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Research Paper

Monitoring Dynamical Behavior and Optical Solutions of Space-Time Fractional Order Double-Chain Deoxyribonucleic Acid Model Considering the Atangana's Conformable Derivative

Samah M. Mabrouk[1], Abdul-Majid Wazwaz[2], Ahmed S. Rashed[1,3]

[1] Department of Physics and Engineering Mathematics, Zagazig University, Faculty of Engineering, 44519, Egypt, Email: samahmm@gmail.com

[2] Department of Mathematics, Saint Xavier University, Chicago, IL 60655, United States, Email: wazwaz@sxu.edu

[3] Department of Basic Science, Faculty of Engineering, Delta University for Science and Technology, 11152, Gamasa, Egypt, Email: ahmed.s.rashed@gmail.com

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Corresponding author: A.S. Rashed (ahmed.s.rashed@gmail.com)

Abstract. DNA, or deoxyribonucleic acid, is found in every single cell and is the cell's primary information storage medium. DNA stores all an organism's genetic information, including the instructions it needs to grow, divide, and live. DNA is made up of four different building blocks called nucleotide bases: adenine (A), thymine (T), cytosine (C), and guanine (G). The genome is sequenced in vitro utilizing encoding strategies such as labelling one bond pair as 0 and the other as 1 to store digital information. In this study, the fractional differential order of double-chain DNA dynamical system was investigated, considering Atangana's conformable fractional derivative. The conformable sub-equation method was applied to the system. The analysis resulted in some interesting new exact solutions of the model. One-soliton kink solution, multiple-soliton solutions, and periodic-wave solutions are the three broad categories that may be used to describe the results. In order to better understanding the solutions found, we have visually investigated a few of them. Both solitary and anti-solitary waves of the DNA strands are seen, attesting to the nonlinear dynamics of the system. The gathered data might be used to conduct application evaluations and draw further scientific findings.

Keywords: Fractional Calculus, Deoxyribonucleic acid system, Conformable sub-equation method, Atangana's conformable derivative.

1. Introduction

Gene mapping is an effective method for locating disease-causing mutations, localizing favorable features, and refining reference genomes across species. From the early discovery of co-inheritance of characteristics and linkage analysis to the rapid advancement of the field brought on by the Human Genome Project and the advent of next-generation sequencing, this area of study has stood the test of time. Gene mapping has been shown to be a high-yield approach with applications in medicine and agriculture [1]. DNA methylation plays a crucial role in maintaining proper gene expression. However, changes in the methylation state of the genome have been seen across a number of organs as a result of the ageing process [2]. The building blocks of all living things are found in the deoxyribonucleic acid (DNA), which is composed of four nucleotides and twenty amino acids (combinations of nucleotides). DNA may be seen as a digital component by considering it with its features and functions, all of which are captured by these distinct elements. When DNA is seen as an organic digital memory, it becomes an attractive data storage medium due to its many advantages over traditional electronic media, such as its greater density, stability, energy efficiency, durability, and lack of predictable technological obsolescence [3]. Since various scientific fields, including Engineering and Biology, have recently made technological strides, scientists from these fields have been able to collaborate on creating effective models that imitate the right nature and its qualities. Industry, medical, engineering, biochemistry, biotechnology, computer science, and other fields may all benefit from these models [4, 5]. Through careful research of DNA's behavior, we can see that its nonlinearity structures are what generate the localized waves that are responsible for the efficient transfer of energy without any of it being lost as heat. DNA replication relies heavily on a series of molecular events known as origins of replication, which initiate the copying of genetic information from one DNA molecule to two [6]. One of the most recent application of DNA is to be used as storage medium. DNA is analogous to a digital media because of the way it is portrayed digitally, with the nucleotide bases functioning as digital symbols. As this area of study develops, it may be possible to store all the world's digital information for a year in a space no larger than four grams. A single gram of DNA has the potential to encode 215 million gigabits of data [7]. DNA is made up of four different building blocks called nucleotide bases: adenine (A), thymine (T), cytosine (C), and guanine (G). The genome is sequenced in vitro utilizing encoding strategies such labelling one bond pair as 0 and the other as 1 to store digital information. Typically, the Shannon

