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Research Paper

Vibration Analysis of Shear Deformable Cylindrical Shells Made of Heterogeneous Anisotropic Material with Clamped Edges

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Abstract. The vibration behavior of moderately-thick inhomogeneous orthotropic cylindrical shells under clamped boundary conditions based on first-order shear deformation theory (FOSDT) is investigated using an analytical approach. The basic relationships for cylindrical shells composed of inhomogeneous orthotropic materials are established, and then partial differential equations of motion are derived in the framework of FOSDT. The analytical expression for frequency is found for the first time using the special approach for clamped boundary conditions. After checking the accuracy of obtained expressions, the effects of shear stress, orthotropy ratio and inhomogeneity on frequency values are examined in detail.

Keywords: Moderately-thick cylinders, clamped edges, inhomogeneity, orthotropy, vibration, frequency.

1. Introduction

Although the simultaneous consideration of inhomogeneity and anisotropy greatly complicates the mathematical solution of stability and vibration problems, ignoring these factors can lead to a significant error, especially in dynamical problems of cylindrical shells. This study was carried out considering that continuous inhomogeneous anisotropic cylindrical shells are widely used in defense industry and other advanced industries. Fundamentals of these theories are presented in the main books on inhomogeneous and anisotropic bodies. Lomakin [1] is devoted to the theoretical solution of fundamental problems of linearly inhomogeneous elastic bodies. On the basis of the constructed elasticity theory of continuous linear elastic bodies, a number of theoretical issues related to the study of the stress-strain state of structural members are solved. In the monograph of Lekhnitsky [2] and Ambartsumyan [3], the theory of homogeneous elastic anisotropic bodies was developed and the solution of static and dynamic problems of some structural elements was discussed. A review of a number of theoretical and experimental issues related to polymeric and inhomogeneous composite materials was carried out in Kravchuk et al. [4]. Babich and Khoroshun [5] investigated the stability and vibration behaviors of shells with varying geometric and mechanical parameters using refined models and the difference method. Pan [6] presented exact solutions for anisotropic, linearly elastic, and functionally graded (FG) composite rectangular laminates under simply supported edge conditions in the three-dimensional formulation. Awrejcewicz and his co-authors [7, 8] devoted to the construction of mathematical models of statics and dynamics for inhomogeneous plates and shells in the two and three-dimensional formulations.

The boundary conditions are practically necessary to define a problem and are also of primary importance in the statics and dynamics of structural elements. Although one of the most frequently used boundary conditions in practical applications is the clamped boundary conditions, the number of publications is limited than simply supported boundary conditions, due to various difficulties that arise during the solution of static and dynamic problems within the framework of shear deformation theories of inhomogeneous structural elements, especially inhomogeneous shells, under clamped boundary conditions. Within the framework of classical and advanced theories, there are some attempts on analytical and numerical solutions of static and dynamic problems of clamped inhomogeneous shells. Sofiyev and co-authors [9, 10] have solved the vibration and stability problems of heterogeneous isotropic and orthotropic shells under clamped boundary conditions in classical shell theory (CST). Shen and Noda [11] studied of stability of FG cylinders in thermal environments using a singular perturbation technique. Brunetti et al. [12] described the class of shallow shells which are bistable after one of their sides is completely clamped. Babaei et al. [13] investigated thermal buckling of FG tubes with the initial imperfection and clamped edges in elastic medium using the two-step perturbation method. Akhmedov [14-17] studied axisymmetric problems of the elasticity for the radially inhomogeneous cylinders and spheres. Li et al. [18] studied forced vibration behaviors of FG cylindrical shells with initial imperfection theory with clamped edges and in the thermal environments. In recent years, Sofiyev and coauthors [19-21] developed a new approach for the solution of stability and vibration problems of heterogeneous shells under clamped boundary conditions and applied it to various shell types. In recent years, the linear and nonlinear behavior of some structural elements has been studied using different numerical and semi-analytical methods [22-26].

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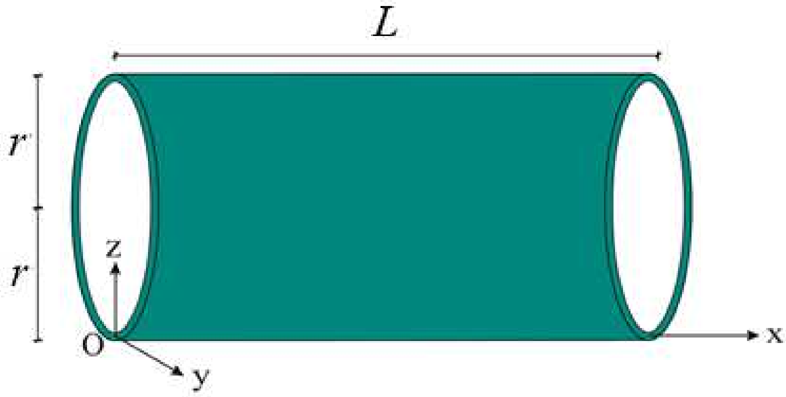

Fig. 1. Configuration of cylindrical shell and coordinate system.

The literature review reveals that the vibration behavior of clamped inhomogeneous orthotropic cylindrical shells has not been adequately studied in the framework of FOSDT. In this study, it is aimed to examine this subject and the analytical expression for frequency is found for the first time using the special approach for clamped boundary conditions. The paper is organized as follows. In Section 2, the problem is formulated and moderately-thick cylindrical shells made of inhomogeneous orthotropic composite materials are described. In Section 3, for cylindrical shells consisting of inhomogeneous orthotropic materials, basic relations are constructed, and then partial differential equations of motion are derived within FOSDT. In Section 4, the analytical expression for the frequency is found using the specific approach for clamped boundary conditions. In Section 5, after checking the accuracy of the obtained expressions, the influences of shear stresses, orthotropy ratio and inhomogeneity on frequency values are examined in detail.

