J. Appl. Comput. Mech., 9(2) (2023) 458-463
DOI: 10.22055/jacm.2022.41600.3779

ISSN: 2383-4536
jacm.scu.ac.ir

Shahid Chamran University of Ahvaz

Journal of Applied and Computational Mechanics

Technical Brief

# A Note on the Discharge over Full-width Rectangular Sharp-crested Weirs and Weirs of Finite Crest Length

Amir H. Azimi[1], Nallamuthu Rajaratnam[2]

[1] Department of Civil Engineering, Lakehead University, Thunder Bay ON, Canada P7B 5E1, Email: azimi@lakeheadu.ca

[2] Department of Civil and Environmental Engineering, University of Alberta, Edmonton AB, Canada T6G 1H9, Email: nr1@ualberta.ca

Received August 10 2022; Revised September 20 2022; Accepted for publication October 08 2022.
Corresponding author: A.H. Azimi (azimi@lakeheadu.ca)
© 2022 Published by Shahid Chamran University of Ahvaz

**Abstract.** The head-discharge relationship for sharp-crested weirs is developed based on the energy consideration upstream and at the crest of the weir. The discharge for weirs of finite crest length is estimated by correlating the critical depth and total energy at the upstream of the weir. In both cases, discharge is linked with the total head. Therefore, prediction of discharge for both sharp-crested weirs and weirs of finite crest length requires an iterative solution method. The present technical note formulates the relationship between the discharge coefficients based on water and total heads to estimate the error associated with implementation of head-discharge equation based on the head. The proposed prediction curves are used to convert the iterative solution method based on the total head to a direct solution strategy based on the water depth at the upstream of sharp-crested weirs and weirs of finite crest length with either a sharped-edge or rounded entrance. Based on the similarity of velocity profiles in gate flow, it is concluded that a distance as short as 2.4 times of the water head is suitable enough to measure the upstream water depth. In sharp-crested weirs, the effect of velocity head is negligible for the approach velocity ratio smaller than 0.1. Different correction curves were developed for weirs of finite crest length based on the ratio of water head to the crest length of weirs.

**Keywords:** Sharp-crested weirs, weirs of finite crest length, discharge coefficient, broad-crested weirs, discharge, free flow, weir.

## 1. Introduction

Weir structures are used for discharge measurement, flow diversion, and water level control and several types of weirs have been designed and tested for different flow conditions and geometries [1]. Amongst all weir models, sharp-crested weirs are frequently used in the field of civil engineering and irrigation canals with relatively small discharges [2]. Sharp-crested weirs are not structurally stable for relatively high discharges and weirs of finite crest length have been employed instead for controlling water level and discharge measurement in rivers and natural streams [3, 4]. Both sharp-crested weirs and weirs of finite crest length perform well in both free flow [5, 6] and in case of floods when the weir structure is submerged [7-10]. A relationship between the performance of weirs of finite crest length in reference to sharp-crested weirs of the same height was formulated by [11]. It was found that in almost all flow conditions, sharp-crested weirs perform better than weirs of finite crest length.

To obtain reliable results, a certain minimum approach length has to be provided for flow measurement of full width rectangular sharp-crested weirs. This length is generally assumed to be about 10 times the width of the channel in which the weir is installed [2]. The flow development upstream of sluice gates in rectangular channels has shown similar pattern of velocity development. The region immediately upstream of vertical sluice gates has some similarity to the region immediately upstream of the vertical sharp-crested weir but there are some differences as well. Velocity measurements on sluice gates in rectangular channel indicated that the velocity profile changes from an almost uniform to a jet-like profile, in a rather relatively short length, approximately equal to 2.4 times the gate opening [12]. For the weir, the corresponding length may be the same with the gate opening replaced by the head on the weir. Particle Image Velocimetry (PIV) measurements of the velocity profiles immediately upstream of a broad-crested weir, appears to indicate roughly the same length [13].