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information is used to evaluate the capacity of a medium to store data. DNA is a heterogeneous polymer made up of a long chain of individual deoxyribonucleotide monomers, each of which may be any one of the four bases (A, T, C, or G) [8]. Moreover, environmental DNA sequencing has given scientists fresh insight into the strategies used by marine bacteria to flourish at varying depths in the ocean. Ancestral environmental data are useful in ocean engineering for gauging the seas' ability to buffer the world from climate change [9]. The dynamical mathematical system describing DNA is considered as an evolution partial differential equation. The evolution equations in one, two and three dimensions have been comprehensively analyzed via numerous techniques including Lie infinitesimals [10-15, 9], Group theoretical method with Lax pair investigation for nonlinear evolution equations and other applications. Moreover, different techniques were employed like the exponential function method [16, 17], Painlevé property analysis [18], singular manifolds method [19-21], the method of sub ODE [22], Hirota Bilinear method [23-25], Elliptic method [26, 27], sin-cos and tan-cot methods, [28-37] and some other numerical techniques [38-42]. Researchers have looked at several promising approaches for dealing with the nonlinear DNA system including solitons, kink and periodic solutions [43-45]. Riccati parameterized factorization techniques were used to a model of DNA created by Peyrard and Bishop in order to find solitary wave solutions for longitudinal and transverse motions [44-46]. The expansion method was used to investigate solutions including solitons, kinks, periodic waves, and multi-soliton waves. Bell-shaped and periodic solitary wave solutions to the coupled DNA nonlinear dynamical equation were characterized numerically [47, 48].

The motivation of the current research is to investigate the fractional order of DNA dynamical system instead of integer version which depends on both of current and historical states to fully describe the behavior of the double-chain Deoxyribonucleic acid. Aiming to benefit of the historical state to help in solving some biological problems especially in developing countries as homeless children and spinal cord donators. The paper is planned as follows. In section 2, the basic properties, and definitions of the Atangana's conformable derivative are offered, beside a general description of the conformable sub-equation method. In section 3, the fractional order of double chain DNA model is introduced and is analytically solved. In section 4, the results are discussed. The paper is terminated by conclusions remarks in section 5. The schematic configuration of the considered system is illustrated in Fig. 1.

2. Preliminaries and Method of Solution

2.1. Definitions and some properties of fractional-order derivatives

In this section, some basic properties, and definitions of the Atangana's conformable fractional order derivatives are provided. Thus, the fractional derivatives of order α is defined as:

Definition 1: Let $f:[0,\infty)\to\mathcal{R}$ and t > 0, then the α^{th} order conformable derivative of f(t) is defined by [49, 50]:

$$ {}_0D_t^{\alpha}\{f(t)\}=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon},\qquad \forall\ t>0,\ \alpha\in(0,1). \tag{1}$$

If f(t) is $\alpha-$differentiable in some (0, a), a > 0, and $\lim_{\varepsilon\to 0^+} f^{(\alpha)}(t)$ exist, then $f^{(\alpha)}(0)=\lim_{\varepsilon\to 0^+} f^{(\alpha)}(t)$.

Definition 2: The Atangana's conformable or $\beta-$conformable derivative is defined as [50]:

$$ {}_0^AD_x^{\alpha}\{f(x)\}=\lim_{\varepsilon\to 0}\frac{f\left(x+\varepsilon\left(x+\frac{1}{\Gamma(\alpha)}\right)^{1-\alpha}\right)-f(x)}{\varepsilon}. \tag{2}$$

Some properties of the Atangana's conformable derivative [51, 50] are as follows:

$$ {}_0^AD_x^{\alpha}(af(x)+bg(x))=a\,{}_0^AD_x^{\alpha}f(x)+b\,{}_0^AD_x^{\alpha}g(x),\ \text{for all a, b}\ \in\mathbb{R}, \tag{3}$$

