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Research Paper

# Thermal Diffusion Responses in an Infinite Medium with a Spherical Cavity using the Atangana-Baleanu Fractional Operator

Doaa Atta[1,2]

[1] Department of Mathematics, College of Science, Qassim University, P.O. Box 6644, Buraydah 51482, Saudi Arabia
[2] Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

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Corresponding author: D. Atta (doaaatta44@gmail.com)


**Abstract.** The main purpose of this study is to deal with a thermoelastic medium containing a spherical cavity within the framework of partial elastic thermal diffusion theory based on the Atangana-Baleanu operator which is characterized by the presence of a non-local single kernel. The chemical potential of the adjacent cavity is taken as a time-dependent function. The governing equations are represented and solved in the Laplace transform domain and the numerical solutions to the Laplace inversion are obtained to address the problem in the physical domain. In the physical field, the expansion of the Fourier approach is also used to obtain the numerical solutions and the stress-strain behavior of the studied medium is graphically illustrated.

**Keywords:** Thermoelasticity; diffusion; fractional operator; heat flux.

## 1. Introduction

In 1695, fractional calculus appeared after classical calculus for a short time in the published studies due to Liouville, Riemann, and Leibniz. Firstly, scientists have treated fractional calculus as a purely mathematical branch without any real-life, physical, or mechanical applications. In recent decades, this discipline of research has had a wide range of applications in a variety of industries, such as biological electrochemistry, fluid mechanics, viscoelasticity, and electromagnetism.

Many researchers have dealt with fractional calculus and its applications, the most famous of which are Riemann and Liouville (RL) who introduced the fractional time derivative, and Kabuto presented partial time derivatives that are fairly accurate models for a variety of physical events. Tripathi et al. [1] presented a theory based on the quasi-invariant partial and temporal thermal conductivity equation that is not related to the thermoelasticity of a thin circular plate model. Other work has been presented in refs [2-4]. In fact, the partial time derivative operator RL has encountered problems in its implementations, such as the RL operator for the constant is not zero; besides its Laplace transform contains components that have no physical meaning. And although the Caputo fractional time derivative addresses these issues, the kernel of the fractional operator is a single non-computable function. In order to address these shortcomings, two fractional time derivatives have been achieved, namely the fractional Caputo-Fabrizio (CF) time derivative [5] which has a non-single kernel, and the Atangana-Baleanu (AB) fractional time derivative [6] with a non-local non-single nucleus. Atangana and Baleanu introduced the partial time derivative AB in the RL and Caputo senses using non-singular Mittag-Leffler kernels. In the fractional temporal derivative AB, smooth and non-localized cores provide a class of models that better define the dynamics of memory effect systems [7-11].

Parveen [12] investigated the thermal interactions for a fractional order for a thick circular plate under the influence of the conduction of heat in the framework of the two-temperature thermoelasticity hypothesis in the frequency domain. Povstenko [13] presented a solution for the equation for fractional conduction of heat for an infinite material with axial symmetry and a penny-shaped fracture so that its surface is subject to a specific heat flow. Abouelregal [14] developed a novel thermoelasticity mathematical model based on a fractional derivative of the traditional heat conductivity Fourier law and applied it to a nonhomogeneous thermoelastic material with a cylindrical hole. Hammouch [15] investigated certain patterns in space and time in a variety of systems of Belousov–Zhabotinskii reaction using Atangana-Baleanu fractional-order time derivative. Al-Refai [16] examined the nonlinear and linear fractional eigenvalue issues including the order's fractional derivative of the Atangana-Baleanu operator, (see, also, the works [17-19]).

It is well known that the thermodynamic effects caused by the deformation of an elastic material are the thermoelastic response. In other words, it is the ability of a substance to contribute to the expansion of temperature fluctuations when any heat source affects it. The field of heat transfer is the field of thermal engineering concerned with the creation, exchange and conversion of heat through various processes, radiation, phase changes, energy transfer through convection, thermal conductivity, and the like. It is known that Fourier's law directly describes the relationship of the heat flow vector to the temperature gradient so that it results in the equivalent thermal conductivity equation when combined with the entropy equilibrium equation.

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One of the most famous models in the field of thermoelasticity is Lord and Schulman [20] who used Maxwell Le Cattaneo's law of thermal conductivity in the theory of expanded thermoelasticity with one relaxation period instead of the traditional Fourier period. But Green-Lindsey's thermoelastic theory [21], which is based on entropy inequality, provides two relaxation times for the thermoelastic process. Green-Wangdy [22-24] similarly proposed three other modified models of thermoelasticity, denoted by the letters GN-I, GN-II, and GN-III. Dhaliwal and Sharif [25] also presented the theory of generalized thermoelasticity of an anisotropic medium using the heat transfer equation method that includes the time required for the process of accelerating heat flow. Sharif et al. refined such a theory to incorporate microparticle effects [26]. In the presence of a heat and gravitational source, Debkumar and Lahiri [27] investigated an anisotropic half-space of 3D using thermoelastic theory. Recently, Abu al-Rabee [28] used partial order theory to explore the thermoplastic vibration of a viscoelastic microbeam based on Winkler, and the damping motion of the viscoelastic material is characterized based on the experimental data. Sadeghi and Kayani [29] used Green-Lindsay and Lord-Shulman theories to solve and construct a generalized thermoplastic–magnetic response layer. Many other research studies on this topic can be found in references [30-41].

Diffusion is one of nature's most surprising and fascinating processes which play an important role in our daily life where molecular transport is also known as diffusion which is the tendency of a system to approach equilibrium when viewed from a connected perspective. For example, the diffusion of a drop of ink in a glass of water, which eventually stains the entire glass, is called chemical diffusion. Also, when the perfume is sprayed in one section of the room, it soon spreads so that it can be smelled all over the place. Diffusion is involved in many electronic industries such as incorporating doping into semiconductors during the manufacturing process to create integrated resistors.