2. Formulation of Problem

Consider the moderately-thick inhomogeneous anisotropic cylindrical shell with the thickness h, length L and radius r (Fig. 1). The $Oxyz$ coordinate system is selected on the middle surface, where Ox and Oy are the axial and circumferential coordinates on the reference surface, respectively, with the z coordinate normal to the Oxy surface and oriented inwards. In the current coordinate system, the three-dimensional region of the cylindrical shell can be mathematically represented as follows:

$$\Omega = \{x, y, z : (x, y, z) \in [0, L] \times [0, 2\pi r] \times [-0.5h, 0.5h]\} \tag{1}$$

The equations of motion for the cylindrical shells that are continuously inhomogeneous in the thickness direction are derived on the assumption that the modulus of elasticity and the density of the orthotropic material are arbitrary functions that vary along the thickness of the structure [19-21]:

$$E_{11}(Z) = \varphi_1(Z)E_{011},\ \ E_{22}(\overline{z}) = \varphi_1(Z)E_{022}, G_{12}(Z) = \varphi_1(Z)G_{012}, G_{13}(Z) = \varphi_1(Z)G_{013},\ G_{23}(Z) = \varphi_1(Z)G_{023},\ \rho(Z) = \varphi_2(Z)\rho_0,\ Z = z/h \tag{2}$$

where E_{0ii} and $G_{0ij}(i = 1,2; j = 2,3)$ symbols define Young and shear moduli and ρ_0 defines the material density. While the inhomogeneity functions $\varphi_i(Z)$ is considered as the general continuous function in the derivation and solution of basic equations, it is used concretely as exponential function, namely $\varphi_i(Z) = e^{k_i(Z-1/2)}$, in the numerical analysis. It is assumed that $k_i(i = 1,2)$ is the inhomogeneity factor and satisfies the condition $-1 \le k_i \le 1$.

3. Governing Equations

The governing relations for inhomogeneous orthotropic cylindrical shells within first order shear deformation theory (FOST) are formed as follows [19]:

$$\begin{bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Y_{11}(Z) & Y_{12}(Z) & 0 \\ Y_{21}(Z) & Y_{22}(Z) & 0 \\ 0 & 0 & Y_{66}(Z) \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{22} \\ \gamma_{12} \end{bmatrix} \tag{3}$$

and

$$\begin{bmatrix} \tau_{13} \\ \tau_{23} \end{bmatrix} = \begin{bmatrix} Y_{55}(Z) & 0 \\ 0 & Y_{55}(Z) \end{bmatrix} \begin{bmatrix} \gamma_{13} \\ \gamma_{23} \end{bmatrix} \tag{4}$$

where $\{\tau\}$ and $\{\varepsilon\}$ are the stress and strain tensors, respectively and $Y_{ij}(Z), (i, j = 1, 2, ,..., 6)$ are defined as:

$$\begin{gathered} Y_{11}(Z) = \frac{E_{11}(Z)}{1-\nu_{12}\nu_{21}},\ Y_{22}(Z) = \frac{E_{22}(Z)}{1-\nu_{12}\nu_{21}},\ Y_{12}(Z) = \frac{\nu_{21}E_{11}(Z)}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_{\theta}(Z)}{1-\nu_{12}\nu_{21}} = Y_{21}(Z), \\ Y_{44}(Z) = G_{23}(Z),\ Y_{55}(Z) = G_{13}(Z),\ Y_{66}(Z) = G_{12}(Z) \end{gathered} \tag{5}$$

In the framework of the transverse shear theory proposed by Ambartsumyan [3] and extended by Sofiyev [21] to inhomogeneous orthotropic shells, the transverse shear stresses can be expressed as a function of the thickness coordinate as follows [3, 21]:

$$\tau_{i3} = f_{,z}(z)\eta_i(x, y)\ (i = 1, 2) \tag{6}$$

where comma characterizes derivative with respect to z and $f(z)$ are posteriori specified shape function of τ_{iz}.

When relations (3), (4) and (6) are taken into account, the strain expressions for inhomogeneous orthotropic cylinders within FOST are expressed as:

$$\begin{pmatrix} e_{11} \\ e_{22} \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} e_{11}^0 - z\dfrac{\partial^2 w}{\partial x^2} + J_1 \dfrac{\partial \eta_1}{\partial x} \\ e_{22}^0 - z\dfrac{\partial^2 w}{\partial y^2} + J_2 \dfrac{\partial \eta_2}{\partial y} \\ \gamma_{12}^0 - 2z\dfrac{\partial^2 w}{\partial x \partial y} + J_1 \dfrac{\partial \eta_1}{\partial y} + J_2 \dfrac{\partial \eta_2}{\partial x} \end{pmatrix} \tag{7}$$

where $e_{ii}^0, \gamma_{ij}^0 (i,j=1,2)$ are strains on the mid-surface and the following definitions are applied:

$$J_1(z) = \int_0^z \frac{1}{Y_{55}(Z)} \frac{df(z)}{dz} dz, \quad J_2(z) = \int_0^z \frac{1}{Y_{44}(Z)} \frac{df(z)}{dz} dz \tag{8}$$