The relationship between the total head at the upstream of the weir and the discharge in sharp-crested weirs is formulated by employing the energy equation between the upstream and downstream of the weir. A relationship between the total head and discharge in weirs of finite crest length is developed by knowing the existence of a control point in the vicinity of the weir structure. The head-discharge formula for weirs of finite crest length is achieved knowing the fact that the total energy at the upstream of the weir in rectangular channels is 1.5 times the critical depth. Discharge coefficients for sharp-crested weirs and weirs of finite crest length are extracted from laboratory measurements by equating the measured discharge with the right-hand-side of the head-discharge equation. Many formulas have been proposed for prediction of discharge coefficients of weirs with different geometries and flow conditions [1, 14-16].

Published online: October 18 2022

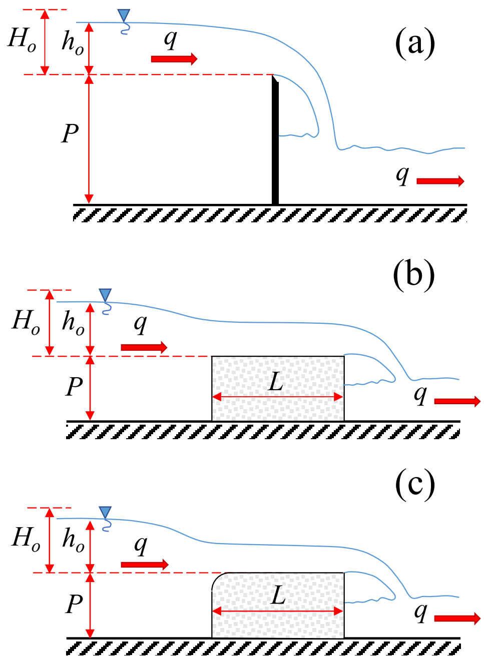


**Fig. 1.** Schematic sketch of free flow over weirs: a) sharp-crested weir; b) weir of finite crest length with sharp upstream edge; c) weir of finite crest length with rounded upstream edge.

The disadvantage of head-discharge equations based on the total head is that the unknown discharge appears in both left and right-hand-sides of the equation and requires an iterative procedure. The present technical note is motivated by finding a relationship between the discharge coefficients based on water and total heads and present conversion curves for these two methods of discharge prediction. The prediction curves provide the range of errors for different types of weirs based on the initial estimation of discharge. The comparison of the discharge coefficients is performed for sharp-crested weirs and weirs of finite crest length with sharped-edge and rounded entrance (Fig. 1).

## 2. Discharge Equation in Sharp-crested Weirs

Consider a rectangular sharp-crested weir of width B located normally across a rectangular channel of width B, with a reasonably long approach channel. The head over this weir, measured some distance upstream is $h_o$ and the total discharge is Q (Fig. 1a). The classical Bernoulli type of analysis, with the nappe pulled-up so that it is horizontal and the pressure distribution across this nappe is hydrostatic, provides the following Weisbach equation for the total discharge:

$$Q = \frac{2}{3}\sqrt{2g}B\left[\left(h_o + \frac{u_a^2}{2g}\right)^{3/2} - \left(\frac{u_a^2}{2g}\right)^{3/2}\right] \tag{1}$$

where $u_a$ is the approach velocity, assumed to be uniform across the full depth of ($h_o$+P), where P is the height of the weir crest above the channel bed and $g$ is the acceleration due to gravity. By expanding the first term inside the brackets using Binomial theorem as:

$$(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \ldots + \binom{n}{n}x^0 y^n \tag{2}$$

The Binomial coefficient in Eq. (2) is obtained by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} \tag{3}$$

By employing the values of x = $h_o$, y = ($u_a^2/2g$), and $n$ = 3/2, it can be shown that the two terms in Eq. (1) reduce to:

$$\left[1 + \frac{1}{2}\left(\frac{u_a^2/2g}{h_o}\right)\right]h_o^{3/2} \tag{4}$$