Nowacki [42-45] has developed the elastic thermal diffusion theory where the thermoplastic model was used. This means that the propagation of thermal elastic waves occurs at infinite speeds. Sherief and Saleh [46] used the theory of thermoplastics diffusion, which deals with one relaxation time for studying the half space issue that is exposed to a heat source. In the framework of Lord and Shulman (LS) theory, Alzahrani [47] demonstrated the mass diffusion impact in a thermoelastic nanoscale beam. Abouelregal et al. [44] studied the effect of the heat and diffusion processes on a medium having a cylindrical hole. Chenlin et al. [49] proposed a comprehensive coupling of thermoelastic diffusion mass diffusion relaxation effects, mechanical deformation, and heat conduction, and solved generalized thermoelastic diffusion difficulties with fractional order strain. Aseem et al. [50] examined a two-dimensional axisymmetric issue for a micropolar circular plate having porous under effects of sources of heat and chemical potential. Davydov and Zemskov [51] explored the propagation of unstable bulk thermoelastic diffusion disturbances. Some other problems are found in the researches [52-58].

We note that the results of Aouadi [59] discussed the problem of an infinite elastic body with a spherical cavity in the ordinary case only, while the results of [60], recently, investigated the same one using the Modified Caputo-Fabrizio's operator with two relaxation parameters only. While, our work in this study discuss a problem of a one dimensional solid having spherical cavity using thermoelastic diffusion with four phase lags, involving Atangana-Baleanu fractional operator with non-singularity kernel. The fractional-order theory is employed in this study to construct the governing equations of thermoelastic diffusion. Heat has been distributed to the spherical cavity's interior surface, which is assumed to be down force. The Laplace transform is used in the transformed domain and the direct approach method are being used to achieve the solution. A comparison is made between the different models of thermoelastic diffusion theory and the effect of phase lag on our problem.

## 2. The Governing Equations

The first step in this study is to establish the fractional-order theory of thermoelastic diffusion's governing equations. In case the body's forces do not exist, the basic equations in modified thermoelastic diffusion theory [61] are as follows:

The equation of motion

$$\rho\ddot{U}_i - \mu\,(U_{i,jj},U_{j,ii}) - \lambda U_{j,ii} + N_1 C_{,i} + N_2 T_{,i} = 0. \tag{1}$$

The entropy equation

$$\dot{\mho}\rho T - (p\eta_i + Q_i)_{,i} = 0\,. \tag{2}$$

The conservation equation of mass

$$\dot{C} + \eta_{i,i} = 0. \tag{3}$$

The constitutive equations

$$\sigma_{ij} - (\lambda\varepsilon_{kk} - N_1 C - N_2 C)\delta_{ij} - 2\mu\varepsilon_{ij} = 0, \tag{4}$$

$$\mho\rho T - T_0(N_2\varepsilon_{kk} + aC) + \rho C_E\Theta = 0, \tag{5}$$

$$\wp + a(T - T_0) - bC + N_1\varepsilon_{kk} = 0, \tag{6}$$

where $\rho$ is the density, $U_i,(i=1,2,3)$ indicate the components of the vector of displacement, of course, Lame's constants are represented by symbols $\mu$, $\lambda$, $\Theta = T - T_0$ refers to the absolute temperature, and C is the diffusion material's concentration in the body, while $N_1$ and $N_2$ are material constants defined as $N_{1,2}$ symbolize the coefficient of linear thermal expansion, and $\varsigma_{c,t}$ denote the coefficient of linear diffusion expansion respectively, where $N_{1,2} = (2\mu + 3\lambda)\varsigma_{c,t}$, $p$ denotes chemical potential, $Q_i$ means heat flux in *i*-th direction, $\eta_i$ implies diffused mass flow vector, $\sigma_{ij}$ is the stress and $\varepsilon_{ij}$ is the strain components, $\mho$ indicates entropy per unit mass, $C_E$ refers to specific heat when the strain is constant, $T_0$ is the temperature that is supposed to satisfy that $|(T - T_0)/T_0| \ll 1$, the thermos-diffusion influence is quantified by $a$, while the diffusion effect is quantified by $b$.

According to the modified Fourier law, the following is the heat conduction equation (see [61]):

$$(\tau_Q D^\alpha + 1)Q = -(\tau_\Theta D^\alpha + 1)\ \nabla k\Theta, \tag{7}$$

where, Q is the heat flux vector.

Likewise, we'll assume an equation of the form for the mass flow vector [61]:

$$\left(\tau_P D^\alpha + 1\right) p_{,i} = -\left(\tau_\eta D^\alpha + 1\right) \eta_i, \tag{8}$$

where $\tau_Q$, $\tau_\Theta, \tau_p$ and $\tau_\eta$ are signify the phase lag of heat flux, the temperature gradient, chemical potential gradient, and the diffusing mass, respectively, $0 < \alpha \leq 1$ is the order of fractional derivatives, $k$ denotes the material thermal conductivity and $D^\alpha$ is the coefficient of diffusion.

Substitute from equation (5) after derivation it with respect to time and also, from equation (3) in the linearized form of equation (2), we get

$$Q_{i,i} = -((N_2 \dot{\varepsilon}_{kk} + a\dot{C})T_0 + C_E \rho\, \Theta). \tag{9}$$

By exploring divergence on both sides of equation (7) and using equation (9), in such case, the equation of heat conduction has been obtained as following:

$$K\left(\tau_\Theta D^\alpha + 1\right)\nabla^2 \Theta = (\tau_Q D^\alpha + 1)((N_2 \varepsilon_{kk} + aC)T_0 + C_E\, \rho\Theta)_{,t}. \tag{10}$$

In the same manner, we can get the equation of diffusion by taking the divergence of all terms of equation (8) and utilizing Eqs. (3) and (6), as well as their time derivatives, i.e.,

$$\left(\tau_p D^\alpha + 1\right)\left(a\Theta_{,ii} - bC_{,ii} + N_1 \varepsilon_{kk,ii}\right) = \frac{\partial}{\partial t}\left(\tau_\eta D^\alpha + 1\right)C. \tag{11}$$

Here, we deal with the Atangana-Baleanu fractional operator (AB), which forms, in Caputo and Riemann-Liouville senses, two fractional derivatives to address the non-localization and non-singularity of kernel that based on the extended the function of Mittag-Leffler. In Caputo's interpretation, the second-order fractional operator Atangana-Baleanu is described as [6]:

$$ABD_t^\alpha f(t) = \frac{1}{1-\alpha}\int_0^t Z_\alpha\left[\frac{\alpha(\ell - t)^\alpha}{1-\alpha}\right]\frac{\partial f(\ell)}{\partial \ell} d\ell, \quad 0 < \alpha \leq 1, \tag{12}$$

where $Z_\alpha\left(-t^\alpha\right) = \sum_{k=1}^{\infty} \frac{(-t)^{\alpha k}}{\Gamma(1+k\alpha)}$, and $Z_\alpha$ is the Mittag-Leffler function in its wider context. In the case of $\alpha = 1$ the constitutive connection can be demonstrated by the material of Kelvin-Voight which has internal linear mechanical energy dissipation.