Using first order shear deformation theory, the motion equations of inhomogeneous cylindrical shells with the strains on the mid-surface, force resultants $\{T\}$, transverse force resultants $\{Q\}$ and moment resultants $\{M\}$ are expressed as [29-31]:

$$\begin{gathered} \frac{\partial M_{11}}{\partial x} + \frac{\partial M_{12}}{\partial y} - Q_1 = 0, \ \frac{\partial M_{21}}{\partial x} + \frac{\partial M_{22}}{\partial y} - Q_2 = 0 \\ \frac{\partial^2 e_{11}^0}{\partial y^2} + \frac{\partial^2 e_{22}^0}{\partial x^2} - \frac{\partial^2 \gamma_{12}^0}{\partial x \partial y} + \frac{1}{r}\frac{\partial^2 w}{\partial x^2} = 0 \\ \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + \frac{T_{22}}{r} - \rho_1 \frac{\partial^2 w}{\partial t^2} = 0 \end{gathered} \tag{9}$$

where $\rho_1 = \int_{-h/2}^{+h/2} \rho(Z) dz$. $\{T\}$, $\{Q\}$ and $\{M\}$ are found from the following integrals [29-31]:

$$\left(T_{ij}, M_{ij}\right) = \int_{-h/2}^{h/2} \tau_{ij}\,[1,z]\,dz\ (i,j=1,2),\ Q_j = \int_{-h/2}^{h/2} \tau_{j3}\,dz\ (j=1,2) \tag{10}$$

in which the components T_{ij} are expressed by the Airy stress function Φ as follows:

$$T_{11} = h\frac{\partial^2 \Phi}{\partial y^2},\ T_{22} = h\frac{\partial^2 \Phi}{\partial x^2},\ T_{12} = -\frac{\partial^2 \Phi}{\partial x \partial y} \tag{11}$$

By using the relations (3), (4), (6), (7), (10) and (11) together, after expressing the force and moment components and the strain components of the mid-surface in terms of $\Phi, \varphi_1, \varphi_2$ and w, the subsequent relations are substituting in the set of equations (9), it transforms into the following form:

$$\begin{gathered} h\left[(p_{11}-p_{31})\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + p_{12}\frac{\partial^4 \Phi}{\partial x^4}\right] - p_{13}\frac{\partial^4 w}{\partial x^4} - (p_{14}+p_{32})\frac{\partial^4 w}{\partial x^2 \partial y^2} + p_{15}\frac{\partial^3 \varphi_1}{\partial x^3} + p_{35}\frac{\partial^3 \varphi_1}{\partial x \partial y^2} + (p_{18}+p_{38})\frac{\partial^3 \varphi_2}{\partial x^2 \partial y} - J_3\frac{\partial \varphi_1}{\partial x} = 0, \\ h\left[p_{21}\frac{\partial^4 \Phi}{\partial y^4} + (p_{22}-p_{31})\frac{\partial^4 \Phi}{\partial x^2 \partial y^2}\right] - (p_{32}+p_{23})\frac{\partial^4 w}{\partial x^2 \partial y^2} - p_{24}\frac{\partial^4 w}{\partial y^4} + p_{35}\frac{\partial^3 \varphi_1}{\partial x \partial y^2} + p_{25}\frac{\partial^3 \varphi_1}{\partial x \partial y^2} + p_{38}\frac{\partial^3 \varphi_2}{\partial x^2 \partial y} + p_{28}\frac{\partial^3 \varphi_2}{\partial y^3} - J_3\frac{\partial \varphi_2}{\partial y} = 0, \\ h\left[q_{11}\frac{\partial^4 \Phi}{\partial y^4} + (q_{12}+q_{21}+q_{31})\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + q_{22}\frac{\partial^4 \Phi}{\partial x^4}\right] - q_{23}\frac{\partial^4 w}{\partial x^4} - (q_{24}+q_{13}-q_{32})\frac{\partial^4 w}{\partial x^2 \partial y^2} - q_{14}\frac{\partial^4 w}{\partial y^4} + \frac{1}{r}\frac{\partial^2 w}{\partial x^2} + q_{25}\frac{\partial^3 \varphi_1}{\partial x^3} + q_{15}\frac{\partial^3 \varphi_1}{\partial x \partial y^2} \\ + q_{35}\frac{\partial^3 \varphi_1}{\partial x \partial y^2} + (q_{28}+q_{38})\frac{\partial^3 \varphi_2}{\partial x^2 \partial y} + q_{18}\frac{\partial^3 \varphi_2}{\partial y^3} = 0, \\ J_3\left(\frac{\partial \varphi_1}{\partial x} + \frac{\partial \varphi_2}{\partial y}\right) + \frac{h}{r}\frac{\partial^2 \Phi}{\partial x^2} - \rho_1\frac{\partial^2 w}{\partial t^2} = 0 \end{gathered} \tag{12}$$

where p_{ij} and q_{ij} are the coefficients and are given in the Appendix.

4. Solution Procedure

The cylindrical shell is subject to the clamped boundary conditions and the mathematical model is as follows [11, 21]:

$$v = w = 0,\ \varphi_1 = 0,\ \varphi_2 = 0, \quad x = 0 \ \text{ and } \ x = L \tag{13}$$

The approximation function satisfying the clamped boundary conditions is sought as follows [19-21]:

$$\begin{gathered} \Phi = \Phi_0(t)\,\sin^2(n_1 x)\sin(n_2 y),\ w = w_0(t)\sin^2(n_1 x)\sin(n_2 y), \\ \varphi_1 = \varphi_{01}(t)\cos(n_1 x)\sin(n_1 x)\sin(n_2 y),\ \varphi_2 = \varphi_{01}(t)\sin^2(n_1 x)\cos(n_1 y) \end{gathered} \tag{14}$$

where $\Phi_0(t), w_0(t), \varphi_{01}(t), \varphi_{02}(t)$ are the time dependent functions, $n_1 = m\pi / L$ and $n_2 = n / r$ where (m,n) is the vibration mode.