By neglecting the smaller higher power terms in the series expansion, it can be shown that the above expression may be reduced to ho3/2 with the error being less than 2.6 % for $h_o/P$ = 1.0. For $h_o/P$ = 5.0, the error increases to 16 %. The range of $h_o/P$ from zero to 1.0 is quite large and in this range, Eq. (1) can be rewritten as:

$$Q \approx \frac{2}{3}\sqrt{2g}Bh_o^{3/2} \tag{5}$$

A discharge coefficient, $C_d$, is added to equate the measured discharge with the right-hand-side of Eq. (5) as:

$$Q = C_d\frac{2}{3}\sqrt{2g}Bh_o^{3/2} \tag{6}$$

Based on laboratory experiments, Kandaswamy and Rouse [17] provided an equation for prediction of discharge coefficient in full-width rectangular sharp-crested weirs as:

$$C_d = 0.605 + 0.08\left(\frac{h_o}{P}\right) \tag{7}$$

The proposed formula by Kandaswamy and Rouse [17] is the simplified version of the Rehbock equation [18]. The head-discharge relationship is often proposed based on the total head, $H_o = h_o+u_a^2/2g$ where the mean approach velocity can be calculated as $u_a = Q/[B(h_o+P)]$, and the resulting discharge equation is expressed as:

$$Q = C_d^* \frac{2}{3}\sqrt{2g}BH_o^{3/2} \tag{8}$$

Since both Eqs. (6) and (8) are used in the literature, a summary of the proposed equations can be found in [1], it is useful to have a direct relationship between $C_d$ and $C_d^*$. The relationship between the discharge coefficients based on water and total heads is given by:

$$C_d^* = C_d\left(\frac{h_o}{H_o}\right)^{3/2} \tag{9}$$

By incorporating the definition of total head and approach velocity into Eq. (9), the relationship between discharge coefficients based on water and total heads is expressed as:

$$C_d^* = C_d\left(\frac{h_o}{h_o + \dfrac{Q^2}{2g[B(h_o+P)]^2}}\right)^{3/2} \tag{10}$$

Including Eqs. (6) for the head-discharge relationship in sharp-crested weirs and Eq. (7) for the definition of discharge coefficient, Eq. (10) is simplified to:

$$\frac{C_d^*}{C_d} = \left(\frac{1}{1+\dfrac{4}{9}\dfrac{[0.605+0.08(h_o/P)]^2}{[1+(P/h_o)]^2}}\right)^{3/2} \tag{11}$$

Eq. (11) shows that the ratio of discharge coefficients based on water and total heads is only a function of $h_o/P$. Eq. (11) was evaluated and the results are shown in Fig. 2 for a range of $h_o/P$ between zero to five. It may be observed that for very small values of $h_o/P$ from zero to 0.1, $C_d^*$ is approximately the same as $C_d$ but as $h_o/P$ increases the ratio of $C_d^*/C_d$ becomes less than unity. The variations of $C_d^*/C_d$ with $h_o/P$ shows that for $h_o/P$ greater than 0.1, this ratio is always less than unity and falls to about 0.93 for $h_o/P$ equal to 1.0 and to approximately 0.85 and 0.67 for $h_o/P$ = 2 and 5, respectively.

## 3. Discharge Equations for Weirs of Finite Crest Length

Weirs of finite crest length are designed for flow measurement and water level control in both free and submerged flow conditions [6, 9]. The head-discharge equation is developed based on occurrence of critical flow on the weir and the discharge equation is developed from the relationship between critical depth and total energy as:

$$Q = \left(\frac{2}{3}\right)^{3/2} C_D^* \sqrt{g}BH_o^{3/2} \tag{12}$$

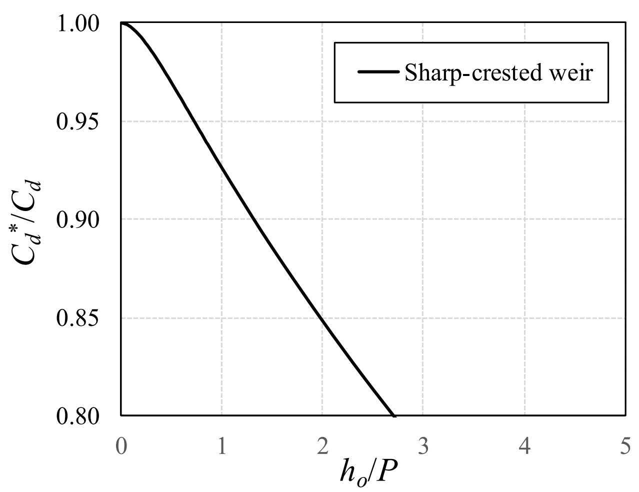


**Fig. 2.** Effect of approach velocity, $h_o/P$, on variations of $C^*_d/C_d$ for sharp-crested weir.

The head-discharge equation for weirs of finite crest length and the proposed formulations for prediction of discharge coefficient were based on the total head, $H_o$, which is also a function of flow discharge [5]. Therefore, an iterative solution strategy is required to estimate and correct the predicted values of discharge, total head, and discharge coefficient. Azimi and Rajaratnam [6] used a number of experimental data on weirs of finite crest length from the literature and proposed a head-discharge equation based on the water head as:

$$Q = \left(\frac{2}{3}\right)^{3/2} C_D \sqrt{g} B h_o^{3/2} \tag{13}$$

Depending on the ratio of water head, $h_o$, to the crest length, $L$, weirs of finite crest length are classified as Long-crested weirs ($0 < h_o/L < 0.1$), Broad-crested weirs ($0.1 \le h_o/L \le 0.4$), Narrow-crested weirs ($0.4 < h_o/L < 2$) and Short-crested weirs ($h_o/L \ge 2$) [6, 19]. A rounded upstream edge in weirs of finite crest length has shown a significant improvement in increasing discharge coefficient. Therefore, separate empirical equations were proposed by Azimi and Rajaratnam [6] for weirs of finite crest length with sharp-edged and rounded entrances (see Figs. 1b and 1c).

The proposed discharge equation for squared-edged Long- and Broad-crested weirs in the study of Azimi and Rajaratnam [6] was only a function of $h_o/(h_o+P)$, therefore, the crest length did not change the variations of normalized discharge coefficient with $h_o/P$. By employing Eq. (10) and the proposed formulation for prediction of discharge coefficient based on water head, $C_D$, the variation of $C_D^*/C_D$ with $h_o/P$ is shown in Fig. 3a. As can be seen, both Long- and Broad-crested weirs behave similarly for $h_o/P \le 1.2$ and the prediction curves deviate afterwards. The variations of $C_D^*/C_D$ with $h_o/P$ shows that for $h_o/P$ greater than 0.2, this ratio is always less than unity and for $h_o/P = 2$ fall to about 0.913 and 0.9 for Long- and Broad-crested weirs, respectively.

The discharge coefficient of Narrow-crested weirs with sharp upstream edge varies linearly with $h_o/L$. Therefore, a family of curves describe the correlation of $C_D^*/C_D$ with $h_o/P$ based on variations of the weir aspect ratio, $P/L$. Figure 3b shows the graphical representation of Eq. (10) for Narrow-crested weirs with sharp upstream edge. As can be seen, the effect of aspect ratio, $P/L$ is negligible for $h_o/P \le 0.4$ and for larger values of $h_o/P$, the difference between discharge coefficient based on water and total heads increases with increasing $P/L$. For example, the reduction of discharge coefficient based on the total head at $h_o/P = 1$ and for $P/L = 0.2$ and 2 were 0.965 and 0.925, respectively.