The relevance of the transform of Laplace operator approach in the differential equations study of is well-known, but, for $0 < \alpha \leq 1$ , Laplace operator may be defined in the following fractional expression [6]:

$$L\left[ABD_t^{(\alpha)} f(t)\right] = \frac{s^\alpha L\left[f(t)\right] - s^{\alpha-1} f(0)}{(1-\alpha)s^\alpha + \alpha}, \quad s > 0. \tag{13}$$

It is obvious that the Laplace transform will be beneficial in the context of Atangana-Baleanu fractional order derivative.

## 3. Problem Formulation

Consider a homogeneous isotropic thermal infinite medium that occupies the area $r_1 \leq R < \infty$ , where $r_1$ is the spherical cavity's radius. Let $(R, \vartheta, \Phi)$ be the spherical polar coordinate system: radial coordinates, common latitude, and longitude, respectively. Because of the symmetry of the spherical cavity, the component of radial displacement $U_r = U(R,t)$ is the only component that does not vanish. The strain tensor's components are as follows:

$$\varepsilon_{rr} = \frac{\partial U}{\partial R}, \qquad \varepsilon_{\vartheta\vartheta} = \varepsilon_{\Phi\Phi} = \frac{U}{R}, \qquad \varepsilon_{R\vartheta} = \varepsilon_{\Phi\vartheta} = \varepsilon_{R\Phi} = 0. \tag{14}$$

The formula for cubic dilatation $\varepsilon$ is as follows:

$$\varepsilon = div(U) = \frac{1}{R^2}\frac{\partial\left(UR^2\right)}{\partial R} = \left(\frac{\partial}{\partial R} + \frac{2}{R}\right)U. \tag{15}$$

According to the previous considerations, equations (1), (4), (6) and equations (9) – (11) can be rewritten as:

$$\frac{\partial^2 U}{\partial t^2}\rho = (\lambda + 2\mu)\frac{\partial \varepsilon}{\partial R} - \frac{\partial}{\partial R}(N_1 C - N_2 T), \tag{16}$$

$$\sigma_{RR} = 2\mu\frac{\partial U}{\partial R} + \lambda\varepsilon - N_1 C - N_2 T, \tag{17}$$

$$\sigma_{\vartheta\vartheta} = \sigma_{\Phi\Phi} = 2\mu\frac{U}{R} + \lambda\varepsilon - N_1 C - N_2 T, \tag{18}$$

$$p = -a\left(T - T_0\right) + bC - N_1\varepsilon, \tag{19}$$

$$K\left(\tau_\Theta D^\alpha + 1\right)\nabla^2\Theta = \frac{\partial}{\partial t}(\tau_Q D^\alpha + 1)((N_2\varepsilon + aC)T_0 + C_E \rho\, \Theta), \tag{20}$$

$$\frac{\partial}{\partial t}\left(\tau_{\eta}D^{\alpha}+1\right)C+\left(\tau_{p}D^{\alpha}+1\right)\nabla^{2}\left(a\Theta-bC+N_{1}\varepsilon\right)=0, \tag{21}$$

where $\nabla^2$ in the spherical coordinates has the form

$$\nabla^{2}=\left(\frac{\partial^{2}}{\partial R^{2}}+\frac{2}{R}\frac{\partial}{\partial R}\right).$$

For more convenience, it would be more appropriate to introduce the non-dimensional variables stated below:

$$\begin{gathered}\left\{r',r_1',u',t'\right\}=\bar{c}\xi\left\{R,r_1,U,\bar{c}t\right\},\qquad \left\{\tau_q',\tau_p',\tau_\theta',\tau_\eta'\right\}=\bar{c}^{-2\alpha}\xi^{\alpha}\left\{\tau_Q,\tau_p,\tau_\Theta,\tau_\eta\right\},P'=\tfrac{p}{N_2},\\ \left\{\sigma_{ij}',q_r',\ C',\ \theta'\right\}=\tfrac{1}{(\lambda+2\mu)}\left\{\sigma_{ij},\tfrac{N_2}{K\bar{c}\xi}Q_r,\bar{c}N_1C,N_2\Theta\right\},\quad \xi K=C_E\rho,\ \bar{c}=\sqrt{(\lambda+2\mu)/\rho}.\end{gathered} \tag{22}$$

The governing Eqs. (13)- (19) adopt the following formulations in regards of the aforementioned dimensionless variables, where the primes have been eliminated for simplicity.

$$\frac{\partial^{2}u}{\partial t^{2}}=\left(\varepsilon-C-\theta\right)_{,r}\ , \tag{23}$$

$$\sigma_{rr}=-\left(\frac{4}{\gamma^{2}}\frac{u}{r}-\varepsilon+C+\theta\right), \tag{24}$$

$$\sigma_{\vartheta\vartheta}=\sigma_{\Phi\Phi}=-\left(\frac{2}{\gamma^{2}}\frac{u}{r}+\left(\frac{2}{\gamma^{2}}-1\right)\varepsilon+C+\theta\right), \tag{25}$$

$$P=-\varepsilon+\Lambda_{1}C-\ \Lambda_{2}\theta, \tag{26}$$

$$\left(\tau_{\theta}D^{\alpha}+1\right)\nabla^{2}\theta=\frac{\partial}{\partial t}\left(\tau_{q}D^{\alpha}+1\right)\left(\varepsilon\wp+\theta+\wp\Lambda_{2}C\right), \tag{27}$$

$$\left(\tau_{p}D^{\alpha}+1\right)\left(\nabla^{2}\varepsilon-\Lambda_{1}\nabla^{2}C+\Lambda_{2}\nabla^{2}\theta\right)+\Lambda_{3}\frac{\partial}{\partial t}(\tau_{\eta}D^{\alpha}+1)C=0, \tag{28}$$

where

$$(\Lambda_{1},\Lambda_{2},\Lambda_{3})=\frac{(\lambda+2\mu)}{N_{1}}\left(\frac{b}{N_{1}},\frac{a}{N_{2}},\frac{1}{\xi N_{2}}\right),\quad \wp=\frac{T_{0}N_{1}^{2}}{\rho(\lambda+2\mu)c_{E}}\quad \text{and}\quad \gamma^{2}=\frac{\lambda+2\mu}{\mu}.$$