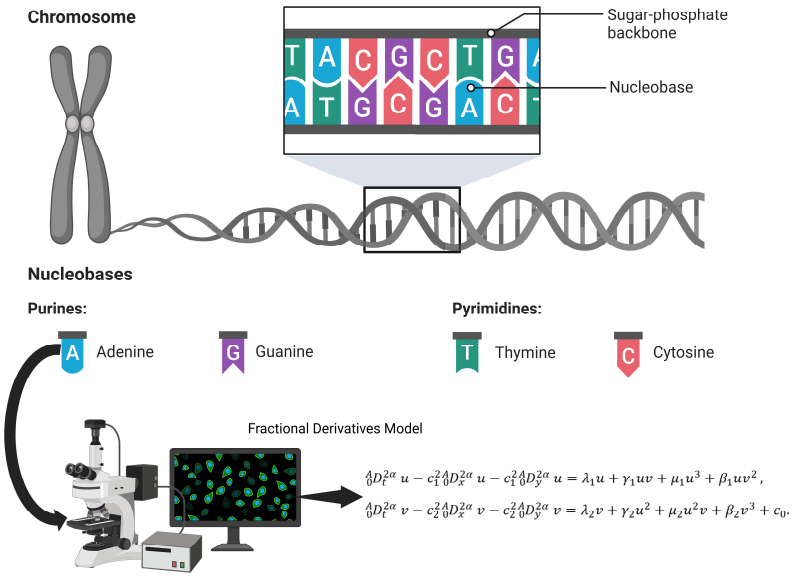


Fig. 1. Schematic configuration of the considered system.

$$ {}_{0}^{A}D_{x}^{\alpha}c = 0, \text{ c is constant,} \tag{4} $$

$$ {}_{0}^{A}D_{x}^{\alpha}\{f(x).g(x)\} = g(x)\,{}_{0}^{A}D_{x}^{\alpha}\{f(x)\} + f(x)\,{}_{0}^{A}D_{x}^{\alpha}\{g(x)\}, \tag{5} $$

$$ {}_{0}^{A}D_{x}^{\alpha}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\,{}_{0}^{A}D_{x}^{\alpha}\{f(x)\} - f(x)\,{}_{0}^{A}D_{x}^{\alpha}\{g(x)\}}{g^{2}(x)}, \tag{6} $$

Recall Eq. (2) and consider $\varepsilon = \left(x + \frac{1}{\Gamma(\alpha)}\right)^{\alpha-1} h$, as $h \to 0$, when $\varepsilon \to 0$, then we have:

$$ {}_{0}^{A}D_{x}^{\alpha}\{f(x)\} = \left(x + \frac{1}{\Gamma(\alpha)}\right)^{1-\alpha} \frac{df(x)}{dx}, \tag{7} $$

$$ \eta = \frac{\zeta}{\alpha}\left(x + \frac{1}{\Gamma(\alpha)}\right)^{\alpha}, \tag{8} $$

where ζ, is a constant, then we have:

$$ {}_{0}^{A}D_{x}^{\alpha}\{f(\eta)\} = \zeta \frac{df(\eta)}{d\eta}. \tag{9} $$

2.2. Conformable sub-equation method

Consider any conformable fractional order differential equation (CFDE):

$$ p\left(u, {}_{0}^{A}D_{t}^{\alpha}u, {}_{0}^{A}D_{x}^{\alpha}u, {}_{0}^{A}D_{y}^{\alpha}u, {}_{0}^{A}D_{t}^{\alpha}\,{}_{0}^{A}D_{x}^{\alpha}u, {}_{0}^{A}D_{t}^{\alpha}\,{}_{0}^{A}D_{y}^{\alpha}u, \ldots\right) = 0, \quad 0 < \alpha < 1. \tag{10} $$

where u is an unknown function and p is a polynomial of u and its Atangana's conformable derivatives. To find the exact solutions of Eq. (10) the fractional sub equation method [50], can be performed as in the following steps:

Step 1:

Utilizing the travelling wave transformation for Eq. (10), by considering:

$$ u(x,y,t) = u(\eta), \eta = \frac{k}{\alpha}\left(x + \frac{1}{\Gamma(\alpha)}\right)^{\alpha} + \frac{l}{\alpha}\left(y + \frac{1}{\Gamma(\alpha)}\right)^{\alpha} + \frac{c}{\alpha}\left(t + \frac{1}{\Gamma(\alpha)}\right)^{\alpha}, \tag{11} $$

where k, l and c are constants to be determined later, the FDE (10) is reduced to a conformable ordinary differential equation in the form:

$$ q\left(u(\eta), ku'(\eta), lu'(\eta), u''(\eta), \ldots\right) = 0. \tag{12} $$

Suppose that the solution of Eq. (12) is in the following series form:

$$ u(\eta) = \sum_{i=0}^{n} a_i \varphi^{i}, \quad \varphi = \varphi(\eta), \tag{13} $$

where a_i, (i =1, 2, ..., n) are constants, the positive integer n is calculated through balancing the highest order derivatives with the order of the highly nonlinear terms appear in (12), in the following manner:

Let the order of $d^{p}u / d\eta^{p}$ to be in the form:

$$ O\left(\frac{d^{p}u}{d\eta^{p}}\right) = n + p, \tag{14} $$

Let the order of $u^{q}(d^{p}u / d\eta^{p})^{s}$ to be in the form:

$$ O\left(u^{q}\left(\frac{d^{p}u}{d\eta^{p}}\right)^{s}\right) = nq + s(p+n). \tag{15} $$

Balancing Eqs. (14) and (15) results in obtaining the proper value of n. Assume that φ satisfies the following Riccati equation:

$$ \frac{d\varphi}{d\eta} = \theta + \varphi^{2}. \tag{16} $$

Step 4:

Substituting (13) along with (16) into (12), leads to an algebraic equation, collect the coefficients of the similar orders of that equation then equating these coefficients by zero, get a system of algebraic equations which can be solved using any mathematical tool. According to the previous results, different forms of analytic solutions can be obtained through the obtained constants values and the back substitution.

3. Atangana's Conformable Fractional Order Double Chain DNA Model

3.1. Mathematical formulation of the problem

The Atangana's conformable fractional order double-chain DNA model in (2+1)-dimensions is:

$$ {}_0^A D_t^{2\alpha} u - c_1^2 {}_0^A D_x^{2\alpha} u - c_1^2 {}_0^A D_y^{2\alpha} u = \lambda_1 u + \gamma_1 uv + \mu_1 u^3 + \beta_1 uv^2, \tag{17} $$

$$ {}_0^A D_t^{2\alpha} v - c_2^2 {}_0^A D_x^{2\alpha} v - c_2^2 {}_0^A D_y^{2\alpha} v = \lambda_2 v + \gamma_2 u^2 + \mu_2 u^2 v + \beta_2 v^3 + c_0. \tag{18} $$

where $u(x, y, t)$ and $v(x, y, t)$ refer to the difference of the longitudinal and the transverse displacements between the bottom and top strands, respectively. The constants $c_1, c_2, \lambda_1, \lambda_2, \gamma_1, \gamma_2, \mu_1, \mu_2, \beta_1, \beta_2$ and c_0 are defined as:

$$ \begin{cases} c_1 = \pm\dfrac{\epsilon}{\rho}, c_2 = \pm\dfrac{F}{\rho}, \lambda_1 = \dfrac{-2\mu}{\rho\sigma h}(c - l_0), \\ \beta_1 = \beta_2 = \dfrac{4\mu l_0}{\rho\sigma h^3}, c_0 = \dfrac{\sqrt{2}\mu(h - l_0)}{\rho\sigma}, \\ \lambda_2 = \dfrac{-2\mu}{\rho\sigma}, \gamma_1 = 2\gamma_2 = \dfrac{2\sqrt{2}\mu l_0}{\rho\sigma h^2}, \mu_1 = \mu_2 = \dfrac{-2\mu l_0}{\rho\sigma h^3}. \end{cases} \tag{19} $$

