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Research Paper

Natural Magneto-velocity Coordinate System for Satellite Attitude Stabilization: Dynamics and Stability Analysis

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Abstract. The paper addresses the problem of attitude stabilization of an artificial Earth satellite with the aid of an electrodynamic control system. Our objective is to stabilize the satellite in a special coordinate system, whose axes are directed along the Lorentz force vector and the geomagnetic induction vector. Thus, natural magneto-velocity coordinate system (NMVCS) is used. We consider the general case of the satellite mass distribution. Therefore, the disturbing action of the gravitation torque is taken into account. The satellite moves along a circular near-Earth orbit. The nonlinear stability analysis based on the Lyapunov direct method is applied in the paper. The proposed approach gives us admissible domains of control parameters for which attitude stabilization in NMVCS is guaranteed without restrictions on the Earth's magnetic field model. Stabilization conditions are formulated in the form of explicit inequalities for the control parameters. As a result, a control strategy for the satellite attitude stabilization in the NMVCS is elaborated.

Keywords: Satellite, attitude stabilization, natural magneto-velocity coordinate system, geomagnetic field, asymptotic stability.

1. Introduction

At present, the problem of controlling the motion of a rigid body is relevant in various applications such as aerospace engineering, robotics, marine engineering. This problem is known to be distinguished by a variety of approaches to the construction of control algorithms [1–6]. Satellite attitude control, which is of the greatest interest to us, is one of the most important applications of this problem. Apparently, all known force factors are used in one or another way in the problem of controlling the motion of a rigid body. Also, magnetoelectro-mechanical coupling, which shown itself to be successful in various topical mechanical problems [7–9], is suitable for satellite attitude control in the geomagnetic field. This method, called electrodynamic attitude control method, is based on the use of Lorentz [10, 11] and magnetic [1] torques simultaneously. It was developed and modified in [12–15] and papers cited therein. Using this method, a number of problems of the satellite attitude control were solved. At this, the two most common coordinate systems were chosen as the basic frames: the orbital [16, 17] and König frames [18, 19].

The orbital coordinate system, one of the axes of which is directed along the local vertical, is very convenient for stabilizing satellites, not only because it allows the satellite to be constantly facing the Earth and ensure the orientation of onboard instruments to ground objects. The local vertical is directed along the force vector of the central gravitational field and therefore the orbital coordinate system is a natural coordinate system in the central gravitational field from the point of view of the satellite attitude dynamics. This coordinate system is convenient for satellites with significantly different principal central moments of inertia. The difference between the real gravitational field of the Earth and the central one in most cases turns out to be non-critical and therefore does not prevent the use of the orbital coordinate system for gravitational stabilization of the satellite. However, for a satellite with equal principal central moments of inertia, the dynamic advantages (i.e., naturalness) of the orbital coordinate system cease to manifest themselves and only kinematic advantages remain.

The König coordinate system, whose origin is located at the center of mass of the satellite, and the axes are in translational motion, is the basic coordinate system for solving navigation problems both for Earth satellites and planets, and for spacecraft designed to operate outside of near space. However, this coordinate system is not naturally related to the force fields in which the satellite or spacecraft moves, which does not contribute to the effective solution of the problem of attitude control.

The Earth's magnetic field, as already noted, is a significant force factor that provides the possibility of attitude stabilization of satellites, regardless of their principal central moments of inertia. Therefore, it is quite expedient to consider the problems of attitude stabilization of satellites in natural coordinate systems associated with the geomagnetic field [1, 20, 21]. Among such coordinate systems, the most natural is NMVCS, introduced in [22] and differing from other known coordinate systems in that it has not one, but two axes associated with the dynamics of the satellite in the geomagnetic field. One of these axes is directed

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along the vector of the magnetic induction of the field [21] and is therefore organically related to the vector of the magnetic torque, and the second axis is directed along the vector of the Lorentz force acting on a charged satellite in the geomagnetic field and therefore is organically related to the Lorentz torque. In this case, the presence of a charge can be a consequence not only of the natural charging of the satellite in the near-Earth plasma, but also of specially provided charged surfaces that play the role of elements of the attitude control system.

Previous successes in the use of electrodynamic control for solving various problems of the attitude dynamics of satellites relative to the orbital [11, 12, 14, 15, 17, 23] and König [13, 18] coordinate systems make it possible to raise a logical question about the use of electrodynamic control to solve similar problems using the coordinate system that best suits the control torques used, namely NMVCS [22]. This article for the first time analyzes exactly this issue from the point of view of attitude dynamics.

Kinematics of NMVCS was investigated in [22]. It is shown in [22], that NMVCS is suitable for attitude stabilization of a satellite operating in the mode of scanning the Earth's surface. Note that in the case of approximation of the geomagnetic field by the field of a magnetic dipole, for a satellite moving in the plane of the geomagnetic equator, NMVCS coincides with the orbital coordinate system. The same coincidence also takes place for an equatorial satellite moving in the field of a "direct" magnetic dipole [1]. In this sense, the approach used in this paper to the formulation of the problem and to the method of its solution generalizes the previously obtained results related to the dynamics of the attitude motion of satellites in the orbital coordinate system. This is the main contribution of this paper as compared to the existing ones in the literature.

2. Coordinate Systems

We consider an artificial Earth satellite. Let its mass center C moves along a circular orbit. The satellite is supplied with an intrinsic magnetic moment $\vec{I}$ and an electric charge $Q = \int_V \sigma dV$. The electric charge is distributed over the volume V with density σ. The radius-vector of the center of charge (point O) is defined by the formula $\overrightarrow{CO} = \vec{\rho}_0 = Q^{-1}\int_V \sigma\vec{\rho}\, dV$. Here the radius-vector $\vec{\rho}$ defines the position of elementary volume dV. We assume that the geomagnetic induction $\vec{B}$ is the same for all points of the satellite. Let $\vec{v}$ be the velocity of the point C in Greenwich frame rigidly connected with the Earth, whose angular velocity is ω_E. Let $\vec{P} = Q\cdot\overrightarrow{CO}$ and $\vec{T} = \vec{v}\times\vec{B}$. While moving in the Earth's magnetic field, the satellite experiences the influence of magnetic and Lorentz torques:

$$\vec{M}_M = \vec{I}\times\vec{B},\quad \vec{M}_L = \vec{P}\times\vec{T}. \tag{1}$$