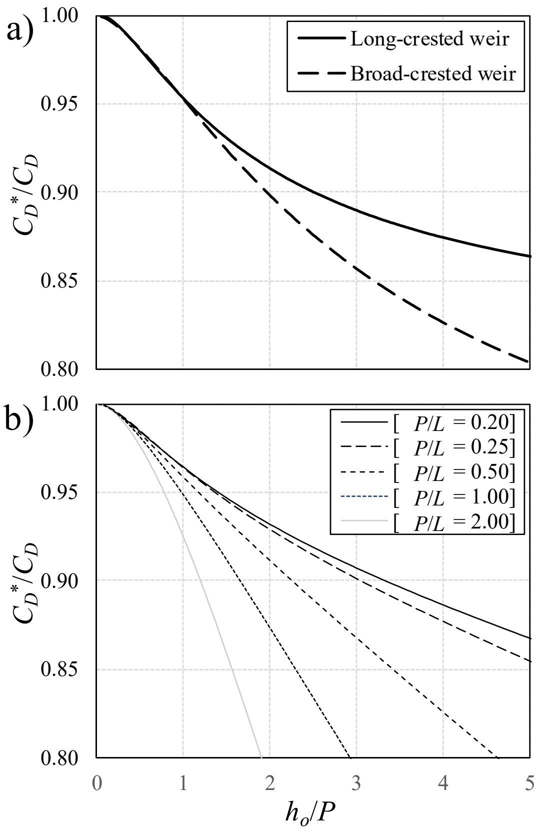


**Fig. 3.** Effect of approach velocity, $h_o/P$, on variations of $C^*_D/C_D$ for weirs of finite crest length with sharp upstream edge: a) Long- and Broad-crested weirs; b) Narrow-crested weirs with different aspect ratio, $P/L$.

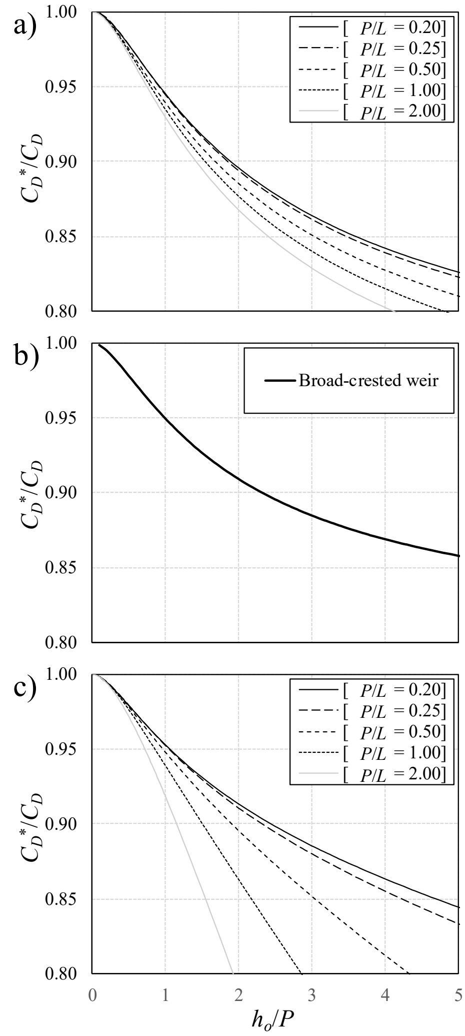


**Fig. 4.** Effect of approach velocity, $h_o/P$, on variations of $C^*_D/C_D$ for weirs of finite crest length with rounded upstream edge: a) Long-crested weir; b) Broad-crested weir; c) Narrow-crested weirs with different aspect ratio, $P/L$.