Applying the divergence towards both sides of Eq. (21), it yields that

$$\frac{\partial^{2}\varepsilon}{\partial t^{2}}-\nabla^{2}\left(\varepsilon-C-\theta\right)=0. \tag{29}$$

Equation (7) also will take the following modified form:

$$(\tau_{q}D^{\alpha}+1)q_{r}+\left(\tau_{\theta}D^{\alpha}+1\right)\frac{\partial\theta}{\partial r}=0. \tag{30}$$

As we will see in the following cases, the issue is completed by adding boundary conditions to the homogeneous initial conditions:

Free traction

$$\sigma_{rr}\left(r_{1},t\right)=0. \tag{31}$$

A thermal shock is applied to the cavity surface

$$q_{0}H\left(t\right)-Q_{r}-g\theta\left(r_{1},t\right)=0. \tag{32}$$

In the case of the chemical potential is function of time

$$p\left(r_{1},t\right)-p_{0}h\left(t\right)=0, \tag{33}$$

where $q_0$ is the applied heat source intensity, $k(.)$ is the unit step of the Heaviside function, $g$ is the non-dimensional convective heat transfer and $p_0$ is the supplied heat source intensity.

## 4. Formulas after the Effect of Laplace Transform

The transforming of Laplace is expressed as follows:

$$\int_{0}^{\infty}e^{-st}\ \ W\left(x,t\right)\,dt=\overline{W}\left(x,s\right). \tag{34}$$

So, after inserting it on Eqs. (21)- (28), we get them in their new configuration as:

$$\nabla^2\left(\bar{\varepsilon}-\bar{C}-\bar{\theta}\right)=s^2\bar{\varepsilon}, \tag{35}$$

$$\nabla^2\bar{\theta}=\wp s\frac{\Omega_q}{\Omega_\theta}\left(\bar{\varepsilon}+\Lambda_2\bar{C}+\frac{\bar{\theta}}{\wp}\right), \tag{36}$$

$$\nabla^2\left(\bar{\varepsilon}-\Lambda_1\bar{C}+\Lambda_2\bar{\theta}\right)=-s\Lambda_3\frac{\Omega_\eta}{\Omega_p}\bar{C}, \tag{37}$$

where $\Omega_i=1+\frac{s^\alpha\ \tau_i}{(1-\alpha)s^\alpha+\alpha}$, $(i=p,q,\theta,\eta)$,

$$\bar{\sigma}_{rr}=-\left(\frac{4}{\gamma^2}\frac{\bar{u}}{r}-\bar{\varepsilon}+\bar{C}+\bar{\theta}\right), \tag{38}$$

$$\bar{\sigma}_{\vartheta\vartheta}=\bar{\sigma}_{\Phi\Phi}=-\left(\frac{2}{\gamma^2}\frac{\bar{u}}{r}+\left(\frac{2}{\gamma^2}-1\right)\bar{\varepsilon}+\bar{C}+\bar{\theta}\right), \tag{39}$$

$$\bar{P}=-\bar{\varepsilon}+\Lambda_1\bar{C}-\Lambda_2\bar{\theta}, \tag{40}$$

$$\Omega_q\bar{q}_r+\Omega_\theta\,\bar{\theta}_{,r}=0. \tag{41}$$

The modified boundary conditions (31) - (33) also, can be transformed to take the form:

$$\begin{gathered}\bar{\sigma}_{rr}(r_1,s)=0,\\ \tfrac{Q_0}{s}-\bar{q}_r-h\bar{\theta}(r_1,t)=0,\\ \bar{P}(r_1,s)-\tfrac{P_0}{s}=0.\end{gathered} \tag{42}$$

By dropping $\bar{e}$ and $\bar{C}$ from Eqs. (35) – (37), this will yield the next equations

$$(\nabla^6-\zeta_1\nabla^4+\zeta_2\nabla^2-\zeta_3)\bar{\theta}=0, \tag{43}$$

$$\begin{gathered}\zeta_1=\tfrac{s}{\Lambda_1-1}\left(\tfrac{\Omega_q}{\Omega_\theta}\left(\Lambda_2\wp((\Lambda_2+2)+\Lambda_1(\wp+1)-1)\right)+\Lambda_3\tfrac{\Omega_\eta}{\Omega_p}+\Lambda_1 s\right),\\ \zeta_2=\tfrac{s^2}{\Lambda_1-1}\left(\tfrac{\Omega_q}{\Omega_\theta}((\Lambda_2^2\wp+\Lambda_1)s+\Lambda_3\tfrac{\Omega_\eta}{\Omega_p}(\wp+1))+\Lambda_3\tfrac{\Omega_\eta}{\Omega_p}s\right),\\ \zeta_3=\tfrac{\Lambda_3 s^4}{(\Lambda_1-1)}\tfrac{\Omega_q\Omega_\eta}{\Omega_p\Omega_\theta}.\end{gathered} \tag{44}$$

Using a similar mechanism, the following equations can be obtained:

$$(\nabla^6-\zeta_1\nabla^4+\zeta_2\nabla^2-\zeta_3)\bar{C}=0, \tag{45}$$

$$(\nabla^6-\zeta_1\nabla^4+\zeta_2\nabla^2-\zeta_3)\bar{\varepsilon}=0. \tag{46}$$

The factorization of equation (43) can be written as in the following:

$$\prod_{i=1}^{3}(\nabla^2-\chi_i^2)\bar{\theta}=0. \tag{47}$$

The characteristic equation of Eq. (47) is written as:

$$\chi^6-\zeta_1\chi^4+\zeta_2\chi^2-\zeta_3=0. \tag{48}$$

In this case $\chi_1^2,\chi_2^2$ and $\chi_3^2$ denote the characteristic equation roots with real components will be in the form:

$$\chi_1=\sqrt{[\zeta_1+2f_1\sin(f_2)\ \ ]/3}, \tag{49}$$

$$\chi_2=\sqrt{[\zeta_1-(\sin(f_2)\ +\sqrt{3}\cos(f_2))\ f_1\ ]/3}, \tag{50}$$

$$\chi_3=\sqrt{[\zeta_1-(\sin(f_2)\ -\sqrt{3}\cos(f_2)\ )f_1\ ]/3}, \tag{51}$$

where $f_1=\sqrt{\zeta_1^2-3\zeta_2}$, $f_2=\frac{\sin^{-1}(f_3)}{3}$, $f_3=-\frac{(2\zeta_1^3-9\zeta_1\zeta_2+27\zeta_3)}{2f_1^3}$.