Substituting the approximation functions (14) into set of Eq. (12) and applying the Galerkin method to the resulting equations, the following ordinary differential equations with second order which depending on the unknown functions $\Phi_0(t)$, $w_0(t)$, $\varphi_{01}(t)$, $\varphi_{02}(t)$ are obtained:

$$\begin{aligned}
&c_{11}\Phi_0(t) - c_{12}w_0(t) + c_{13}\varphi_{01}(t) + c_{14}\varphi_{02}(t) = 0\\
&c_{21}\Phi_0(t) - c_{22}w_0(t) + c_{23}\varphi_{01}(t) + c_{24}\varphi_{02}(t) = 0\\
&c_{31}\Phi_0(t) - c_{32}w_0(t) + c_{33}\varphi_{01}(t) + c_{34}\varphi_{02}(t) = 0\\
&c_{41}\Phi_0(t) + c_{\rho_1}\frac{d^2w_0(t)}{dt^2} + c_{43}\varphi_{01}(t) + c_{44}\varphi_{02}(t) = 0
\end{aligned} \tag{15}$$

where

$$\begin{gathered}
c_{11} = \left[(p_{11} - p_{31})n_1^2n_2^2 + p_{12}n_1^4\right]h,\ c_{12} = (p_{14} + p_{32})n_1^2n_2^2 + p_{13}n_1^4,\\
c_{13} = p_{15}n_1^3 + p_{35}n_1n_2^2 + J_3n_1, c_{14} = (p_{18} + p_{38})n_1^2n_2,\ \ c_{21} = \left[p_{21}n_2^4 + (p_{22} - p_{31})n_1^2n_2^2\right]h,\\
c_{22} = (p_{32} + p_{23})n_1^2n_2^2 + p_{24}n_2^4,\ c_{23} = (p_{25} + p_{35})n_1n_2^2,\ c_{24} = p_{28}n_2^3 + p_{38}n_1^2n_2 + J_3n_2,\\
c_{31} = h\left[q_{22}n_1^4 + (q_{12} + q_{21} + q_{31})n_1^2n_2^2 + q_{11}n_2^4\right], c_{32} = q_{23}n_1^4 + (q_{24} + q_{13} + q_{32})n_1^2n_2^2 + q_{14}n_2^4 + n_1^2/r,\\
c_{33} = q_{25}n_1^3 + (q_{15} + q_{35})n_1n_2^2,\ c_{34} = (q_{28} + q_{38})n_1^2n_2 + q_{18}n_2^3,\ c_{41} = n_1^2h/r,\ c_{43} = J_3n_1,\ \ c_{44} = J_3n_2
\end{gathered} \tag{16}$$

If the variables $\Phi_0(t)$, $\varphi_{01}(t)$, $\varphi_{02}(t)$ are eliminated from the system of Eq. (15), the following time dependent ordinary differential equation is obtained:

$$\frac{d^2w_{01}(t)}{dt^2} + \frac{b_{22}b_{11} - b_{12}b_{21}}{b_{11}c_{\rho_1}}w_{01}(t) = 0 \tag{17}$$

where

$$\begin{gathered}
b_{11} = d_{21} - \frac{d_{11}d_{23}}{d_{13}},\ b_{12} = d_{22} - \frac{d_{12}d_{23}}{d_{13}},\ \ b_{21} = d_{31} - \frac{d_{11}d_{33}}{d_{13}},\ b_{22} = d_{32} - \frac{d_{12}d_{23}}{d_{13}},\\
d_{11} = c_{21} - \frac{c_{11}c_{24}}{c_{14}},\ \ d_{12} = \frac{c_{12}c_{24}}{c_{14}} - c_{22},\ d_{13} = c_{23} - \frac{c_{24}c_{13}}{c_{14}},\ d_{21} = c_{31} - \frac{c_{11}c_{34}}{c_{14}},\\
d_{22} = \frac{c_{12}c_{34}}{c_{14}} - c_{32}, d_{C23} = c_{33} - \frac{c_{13}c_{34}}{c_{14}}, d_{31} = c_{41} - \frac{c_{11}c_{44}}{c_{14}}, d_{C32} = \frac{c_{12}c_{44}}{c_{14}}, d_{33} = c_{43} - \frac{c_{13}c_{44}}{c_{14}}.
\end{gathered} \tag{18}$$

The expression for the frequency of clamped inhomogeneous orthotropic cylindrical shells within FOST is easily found from Eq. (17) without any intermediate mathematical operations as follows:

$$\omega_{sdt} = \sqrt{\frac{b_{22}b_{11} - b_{12}b_{21}}{b_{11}c_{\rho_1}}} \tag{19}$$

When the transverse shear stresses are not taken into account in the fundamental relations, that is, within the framework of classical shell theory, the expression of the frequency of inhomogeneous orthotropic cylindrical shells with clamped boundary conditions is obtained as follows:

$$\begin{aligned}
\omega_{cst} = \sqrt{\frac{8}{3\rho_1}}\Bigg\{&2p_{13}n_1^4 + \frac{1}{2}n_1^2n_2^2(p_{14} + 2p_{32} + p_{23}) + \frac{3}{8}p_{24}n_2^4 - \Bigg[\frac{1}{2}n_1^2n_2^2(p_{11} - 2p_{31} + p_{22}) +\\
&2p_{12}n_1^4 + \frac{3}{8}p_{21}n_2^4 - \frac{1}{2}\frac{n_1^2}{r}\Bigg] \times \frac{2q_{23}n_1^4 + \frac{1}{2}n_1^2n_2^2(q_{13} - q_{32} + q_{24}) + \frac{3}{8}q_{14}n_2^4 + \frac{1}{2}\frac{n_1^2}{r}}{2q_{22}n_1^4 + \frac{1}{2}n_1^2n_2^2(q_{12} + q_{31} + q_{21}) + \frac{3}{8}q_{11}n_2^4}\Bigg\}^{0.5}
\end{aligned} \tag{20}$$

The nondimensional frequency parameter of inhomogeneous orthotropic cylindrical shells based on FOSDT and CST are used as:

$$\bar{\omega} = \omega r\sqrt{\frac{\rho_0(1 - \nu_{12}\nu_{21})}{E_{022}}} \tag{21}$$

5. Results and Discussion

5.1. Comparisons

In this subsection, we performed comparisons to test the accuracy of the analytical expression for the frequency parameter that found in this study. Two examples of comparison are presented with the results obtained within the framework of classical and shear deformation theories.