ρ, is the mass density, σ, is the area of the transverse cross-section, ϵ, is the Young's modulus, F, is the tension density of the strand, μ, is the rigidity of the elastic membrane, h, is the distance between the two strands and l_0, is the height of the membrane in the equilibrium position. Now, the following transformation is used:

$$ v = au + b. \tag{20} $$

Equation (20) reduces Eq. (17) to:

$$ {}_0^A D_t^{2\alpha} u - c_1^2 {}_0^A D_x^{2\alpha} u - c_1^2 {}_0^A D_y^{2\alpha} u = u^3\left(\mu_1 + \beta_1 a^2\right) + u^2\left(\gamma_1 a + 2\beta_1 ab\right) + u\left(\lambda_1 + \gamma_1 b + \beta_1 b^2\right) \tag{21} $$

and Eq. (18) is reduced to:

$$ a\,{}_0^A D_t^{2\alpha} u - ac_2^2 {}_0^A D_x^{2\alpha} u - ac_2^2 {}_0^A D_y^{2\alpha} u = u^3\left(a\mu_2 + \beta_2 a^3\right) + u^2\left(\gamma_2 + \mu_2 b + 3\beta_2 a^2 b\right) + u\left(\lambda_2 a + 3\beta_2 ab^2\right) + \lambda_2 b + \beta_2 b^3 + c_0. \tag{22} $$

Equations (21) and (22) are similar for:

$$ b = \frac{h}{\sqrt{2}} \quad , \quad F = \epsilon. \tag{23} $$

Finally, the system of equations (17) and (18) is reduced to a single fractional order equation:

$$ {}_0^A D_t^{2\alpha} u - C_1^2 {}_0^A D_x^{2\alpha} u - C_1^2 {}_0^A D_y^{2\alpha} u - Au^3 - Bu^2 - Du = 0, \tag{24} $$

where,

$$ A = \frac{\Omega}{h^3}\left(4a^2 - 2\right), \; B = \frac{6\sqrt{2}a}{h^2}\Omega, \;\; D = \frac{6\Omega}{h} - \frac{2\Omega}{l_0}, \;\; \Omega = \frac{\mu l_0}{\rho\sigma} \text{ and } C_1 = C_2. \tag{25} $$

3.2. Analytical solution of the problem

The conformable sub-equation method will be applied to find analytic solutions of equation (24). Inserting equation (11) into equation (24), considering the fractional derivatives' properties (3)-(9), obtaining a conformable order ordinary differential equation:

$$ c^2 u'' - C_1^2 k^2 u'' - C_1^2 l^2 u'' - Au^3 - Bu^2 - Du = 0 \tag{26} $$

balancing the highest order derivatives with the order of the highly nonlinear terms in (26), according to (14) and (15):

$$ n + 2 = 3n \;\Rightarrow\; n = 1 \tag{27} $$

Setting $n = 1$ in Eq. (13) yields:

$$ u = a_0 + a_1\varphi. \tag{28} $$

Inserting Eqs. (28) and (16) into Eq. (26), considering Eq. (11) and the properties of Atangana's conformable fractional derivatives (3)-(9), getting the algebraic equation:

$$ c^{2\alpha}\left(2a_1\theta\varphi + 2a_1\varphi^3\right) - C_1^2 k^{2\alpha}\left(2a_1\theta\varphi + 2a_1\varphi^3\right) - C_1^2 l^{2\alpha}\left(2a_1\theta\varphi + 2a_1\varphi^3\right) - A\left(a_0^3 + 3a_0^2 a_1\varphi + 3a_0 a_1^2\varphi^2 + a_1^3\varphi^3\right) - B\left(a_0^2 + 2a_0 a_1\varphi + a_1^2\varphi^2\right) - Da_0 - Da_1\varphi = 0. \tag{29} $$

