2 + C^2c^2 - A^2 - AC,$$

$$a_0 = \frac{-2cAB_2C(cK_{11}+k_3)+[(A-C)(A^2+2AC-C^2)c^4-2A(A^2+AC+C^2)c^2+A^3+A^2C]K_{33}}{2B_1B_3C}.$$

-The components of the gyrostatic moment are

$$k_1 = \frac{cAB_2K_{13}}{B_3},\ k_2 = 0. \tag{15}$$

-The elements of the matrices $\mathbf{K}$, $\bar{\mathbf{K}}$ are

$$\begin{aligned} &K_{12}=0, K_{23}=0,\ K_{22}=K_{11},\\ &\bar{K}_{11}=K_{33}/2,\ \bar{K}_{22}=K_{33}/2,\ \bar{K}_{33}=K_{11}-K_{33}/2,\\ &\bar{K}_{12}=0,\ \bar{K}_{23}=0,\ \bar{K}_{13}=-K_{13}. \end{aligned} \tag{16}$$

- The elements of the matrix J is given by

$$\begin{aligned} &J_{12}=0,\ J_{23}=0,\\ &J_{22}=J_{11}+\frac{C(B_2+2C)^2K_{13}^2}{B_3^2},\\ &J_{13}=\{-2cAB_2C(cK_{11}+k_3)+[(A-C)(A^2+2AC-C^2)c^4-2A(A^2+AC+C^2)c^2+A^3+A^2C]K_{33}\}K_{13}/(2B_1B_3C). \end{aligned} \tag{17}$$

- The two parameters f, h are

$$f = \frac{(Ac^4-Cc^4-2Ac^2-Cc^2+A)(AK_{33}-CK_{11})+4cACk_3}{2B_1C}, \tag{18}$$

$$\begin{aligned} h=\{&-4B_1^2B_3^2C^2[c(Ac^4-Cc^4-2Ac^2+A)J_{33}+(Ac^4-Cc^4-2Ac^2-Cc^2+A)s_3]+B_3^2(B_2+2C)^2[(c(B_2+2C)K_{33}+2Ck_3)((B_1-2Ac^2)K_{33}+2cCk_3)\\ &+2cC((Ac^4-Cc^4-2Ac^2-Cc^2+A)K_{33}+4cCk_3)K_{11}+4c^3C^2K_{11}^2]+4cB_1^2C^2(c^2-1)(B_2+C)[B_3^2J_{11}+C(B_2+2C)^2K_{13}^2]\}/(8cB_1^2B_3^2C^3), \end{aligned} \tag{19}$$

and s_3, k_3, K_{11}, K_{33}, K_{13}, J_{11}, J_{33} are arbitrary constants.

5. Complete Solution for Euler-Poisson's Equations

As described in section 3, we will now obtain the components of the vectors $\boldsymbol{\omega}$, $\boldsymbol{\gamma}$ as functions of time. By employing of the values of the parameters (12)-(19) in addition to the solution (11), γ_2 takes the form

$$\gamma_2^2 = 1-\gamma_1^2-c^2, \tag{20}$$

equation (8) can be written in three different forms and the solution for each is obtained in terms of time t as follow:

1- The first form is

$$\int_{\gamma_1}^{a}\frac{d\gamma_1}{\sqrt{(a^2-\gamma_1^2)(\gamma_1^2-b^2)}} = \frac{(B_2+2C)K_{13}}{B_3}t,$$

and integrating we have

$$\gamma_1 = a\ dn(\frac{a(B_2+2C)K_{13}}{B_3}t). \tag{21}$$

2- The second form is

$$\int_{b}^{\gamma_1}\frac{d\gamma_1}{\sqrt{(a^2-\gamma_1^2)(\gamma_1^2-b^2)}} = \frac{(B_2+2C)K_{13}}{B_3}t,$$

and the solution is

$$\gamma_1 = \frac{b}{dn(\frac{a(B_2+2C)K_{13}}{B_3}t)}. \tag{22}$$

For the last two forms $\{a^2=1-c^2,\ b^2=-a_1/a_2\}$ or $\{a^2=-a_1/a_2,\ b^2=1-c^2\}$ where