The unit vectors $\vec{w}, \vec{b}, \vec{l}$ of the orthogonal magneto-velocity coordinate system are introduced as follows [22]:

$$\vec{w} = \frac{\vec{B}\times\vec{T}}{|\vec{B}|\cdot|\vec{T}|},\quad \vec{b} = \frac{\vec{B}}{|\vec{B}|},\quad \vec{l} = \frac{\vec{T}}{|\vec{T}|}.$$

The advantage of choosing such a coordinate system as the basic one is described in [22] and manifests itself precisely when using the electrodynamic stabilization system, as will be shown below. Alongside with the magneto-velocity coordinate system, we use the satellite-fixed coordinate system $Cxyz$ with unit vectors $\vec{i}, \vec{j}, \vec{k}$ and the orbital coordinate system $C\xi\eta\zeta$ whose axis $C\xi$ (unit vector $\vec{\xi}_0$) is along the tangent to the orbit in direction of motion, axis $C\eta$ (unit vector $\vec{\eta}_0$) is orthogonal to the orbital plane, axis $C\zeta$ (unit vector $\vec{\zeta}_0$) is along the radius-vector $\vec{R} = \overrightarrow{O_E C} = R\vec{\zeta}_0$ of the satellite mass centre with respect to the Earth centre O_E (Fig. 1). The angular velocity of the orbital coordinate system with respect to the König coordinate system is $\vec{\omega}_0 = \omega_0\vec{\eta}_0$. Introducing the orbital inclination i and the argument of latitude $u = \omega_0 t$, we have $\vec{v} = R(\omega_0 - \omega_E \cos i)\vec{\xi}_0 + R\omega_E \sin i \cos u\, \vec{\eta}_0$.

Let $\vec{\omega}$ be the absolute angular velocity of the satellite, $\vec{\omega}'$ be the angular velocity of the satellite with respect to the magneto-velocity frame, $\vec{\omega}_0'$ be the angular velocity of the satellite with respect to the orbital frame, $\vec{\omega}_b$ be the angular velocity of the magneto-velocity frame with respect to the König frame, $\vec{\omega}_{b0}$ be the angular velocity of the magneto-velocity frame with respect to the orbital frame. Since $\vec{\omega} = \vec{\omega}_0 + \vec{\omega}_0'$ and $\vec{\omega}_0' = \vec{\omega}_{b0} + \vec{\omega}'$, we obtain the equality $\vec{\omega} = \vec{\omega}_0 + \vec{\omega}_{b0} + \vec{\omega}'$. Let $\omega_x', \omega_y', \omega_z'$ be components of the vector $\vec{\omega}'$ in the system $Cxyz$.

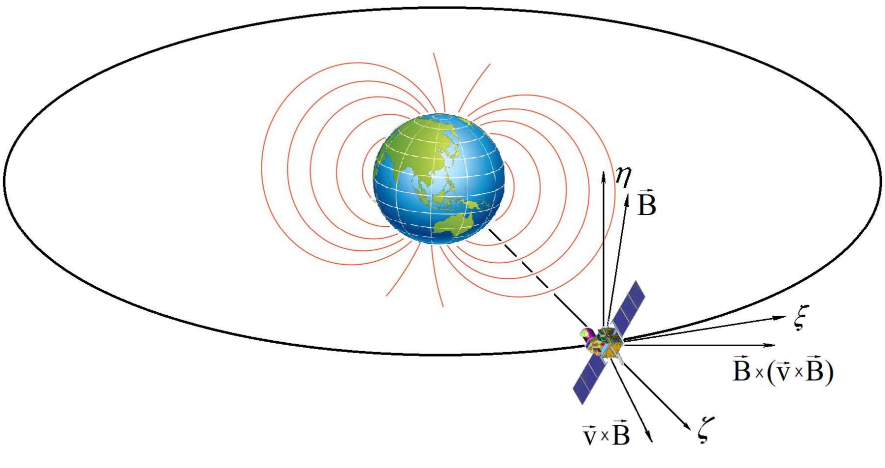

Fig. 1. Natural magneto-velocity and orbital coordinate systems.

3. Kinematic Equations

Let us define the position of the coordinate system $Cxyz$ with respect to $C\xi\eta\zeta$ by means of direction cosine matrix $\mathbf{A}$ with elements $\alpha_i, \beta_i, \gamma_i$ (i = 1,2,3) in accordance with the equations $\vec{\xi}_0 = \alpha_1\vec{i} + \alpha_2\vec{j} + \alpha_3\vec{k}$, $\vec{\eta}_0 = \beta_1\vec{i} + \beta_2\vec{j} + \beta_3\vec{k}$, $\vec{\zeta}_0 = \gamma_1\vec{i} + \gamma_2\vec{j} + \gamma_3\vec{k}$.

Also, we introduce the orthogonal matrix $\mathbf{B} = (b_{ij})$, which defines the position of the orbital frame with respect to the magneto-velocity frame in accordance with the equations $\vec{w} = b_{11}\vec{\xi}_0 + b_{12}\vec{\eta}_0 + b_{13}\vec{\zeta}_0$, $\vec{b} = b_{21}\vec{\xi}_0 + b_{22}\vec{\eta}_0 + b_{23}\vec{\zeta}_0$, $\vec{l} = b_{31}\vec{\xi}_0 + b_{32}\vec{\eta}_0 + b_{33}\vec{\zeta}_0$.

The entries of the matrix $\mathbf{B}$ can be derived in the explicit form as soon as the parameters of the satellite orbit are given. For this derivation we make use of the known expansions of vectors $\vec{B}$ and $\vec{v}$ in the orbital frame. Next, we use equalities:

$$\vec{w} = [(B_\eta T_\zeta - B_\zeta T_\eta)\vec{\xi}_0 + (B_\zeta T_\xi - B_\xi T_\zeta)\vec{\eta}_0 + (B_\xi T_\eta - B_\eta T_\xi)\vec{\zeta}_0] / (|\vec{B}||\vec{T}|),$$
$$\vec{b} = (B_\xi\vec{\xi}_0 + B_\eta\vec{\eta}_0 + B_\zeta\vec{\zeta}_0)/|\vec{B}|, \quad \vec{l} = (T_\xi\vec{\xi}_0 + T_\eta\vec{\eta}_0 + T_\zeta\vec{\zeta}_0)/|\vec{T}|,$$

from which we obtain elements b_{ij} as known functions of time:

$$b_{11} = (B_\eta T_\zeta - B_\zeta T_\eta)/(|\vec{B}||\vec{T}|), \quad b_{12} = (B_\zeta T_\xi - B_\xi T_\zeta)/(|\vec{B}||\vec{T}|), \quad b_{13} = (B_\xi T_\eta - B_\eta T_\xi)/(|\vec{B}||\vec{T}|),$$
$$b_{21} = B_\xi/|\vec{B}|, \quad b_{22} = B_\eta/|\vec{B}|, \quad b_{23} = B_\zeta/|\vec{B}|,$$
$$b_{31} = T_\xi/|\vec{T}|, \quad b_{32} = T_\eta/|\vec{T}|, \quad b_{33} = T_\zeta/|\vec{T}|.$$