Equation (43) has the appropriate solution:

$$\bar{\theta}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}J_{1/2}(\chi_i r)\ \mu_i(s). \tag{52}$$

here, $J_{1/2}(.)$ indicates the Bessel functions of second type of order $1/2$ in its modified form Eqs. (45) and (46) have similar solutions, which may be expressed as:

$$\bar{C}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}J_{1/2}(\chi_i r)\ \mu_i'(s), \tag{53}$$

$$\bar{\varepsilon}(r,s)=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}J_{1/2}(\chi_i r)\ \mu_i''(s), \tag{54}$$

where, $\mu_i$, $\mu_i'$ and $\mu_i''$ are parameters that only depending on $s$ .

Subrogating Eqs. (52) – (54) in Eqs. (35) – (37) produces:

$$\mu_i'(s)=\frac{\chi_i^4\Omega_\theta-\chi_i^2\left[s^2\Omega_\theta+(1+\wp)s\Omega_q\right]+s^3\Omega_q}{\wp s\Omega_q[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2]}\mu_i(s),$$

$$\mu_i''(s)=\frac{\chi_i^2[\chi_i^2\Omega_\theta-(1-\wp\Lambda_2)s\Omega_q]}{\wp s\Omega_q[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2]}\mu_i(s).$$

Now, after substituting of the parameters $C_i$ and $\varepsilon_i$ in equations (53) and (45), the modified equations take the form:

$$\bar{C}=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}\frac{\chi_i^4\Omega_\theta-\chi_i^2\left[s^2\Omega_\theta+(1+\wp)s\Omega_q\right]+s^3\Omega_q}{\wp s\Omega_q\left[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2\right]}J_{1/2}(\chi_i r)\ \mu_i(s), \tag{55}$$

$$\bar{\varepsilon}=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}\frac{\chi_i^2\left[\chi_i^2\Omega_\theta-(1-\wp\Lambda_2)s\Omega_q\right]}{\varepsilon s\Omega_q\left[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2\right]}J_{1/2}(\chi_i r)\ \mu_i(s). \tag{56}$$

We get at by differentiating the two sides of equations (52), (55) and (56) in terms of $r$ and inserting the resultant expressions to Eqs. (23) - (26) in the non-dimensional form of transformed domain:

$$\bar{u}=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}\frac{\chi_i\left[-\chi_i^2\Omega_\theta+(1-\wp\Lambda_2)s\Omega_q\right]}{\wp s\Omega_q\left[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2\right]}J_{3/2}(\chi_i r)\ \mu_i(s), \tag{57}$$

$$\bar{\sigma}_{rr}=\frac{-1}{\sqrt{r}}\sum_{i=1}^{3}\frac{s\left[-\chi_i^2\Omega_\theta+(1-\wp\Lambda_2)s\Omega_q\right]}{\wp\Omega_q\left[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2\right]}\mu_i(s)[J_{1/2}(\chi_i r)+\frac{\Psi_1}{r}J_{3/2}(\chi_i r)], \tag{58}$$

$$\bar{\sigma}_{\vartheta\vartheta}=\frac{-1}{\sqrt{r}}\sum_{i=1}^{3}\frac{\left[-\chi_i^2\Omega_\theta+(1-\wp\Lambda_2)s\Omega_q\right]}{\wp s\Omega_q\left[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2\right]}\mu_i(s)\left(\Psi_2 J_{1/2}(\chi_i r)+\frac{\Psi_3}{r}J_{3/2}(\chi_i r)\right), \tag{59}$$

$$\bar{P}=\frac{\Omega_\eta\alpha_3}{\Omega_p\sqrt{r}}\sum_{i=1}^{3}\frac{\chi_i^4\Omega_\theta-\chi_i^2\left[s^2\Omega_\theta+(1+\wp)s\Omega_q\right]+s^3\Omega_q}{\wp\Omega_q\chi_i^2\left[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2\right]}\mu_i(s)J_{1/2}(\chi_i r), \tag{60}$$

$$\Psi_1=\frac{4\chi_i}{\gamma^2 s^2},\ \Psi_2=\left(s^2-\frac{2\chi_i^2}{\gamma^2}\right),\ \Psi_3=-\frac{2\chi_i}{\gamma^2}.$$

The following features of modified Bessel functions were used to reach the aforementioned answer [62]

$$\frac{dJ_{1/2}(\chi r)}{dr}=-\chi J_{3/2}(\chi r)+\frac{1}{2r}J_{1/2}(\chi r). \tag{61}$$

Applying the transforming of Laplace on the boundary conditions (42) and then sing Eqs. (52), (58) and (60) to assess the unknown parameters $\mu_i$, $(i=1,2,3)$ . As a consequence, the set of linear equations shown below develops:

$$\sum_{i=1}^{3}\frac{s\left[\chi_i^2\Omega_\theta-(1-\wp\Lambda_2)s\Omega_q\right]}{\wp\Omega_q\left[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2\right]}(J_{1/2}(\chi_i r_1)+\frac{\Psi_1}{r_1}J_{3/2}(\chi_i r_1))\mu_i(s)=0, \tag{62}$$

$$\frac{q_0\Omega_q\sqrt{r_1}}{s}=\sum_{i=1}^{3}(\Omega_\theta\chi_i J_{3/2}(\chi_i r_1)+\Omega_q g J_{1/2}(\chi_i r_1))\mu_i(s), \tag{63}$$

$$\frac{p_0\wp\Omega_p\Omega_q\sqrt{r_1}}{s\Lambda_3\Omega_\eta}=\sum_{i=1}^{3}\frac{\chi_i^4\Omega_\theta-\chi_i^2\left[s^2\Omega_\theta+(1+\wp)s\Omega_q\right]+s^3\Omega_q}{\chi_i^2\left[(1+\Lambda_2)\chi_i^2-\Lambda_2 s^2\right]}J_{1/2}(\chi_i r_1)\mu_i(s). \tag{64}$$

After the linear system of Eqs. (62)- (64) is being solved, the unknown parameters $\mu_1$, $\mu_2$ and $\mu_3$ can indeed be derive and our problem in framework of the transforming of Laplace is now complete.

