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Research Paper

Free Vibration Analysis of 2D Functionally Graded Strip Beam using Finite Element Method

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Abstract. This article aims to investigate the free vibration of axial and bi-directional functionally graded (2D-FG) two-dimensional plane stress strip by using finite element method. The rule of mixture based on Vogit model is proposed to describe the change in the volume fractions of metal and ceramics constituents. The materials are graded continuously and smoothly in both axial and thickness directions according to the power law formula. Two-dimensional plane stress constitutive equations are proposed to describe the stress and strain across the beam domain. Finite element model using ANSYS software is developed to discretize the spatial domain of strip and modal solution is exploited to evaluate the eigenvalues (natural frequencies) and mode shapes of 2D FG strip beam. The effects of materials gradation in axial and bi-directional and boundary conditions on the natural frequencies are investigated. The proposed model can be used in design and analysis of 2D-FG structures manufactured from two different constituents and selecting the optimum gradation parameter based on the natural frequency's constraints, such as naval, nuclear and aerospace structures.

Keywords: Functionally Graded Strip; Plane stress problem; Free vibration analysis; Axial gradient; 2D gradient; Finite element analysis.

1. Introduction

Functionally graded material refers to the composite material, where the material composition varied from layer to layer or varied in a certain direction in which the material achieved the required material properties. The FGM can be designed for specific applications and functions. The term FGM was discovered in the last of the 20th century by two experts in Japan. After that, the power to continue the invention and discover new materials with high resistance using FGM.

The FGM is made from a mixture of ceramic and metal, according to that gradation of material the FGM enhanced thermal resistance with low thermal conductivity due to ceramic material and improve mechanical properties due to metal material. The FGM is fabricated by a certain process like bulk processing. The FGM is commonly used in aerospace and nuclear applications, cutting tools, and engine components [1-3]. According to the variety used of FGM, many researchers studied the FGM statically and dynamically as plate [4-7], shells [8-11], microbeam [12-16], and nanobeams [17-18] shapes. The effort of studying the FGM as a beam structure is limited compared with other shapes, so beam shape will be considered in this paper. FGM properties could be changed through the thickness or longitudinal axis, in recent studies they employed the material gradation in two dimensional (2D).

Many studies utilized the gradation of material as a layered beam or through the thickness [18-28]. For the thermal application and layered FG beam, Bashiri et al. [19] have introduced the dynamic response of a multilayered FG beam where the beam is proposed to temperature change. Power law formulation was used to define the material gradation through the thickness of each layer. The stress continuum model was also used and the equation of motion was derived. The Newmark method was applied to solve the time domain incrementally. Alnujaie et al. [20] introduced the dynamic response of a porous FG beam under a sinusoidal point load. Asiri et al. [21] analyzed the dynamic viscoelastic response of FG thick beam under a dynamic load. Jena et al. [22] introduced the vibration of FG nanobeam which contains a porosity in beam structure. Shabani and Cunedioglu [23] introduced free vibration by using Timoshenko beam theory of multilayered symmetric sandwich beam. The beam consisted of 50 layers and each layer had a different material than the other. The FG beam cracked on two edges and they used power-law exponential formula for material distribution of the beam. To obtain the natural frequency of cracked beam, they used a developed code showing good agreement compared with the results in the literature. Su et al. [24] studied the multiple-stepped FG beam and assumed the material gradation by power-law formulation by using the first-order shear deformation theory. Lee and Lee [25] analyzed free vibration of FG beam by applying Bernoulli beam theory using

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an exact transfer matrix for obtaining the natural frequencies. Jing et al. [26] studied the static and free vibrations of FG beam and used a combination of finite volume and Timoshenko beam theories. The FG beam equation of motion was derived by Hamilton's principle. Li et al. [27] studied the free vibration of the FG beam by using both classical and the first-order shear deformation beam theories. Liu and Shu [28] carried out the free vibration of the FG beam which the material properties had the exponential gradation.

In the case of longitudinal (Axial) gradation, more studies were conducted in recent years [29-33]. Cao et al. [29] carried out the free vibration analysis of the FG beam and utilized the asymptotic development method to investigate the dynamic behavior of FG beam. Li et al. [30] investigated the free vibration of the FG beam where the beam properties changed according to the exponential formulation. Akgöz and Civalek [31] analyzed free vibration of FG strain bar and employed the Rayleigh-Ritz method for determining free vibrations mode shapes of the bar. Shahba and Rajasekaran [32] studied the free vibration of tapered Bernoulli FG beam. They used finite element method for obtaining the natural frequencies. Alshorbagy et al. [33] studied the dynamic characteristics of FG beams. The beam condition was simply supported adopting the Euler-Bernoulli's theory of beams and the principle of virtual work was used to achieve the system's governing equations.

In recent years, the material gradient in both directions means the material properties change simultaneously in axial and thickness directions. A few researchers carried out that type of material change [34-37]. Fariborz and Batra [34] analyzed the free vibration of curved beams by applying the shear deformation theory. They used the Hamilton's principle to derive the equation of motion. Ahlawat [35] studied the vibrations of bi-directional circular plate by assuming the Kirchhoff's plate theory. The equation of motion was derived by a differential quadratic method. The power-law formula was also used to define the material changes through the plate's thickness. Pydah and Sabale [36] analyzed the static behavior of a circular beam subjected to various tip loads.