Collect the coefficients of the similar orders in Eq. (29), reveals that:

$$ \text{Coefficient of } \varphi^0 = -Aa_0^3 - Ba_0^2 - Da_0 = 0, \tag{30} $$

$$ \text{Coefficient of } \varphi = 2a_1\theta c^{2\alpha} - 2a_1\theta C_1^2 k^{2\alpha} - 2a_1\theta C_1^2 l^{2\alpha} - 3a_0^2 a_1 A - 2a_0 a_1 B - Da_1 = 0, \tag{31} $$

$$\text{Coefficient of } \varphi^2 = -3a_0a_1^2A - a_1^2B = 0, \tag{32}$$

$$\text{Coefficient of } \varphi^3 = 2a_1c^{2\alpha} - 2a_1C_1^2k^{2\alpha} - 2a_1C_1^2l^{2\alpha} - a_1^3A = 0. \tag{33}$$

By solving the system of equations (30)-(33), the values of the constants are as follows:

$$a_0 = \frac{-B}{3A},\ \ a_1 = \sqrt{\frac{2}{A}\left(c^{2\alpha} - C_1^2k^{2\alpha} - C_1^2l^{2\alpha}\right)},\ \ D = \frac{2B^2}{9A} \text{ and } \theta = \frac{Ba_0 + D}{Aa_1^2} \equiv \frac{-B^2 + 3AD}{3A^2a_1^2}. \tag{34}$$

4. Results and Discussions

Referring to the solutions of fractional Riccati equation (16), by Zhang et al. [52], the function $\varphi(\eta)$ is expressed in the form:

$$\varphi(\eta) = \begin{cases} -\sqrt{-\theta}\tanh_\alpha\left(\sqrt{-\theta}\eta\right), & \theta < 0 \\ -\sqrt{-\theta}\coth_\alpha\left(\sqrt{-\theta}\eta\right), & \theta < 0 \\ \sqrt{\theta}\tan_\alpha\left(\sqrt{\theta}\eta\right), & \theta > 0 \\ -\sqrt{\theta}\cot_\alpha\left(\sqrt{\theta}\eta\right), & \theta > 0 \\ -\dfrac{\Gamma(1+\alpha)}{\xi^\alpha + \omega}, & \omega = \text{const}, \ldots \theta = 0. \end{cases} \tag{35}$$

Then the solutions of the DNA system of Eqs. (17) and (18), considering Eqs. (20), (28) and (35) are as follows:

$$u_1 = \frac{-B}{3A} - \sqrt{\frac{B^2 - 3AD}{3A^2}}\ \tanh_\alpha\left(\sqrt{\frac{B^2 - 3AD}{6A\left(c^{2\alpha} - C_1^2k^{2\alpha} - C_1^2l^{2\alpha}\right)}}(kx + ly + ct)\right),\ B^2 - 3AD < 0, A \neq 0. \tag{36}$$