## 5. The Inverse of Laplace Transforms

Next, we face a lot of long and complex expressions when obtaining answers in the transformed domain, therefore, we use a numerical approach used by Durbin [63] to reverse the direction of Laplace twists to get solutions in a physical field. The inversion solutions may be obtained using the following formula in this technique:

$$M(t)=\left(2\,\mathrm{Re}\sum_{k=1}^{m}M(c_1+ic_2)e^{-ic_2t}+M(c_1)\right)\frac{e^{c_1t}}{2t_1},$$

$$c_2=\frac{\pi k}{t_1}, \tag{65}$$

where $m$ is an integer large enough that it denotes the parts number in the infinite Fourier series which has been truncated and it should be selected as this

$$\mathrm{Re}(M(c_1+i\pi m/t_1)\ e^{-i\pi mt/t_1})e^{c_1t}\le\varepsilon,$$

where $c_1$ is a real integer bigger than the sum of the real parts of all the singularities, and $\varepsilon$ is a very little positive number corresponding to the level of accuracy necessary. For speeding up the series convergence in Equation (61), the $\varepsilon$ -algorithm [63] has been used.

## 6. Numerical Results

The homogeneous isotropic thermal infinite medium is taken to be made of copper material. Air is assumed to be the penetrating material that comes into touch with the spherical cavity's interior surface. Material factors in the problem are thus stated in SI units [64, 65] as:

$$C_E=383.1\ kg^{-1}K^{-1},\quad K=386\ W\ m^{-1}K^{-1},\quad \mu=3.86\times10^{10}N\ m^{-2}, r=8954\ kg\ m^{-3}$$

$$\lambda=7.76\times10^{10}N\ m^{-2},\ N_1=1.98\times10^{-4}m^3kg^{-1},\quad N_2=1.78\times10^{-5}\ K^{-1},\ \tau_q=0.2,$$

$$\tau_\theta=0.1\ ,\ \tau_\eta=0.05\ ,\ \tau_p=0.03,\ a=1.2\times10^4m^2K^{-1}s^{-2},\quad b=0.9\times10^6m^5kg^{-1}s^{-2},$$

$$T_0=293\ K,\quad D=0.85\times10^{-8}kg\ s\ m^{-2}.$$

Applying these data, it is easily obtained that

$$\xi=8886.73,\quad \wp=0.0168,\quad \gamma=2,\quad \Lambda_1=0.53,\quad \Lambda_2=5.43,\ \text{and}\ \ \Lambda_3=36.24$$

where $r_1=1$, $q_0=1$ and $g=1$ are the radius and non-dimensional convective heat transfer coefficients of a spherical cavity.

### 6.1. Comparison between different thermoelastic diffusion models

In this section, a comparison will be produced between the different models of thermoelastic diffusion theory, which can be obtained as a special case of the proposed model. The following numerical results were for dimensionless time $t=0.05$ for physical quantities using this data; the problem is solved by using Laplace transform and Laplace inversion. The distributions of dimensionless temperature $\theta(r,t)$, the radial displacement $u(r,t)$, radial thermal stress $\sigma_{rr}(r,t)$, hoop thermal stress $\sigma_{\theta\theta}(r,t)$ and concentration of mass $C$ were evaluated and shown graphically. The comparison of the different models using the operator of Atangana-Baleanu will be performed in case of the fractional is absence, the used models are the classical model of thermoelastic diffusion (CTED) proposed by Nowacki [42] that can be obtained when $(\tau_\theta=\tau_q=\tau_\eta=\tau_p=0)$, thermal elastic diffusion of Lord and Shulman (TDLS) proposed by Sherif [64] which is acquired when $(\tau_\theta=\tau_p=0)$, thermoelastic diffusion with phase lag (TDDPL) proposed by Aloudi [65]. While in the presence of fractional, we used fractional thermal elastic diffusion of Lord and Shulman (FTDLS), and fractional thermoelastic diffusion with phase lag (FTDDPL) at $\alpha=0.5$.

Figures 1 to 5 are plotted for the mentioned physical quantities in the range $1\le r\le 3$. Figure 1 presents the distribution of the temperature $\theta(r,t)$ with the radius r; it can be shown that the temperature distribution takes its highest value in the CTED model, while the heat distribution decreases in the case of the FTDDPL model, this means that the fraction deduce the heat wave. It is clear also that there is an inverse relationship between the heat distribution and the radius and. All the models in figure 2 show that the displacements distribution gradually decrease to reach zero at $r=1.25$ then take negative values to rise slightly again to the opposite side until they reach zero again near $r=1.8$. It is noticeable that the fracture has an obvious impact on the displacement distribution. The stress maximum values rightward shift as speed rises, as seen in figure 3, demonstrate the influence of the different models on the radial stress $\sigma_{rr}(r,t)$ against radial distance $r$, the stress decreases in value throughout the medium $1\le r\le 1.2$ after that it increases tell reaches $r=3$, the radial stress the minimum values occurs when CTED model applies while it reaches its maximum one occurs at the duplicated one TDDPL. Figure 4 depicts the models effects of on the $\sigma_{\theta\theta}(r,t)$ along the radial distance $r$, the wave of the stress starts with negative values and increase gradually to intersect with the radius line near $r=1.25$ then rises to take positive values, but soon it goes down again to reach its zero value and settle on this value tell $r=3$. The inclusion of a fractional order parameter $\alpha$ has a meaningful impact on stress distribution.

In figure 5, a comparison of various models on mass concentration $C$ is observed. This figure clarifies that in the case of CTED model, the concentration of the diffusion material is greater than in the case of FTDDPL, this is a normal results, as the existence of fraction in FTDDPL reduces the values of the physical quantities. The fractional impact is entirely apparent on the mass concentration $C$.