In this study, free vibration of 2D gradation FG strip beam is studied where the material properties change according to the power-law formulation. Finite element analysis by ANSYS software is exploited to evaluate the natural frequencies and mode shapes of bi-directional functionally graded material. This research carries out the material gradation of the 2D plane stress strip with FG gradation through two dimensions simultaneously. The obtained results are compared with the current results in the literature. Finally, the natural frequencies and the effect of boundary conditions are comprehensively investigated.

2. Functionally Graded Beam

A FGM beam with uniform cross-section has been considered in this study. The beam's dimensions are denoted by L for length, B for width and H for height with a coordinate system (Oxyz) as shown in Fig. 1.

The material properties of the beam such as Young's modulus (E) and density (ρ) changing smoothly through the axial (-x/L to x/L) and thickness directions (-y/H to y/H) (2D direction), as shown in Fig. 2. The material gradient is according to the power law index [35], so the material properties of the beam are given as:

$$P(x,y) = \left[(P_c - P_m)(\frac{x}{L}+\frac{1}{2})^{\alpha_x} + P_m\right] e^{\alpha_y \frac{y}{H}} \tag{1}$$

where (P_c, P_m) is the material properties (Young's modulus and density) of the ceramic and metal segments, respectively, α_x and α_y are the non-negative material gradient indexes (0, 0.2, 0.4 …10) in the axial and thickness directions, respectively.

The variation of the beam material along the axial axis is shown in Fig. 3. In this situation, the material gradient is changed from left (-x/L) to the right (x/L) (axial gradient). The variation of Young modulus is along the beam axis with different gradient index (α) as presented in Fig. 4. Therefore, the thickness gradient will not be considered in this case ($\alpha_y = 0$) and the material properties of the beam is given by:

$$P(x,y) = \left[(P_c - P_m)(\frac{x}{L}+\frac{1}{2})^{\alpha_x} + P_m\right] \tag{2}$$

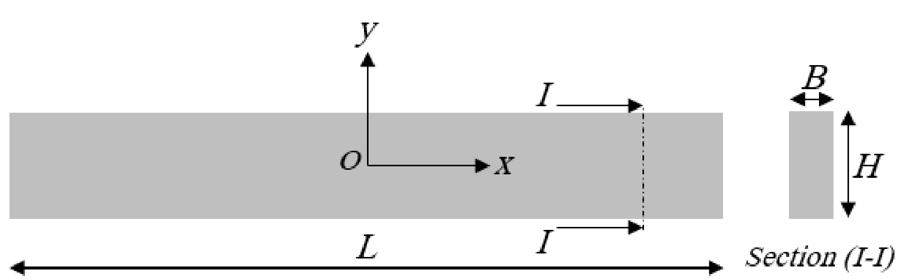

Fig. 1. Geometry and coordinate systems of beam.

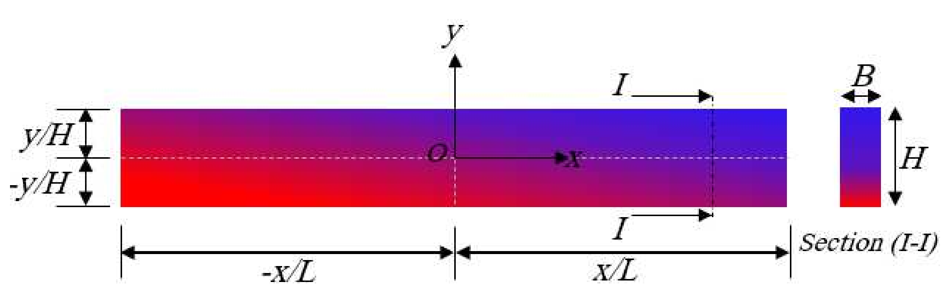

Fig. 2. FG beam (2D gradient).

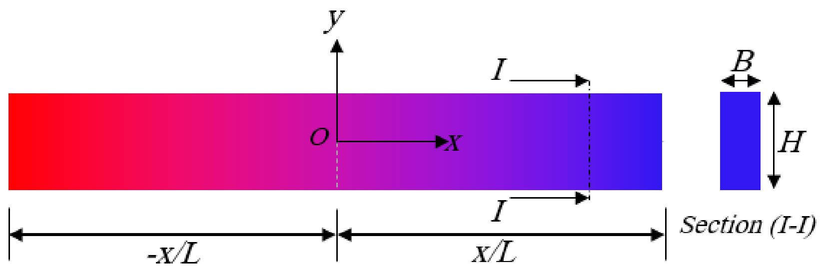

Fig. 3. FG beam (axial gradient).