To define the attitude position of the satellite with respect to the magneto-velocity frame, considered as the basic one, we introduce the orthogonal matrix $\mathbf{N} = (n_{ij}) = \mathbf{BA}$. The programmed attitude of the satellite in the basic coordinate frame is the equilibrium position corresponding to the identity matrix $\mathbf{N} = \mathbf{E}$. In the programmed attitude the unit vectors $\vec{i}, \vec{j}, \vec{k}$ coincide with the unit vectors $\vec{w}, \vec{b}, \vec{l}$ of the basic coordinate frame. Let $\vec{\sigma}_1, \vec{\sigma}_2, \vec{\sigma}_3$ be the unit vectors $\vec{w}, \vec{b}, \vec{l}$ of the basic coordinate frame, expressed in the basis $\vec{i}, \vec{j}, \vec{k}$:

$$\vec{\sigma}_1 = \mathbf{N}^T\vec{w} = (n_{11}, n_{12}, n_{13})^T, \quad \vec{\sigma}_2 = \mathbf{N}^T\vec{b} = (n_{21}, n_{22}, n_{23})^T, \quad \vec{\sigma}_3 = \mathbf{N}^T\vec{l} = (n_{31}, n_{32}, n_{33})^T$$

Then the programmed attitude motion of the satellite is defined by equations:

$$\vec{\sigma}_1 = \vec{\rho}_1 = (1,0,0)^T, \quad \vec{\sigma}_2 = \vec{\rho}_2 = (0,1,0)^T, \quad \vec{\sigma}_3 = \vec{\rho}_3 = (0,0,1)^T, \quad \vec{\omega}' = \vec{0}. \tag{2}$$

Kinematic differential equations governing the variations of the unit vectors $\vec{\sigma}_1, \vec{\sigma}_2, \vec{\sigma}_3$ can be written in the form of Poisson equations

$$\dot{\vec{\sigma}}_1 = -\vec{\omega}' \times \vec{\sigma}_i \quad (i = 1,2,3), \tag{3}$$

where the dot denotes the local derivative in the satellite-fixed frame $Cxyz$.

In fact, the programmed attitude motion (2) is not only the equilibrium position of the satellite in the basic coordinate frame, but also a special attitude motion of the satellite in the König frame and in the orbital frame. For mathematical modeling the programmed attitude motion we will use the expression for the angular velocity $\vec{\omega}_b = \omega_{b1}\vec{\sigma}_1 + \omega_{b2}\vec{\sigma}_2 + \omega_{b3}\vec{\sigma}_3$, derived in [22]:

$$2\vec{\omega}_b = (\vec{b}_3\dot{\vec{b}}_2 - \vec{b}_2\dot{\vec{b}}_3 + 2\omega_0 b_{12})\vec{\sigma}_1 + (\vec{b}_1\dot{\vec{b}}_3 - \vec{b}_3\dot{\vec{b}}_1 + 2\omega_0 b_{22})\vec{\sigma}_2 + (\vec{b}_2\dot{\vec{b}}_1 - \vec{b}_1\dot{\vec{b}}_2 + 2\omega_0 b_{32})\vec{\sigma}_3.$$

Here $\vec{b}_i = (b_{i1}, b_{i2}, b_{i3})^T$, $\dot{\vec{b}}_i = (\dot{b}_{i1}, \dot{b}_{i2}, \dot{b}_{i3})^T$ $(i = 1,2,3)$. Based on the properties of the vectors $\vec{b}_i$ we can simplify the expression for $\vec{\omega}_b$ and rewrite it in the form

$$\vec{\omega}_b = (\vec{b}_3\dot{\vec{b}}_2 + \omega_0 b_{12})\vec{\sigma}_1 + (\vec{b}_1\dot{\vec{b}}_3 + \omega_0 b_{22})\vec{\sigma}_2 + (\vec{b}_2\dot{\vec{b}}_1 + \omega_0 b_{32})\vec{\sigma}_3. \tag{4}$$

Thus, for a given satellite orbit, the angular velocity $\vec{\omega}_b$ is expressed as a known vector function of time.

In the programmed attitude motion the angular velocity $\vec{\omega}$ of the satellite becomes equal to $\vec{\omega}_b$, and vectors $\vec{T}$ and $\vec{B}$ take on the values:

$$\vec{T}_0 = |\vec{T}|(b_{31}\vec{r}_1 + b_{32}\vec{r}_2 + b_{33}\vec{r}_3), \quad \vec{B}_0 = |\vec{B}|(b_{21}\vec{r}_1 + b_{22}\vec{r}_2 + b_{23}\vec{r}_3),$$

where $\vec{r}_i = (b_{1i}, b_{2i}, b_{3i})^T$ $(i = 1,2,3)$ are the known functions of time.

In the next section we will use the projections of the $\vec{\omega}_b$ on the axes x, y, z:

$$\omega_{bx} = \sum_{i=1}^{3}\omega_{bi}n_{i1}, \quad \omega_{by} = \sum_{i=1}^{3}\omega_{bi}n_{i2}, \quad \omega_{bx} = \sum_{i=1}^{3}\omega_{bi}n_{i3}$$

and the local derivative of $\vec{\omega}_b$ over time calculated in the coordinate system $Cxyz$ in accordance with derivative transformation formula (kinematic theorem [24]):

$$\left(\frac{d\vec{\omega}_b}{dt}\right)_{xyz} = \left(\frac{d\vec{\omega}_b}{dt}\right)_{wbl} + (-\vec{\omega}') \times \vec{\omega}_b = \begin{pmatrix}\dot{\omega}_{b1}\\ \dot{\omega}_{b2}\\ \dot{\omega}_{b3}\end{pmatrix} + \begin{pmatrix}\omega_{by}\omega_z' - \omega_{bz}\omega_y'\\ \omega_{bz}\omega_x' - \omega_{bx}\omega_z'\\ \omega_{bx}\omega_y' - \omega_{by}\omega_x'\end{pmatrix}.$$

The components of this vector will be denoted by $\varepsilon_{bx}, \varepsilon_{by}, \varepsilon_{bz}$.