$$\begin{aligned} a_1 = B_2^2\{&B_3^2[4cAC^2k_3^2+2C((B_3c^2+A^2-ACc^2-A^2c^2)K_{33}+4c^2ACK_{11})k_3-4B_1^2C^2s_3-c\{4C^2B_1^2(J_{33}-J_{11})+(B_3K_{33}\\ &+2ACK_{11})((2Ac_2-B_1)K_{33}-2c^2CK_{11})\}]+4cB_1^2C^3(B_1-2A)^2K_{13}^2\}, \end{aligned}$$

$$a_2 = 4cAB_1^2B_2^2C^4K_{13}^2,$$

The modulus of the elliptic integral is $\sqrt{1-b^2/a^2}$, and in condition that $a_1/a_2<0,\ 0<b<\gamma_1<a$.

3- The third form is

$$\int_{\gamma_1}^{b} \frac{d\gamma_1}{\sqrt{(a^2+\gamma_1^2)(b^2-\gamma_1^2)}} = \frac{(B_2+2C)K_{13}}{B_3} t,$$

and integrating, we obtain

$$\gamma_1 = b\, cn\Big(\frac{\sqrt{a^2+b^2}(B_2+2C)K_{13}}{B_3} t\Big), \tag{23}$$

with modulus of the elliptic integral is $b/\sqrt{a^2+b^2},\ b^2 = 1-c^2,\ a^2 = a_1/a_2,$ and in condition that $a_1/a_2 > 0,\ 0 < \gamma_1 < b.$

Using the equation (9), we get the components of the angular velocity of the body

$$p = \{(B_1-2A)\sqrt{1-c^2-\gamma_1^2}[B_3\{2cCk_3+(B_1-2c^2A)K_{33}+2c^2CK_{11}\}+2cB_1C^2K_{13}\gamma_1]+2cB_1B_3C^2\dot{\gamma}_1\}\gamma_1 / (2B_1B_3C(B_1-2A)\sqrt{1-c^2-\gamma_1^2}),$$

$$q = \{(B_1-2A)\sqrt{1-c^2-\gamma_1^2}[B_3\{2cCk_3+(B_1-2c^2A)K_{33}+2c^2CK_{11}\}+2cB_1C^2K_{13}\gamma_1]+2cB_1B_3C^2\dot{\gamma}_1\} / (2B_1B_3(B_1-2A)), \tag{24}$$

$$r = \{c(B_1-2A)\sqrt{1-c^2-\gamma_1^2}[B_3\{2cCk_3+2A(1-c^2)K_{33}+2c^2CK_{11}\}+2cC^2B_1K_{13}\gamma_1]-2(1-c^2)AB_1B_3C\dot{\gamma}_1\} / (2B_1B_3C(B_1-2A)\sqrt{1-c^2-\gamma_1^2}).$$

From (21)-(23) into (20) and (24), one can express γ_2, p, q, r as elliptic functions in the time t. Finally, one may now verify that the expressions (11), (20)-(24) form a new solution for the equations (1) under the conditions (11)-(18).

The majority of the solutions for the problem under consideration has been summarized and collected in [10]. In a comparison with these solutions, the obtained solution is new.

From the above, it becomes clear to us the importance of the reduced equation in facilitating finding a solution to the problem of the motion of a rigid body with a multi-connected surface moving in an incompressible ideal infinite fluid. It is noted that depending on the imposition of different solutions to the reduced equation, we can get a large number of solutions to the equations of motion of the body depending on the values of the parameters of the issue.

6. Conclusion

The problem of the motion of a rigid body with a multi-connected surface in an incompressible ideal infinite fluid has been considered. This problem has been proved by Yehia [2] is equivalent to the problem of the motion of a rigid body rotating under the influence of potential forces (quadratic in the components of the vector γ which is fixed in the space) and gyroscopic forces (due to the attached rotors in the body which rotating with a constant angular velocity about their axes) and he also reduced the equations of the motion to the orbital equation. Thus, the solution of one problem is a solution for the other problem. In this work, we have assumed the angle between the body axis of symmetry and the vertical axis in space is constant. Utilizing this assumption, we have found a solution for the orbital equation and consequently, to the Euler-Poisson equation describing the motion. This solution is expressed in terms of Jacobi elliptic functions of the time t. In a comparison with the previous results in the literature, the obtained solution is new.

Author Contributions

These authors contributed equally to this work. All authors discussed the results, reviewed, and approved the final version of the manuscript.

Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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