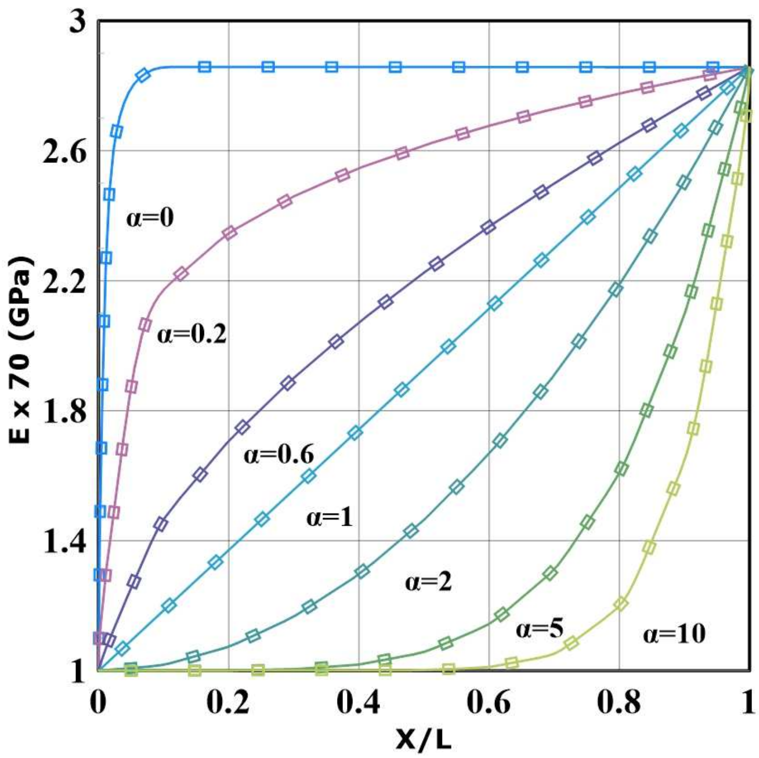

Fig. 4. Variation of the material properties (axial gradient).

3. Results and Discussion

In this section, two cases of material gradient results have discovered respectively. The FG beam consists of two materials (Aluminum (AL) and Zirconia (ZrO_2)). The left side of the beam is pure aluminum as a metal segment whereas the right side of the beam is pure zirconia as a ceramic part. The material properties of FG beam are shown in Table 1. Beam dimensions in this study are L = 0.2 m, H = 0.02 m and B = 0.001 m.

3.1. FGM through the axial direction

Free vibrations of the FG beam are analyzed in this work by computing the natural frequencies of a uniform beam. Different boundary conditions of the beam are considered by performing a finite element analysis using the program ANSYS (APDL). The beam element type is based on (SHELL-181). The definition of material properties such as Young's modulus and density are based on ("*DO" Command) [29]. The beam is divided into small segments in which each one has a different material property according to the material gradient stated in Eq. (2) as shown in Fig. 5. The number of elements that will be used in this case is 50 to enhance the convergence and mesh refinement elements. The obtained non-dimensional frequencies of the FG beam for two classical boundary conditions (Clamped-Pinned (**C-P**) and Simply-Supported (**S-S**)) are compared with the results reported in the literature [29]. The computed frequencies with gradient index (α = 3) show a satisfactory agreement with the previous findings as tabulated in Table 2.

Table 1. The beam's material properties.

Properties	Unit	Aluminum (AL)	Zirconia (ZrO_2)
E	GPa	70	200
ρ	kg/m^3	2702	5700

Table 2. The first four non-dimensional frequencies λ_i of FG beam when α = 3.

N	S-S		C-P	
	Ref. [27]	Present analysis	Ref. [27]	Present analysis
1	10.37	9.89	15.718	15.542
2	41.97	39.66	52.807	50.482
3	94.51	89.63	110.611	105.75
4	168	160.29	189.356	181.827

Table 3. First four non-dimensional frequencies λ_i of FG beam with C-F conditions and gradient index (α).

BCs	N	α (gradient index)					
		0.2	**0.5**	**1**	**2**	**4**	**10**
C-F	1	4.119	3.965	3.826	3.601	3.542	3.539
	2	25.803	24.840	23.973	22.562	22.192	22.172
	3	72.294	69.595	67.165	63.210	62.179	62.120
	4	141.859	136.562	131.793	124.034	122.009	121.894

Table 4. The first four non-dimensional frequencies λ_i of FG beam with C-C conditions and gradient index (α).

BCs	N	α (gradient index)					
		0.2	**0.5**	**1**	**2**	**4**	**10**
C-C	1	26.347	25.363	24.477	22.938	22.661	22.639
	2	72.611	69.900	67.459	63.487	62.451	62.392
	3	142.433	137.116	132.325	124.537	122.501	122.388
	4	235.725	226.933	218.996	206.099	202.746	202.558

Table 5. The first four non-dimensional frequencies λ_i of FG beam with C-P conditions and gradient index (α).

BCs	N	α (gradient index)					
		0.2	**0.5**	**1**	**2**	**4**	**10**
C-P	1	18.058	17.384	16.777	15.789	15.532	15.501
	2	58.557	56.371	54.401	51.199	50.362	50.256
	3	122.340	117.772	113.658	106.968	105.220	104.837
	4	209.595	201.771	194.724	183.263	180.270	179.757

Table 6. The first four non-dimensional frequencies λ_i of FG beam at S-S with gradient index (α).