4. Control Torques and Compensation of Disturbing Torques

It was revealed in the papers [23, 12–15] that the magnetic and Lorentz torques (1) can be used simultaneously for satellite attitude stabilization in the orbital frame as well as in the König one. In this paper we justify the usage of these torques for satellite attitude stabilization in the magneto-velocity frame. Following [23, 12–15], we choose each of the controlled parameters $\vec{P}$ and $\vec{I}$ as the sum of two components:

$$\vec{P} = \vec{P}_R + \vec{P}_D, \quad \vec{I} = \vec{I}_R + \vec{I}_D,$$

where $\vec{P}_R = Qk_L\vec{T}_0$ and $\vec{I}_R = k_M\vec{B}_0$ ensure the restoring components of control torques (1), and $\vec{P}_D = Qh_L\vec{\omega}' \times \vec{T}$, $\vec{I}_D = h_M\vec{\omega}' \times \vec{B}$ ensure the dissipative components of control torques (1). Here k_L, k_M, h_L, h_M are scalar coefficients, which can be functions of time.

However, the satellite in the process of stabilization is under the influence of the gravity gradient torque [1]:

$$\vec{M}_G = 3\omega_0^2(b_{13}\vec{\sigma}_1 + b_{23}\vec{\sigma}_2 + b_{33}\vec{\sigma}_3) \times (\mathbf{J}(b_{13}\vec{\sigma}_1 + b_{23}\vec{\sigma}_2 + b_{33}\vec{\sigma}_3)). \tag{5}$$

Here $\mathbf{J} = \mathrm{diag}(A,B,C)$ is the satellite inertia tensor in the central principal axes of inertia $Cxyz$. The components of this torque in the satellite-fixed frame $Cxyz$ are:

$$\begin{aligned}
M_{Gx} &= 3\omega_0^2(C-B)(b_{13}n_{12} + b_{23}n_{22} + b_{33}n_{32})(b_{13}n_{13} + b_{23}n_{23} + b_{33}n_{33}),\\
M_{Gy} &= 3\omega_0^2(A-C)(b_{13}n_{13} + b_{23}n_{23} + b_{33}n_{33})(b_{13}n_{11} + b_{23}n_{21} + b_{33}n_{31}),\\
M_{Gz} &= 3\omega_0^2(B-A)(b_{13}n_{11} + b_{23}n_{21} + b_{33}n_{31})(b_{13}n_{12} + b_{23}n_{22} + b_{33}n_{32}).
\end{aligned}$$

Also, we will use the components of $\vec{M}_G$ in the basic frame (NMVCS):

$$\begin{aligned}
M_{Gw} &= 3\omega_0^2[A(b_{23}n_{31} - b_{33}n_{21})(b_{13}n_{11} + b_{23}n_{21} + b_{33}n_{31}) + B(b_{23}n_{32} - b_{33}n_{22})(b_{13}n_{12} + b_{23}n_{22} + b_{33}n_{32}) + C(b_{23}n_{33} - b_{33}n_{23})(b_{13}n_{13} + b_{23}n_{23} + b_{33}n_{33})],\\
M_{Gb} &= 3\omega_0^2[A(b_{33}n_{11} - b_{13}n_{31})(b_{13}n_{11} + b_{23}n_{21} + b_{33}n_{31}) + B(b_{33}n_{12} - b_{13}n_{32})(b_{13}n_{12} + b_{23}n_{22} + b_{33}n_{32}) + C(b_{33}n_{13} - b_{13}n_{33})(b_{13}n_{13} + b_{23}n_{23} + b_{33}n_{33})],\\
M_{Gl} &= 3\omega_0^2[A(b_{13}n_{21} - b_{23}n_{11})(b_{13}n_{11} + b_{23}n_{21} + b_{33}n_{31}) + B(b_{13}n_{22} - b_{23}n_{12})(b_{13}n_{12} + b_{23}n_{22} + b_{33}n_{32}) + C(b_{13}n_{23} - b_{23}n_{13})(b_{13}n_{13} + b_{23}n_{23} + b_{33}n_{33})],
\end{aligned}$$

In general case, the gravity gradient torque (5) takes on the following nonzero value in the programmed satellite motion (2):

$$\vec{M}_G = 3\omega_0^2\vec{r}_3 \times (J\vec{r}_3) = 3\omega_0^2[(C-B)b_{23}b_{33}\vec{\rho}_1 + (A-C)b_{13}b_{33}\vec{\rho}_2 + (B-A)b_{13}b_{23}\vec{\rho}_3]. \tag{6}$$

Therefore, the construction of an electrodynamic control for the satellite attitude stabilization should also provide us the solution of the problem of compensating a permanently acting disturbing torque $\vec{M}_G$.

Consider the Euler differential equations governing the satellite attitude motion:

$$\mathbf{J}(\dot{\vec{\omega}}_b + \dot{\vec{\omega}}')_{xyz} + (\vec{\omega}_b + \vec{\omega}') \times (\mathbf{J}(\vec{\omega}_b + \vec{\omega}')) = \vec{M}_L + \vec{M}_M + \vec{M}_G. \tag{7}$$

It can be easily seen that the left-hand side of equations (7) is equal to $\mathbf{J}(\dot{\vec{\omega}}_b)_{xyz} + \vec{\omega}_b \times (\mathbf{J}\vec{\omega}_b)$ in the programmed attitude motion (2). This means that we can classify the torque:

$$\vec{M}_d = \vec{M}_G - \mathbf{J}(\dot{\vec{\omega}}_b)_{xyz} - \vec{\omega}_b \times (\mathbf{J}\vec{\omega}_b)$$

as disturbing one, and design attitude control in such a way that this disturbing torque should be compensated.