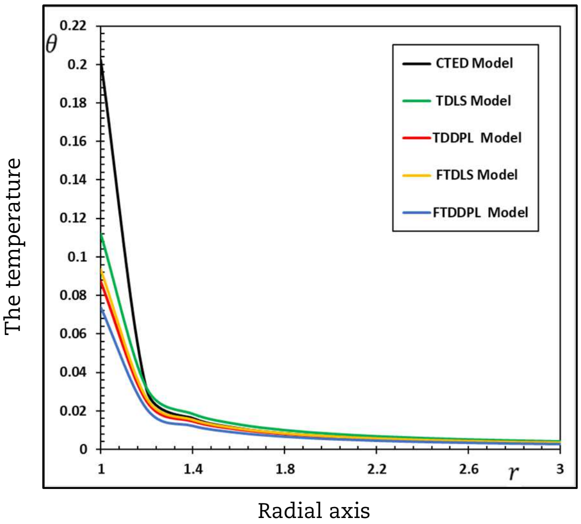


**Fig. 1.** The temperature $\theta$ for different models

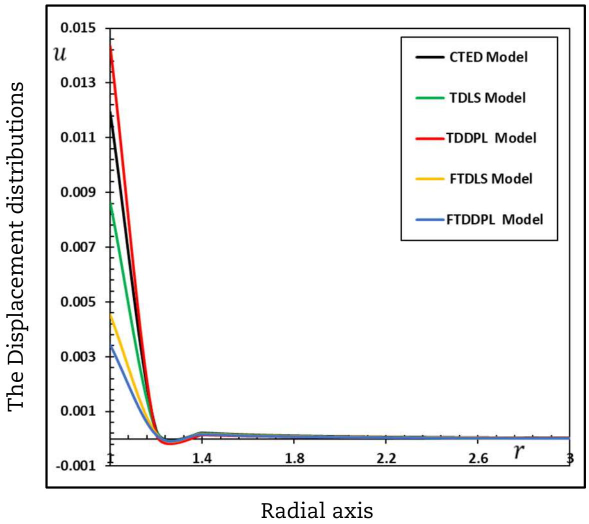


**Fig. 2.** The Displacement distributions $u$ for different models

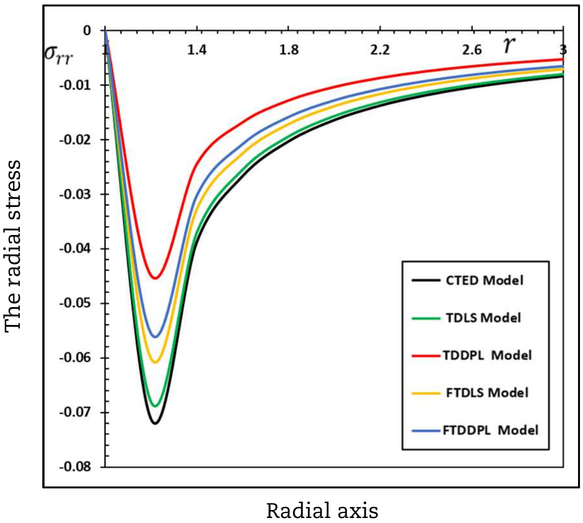


**Fig. 3.** The radial stress $\sigma_{rr}$ for different models

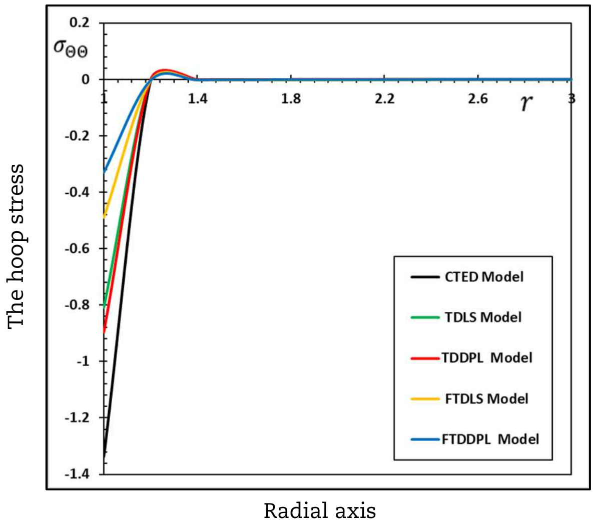


**Fig. 4.** The hoop stress $\sigma_{\theta\theta}$ for different models

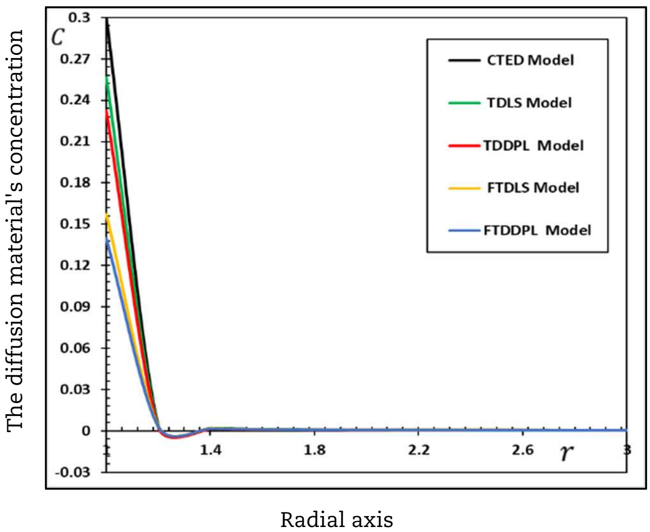


**Fig. 5.** The Concentration $C$ for different models

### 6.2. The effect of fraction order

Figures 6 to 9 depict the effect of the fractional order on the temperature change $\theta(r,t)$, the radial displacement $u(r,t)$, thermal stress $\sigma_{rr}(r,t)$ and $\sigma_{\theta\theta}(r,t)$. For the fractional order parameter that has a large impact FDDPL model that including Atangana–Baleanu operator which has no singularity point. Figure 6 shows the temperature change against radial radius r for various fractional parameter values. It can be shown that the decay rate of the thermal distribution be faster for the fractional order $\alpha$=0.5 than its counterpart $(\alpha = 0.7, 0.8, 0.9, 1)$. Figure 7 displays the displacement field $u$ inside the medium against radius $r$ fluctuation for FDDPL Model and with different values of $\alpha$. It can be shown that in the interval $1 \le r \le 3$, as the value of the fractional order parameter grows, so does the peak of thermal displacement. The inclusion of fractional parameter $\alpha$ has a substantial influence on the solution of the stress distribution $\sigma_{rr}$, as shown in figure 8, for constant values of $r$, the increase for $\alpha$ leads to a decrease in the stress distribution. Figure 9 illustrates that the curves of hoop stress $\sigma_{\theta\theta}$ change starting from negative values at r=1 to intersect all at r=1.2 to reach their values at zero and then rise a little above a radius line taking positive values and after a not long time the values decrease again to reach zero and this zero situation continues until the end of the wave. The angular stress is reduced in magnitude across the medium when fractional parameter is increased.