BCs	N	α (gradient index)					
		0.2	**0.5**	**1**	**2**	**4**	**10**
S-S	1	11.492	11.063	10.676	10.048	9.884	9.875
	2	46.035	44.316	42.768	40.202	39.593	39.547
	3	103.800	99.924	96.435	96.435	89.275	89.098
	4	185.008	178.098	171.750	171.750	159.120	158.973

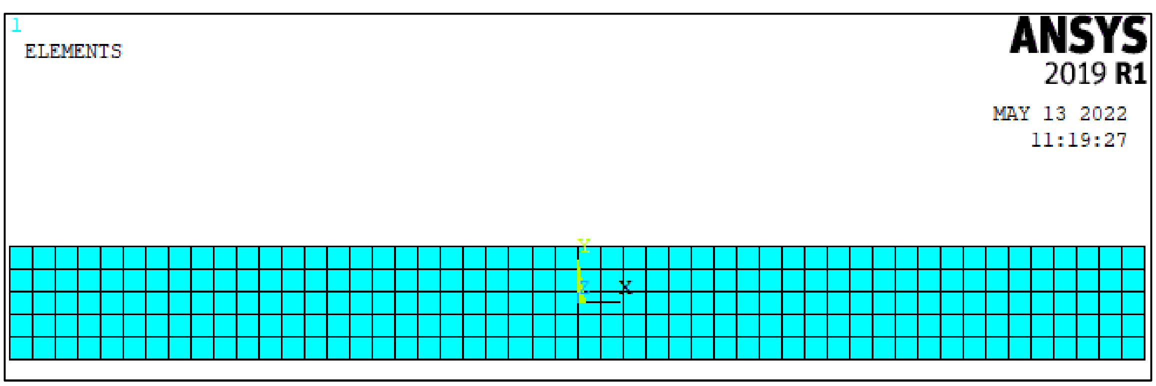

Fig. 5. Finite segment model of the FG beam.

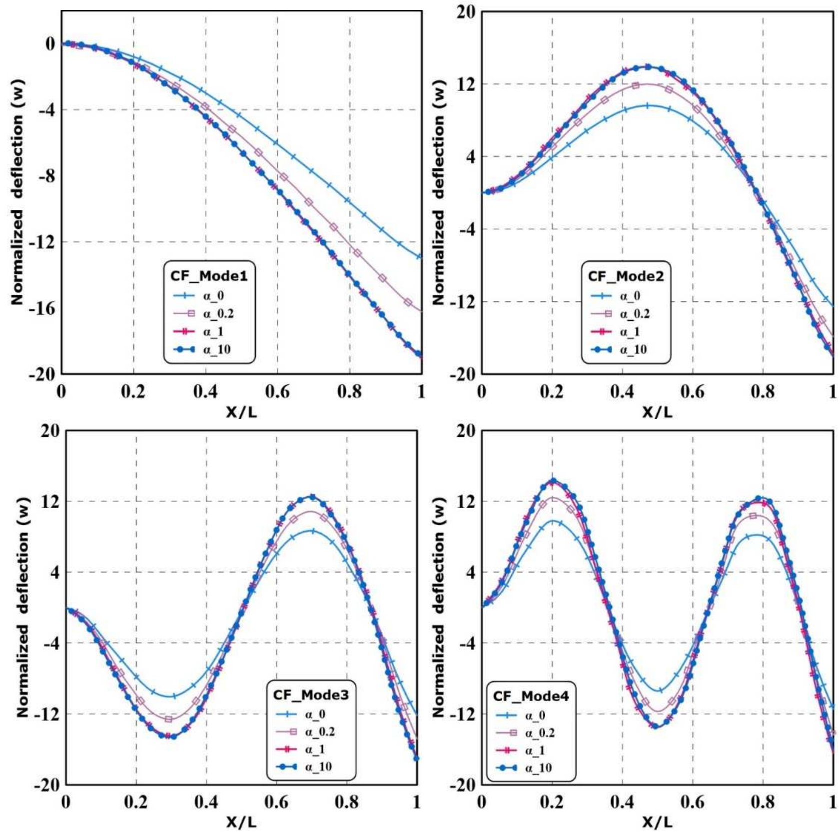

Fig. 6. The first four mode shapes, w, versus X/L for axial graded beams (Clamped-Free (CF)).