In the first example, the cyclic frequency ($\varpi_{cst} = \omega_{cst}/2\pi$) values (in Hz) for clamped-clamped cylindrical shells obtained in this study are compared with the results obtained by FEM [27] and wave propagation method [28], and the results are listed in Table 1. The formula (20) is used in the comparison and it is taken into account that $k_1 = k_2 = 0$, $E_{011} = E_{022} = E_0$, $\nu_{12} = \nu_{21} = \nu_0$ for homogeneous isotropic material. In the comparison, the shell characteristics and material properties are taken from Xuebin [28] and are as follows: $L = 0.8$ m, $r = 0.3048$ m, $h = 1.1016$ mm, $E_0 = 64.73$ GPa, $\upsilon_0 = 0.3285$, $\rho_0 = 2700\,\text{kg/m}^3$. The current results for the clamped boundary condition are reliable and are among the results obtained by FEM and wave propagation approach method.

Table 1. Comparison of cyclic frequency values (in Hz) with the results obtained from FEM and wave propagation approach method.

n	$\varpi_{cst} = \omega_{cst} / 2\pi$ (in Hz)		
	Santiago and Wisniewski [27]	Xuebin [28]	Present
1	1206.835	1643.599	1229.979
2	632.204	759.714	751.742
3	368.149	406.966	435.504
4	272.719	275.872	273.733

Table 2. Comparison of nondimensional frequency parameter values of homogeneous cylindrical shells within FOSDT.

	ω_{1sdt}	
	Sofiyev and Fantuzzi [19]	Present study
r/h	$f_{,z} = 1 - 4Z^2$	
20	0.605 (1)	0.6052 (1)
30	0.451 (1)	0.4511(1)
50	0.335 (2)	0.3348 (2)
75	0.274 (6)	0.27398 (6)
r/h	$f_{,z} = 1$	
20	0.610 (1)	0.6097 (1)
30	0.452 (1)	0.4524 (1)
50	0.335 (2)	0.3349 (2)
75	0.274 (6)	0.2741 (6)

Table 3. Distribution of nondimensional frequency parameter of cylindrical shells with HOM and INH-orthotropic profiles against *r/h* within CST and FOSDT (*L/r* = 0.25).

r/h	$k_1 = k_2 = 0$	$k_1 = 1, k_2 = 0$	$k_1 = k_2 = 1$	$k_1 = k_2 = 0$	$k_1 = 1, k_2 = 0$	$k_1 = k_2 = 1$
	Hom	INH-exp		Hom	INH-exp	
	ϖ_{ST}, (n)			ϖ_{CT}, (n)		
20	6.527 (1)	5.086 (1)	6.397 (1)	9.140 (1)	7.091 (1)	8.918 (1)
25	5.763 (1)	4.486 (1)	5.642 (1)	7.320 (1)	5.679 (1)	7.143 (1)
30	5.117 (1)	3.981 (1)	5.007 (1)	6.108 (1)	4.739 (1)	5.961 (1)
40	4.132 (1)	3.212 (1)	4.040 (1)	4.597 (1)	3.567 (1)	4.487 (1)
50	3.443 (1)	2.676 (1)	3.365 (1)	3.693 (1)	2.867 (1)	3.606 (1)
60	2.945 (1)	2.289 (1)	2.878 (1)	3.094 (1)	2.402 (1)	3.021 (1)
70	2.573 (1)	2.000 (1)	2.515 (1)	2.668 (1)	2.072 (1)	2.606 (1)
80	2.287 (1)	1.778 (1)	2.236 (1)	2.351 (1)	1.826 (1)	2.297 (1)
90	2.061 (1)	1.602 (1)	2.015 (1)	2.106 (1)	1.637 (1)	2.058 (1)
100	1.879 (1)	1.461 (1)	1.838 (1)	1.912 (1)	1.486 (1)	1.869 (1)

In the second example, the nondimensional frequency parameter $\omega_{1sdt} = \omega_{sdt} r\sqrt{(1-\nu_{st}^2)\rho_{st} / E_{st}}$ values of homogeneous isotropic cylindrical shells obtained by using different shape functions within the framework of FOSDT are compared with the results of Sofiev and Fantuzzi [19], and listed in Table 2. The formula (19) is used for the frequency in the comparison and it is taken into account that $k_1 = k_2 = 0, E_{011} = E_{022} = E_0, \nu_{12} = \nu_{21} = \nu_0$ for homogeneous isotropic material. The cylindrical shell characteristics and isotropic material properties used in the comparisons are taken from Sofiyev and Fantuzzi [19] and are as follows: $L/r = 0.5$, $r/h = 20,30,50,75$, $E_0 = 207.7877$ GPa, $\nu_0 = 0.317756$, $\rho_0 = 8166$, $E_{st} = 322.27$ GPa, $\nu_{st} = 0.24$, $\rho_{st} = 2370$. It is seen that the current results obtained in the framework of FOSDT for different shape functions are in good agreement with the results of Sofiyev and Fantuzzi [19].

5.2. Vibration behaviors

Two types of inhomogeneity are discussed; in the first case, $k_1 = 1, k_2 = 0$ or INH$_1$ that is, Young and shear moduli change depending on the thickness coordinate, whereas the density remains constant. In the second case, it is considered as $k_1 = k_2 = 1$ or INH$_2$, that is, Young's moduli, shear moduli and density vary depending on the thickness coordinate together. In the inhomogeneity functions $\varphi_i(Z) = e^{k_i(Z-1/2)}$, when $k_1 = k_2 = 0$, the inhomogeneous orthotropic material turns into the homogeneous orthotropic material. The transverse shear stresses are distributed as parabolic functions, that is $f_{,z} = 1 - Z^2$ [3, 30].