The problem of electrodynamic compensation of an arbitrary disturbing torque was solved in [25] for the case when the basic coordinate system is König. In this case, the whole procedure and final formulas of the work [25] are valid up to designations when constructing compensating electrodynamic control in the NMVCS. In this way, we come to the following result: additional components $\vec{P}_1$ and $\vec{I}_1$ should be added in controlled vectors $\vec{P}$ and $\vec{I}$, respectively. The projections of $\vec{P}_1$ and $\vec{I}_1$ on the axes of the basic coordinate system are as follows:

$$P_{1w} = \frac{1}{|\vec{T}|^2}\left[(\vec{M}_d \times \vec{T})_w - \frac{B_w B_l}{|\vec{B}|^2}(\vec{M}_d \times \vec{T})_l\right], \quad P_{1b} = \frac{1}{|\vec{T}|^2}\left[(\vec{M}_d \times \vec{T})_b - \frac{B_b B_l}{|\vec{B}|^2}(\vec{M}_d \times \vec{T})_l\right], \quad P_{1l} = \frac{B_w^2 + B_b^2}{|\vec{B}|^2|\vec{T}|^2}(\vec{M}_d \times \vec{T})_l, \tag{8}$$

$$I_{1w} = \frac{1}{|\vec{B}|^2}\left[(\vec{M}_d \times \vec{B})_w - \frac{T_w T_l}{|\vec{T}|^2}(\vec{M}_d \times \vec{B})_l\right], \quad I_{1b} = \frac{1}{|\vec{B}|^2}\left[(\vec{M}_d \times \vec{B})_b - \frac{T_b T_l}{|\vec{T}|^2}(\vec{M}_d \times \vec{B})_l\right], \quad I_{1l} = \frac{T_w^2 + T_b^2}{|\vec{B}|^2|\vec{T}|^2}(\vec{M}_d \times \vec{B})_l. \tag{9}$$

In the basic reference frame $\vec{T} = |\vec{T}|\vec{l}$, $\vec{B} = |\vec{B}|\vec{b}$ and the disturbing torque $\vec{M}_d$ can be written in the form $\vec{M}_d = M_{dw}\vec{w} + M_{db}\vec{b} + M_{dl}\vec{l}$. Here

$$\begin{aligned}
M_{dw} &= M_{Gw} - A\varepsilon_{bx}n_{11} - B\varepsilon_{by}n_{12} - C\varepsilon_{bz}n_{13} + A\omega_{bx}(\omega_{by}n_{13} - \omega_{bz}n_{12}) + B\omega_{by}(\omega_{bz}n_{11} - \omega_{bx}n_{13}) + C\omega_{bz}(\omega_{bx}n_{12} - \omega_{by}n_{11}),\\
M_{db} &= M_{Gb} - A\varepsilon_{bx}n_{21} - B\varepsilon_{by}n_{22} - C\varepsilon_{bz}n_{23} + A\omega_{bx}(\omega_{by}n_{23} - \omega_{bz}n_{22}) + B\omega_{by}(\omega_{bz}n_{21} - \omega_{bx}n_{23}) + C\omega_{bz}(\omega_{bx}n_{22} - \omega_{by}n_{21}),\\
M_{dl} &= M_{Gl} - A\varepsilon_{bx}n_{31} - B\varepsilon_{by}n_{32} - C\varepsilon_{bz}n_{33} + A\omega_{bx}(\omega_{by}n_{33} - \omega_{bz}n_{32}) + B\omega_{by}(\omega_{bz}n_{31} - \omega_{bx}n_{33}) + C\omega_{bz}(\omega_{bx}n_{32} - \omega_{by}n_{31}).
\end{aligned}$$

Substituting $\vec{T}$, $\vec{B}$, $\vec{M}_d$ in (8) and (9), we obtain:

$$\vec{P}_1 = |\vec{T}|^{-1}(M_{db}\vec{w} - M_{dw}\vec{b}), \quad \vec{I}_1 = -|\vec{B}|^{-1}M_{dl}\vec{w}.$$

The same vectors in the satellite-fixed reference frame have the following form for the programmed attitude motion:

$$\vec{P}_1 = |\vec{T}|^{-1}(M_{db}\vec{\rho}_1 - M_{dw}\vec{\rho}_2), \quad \vec{I}_1 = -|\vec{B}|^{-1}M_{dl}\vec{\rho}_1.$$

Adding $\vec{P}_1$ and $\vec{I}_1$ to $\vec{P}=\vec{P}_R+\vec{P}_D$ and $\vec{I}=\vec{I}_R+\vec{I}_D$, respectively, we obtain the following control torques:

$$\vec{M}_L=(Qk_L\vec{T}_0+\vec{P}_1)\times\vec{T}+Qh_L(\vec{\omega}'\times\vec{T})\times\vec{T}=k_{L0}\,\vec{\rho}_3\times\vec{\sigma}_3+M_{db}\,\vec{\rho}_1\times\vec{\sigma}_3-M_{dw}\,\vec{\rho}_2\times\vec{\sigma}_3-h_{L0}((\vec{\omega}'\vec{\sigma}_1)\vec{\sigma}_1+(\vec{\omega}'\vec{\sigma}_2)\vec{\sigma}_2),$$

$$\vec{M}_M=(k_M\vec{B}_0+\vec{I}_1)\times\vec{B}+h_M(\vec{\omega}'\times\vec{B})\times\vec{B}=k_{M0}\,\vec{\rho}_2\times\vec{\sigma}_2-M_{dl}\,\vec{\rho}_1\times\vec{\sigma}_2-h_{M0}((\vec{\omega}'\vec{\sigma}_1)\vec{\sigma}_1+(\vec{\omega}'\vec{\sigma}_3)\vec{\sigma}_3)\,.$$

Coefficients k_L,k_M,h_L,h_M are taken in the form:

$$k_L=\frac{k_{L0}}{Q\,|\vec{T}|^2},\quad h_L=\frac{h_{L0}}{Q\,|\vec{T}|^2},\quad k_M=\frac{k_{M0}}{|\vec{B}|^2},\quad h_M=\frac{h_{M0}}{|\vec{B}|^2},$$

where $k_{L0},k_{M0},h_{L0},h_{M0}$ are positive constants that are at our disposal. As a result, the Euler equations (7) take on the form:

$$\begin{aligned}&J\dot{\vec{\omega}}'+\vec{\omega}_b\times(J\vec{\omega}')+\vec{\omega}'\times(J\vec{\omega}_b)+\vec{\omega}'\times(J\vec{\omega}')=k_{L0}\,\vec{\rho}_3\times\vec{\sigma}_3+k_{M0}\,\vec{\rho}_2\times\vec{\sigma}_2+M_{dw}\vec{\sigma}_1+M_{db}\vec{\sigma}_2+M_{dl}\vec{\sigma}_3\\&+M_{db}\,\vec{\rho}_1\times\vec{\sigma}_3-M_{dw}\,\vec{\rho}_2\times\vec{\sigma}_3-M_{dl}\,\vec{\rho}_1\times\vec{\sigma}_2-h_{L0}((\vec{\omega}'\vec{\sigma}_1)\vec{\sigma}_1+(\vec{\omega}'\vec{\sigma}_2)\vec{\sigma}_2)-h_{M0}((\vec{\omega}'\vec{\sigma}_1)\vec{\sigma}_1+(\vec{\omega}'\vec{\sigma}_3)\vec{\sigma}_3),\end{aligned}\tag{10}$$

where $\vec{\omega}_b$ is determined by the formula (4).