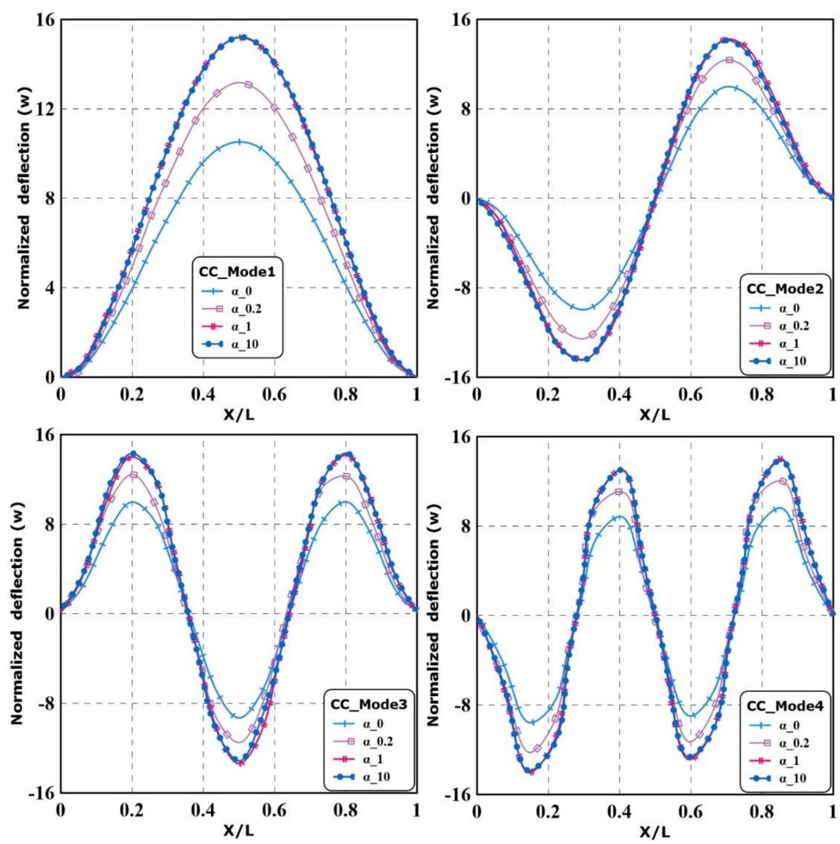

Fig. 7. The first four mode shapes, w, versus X/L for axial graded beams (Clamped-Clamped (CC)).

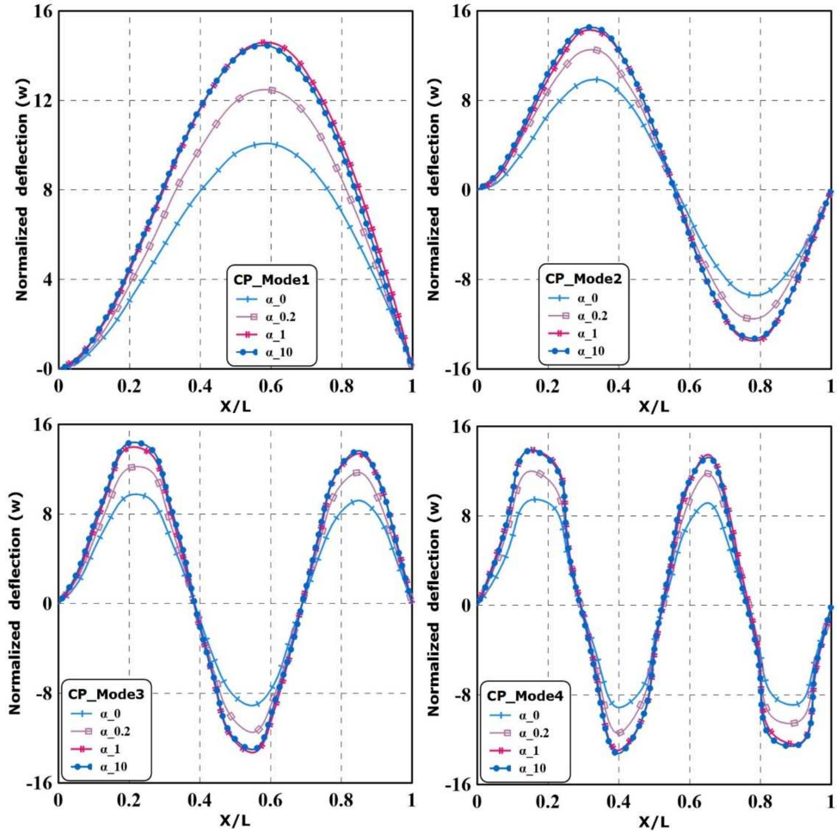

Fig. 8. The first four mode shapes, w, versus X/L for axial graded beams (Clamped-Pinned (CP)).