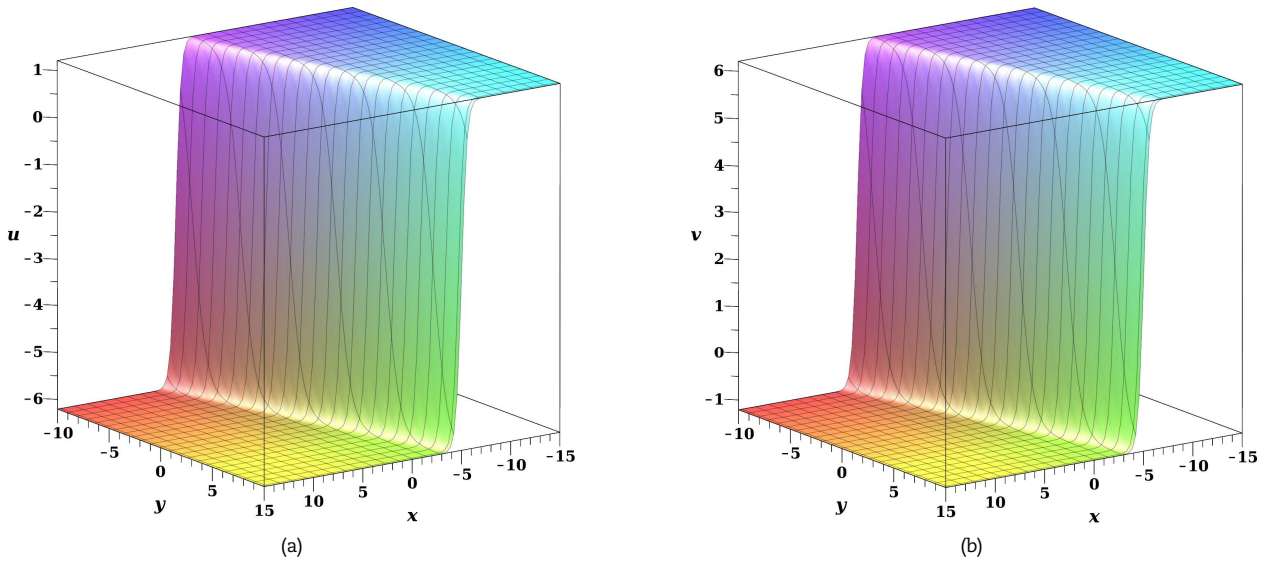
$$v_1 = a\left(\frac{-B}{3A} - \sqrt{\frac{B^2 - 3AD}{3A^2}}\ \tanh_\alpha\left(\sqrt{\frac{B^2 - 3AD}{6A\left(c^{2\alpha} - C_1^2k^{2\alpha} - C_1^2l^{2\alpha}\right)}}(kx + ly + ct)\right)\right) + b,\ B^2 - 3AD < 0, A \neq 0. \tag{37}$$

Equations (36) and (37) reveal the kink solitary waves for the longitudinal and the transversal motions as shown in Fig. 2, for $\alpha = 0.2$, $B = 15$, $A = 2$, $D = 10$, $k = 0.5$, $l = 0.2$, $c = 1$, $C_1 = 0.1$, $t = 0.5$, $a = 1$ and $b = 5$.

$$u_2 = \frac{-B}{3A} - \sqrt{\frac{B^2 - 3AD}{3A^2}}\ \coth_\alpha\left(\sqrt{\frac{B^2 - 3AD}{6A\left(c^{2\alpha} - C_1^2k^{2\alpha} - C_1^2l^{2\alpha}\right)}}(kx + ly + ct)\right),\ B^2 - 3AD < 0, A \neq 0. \tag{38}$$

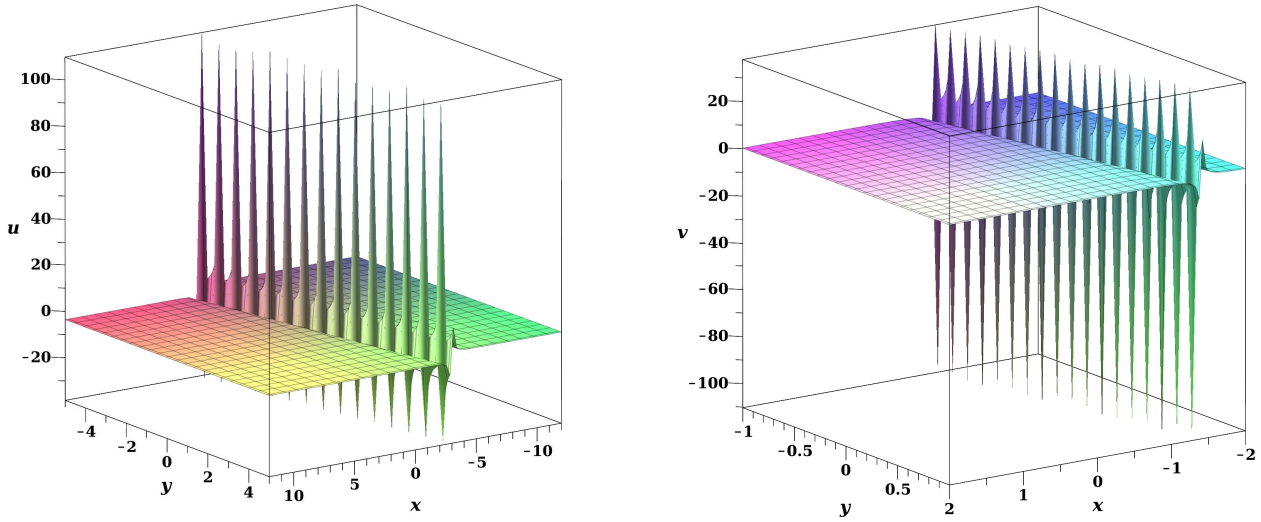
$$v_2 = a\left(\frac{-B}{3A} - \sqrt{\frac{B^2 - 3AD}{3A^2}}\ \coth_\alpha\left(\sqrt{\frac{B^2 - 3AD}{6A\left(c^{2\alpha} - C_1^2k^{2\alpha} - C_1^2l^{2\alpha}\right)}}(kx + ly + ct)\right)\right) + b,\ B^2 - 3AD < 0, A \neq 0. \tag{39}$$