5. Stabilization Conditions

To derive conditions on control parameters guaranteeing the asymptotic stability of the prescribed satellite programmed motion, we will apply the approaches developed in [2, 3, 26, 27].

Rewrite the Euler equations (10) in the form:

$$\begin{aligned}J\dot{\vec{\omega}}'+\vec{\omega}_b^{(\pi)}\times(J\vec{\omega}')+\vec{\omega}'\times(J\vec{\omega}_b^{(\pi)})&=k_{L0}\,\vec{\rho}_3\times\vec{\sigma}_3+k_{M0}\,\vec{\rho}_2\times\vec{\sigma}_2+M_{dw}^{(\pi)}(\vec{\sigma}_2-\vec{\rho}_2)\times\vec{\rho}_3+M_{db}^{(\pi)}(\vec{\sigma}_2-\vec{\rho}_2)+M_{dl}^{(\pi)}(\vec{\sigma}_3-\vec{\rho}_3)\\&+M_{db}^{(\pi)}\,\vec{\rho}_1\times(\vec{\sigma}_3-\vec{\rho}_3)-M_{dl}^{(\pi)}\vec{\rho}_1\times(\vec{\sigma}_2-\vec{\rho}_2)-h_{L0}\begin{pmatrix}\omega_x'\\\omega_y'\\0\end{pmatrix}-h_{M0}\begin{pmatrix}\omega_x'\\0\\\omega_z'\end{pmatrix}+\vec{\mu}(\vec{\omega}',\vec{\sigma}_2,\vec{\sigma}_3)\end{aligned}\tag{11}$$

Here the superscript (π) means that the corresponding terms are calculated in the programmed attitude motion $\mathbf{N}=\mathbf{E}$. It is worth noting that these terms with superscript (π) are known and they are all bounded functions of time. Vector function $\vec{\mu}(\vec{\omega}',\vec{\sigma}_2,\vec{\sigma}_3)$ includes nonlinear terms with respect to deviations from the programmed motion (2) and satisfies the estimate:

$$\|\vec{\mu}(\vec{\omega}',\vec{\sigma}_2,\vec{\sigma}_3)\|\le c_0(\|\vec{\omega}'\|^2+\|\vec{\sigma}_2-\vec{\rho}_2\|^2+\|\vec{\sigma}_3-\vec{\rho}_3\|^2),\quad c_0=\text{const}>0.$$

Next, define a Lyapunov function candidate as follows:

$$V(\vec{\omega}',\vec{\sigma}_2,\vec{\sigma}_3)=\frac{1}{2}\vec{\omega}'^{T}J\vec{\omega}'+\frac{k_{M0}}{2}\|\vec{\rho}_2-\vec{\sigma}_2\|^2+\frac{k_{L0}}{2}\|\vec{\rho}_3-\vec{\sigma}_3\|^2-\lambda(k_{L0}\,\vec{\rho}_3\times\vec{\sigma}_3+k_{M0}\,\vec{\rho}_2\times\vec{\sigma}_2)^{T}J\vec{\omega}',$$

where $\lambda>0$ is a tuning parameter. Then:

$$\begin{aligned}&\frac{1}{2}\|J^{1/2}\vec{\omega}'\|^2+\frac{k_{M0}}{2}\|\vec{\rho}_2-\vec{\sigma}_2\|^2+\frac{k_{L0}}{2}\|\vec{\rho}_3-\vec{\sigma}_3\|^2-\lambda\sqrt{c_1}(k_{L0}\|\vec{\rho}_3-\vec{\sigma}_3\|+k_{M0}\|\vec{\rho}_2-\vec{\sigma}_2\|)\|J^{1/2}\vec{\omega}'\|\\&\le V(\vec{\omega}',\vec{\sigma}_2,\vec{\sigma}_3)\le\frac{1}{2}\|J^{1/2}\vec{\omega}'\|^2+\frac{k_{M0}}{2}\|\vec{\rho}_2-\vec{\sigma}_2\|^2+\frac{k_{L0}}{2}\|\vec{\rho}_3-\vec{\sigma}_3\|^2+\lambda\sqrt{c_1}(k_{L0}\|\vec{\rho}_3-\vec{\sigma}_3\|+k_{M0}\|\vec{\rho}_2-\vec{\sigma}_2\|)\|J^{1/2}\vec{\omega}'\|.\end{aligned}$$

Here $c_1=\max\{A,B,C\}$. With the aid of the Silvester criterion, it is easy to verify that if

$$c_1\lambda^2(k_{L0}+k_{M0})<1\,,\tag{12}$$

then the function $V(\vec{\omega}',\vec{\sigma}_2,\vec{\sigma}_3)$ is positive definite.

Differentiating the constructed Lyapunov function along the solutions of the system (3), (11), we obtain:

$$\begin{aligned}&\dot{V}\le-\lambda\|\vec{\psi}\|^2-h_{L0}(\omega_x'^2+\omega_y'^2)-h_{M0}(\omega_x'^2+\omega_z'^2)+c_2\|\vec{\omega}'\|^2+(\|\vec{\omega}'\|+\lambda\|\vec{\psi}\|)(c_3\|\vec{\sigma}_2-\vec{\rho}_2\|+c_4\|\vec{\sigma}_3-\vec{\rho}_3\|)\\&+\lambda\|\vec{\psi}\|\|\vec{\omega}'\|(h_{L0}+h_{M0}+c_5)+\lambda c_1(k_{L0}+k_{M0})\|\vec{\omega}'\|^2+c_0(\|\vec{\omega}'\|+\lambda\|\vec{\psi}\|)(\|\vec{\omega}'\|^2+\|\vec{\sigma}_2-\vec{\rho}_2\|^2+\|\vec{\sigma}_3-\vec{\rho}_3\|^2).\end{aligned}$$