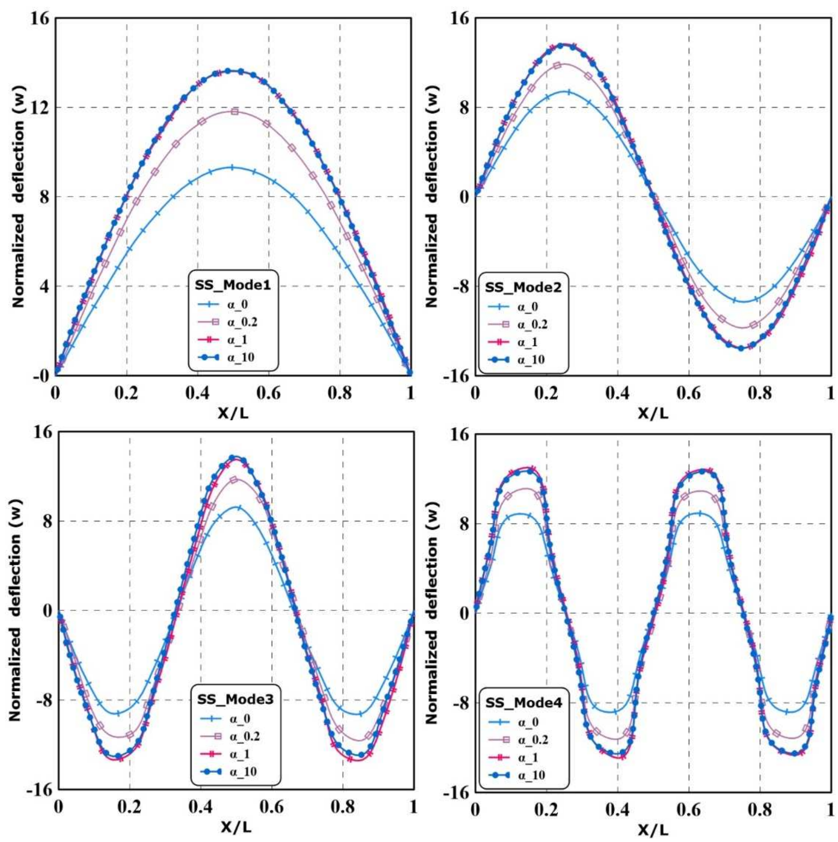

Fig. 9. The first four mode shapes, w, versus X/L for axial graded beams (Simply-Supported (SS)).

Tables 3 to 6 show the non-dimensional frequencies of functionally graded beam at classical boundary conditions (Clamped-Free (**C-F**), Clamped-Clamped (**C-C**), Clamped-Pinned (**C-P**), and Simply-Supported (**S-S**)) with the variation of material gradient index (α). When the gradient index (α) increase, the non-dimensional frequencies decrease until the gradient index become (α = 10) which the material considered as a homogenous material.

The equation used to derive a non-dimensional equation is ($\lambda_i = \omega_i L^2\sqrt{\rho_L A/E_L I}$) where ω_i is the angular frequency for each case of boundary conditions with gradient index (α). Four mode shapes have been computed for different boundary conditions and different material gradients along the FG beam. The mode shape changes according to the material gradient index, which changes smoothly from aluminum to zirconia by increasing the gradient parameter. It is noticed that the deflection of the beam changed as the material changed. When α = 0, the beam material is considered as a homogenous beam because it contains more constituents of zirconia material than aluminum material, which gives the small value of beam deflection (w) because it has a large stiffness compared to the aluminum side. As the material gradient increases, the beam deflection (w) also increases as much as the beam stiffness decreased, which affects the mode shapes of the FG beam. However, the axial gradient has an obvious effect on FG beam mode shapes, as shown in Figures 6 through 9.

3.2. FGM through the axial & thickness directions (2D direction)

In Tables 7 to 10, the results are computed for 2D direction beam with a variety of gradient index in the axial (α_x) and thickness directions (α_y), also the beam studied at different boundary conditions (Clamped-Free (CF), Clamped-Clamped (CC), Clamped-Pinned (CP) and Simply-Supported (SS) conditions). The non-dimensional frequency decreases as much as the axial gradient (α_x) of the FG beam increases. The material gradient through the thickness (α_y) has an obvious effect on the beam's frequency. As can be seen, the material gradient in the axial direction has a much greater influence than that of the thickness gradient. Due to material behavior changing from left to right (Zirconia (ZrO_2) side), the material properties became softer and the stiffness of the FG beam started to decrease as much as the axial gradient increased.

Table 7. Non-dimensional natural frequency (λ_i) of 2D material gradient at C-F beam.

BCs	α_x	α_y						
		0.2	**0.4**	**0.6**	**0.8**	**1**	**2**	**5**
C-F	**0**	4.207	4.186	4.168	4.154	4.143	4.111	4.093
	0.2	3.909	3.889	3.873	3.860	3.849	3.819	3.802
	0.4	3.725	3.706	3.691	3.678	3.668	3.640	3.624
	0.6	3.651	3.632	3.617	3.605	3.595	3.567	3.552
	0.8	3.626	3.608	3.593	3.581	3.571	3.543	3.528
	1	3.619	3.600	3.585	3.573	3.563	3.535	3.520
	2	3.615	3.597	3.582	3.570	3.560	3.532	3.517
	5	3.612	3.594	3.579	3.566	3.556	3.529	3.514

Table 8. Non-dimensional natural frequency (λ_i) of 2D material gradient at C-C beam.

BCs	α_x	α_y						
		0.2	**0.4**	**0.6**	**0.8**	**1**	**2**	**5**
C-C	**0**	26.848	26.698	26.576	26.477	26.397	26.085	26.042
	0.2	24.944	24.804	24.691	24.599	24.523	24.311	24.195
	0.4	23.773	23.639	23.531	23.444	23.372	23.170	23.059
	0.6	23.299	23.169	23.063	22.977	22.907	22.709	22.599
	0.8	23.141	23.011	22.905	22.822	22.751	22.555	22.447
	1	23.092	22.962	22.857	22.772	22.704	22.507	22.399
	2	23.070	22.941	22.837	22.751	22.681	22.486	22.379
	5	23.064	22.936	22.832	22.746	22.676	22.481	22.353

Table 9. Non-dimensional natural frequency (λ_i) of 2D material gradient at C-P beam.