The homogeneous orthotropic material type used in numerical calculations (except Fig. 4.) is glass-epoxy and its mechanical properties are as follows [30]: $E_{011} = 53.78$ GPa, $E_{022} = 17.93$ GPa, $G_{012} = G_{013} = 8.96$ GPa, $G_{023} = 3.45$ GPa, $\nu_{12} = 0.25$, $\rho_0 = 1950$ kg/m^3.

The nondimensional frequency parameter of inhomogeneous orthotropic cylindrical shells within FOSDT and CST are compared among themselves and with those of homogeneous orthotropic cases. $(\omega_{1cst} - \omega_{1sdt}) / \omega_{1cst} \times 100\%$ and $(\omega^{INH} - \omega^{Hom}) / \omega^{Hom} \times 100\%$. Since the number of longitudinal waves corresponding to the minimum values of the nondimensional frequency parameter is equal to one, it is not written in the Tables.

The distributions of magnitudes of nondimensional frequency parameter of cylindrical shells with homogeneous and two inhomogeneous-exponential profiles versus the radius-to- thickness ratio within CST and FOSDT are listed in Table 3 and Fig. 2. The values of nondimensional frequency parameter for clamped homogeneous and both inhomogeneous orthotropic cylindrical shells reduced, whereas the corresponding circumferential wave numbers does not change and is equal to one depending on the increase of r/h within FOSDT and CST. The influence of transverse shear strains on the nondimensional frequency parameter reaches 28.59%, 28.28 and 28.7% as $r/h = 20$, while it reduces to 1.73%, 1.68% and 1.66%, respectively, as $r/h = 100$.

At the same time, the influence of inhomogeneity on the nondimensional frequency parameter is remarkable and slightly increases from 22.08% to 22.28 within FOSDT and slightly reduces from 22.42% to 22.28% within CST for the INH$_1$-exponential profile, while the INH$_2$-exponential profile effect is less pronounced than in the other case, increases from 1.99% to 2.28% within FOSDT and reduces from 2.43% to 2.36% within CST as the r/h increment from 20 to 60, it is reduced by up to 2.18% with the next increase of r/h.

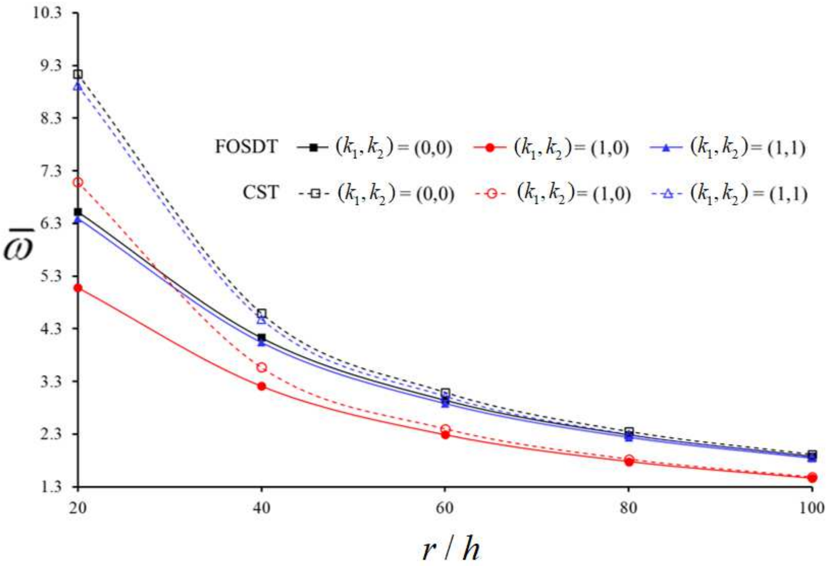

Fig. 2. Distribution of nondimensional frequency parameter of homogeneous and inhomogeneous cylindrical shells against *r/h* within CST and FOSDT (*L/r* = 0.25).

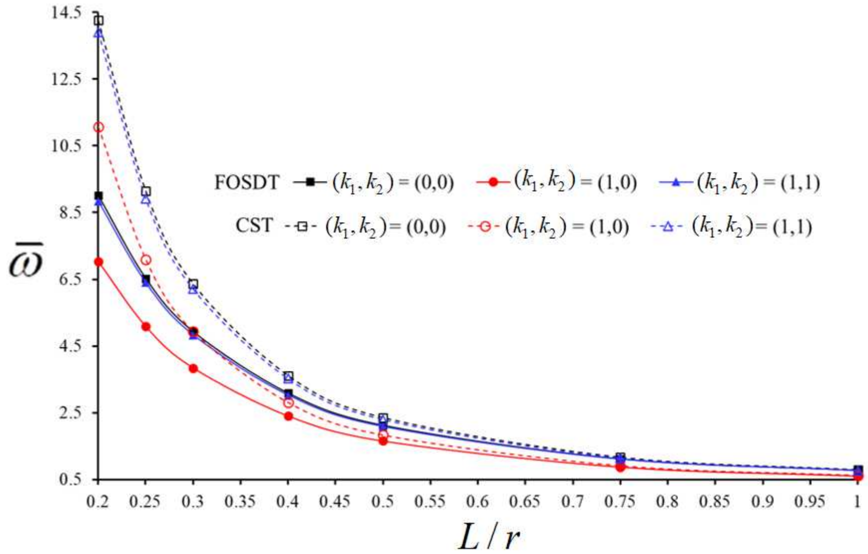

Fig. 3. Variation of nondimensional frequency parameter of HOM and INH-orthotropic cylindrical shells against *L/r* within CST and FOSDT (*r/h* = 20).

As can be seen, when the *r/h* ratio approaches 100, the effects of transverse shear deformations on the frequency weaken considerably and it becomes advantageous to use the classical shell theory for thin shells.