The variations of discharge coefficient ratio with $h_o/P$ are evaluated for weirs of finite crest length with rounded upstream entrance. Azimi and Rajaratnam [6] proposed a power law equation for prediction of Long-crested weirs with an upstream round edge which was a function of both $h_o/(h_o+P)$ and $h_o/L$. Figure 4a shows the correlation of normalized discharge coefficient, $C_D^*/C_D$ with $h_o/P$ for Long-crested weirs with a rounded upstream (see Fig. 1c). The variations of discharge coefficient ratios between $P/L$ = 0.2 and 2 for $h_o/P$ = 1 and 2 were −1.6% and −3.1%, respectively. The discharge coefficient of Broad-crested weirs with a rounded upstream is linearly correlated with $h_o/(h_o+P)$. The estimated discharge based on the water head, and discharge coefficient prediction formula by Azimi and Rajaratnam [6] were employed in Eq. (10) and the results are plotted in Fig. 4b. The results show that the difference between discharge predictions based on water and total head for $h_o/P \leq 0.4$ is less than 2% and a 5% underestimation occurs for $h_o/P$ = 1. Azimi and Rajaratnam [6] proposed a linear formula for prediction of discharge coefficient with variations of $h_o/L$ for Narrow-crested weirs with rounded upstream. Figure 4c shows the effect of weir's aspect ratio, $P/L$, on variations of normalized discharge coefficient. As can be seen, for $h_o/P$ = 1 the discharge coefficient ratio varies between 0.953 and 0.92 for aspect ratios of $P/L$ = 0.2 and 2, respectively.

## 4. Conclusions

A theoretical approach was utilized to formulate a correlation between predictions of discharge coefficient based on the total head as a function of discharge prediction based on the water head measured at the upstream of the weir structure. In sharp-crested weirs, the difference between estimated discharge from water head and total head is negligible for $h_o/P \leq 0.1$. Therefore, no iteration is needed for such a low values of $h_o/P$. However, the error in estimation of discharge using the water depth increases up to 7% for $h_o/P$ = 1. The difference between the first estimation of discharge using water depth and the final prediction of discharge in sharped-edge Long- and Broad-crested weirs is 5% for $h_o/P$ = 1 and negligible for $h_o/P \leq 0.2$. In Narrow-crested weirs with sharp-edged entrance, the accuracy of employing water depth instead of total head depends on the weir's aspect ratio and less error was achieved in small aspect ratio of $P/L$ = 0.2. A comparison between the prediction curves of discharge coefficients for weirs of finite crest length with sharp and rounded upstream entrance indicated that employing water head in head-discharge formulation (i.e., Eq. (8)) caused a smaller error in weirs with a sharp-edged upstream.

## Author Contributions

The manuscript was written through the contribution of both authors. Both authors discussed the results, reviewed and approved the final version of the manuscript.

## Acknowledgments

Not applicable.

## Conflict of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

## Funding

The authors received no financial support for the research, authorship, and publication of this article.

## Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

## Nomenclature

| | |
|---|---|
| $B$ | Weir width (m); |
| $C_d$ | Discharge coefficient in sharp-crested weirs based on water head; |
| $C_d^*$ | Discharge coefficient in sharp-crested weirs based on total head; |
| $C_D$ | Discharge coefficient in weirs of finite crest length based on water head; |
| $C_D^*$ | Discharge coefficient in weirs of finite crest length based on total head; |
| $g$ | Gravitational acceleration (m/s$^2$); |
| $h_o$ | Water head upstream of the weir (m); |
| $H_o$ | Total head upstream of the weir (m); |
| $L$ | Crest length (m); |
| $P$ | Weir height (m); |
| $Q$ | Flow discharge (m$^3$/s); |
| $u_a$ | Approach velocity (m/s); |