BCs	α_x	α_y						
		0.2	**0.4**	**0.6**	**0.8**	**1**	**2**	**5**
C-P	**0**	18.422	18.335	18.263	18.205	18.159	18.022	17.947
	0.2	17.115	17.034	16.968	17.085	16.870	16.744	16.673
	0.4	16.312	16.234	16.171	16.120	16.078	15.958	15.891
	0.6	15.987	15.911	15.849	15.799	15.757	15.640	15.574
	0.8	15.878	15.803	15.742	15.691	15.651	5.271	15.469
	1	15.845	15.769	15.708	15.658	15.617	15.501	15.436
	2	15.830	15.755	15.694	15.644	15.603	15.486	15.421
	5	15.829	15.754	15.693	15.643	15.602	15.485	15.398

Table 10. Non-dimensional natural frequency (λ_i) of 2D material gradient at S-S beam.

BCs	α_x	α_y						
		0.2	0.4	0.6	0.8	1	2	5
S-S	0	11.702	11.664	11.633	11.607	11.586	11.524	11.488
	0.2	10.872	10.837	10.808	10.784	10.764	10.706	10.673
	0.4	10.362	10.328	10.300	10.277	10.258	10.203	10.172
	0.6	10.155	10.122	10.095	10.073	10.054	10.000	9.969
	0.8	10.086	10.054	10.027	10.004	9.986	9.932	9.901
	1	10.065	10.032	10.005	9.983	9.965	9.911	9.880
	2	10.056	10.023	9.996	9.974	9.955	9.902	9.871
	5	10.032	9.996	9.962	9.941	9.922	9.884	9.857

4. Conclusions

Free vibration of a functionally graded strip beam was analyzed in this work. The Finite element model using ANSYS software was used to evaluate the natural frequencies of the 2D FG strip beam. The FG beam was made of two materials (Aluminum and Zirconia). Two types of material gradation were studied (Axial and 2D gradation) with different material gradient index (α = 0, 0.1, 0.3, 10). The non-dimensional frequencies were derived at different boundary conditions and material gradient index. It was shown that the axial gradient has more effect on the FG beam's natural frequency than the thickness gradient based on the rule of mixture equation. The convergence of results increased as much as the number of divided segments of the FG beam increased.

Author Contributions

Conceptual and proposal developed by Mohamed A. Eltaher and S. Asiri. M.A. Al-Zahrani and M.A. Eltaher developed the mathematical modeling and examined the theory; K.I. Ahmed developed the ANSYS command and validated the code with literature. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

References

[1] Koizumi, M., The concept of FGM, *Ceramic Transactions*, 34, 1993, 3–10.
[2] Suresh, S., Mortensen, A., *Fundamentals of Functionally Graded Materials,* IOM Communications, London, 1998.
[3] Kieback, B., Neubrand, A., Riedel, H., Processing techniques for functionally graded materials, *Materials Science and Engineering: A*, 362, 2003, 81-106.
[4] Pourabdy, M., Shishesaz, M., Shahrooi, S., Roknizadeh, S.A.S., Analysis of Axisymmetric Vibration of Functionally-Graded Circular Nano-Plate Based on the Integral Form of the Strain Gradient Model, *Journal of Applied and Computational Mechanics*, 7(4), 2021, 2196-2220.
[5] Karsh, P.K., Mukhopadhyay, T., Dey, S., Stochastic dynamic analysis of twisted functionally graded plates, *Composites Part B: Engineering*, 147, 2018, 259-278.
[6] Gupta, A., Talha, M., Singh, B.N., Vibration characteristics of functionally graded material plate with various boundary constraints using higher order shear deformation theory, *Composites Part B: Engineering*, 94, 2016, 64-74.
[7] Fazzolari, F.A., Natural frequencies and critical temperatures of functionally graded sandwich plates subjected to uniform and non-uniform temperature distributions, *Composite Structures*, 121, 2014, 197-210.
[8] Zhang, W., Liu, T., Xi, A., Wang, Y.N., Resonant responses and chaotic dynamics of composite laminated circular cylindrical shell with membranes, *Journal of Sound and Vibration*, 423, 2018, 65-99.
[9] Wang, Y., Ye, C., Zu, J.W., Identifying the temperature effect on the vibrations of functionally graded cylindrical shells with porosities, *Applied Mathematics and Mechanics*, 39(11), 2018, 1587-1604.
[10] Liu, Y.Z., Hao, Y.X., Zhang, W., Chen, J., Li, S.B., Nonlinear dynamics of initially imperfect functionally graded circular cylindrical shell under complex loads, *Journal of Sound and Vibration*, 348, 2015, 294-328.
[11] Strozzi, M., Pellicano, F., Nonlinear vibrations of functionally graded cylindrical shells, *Thin-Walled Structures*, 67, 2013, 63-77.
[12] Shafiei, N., Kazemi, M., Ghadiri, M., Nonlinear vibration of axially functionally graded tapered microbeams, *International Journal of Engineering Science*, 102, 2016, 12-26.
[13] Simsek, M., Nonlinear static and free vibration analysis of microbeams based on the nonlinear elastic foundation using modified couple stress theory and He's variational method, *Composite Structures*, 112, 2014, 264-272.