The variation of nondimensional frequency parameter of HOM, INH_1 and INH_2 –orthotropic cylindrical shells against the length-to-radius ratio in the framework of two theories are presented in Table 4 and Fig. 3. The values of nondimensional frequency parameter for clamped HOM and INH-orthotropic cylindrical shells reduce, while the corresponding circumferential wave numbers increase as increment *L/r* within FOSDT and CST. The influence of material inhomogeneity on the nondimensional frequency parameter is remarkable and slightly decreases from 22.01% to 21.68% and from 22.42% to 21.77% within FOSDT and CST, respectively, for the INH_1-exponential profile, while the influence of INH_2 profile is less pronounced than in the other case, reduced from 1.91% to 1.55% and from 2.43% to 1.52% within FOSDT and CST, respectively, as the length-to-radius ratio increment from 0.25 to 1. The influence of shear strains on the nondimensional frequency parameter reduced from 36.84%, 36.5% and 36.5% to 1.9%, 1.78% and 1.93%, respectively, as *L/r* increase from 0.25 to 1. It should be emphasized that when the *L/r* ratio approaches one, the effects of transverse shear deformations on the frequency weaken significantly.

Table 4. Distribution of nondimensional frequency parameter of cylindrical shells with HOM and INH-orthotropic profiles versus *L/r* within CST and FOSDT (*r/h* = 20).

L/r	$k_1 = k_2 = 0$	$k_1 = 1, k_2 = 0$	$k_1 = k_2 = 1$	$k_1 = k_2 = 0$	$k_1 = 1, k_2 = 0$	$k_1 = k_2 = 1$
	Hom	INH-exp		Hom	INH-exp	
	ϖ_{ST}, (n)			ϖ_{CT}, (n)		
0.20	9.008 (1)	7.025 (1)	8.836 (1)	14.262 (1)	11.063 (1)	13.915 (1)
0.25	6.527 (1)	5.086 (1)	6.397 (1)	9.140 (1)	7.091 (1)	8.918 (1)
0.30	4.933 (1)	3.841 (1)	4.831 (1)	6.362 (1)	4.936 (1)	6.209 (1)
0.40	3.091 (1)	2.405 (1)	3.025 (1)	3.612 (1)	2.804 (1)	3.526 (1)
0.50	2.127 (1)	1.656 (1)	2.083 (1)	2.355 (1)	1.829 (1)	2.301 (3)
0.75	1.120 (1)	0.875 (1)	1.101 (1)	1.165 (1)	0.909 (1)	1.143 (1)
1.0	0.775 (3)	0.607 (3)	0.763 (3)	0.790 (3)	0.618 (3)	0.778 (3)

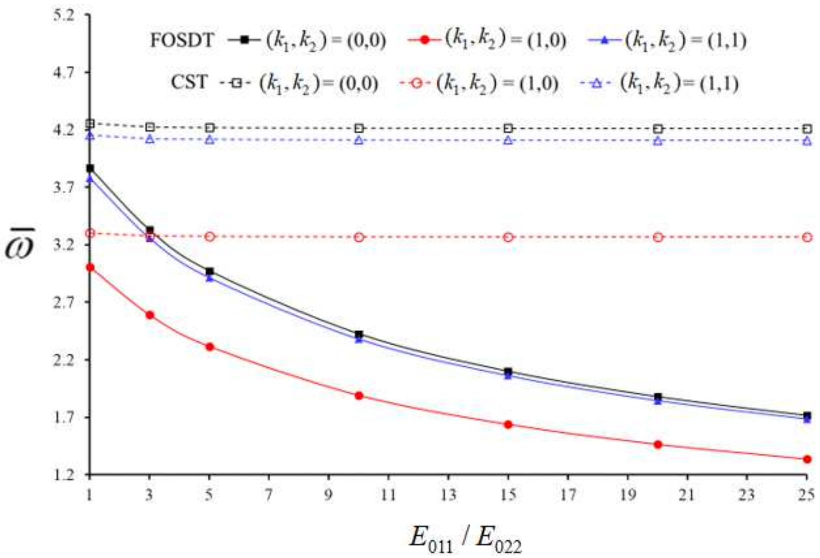

Fig. 4. Variation of nondimensional frequency parameter of cylindrical shells with HOM and INH-orthotropic profiles versus E_{011}/E_{022}.

The variation of nondimensional frequency parameter of HOM, INH_1 and INH_2 –orthotropic cylindrical shells against the orthotropy ratio in the framework of FOSDT and CST are plotted in Fig. 4. The following data is used in the drawing of Fig. 4: $E_{011}=1.38\times10^5$ MPa, $E_{022}=E_{011}/i$, $i=1,3,5,10,15,20,25$, $G_{012}=0.5E_{022}$, $G_{013}=0.5E_{022}$, $G_{023}=0.2E_{022}$, $\upsilon_{12}=0.25$, $\rho_0=1$, $\upsilon_{21}=\upsilon_{12}E_{022}/E_{011}$, $r/h=20$, $L/r=0.25$. It is observed that the influence of transverse shear deformations on the nondimensional frequency parameter increased significantly when the orthotropy ratio E_{011}/E_{022} increased. For example, when the orthotropy ratio is one, the influence of shear deformation on the frequency is around 9%, while this effect increases to around 59% when $E_{011}/E_{022}=25$. In the homogeneous orthotropic crust, those effects are slightly more pronounced than in the inhomogeneous crusts. When the E_{011}/E_{022} ratio increases from 1 to 25 within the framework of FOSDT, the effect of inhomogeneity on the frequency values, although a slight decrease in the INH_1 profile, remains significant and is around 22%, while the effect of the INH_2 profile on the nondimensional frequency parameter decreases from 2.22% to 1.69%.