Here

$$c_2=c_1\sup_{t\ge0}\|\vec{\omega}_b^{(\pi)}\|,\quad c_3=\sup_{t\ge0}(|M_{db}^{(\pi)}|+|M_{dw}^{(\pi)}|+|M_{dl}^{(\pi)}|),\quad c_4=\sup_{t\ge0}(|M_{db}^{(\pi)}|+|M_{dl}^{(\pi)}|),\quad c_5=c_2+\sup_{t\ge0}\|J\vec{\omega}_b^{(\pi)}\|,\quad \vec{\psi}=k_{L0}\,\vec{\rho}_3\times\vec{\sigma}_3+k_{M0}\,\vec{\rho}_2\times\vec{\sigma}_2.$$

Assume that $h_{L0}=h\tilde{h}_L, h_{M0}=h\tilde{h}_M, k_{L0}=k\tilde{k}_L, k_{M0}=k\tilde{k}_M$, where $h>0$ and $k>0$ are parameters, whereas $\tilde{h}_L,\tilde{h}_M,\tilde{k}_L,\tilde{k}_M$ are fixed positive constants. Then:

$$\begin{aligned}&\dot{V}\le-\lambda\|\vec{\psi}\|^2-(c_6h-c_2-\lambda kc_1(\tilde{k}_L+\tilde{k}_M))\|\vec{\omega}'\|^2+(\|\vec{\omega}'\|+\lambda\|\vec{\psi}\|)(c_3\|\vec{\sigma}_2-\vec{\rho}_2\|+c_4\|\vec{\sigma}_3-\vec{\rho}_3\|)\\&+\lambda\|\vec{\psi}\|\|\vec{\omega}'\|(h(\tilde{h}_L+\tilde{h}_M)+c_5)+c_0(\|\vec{\omega}'\|+\lambda\|\vec{\psi}\|)(\|\vec{\omega}'\|^2+\|\vec{\sigma}_2-\vec{\rho}_2\|^2+\|\vec{\sigma}_3-\vec{\rho}_3\|^2),\end{aligned}$$

where $c_6=\min\{\tilde{h}_L,\tilde{h}_M\}$.

In [15], it was proved that, for any $\chi \in (0,1)$, there exists a positive number δ such that:

$$\|\vec{\psi}\|^2 \geq \chi k^2(\tilde{k}_M^2 \|\vec{\sigma}_2 - \vec{\rho}_2\|^2 + \tilde{k}_L^2 \|\vec{\sigma}_3 - \vec{\rho}_3\|^2)$$

for $\|\vec{\sigma}_2 - \vec{\rho}_2\| + \|\vec{\sigma}_3 - \vec{\rho}_3\| < \delta$. Hence, the inequalities:

$$\dot{V} \leq -\lambda\chi k^2(\tilde{k}_M^2 \|\vec{\sigma}_2 - \vec{\rho}_2\|^2 + \tilde{k}_L^2 \|\vec{\sigma}_3 - \vec{\rho}_3\|^2) - (c_6 h - c_2 - \lambda k c_1(\tilde{k}_L + \tilde{k}_M))\|\vec{\omega}'\|^2 + \frac{1}{2}(c_3 + c_4 + \lambda k(\tilde{k}_M + \tilde{k}_L)(h(\tilde{h}_L + \tilde{h}_M) + c_5))\|\vec{\omega}'\|^2$$
$$+\frac{1}{2}(c_3 + \lambda k(2\tilde{k}_M c_3 + \tilde{k}_M c_4 + \tilde{k}_L c_3 + \tilde{k}_M(h(\tilde{h}_L + \tilde{h}_M) + c_5)))\|\vec{\sigma}_2 - \vec{\rho}_2\|^2 + \frac{1}{2}(c_4 + \lambda k(2\tilde{k}_L c_4 + \tilde{k}_M c_4 + \tilde{k}_L c_3 + \tilde{k}_L(h(\tilde{h}_L + \tilde{h}_M) + c_5)))\|\vec{\sigma}_3 - \vec{\rho}_3\|^2$$
$$+c_0(\|\vec{\omega}'\| + \lambda k(\tilde{k}_M \|\vec{\sigma}_2 - \vec{\rho}_2\| + \tilde{k}_L \|\vec{\sigma}_3 - \vec{\rho}_3\|))(\|\vec{\omega}'\|^2 + \|\vec{\sigma}_2 - \vec{\rho}_2\|^2 + \|\vec{\sigma}_3 - \vec{\rho}_3\|^2)$$

hold for $\|\vec{\sigma}_2 - \vec{\rho}_2\| + \|\vec{\sigma}_3 - \vec{\rho}_3\| < \delta$. Let $\beta = \lambda k$. Then (12) can be rewritten as follows:

$$c_1\beta^2(\tilde{k}_L + \tilde{k}_M) < k. \quad (13)$$

It is easy to verify that under the conditions

$$2\beta k\tilde{k}_M^2 > c_3 + \beta(2\tilde{k}_M c_3 + \tilde{k}_M c_4 + \tilde{k}_L c_3 + \tilde{k}_M(h(\tilde{h}_L + \tilde{h}_M) + c_5)), \quad (14)$$

$$2\beta k\tilde{k}_L^2 > c_4 + \beta(2\tilde{k}_L c_4 + \tilde{k}_M c_4 + \tilde{k}_L c_3 + \tilde{k}_L(h(\tilde{h}_L + \tilde{h}_M) + c_5)), \quad (15)$$

$$c_6 h > c_2 + \beta c_1(\tilde{k}_L + \tilde{k}_M) + \frac{1}{2}(c_3 + c_4 + \beta(\tilde{k}_L + \tilde{k}_M)(h(\tilde{h}_L + \tilde{h}_M) + c_5)) \quad (16)$$

one can choose positive numbers $\tilde{\delta}$ and $\tilde{\alpha}$ such that:

$$\dot{V} \leq -\tilde{\alpha}(\|\vec{\omega}'\|^2 + \|\vec{\sigma}_2 - \vec{\rho}_2\|^2 + \|\vec{\sigma}_3 - \vec{\rho}_3\|^2) \quad \text{for} \quad \|\vec{\omega}'\| + \|\vec{\sigma}_2 - \vec{\rho}_2\| + \|\vec{\sigma}_3 - \vec{\rho}_3\| < \tilde{\delta}.$$

Thus, the fulfilment of the inequalities (13)–(16) implies the asymptotic stability of the programmed motion.