## References

[1] Azimi, A.H., *An introduction to hydraulic structures*, Chapter 16, In: Water Engineering Modeling and Mathematic Tools, Elsevier, Amsterdam, 2021.
[2] Ackers, P., White, W.R., Perkins, J.A., Harrison, A.J.M., *Weirs and Flumes for Flow Measurement*, Wiley, 1978.
[3] Bos, M.G., *Long throated flumes and broad crested weirs*, Martinus Nijhoff/Dr W. Junk Publishers, Dordrecht, The Netherlands, 1985.
[4] Ramamurthy, A.S., Tim, U.S., Rao, M.V.J., Characteristics of square-edged and round-nosed broad-crested weirs, *J. Irrig. Drain. Eng.*, 114, 1988, 61–73.
[5] Hager, W.H., Schwalt, M., Broad-crested weir, *J. Irrig. Drain Eng.*, 120(1), 1994, 13-26.
[6] Azimi, A.H., Rajaratnam, N., Discharge characteristics of weirs of finite crest length, *J. Hydraul. Eng.*, 135(12), 2009, 1081-1085.
[7] Rajaratnam, N., Muralidhar, D., Flow below deeply submerged rectangular weirs, *J. Hydraul. Res.*, 7(3), 1969, 355–374.
[8] Wu, S., Rajaratnam, N., Submerged flow regimes of rectangular sharp-crested weirs, *J. Hydraul. Eng.*, 122, 1996, 412–414.
[9] Azimi, A.H., Rajaratnam, N., Zhu, D.Z., Submerged flows over rectangular weirs of finite crest length, *J. Irrig. Drain. Eng.*, 140(5), 2014, 06014001.
[10] Azimi, A.H., Rajaratnam, N., Zhu, D.Z., Water surface characteristics of submerged rectangular sharp-crested weirs, *J. Hydraul. Eng.*, 142(5), 2016, 06016001.
[11] Azimi, A.H., Rajaratnam, N., Zhu, D.Z., A Note on Sharp-crested Weirs and Weirs of Finite Crest Length, *Canad. J. Civil Eng.*, 39(11), 2012, 1234-1237.
[12] Rajaratnam, N., Humphries, J.A., Free flow upstream of vertical sluice gates, *J. Hydraul. Res.*, 20(5), 1982, 427-437.
[13] Kirkgoz, M.S, Akoz, M.S., Oner, A.A., Experimental and theoretical analyses of two- dimensional flows upstream of broad-crested weirs, *Canad. J. Civil Eng.*, 35, 2008, 975-986.
[14] Akoz, M.S., Gumus, V., Kirkgoz, M.S., Numerical simulation of flow over a semi cylinder weir, *J. Irrigat. Drain. Eng.*, 140(6), 2014, 04014016.
[15] Dizabadi, S., Hakim, S.S., Azimi, A.H., Discharge characteristics and structure of flow in labyrinth weirs with a downstream pool, *Flow Meas. Instrum.*, 71, 2020, 101683.

[16] Zhang, J., Chang, Q., Zhang, Q., Li, S., Experimental study on discharge coefficient of a gear-shaped weir, *Water Sci. Eng.*, 11(3), 2018, 258-264.
[17] Kandaswamy, A.M., Rouse, H., Characteristics of terminal weirs and sills, *J. Hydraul. Div.*, 83(4), 1957, 1345.
[18] Rehbock, T., Discussion of precise weir measurements, *Transmission*, 93, 1929, 1143-1162.
[19] Govinda Rao, N.S., Muralidhar, D., Discharge characteristics of weirs of finite crest width, *Houille Blanche*, 18(5), 1963, 537–545.

## ORCID iD

Amir H. Azimi https://orcid.org/0000-0003-0166-8830
Nallamuthu Rajaratnam https://orcid.org/0000-0002-1867-4893

© 2022 Shahid Chamran University of Ahvaz, Ahvaz, Iran. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC 4.0 license) (http://creativecommons.org/licenses/by-nc/4.0/).

**How to cite this article:** Azimi A.H., Rajaratnam N. A Note on the Discharge over Full-width Rectangular Sharp-crested Weirs and Weirs of Finite Crest Length, *J. Appl. Comput. Mech.*, 9(2), 2023, 458–463. https://doi.org/10.22055/jacm.2022.41600.3779

**Publisher's Note** Shahid Chamran University of Ahvaz remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.