6. Conclusion

In the framework of FOSDT, the vibration behavior of inhomogeneous orthotropic composite cylindrical shells under clamped boundary conditions was investigated with a unique approach. After deriving the basic relations and equations of the inhomogeneous orthotropic cylindrical shell in the framework of FOSDT, the analytical formula for the frequency in clamped boundary conditions was obtained. After confirming the accuracy of the obtained formula for frequency, detailed analyses and comments were carried out in comparison with the results in classical shell theory, and the effects of inhomogeneity and orthotropy were analyzed.

Author Contributions

The author discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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Appendix

Here are the following definitions applied in the present work:

$$
\begin{aligned}
&p_{11}=K_{11}^1 q_{11}+K_{12}^1 q_{12},\ p_{12}=K_{11}^1 q_{12}+K_{12}^1 q_{21},\ p_{13}=K_{11}^1 q_{13}+K_{12}^1 q_{23}+K_{11}^2,\\
&p_{14}=K_{11}^1 q_{14}+K_{12}^1 q_{24}+K_{12}^2,\ p_{15}=K_{11}^1 q_{15}+K_{12}^1 q_{25}+K_{15}^1,\\
&p_{18}=K_{11}^1 q_{18}+K_{12}^1 q_{28}+K_{18}^1,\ p_{21}=K_{21}^1 q_{11}+K_{22}^1 q_{21},\ p_{22}=K_{21}^1 q_{12}+K_{22}^1 q_{22},\\
&p_{23}=K_{21}^1 q_{13}+K_{22}^1 q_{23}+K_{21}^2,\ p_{24}=K_{21}^1 q_{14}+K_{22}^1 q_{24}+K_{22}^2,\\
&p_{25}=K_{21}^1 q_{15}+K_{22}^1 q_{25}+K_{25}^2,\ p_{28}=K_{21}^1 q_{18}+K_{22}^1 q_{28}+K_{28}^1,\\
&p_{31}=K_{66}^1 q_{31},\ p_{32}=K_{66}^1 q_{32}+2K_{66}^2,\ p_{35}=K_{35}^1-K_{66}^1 q_{35},\ p_{38}=K_{38}^1-K_{66}^1 q_{38},\\
&q_{11}=\frac{K_{22}^0}{\Delta},\ q_{12}=-\frac{K_{12}^0}{\Delta},\ q_{13}=\frac{K_{12}^0K_{21}^1-K_{11}^1K_{22}^0}{K_{11}^0K_{22}^0-K_{12}^0K_{21}^0},\ q_{14}=\frac{K_{12}^0K_{22}^1-K_{12}^1K_{22}^0}{K_{11}^0K_{22}^0-K_{12}^0K_{21}^0},\\
&q_{15}=\frac{K_{25}^0K_{12}^0-K_{15}^0K_{22}^0}{K_{11}^0K_{22}^0-K_{12}^0K_{21}^0},\ q_{18}=\frac{K_{28}^0K_{12}^0-K_{18}^0K_{22}^0}{K_{11}^0K_{22}^0-K_{12}^0K_{21}^0},\ q_{21}=-\frac{K_{21}^0}{K_{11}^0K_{22}^0-K_{12}^0K_{21}^0},\\
&q_{22}=\frac{K_{11}^0}{K_{11}^0K_{22}^0-K_{12}^0K_{21}^0},\ q_{23}=\frac{K_{11}^1K_{21}^0-K_{21}^1K_{11}^0}{\Delta}K_{11}^0K_{22}^0-K_{12}^0K_{21}^0,\ q_{24}=\frac{K_{12}^1K_{21}^0-K_{22}^1K_{11}^0}{K_{11}^0K_{22}^0-K_{12}^0K_{21}^0},\\
&q_{25}=K_{11}^0K_{22}^0-K_{12}^0K_{21}^0,\ q_{28}=\frac{K_{18}^0K_{21}^0-K_{28}^0K_{11}^0}{K_{11}^0K_{22}^0-K_{12}^0K_{21}^0},\ q_{31}=\frac{1}{K_{66}^0},\ q_{32}=-\frac{2K_{66}^1}{K_{66}^0},\\
&q_{35}=\frac{K_{35}^0}{K_{66}^0},\ q_{38}=\frac{K_{38}^0}{K_{66}^0},\ J_3=f(h/2)-f(-h/2).
\end{aligned}
\tag{A1}
$$

in which

$$
K_{11}^{i_1}=\int_{-h/2}^{h/2} Y_{11}(Z)z^{i_1}\,dz,\ K_{12}^{i_1}=\int_{-h/2}^{h/2} Y_{12}(Z)z^{i_1}\,dz=\int_{-h/2}^{h/2} Y_{21}(Z)z^{i_1}\,dz,
\tag{A2}
$$

$$\begin{aligned}
&K_{22}^{i_1}=\int_{-h/2}^{h/2}Y_{22}(Z)z^{i_1}\,dz,\ K_{66}^{i_1}=\int_{-h/2}^{h/2}G_{12}(Z)z^{i_1}\,dz;\ i_1=0,1,2,\\
&K_{15}^{i_2}=\int_{-h/2}^{h/2}z^{i_2}J_1(z)Y_{11}(Z)dz,\ K_{18}^{i_2}=\int_{-h/2}^{h/2}z^{i_2}J_2(z)Y_{12}(Z)dz,\\
&K_{25}^{i_2}=\int_{-h/2}^{h/2}z^{i_2}J_1(z)Y_{21}(Z)dz,\ K_{28}^{i_2}=\int_{-h/2}^{h/2}z^{i_2}J_2(z)Y_{22}(Z)dz,\\
&K_{35}^{i_2}=\int_{-h/2}^{h/2}z^{i_2}J_1(z)Y_{66}(Z)dz,\ K_{38}^{i_2}=\int_{-h/2}^{h/2}z^{i_2}J_2(z)Y_{66}(Z)dz,\ i_2=0,1.
\end{aligned}\tag{A2-cont.}$$

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