Eliminating the auxiliary parameter β from (13)–(16), we obtain:

$$\sqrt{k}(2k\tilde{k}_M^2 - 2\tilde{k}_M c_3 - \tilde{k}_M c_4 - \tilde{k}_L c_3 - \tilde{k}_M(h(\tilde{h}_L + \tilde{h}_M) + c_5)) > c_3\sqrt{c_1}\sqrt{\tilde{k}_L + \tilde{k}_M}, \quad (17)$$

$$\sqrt{k}(2k\tilde{k}_L^2 - 2\tilde{k}_L c_4 - \tilde{k}_M c_4 - \tilde{k}_L c_3 - \tilde{k}_L(h(\tilde{h}_L + \tilde{h}_M) + c_5)) > c_4\sqrt{c_1}\sqrt{\tilde{k}_L + \tilde{k}_M}, \quad (18)$$

$$(2k\tilde{k}_M^2 - 2\tilde{k}_M c_3 - \tilde{k}_M c_4 - \tilde{k}_L c_3 - \tilde{k}_M(h(\tilde{h}_L + \tilde{h}_M) + c_5))(2c_6 h - 2c_2 - c_3 - c_4) > c_3(\tilde{k}_L + \tilde{k}_M)(2c_1 + c_5 + h(\tilde{h}_L + \tilde{h}_M)), \quad (19)$$

$$(2k\tilde{k}_L^2 - 2\tilde{k}_L c_4 - \tilde{k}_M c_4 - \tilde{k}_L c_3 - \tilde{k}_L(h(\tilde{h}_L + \tilde{h}_M) + c_5))(2c_6 h - 2c_2 - c_3 - c_4) > c_4(\tilde{k}_L + \tilde{k}_M)(2c_1 + c_5 + h(\tilde{h}_L + \tilde{h}_M)). \quad (20)$$

As a result, we arrive at the following theorem.

Theorem. If the control parameters k and h satisfy the conditions (17)–(20), then the programmed motion (2) of the system (3), (11) is asymptotically stable.

6. Computer Modeling

The aim of the present paper is to provide a constructive approach to stability analysis in the problem of satellite attitude stabilization in the NMVCS in the presence of the disturbing effect of the gravitational torque. The suggested approach is based on the Lyapunov direct method for the system governing the satellite attitude dynamics. The theorem proved in the section 5 ensures specific estimates for the control parameters that provide a solution to the problem of satellite attitude stabilization in NMVCS without any limitations on the geomagnetic field model. In this section, we illustrate the theorem by means of a numerical simulation.

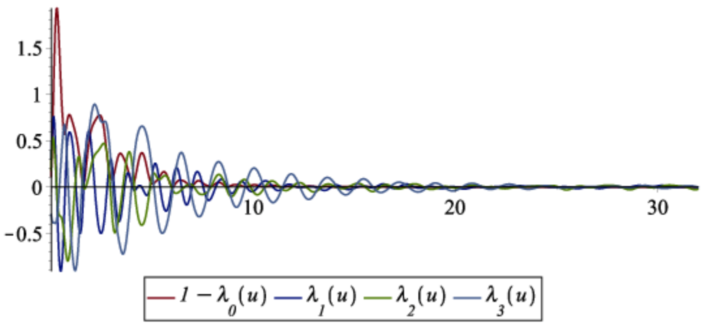

Fig. 2. The quaternion components time history.

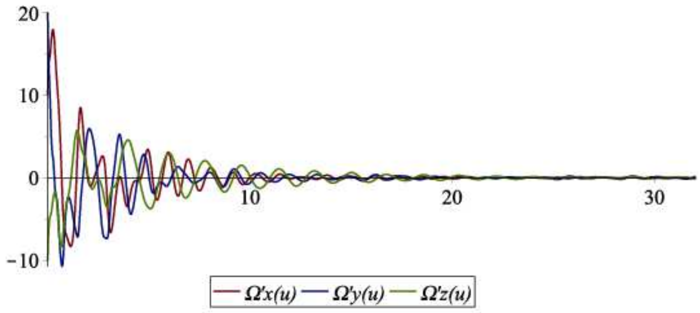

Fig. 3. The relative angular velocity time history.

Consider a typical satellite which moves in orbit with radius $R = 7\cdot 10^6$ and inclination $i = 30^\circ$. Here and in what follows all parameters are taken in International System of Units. Let the inertial parameters of a satellite are given as: $A = 1000$, $B = 600$, $C = 1400$. The electric charge $Q = 5\cdot 10^{-3}$.

The control torques $\vec{M}_L$ and $\vec{M}_M$ are characterized by coefficients $k_{L0} = 0.02$, $k_{M0} = 0.02$, $h_{L0} = 0.5$, $h_{M0} = 0.5$ that satisfy the theorem conditions. In accordance with the theorem, the asymptotic stability of the satellite attitude in the NMVCS is guaranteed with some attraction region. Let us take the following initial conditions for the satellite attitude motion: $\varphi(0) = 0.5$, $\psi(0) = -0.5$, $\theta(0) = 0.5$, $\omega_x^{'}(0) = 10\omega_0$, $\omega_y^{'}(0) = 20\omega_0$, $\omega_z^{'}(0) = -10\omega_0$ ($\omega_0 = 0.001078$ for the chosen R).

The asymptotic stability is confirmed by computer modeling. The convergence process is shown in Fig. 2 (quaternion components) and Fig. 3 (relative angular velocity components).

Here the dimensionless time $u = \omega_0 t$ is along the horizontal axes, $\Omega' x(u) = \omega_x^{'}(u)/\omega_0$, $\Omega' y(u) = \omega_y^{'}(u)/\omega_0$, $\Omega' z(u) = \omega_z^{'}(u)/\omega_0$.

7. Conclusion

In the present paper, we have studied satellite attitude stabilization in natural magneto-velocity coordinate system (NMVCS) whose axes are associated with the geomagnetic induction and Lorentz force. Electrodynamic attitude stabilization system based on simultaneously applied Lorentz and magnetic torques has been used. Since the Lorentz force is orthogonal both to the geomagnetic induction and relative velocity of the satellite, NMVCS is the most convenient for electrodynamic attitude stabilization of a satellite that moves in an orbit with low or medium inclination. The nonlinear stability analysis based on the Lyapunov direct method [2, 28] is applied to solve the problem. Using an original construction of Lyapunov function, sufficient conditions guaranteeing the asymptotic stability of the satellite programmed motion are obtained. Thus, the problem of satellite attitude stabilization in NMVCS is solved without any limitations on the model of the geomagnetic field and the orbit inclination.

Author Contributions

All authors contribute equally in preparation of this manuscript. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

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