[14] Abouelregal, A.E., Mohammad-Sedighi, H., Faghidian, S.A., Shirazi, A.H., Temperature-dependent physical characteristics of the rotating nonlocal nanobeams subject to a varying heat source and a dynamic load, *Facta Universitatis, Series: Mechanical Engineering*, 19(4), 2021, 633-656.
[15] Sedighi, H.M., Shirazi, K.H., Noghrehabadi, A.R., Yildirim, A., Asymptotic investigation of buckled beam nonlinear vibration, *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering*, 36(M2), 2012, 107-116.
[16] Jena, S.K., Chakraverty, S., Malikan, M., Sedighi, H.M., Implementation of Hermite–Ritz method and Navier's technique for vibration of functionally graded porous nanobeam embedded in Winkler–Pasternak elastic foundation using bi-Helmholtz nonlocal elasticity, *Journal of Mechanics of Materials and Structures*, 15(3), 2020, 405-434.
[17] Eltaher, M.A., Abdelrahman, A.A., Al-Nabawy, A., Khater, M., Mansour, A., Vibration of nonlinear graduation of nano-Timoshenko beam considering the neutral axis position, *Applied Mathematics and Computation*, 235, 2014, 512-529.
[18] Eltaher, M.A., Alshorbagy, A.E., Mahmoud, F.F., Vibration analysis of Euler–Bernoulli nanobeams by using finite element method, *Applied Mathematical Modelling*, 37(7), 2012, 4787-4797.
[19] Bashiri, A.H., Akbas, S.D., Abdelrahman, A.A., Assie, A., Eltaher, M.A., Mohamed, E.F., The vibration of multilayered functionally graded deep beams under thermal load, *Geomechanics and Engineering*, 24(6), 2021, 545-557.
[20] Alnujaie, A., Akbas, S.D., Eltaher, M.A., Assie, A.E., Damped forced vibration analysis of layered functionally graded thick beams with porosity, *Smart Structures and Systems*, 27(4), 2021, 679-689.
[21] Asiri, S.A., Akbaş, S.D., Eltaher, M.A., Dynamic analysis of layered functionally graded viscoelastic deep beams with different boundary conditions due to a pulse load, *International Journal of Applied Mechanics*, 12(05), 2020, 2050055.
[22] Jena, S.K., Chakraverty, S., Malikan, M., Sedighi, H.M., Implementation of Hermite–Ritz method and Navier's technique for vibration of functionally graded porous nanobeam embedded in Winkler–Pasternak elastic foundation using bi-Helmholtz nonlocal elasticity, *Journal of Mechanics of Materials and Structures*, 15(3), 2020, 405-434.
[23] Shabani, S., Cunedioglu, Y., Free vibration analysis of functionally graded beams with cracks, *Journal of Applied and Computational Mechanics*, 6(4), 2020, 908-919.
[24] Su, Z., Jin, G., Ye, T., Vibration analysis of multiple-stepped functionally graded beams with general boundary conditions, *Composite Structures*, 186, 2018, 315-323.
[25] Lee, J.W., Lee, J.Y., Free vibration analysis of functionally graded Bernoulli-Euler beams using an exact transfer matrix expression, *International Journal of Mechanical Sciences*, 122, 2017, 1-17.
[26] Jing, L.L., Ming, P.J., Zhang, W.P., Fu, L.R., Cao, Y.P., Static and free vibration analysis of functionally graded beams by combination Timoshenko theory and finite volume method, *Composite Structures*, 138, 2015, 192-213.
[27] Li, S.R., Wan, Z.Q., Zhang, J.H., Free vibration of functionally graded beams based on both classical and first-order shear deformation beam theories, *Applied Mathematics and Mechanics*, 35(5), 2014, 591-606.
[28] Liu, Y., Shu, D.W., Free vibration analysis of exponential functionally graded beams with a single delamination, *Composites Part B: Engineering*, 59, 2014, 166-172.
[29] Cao, D., Gao, Y., Yao, M., Zhang, W., Free vibration of axially functionally graded beams using the asymptotic development method, *Engineering Structures*, 173, 2018, 442-448.
[30] Li, X.F., Kang, Y.A., Wu, J.X., Exact frequency equations of free vibration of exponentially functionally graded beams, *Applied Acoustics*, 74(3), 2013, 413-420.
[31] Akgöz, B., Civalek, Ö., Longitudinal vibration analysis of strain gradient bars made of functionally graded materials (FGM), *Composites Part B: Engineering*, 55, 2013, 263-268.
[32] Shahba, A., Rajasekaran, S., Free vibration and stability of tapered Euler–Bernoulli beams made of axially functionally graded materials, *Applied Mathematical Modelling*, 36(7), 2012, 3094-3111.
[33] Alshorbagy, A.E., Eltaher, M.A., Mahmoud, F.F., Free vibration characteristics of a functionally graded beam by finite element method, *Applied Mathematical Modelling*, 35(1), 2011, 412-425.
[34] Fariborz, J., Batra, R.C., Free vibration of bi-directional functionally graded material circular beams using shear deformation theory employing logarithmic function of radius, *Composite Structures*, 210, 2019, 217-230.
[35] Ahlawat, N., Numerical solution for buckling and vibration of bi-directional FGM circular plates, *AIP Conference Proceedings*, 2061, 2019, 020020.
[36] Pydah, A., Sabale, A., Static analysis of bi-directional functionally graded curved beams, *Composite Structures*, 160, 2017, 867-876.

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