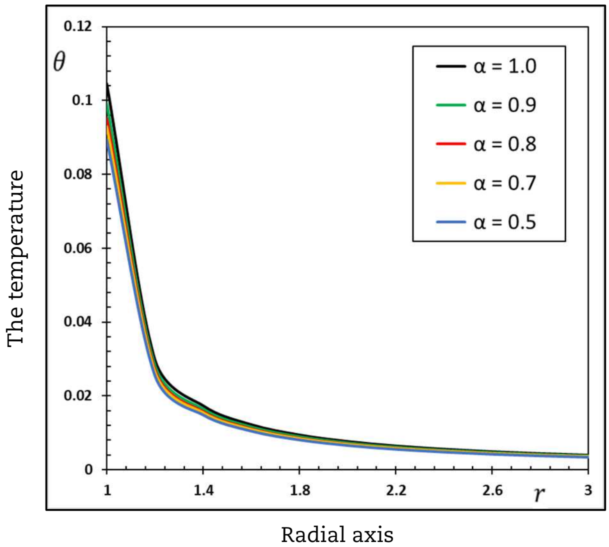


**Fig. 6.** The temperature $\theta$ versus fractional parameter $\alpha$

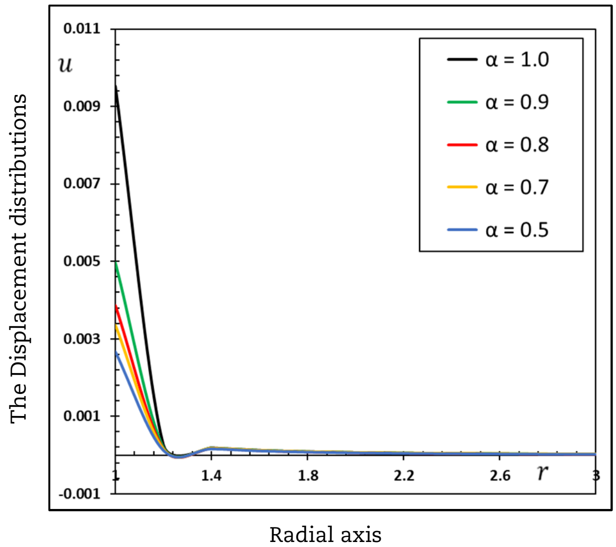


**Fig. 7.** The Displacement distributions $u$ versus fractional parameter $\alpha$

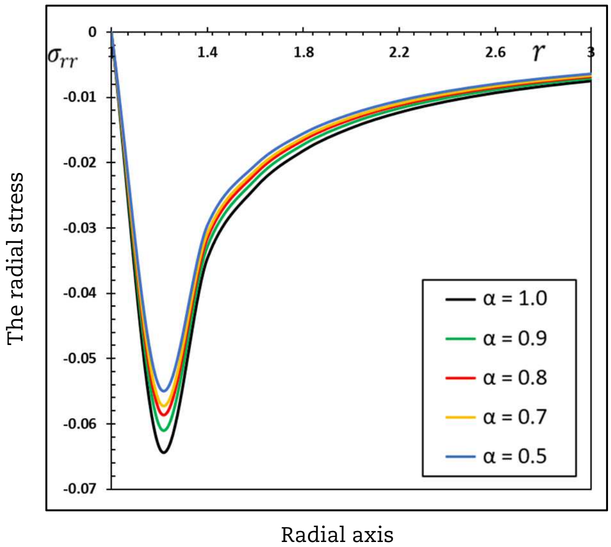


**Fig. 8.** The radial stress $\sigma_{rr}$ versus fractional parameter $\alpha$

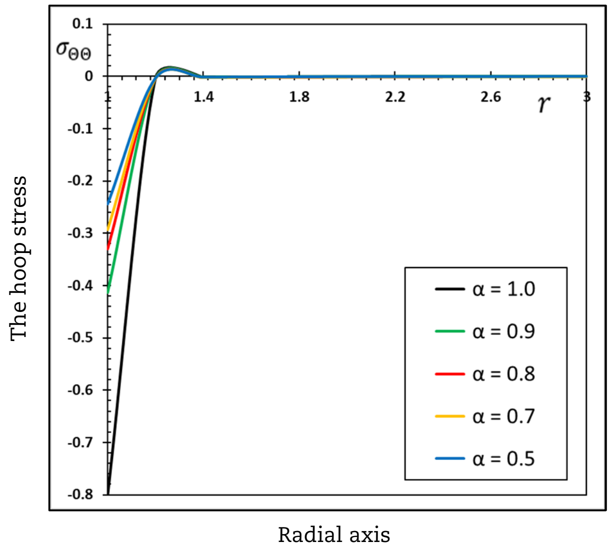


**Fig. 9.** The hoop stress $\sigma_{\theta\theta}$ versus fractional parameter $\alpha$

## 7. Conclusions

This article presented a thermoelastic partial diffusion model including the Atangana-Baleanu operator. This operator has all the properties of the other operators however; it is distinguished from them by the presence of a non-singular kernel. This operator was applied to a thermoplastic solid with a spherical cavity whose surfaces are controlled by heat flow. The Laplace transform has been used to deal the governing equations of our problem. We from the main results we derive the following:

- Atangana-Baleanu model has a prominent influence on different distributions
- The initial and boundary conditions are fulfilled by all physical quantities.
- As the radial axis $r$ is increased, the values of all physical quantities' distributions converge to zero.
- There is a clear difference in the results in the case of presence and absence of fractional.

- When the modulus of the fractional order is reduced, it causes the expected wave propagation velocity to increase, which means that the fracture reduces thermal and mechanical waves.
- The theory of elastic thermal diffusion with four phase lags using the Atangana-Baleanu model overcomes the disadvantages that others have encountered in solving this issue by traditional methods.

It is interesting to generalize the results of this paper to be in two dimensions

## Author Contributions

This article has one author.

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There are no competing interests

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Not applicable

## Nomenclature

| | | | |
|---|---|---|---|
| $A_c$ | The cross-sectional area of micro-channel [$m^2$] | $T$ | Local mean temperature [K] |
| $L_x$ | Length of heat sink [m] | $U_i$ | Mean velocity components ($i$ = 1, 2, 3) |
| $R_{th}$ | Thermal resistance [°C/W] | ⸜, ⸝ | Design variables, $W_c/H_c$ and $W_w/H_c$ |

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## ORCID iD

Doaa Atta https://orcid.org/0000-0002-7802-2299



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