The solitary wave for the longitudinal motion and the transversal motions u_2 and v_2 are plotted as Fig. 3, for the following arbitrary constants $\alpha = 0.5$, $B = 15$, $A = 2$, $D = 10$, $k = 0.5$, $l = 0.2$, $c = 1$, $t = 0.5$, $a = 0.2$ and $b = 5$.



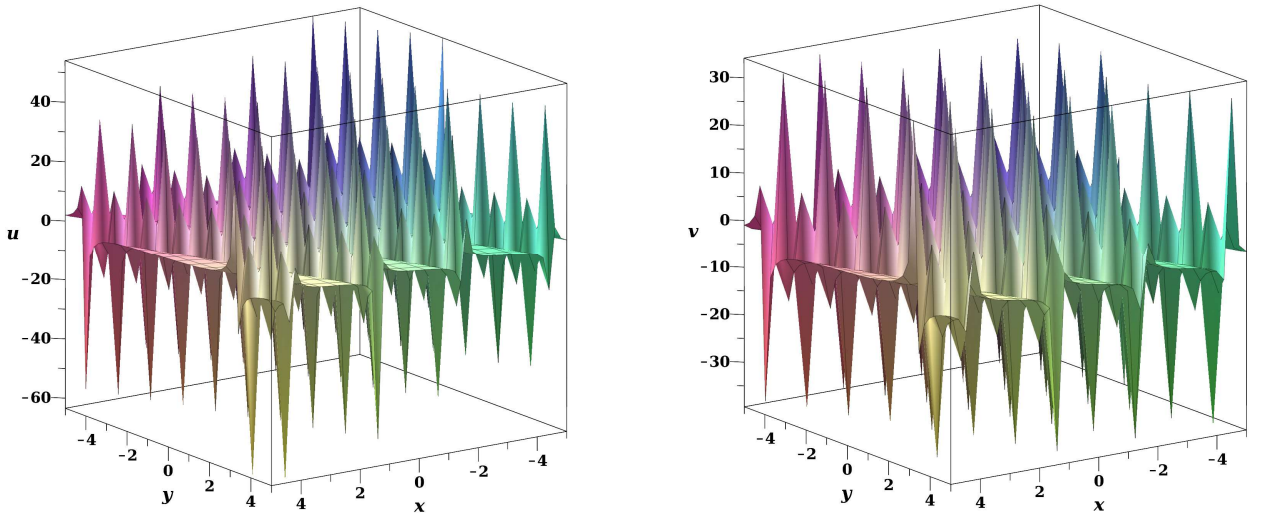
(a) (b)

Fig. 2. (a) The kink solitary wave for the longitudinal motion, (b) the anti-kink solitary wave for the transversal motion at $\alpha = 0.2$, $B = 15$, $A = 2$, $D = 10$, $k = 0.5$, $l = 0.2$, $c = 1$, $C_1 = 0.1$, $t = 0.5$, $a = 1$ and $b = 5$.



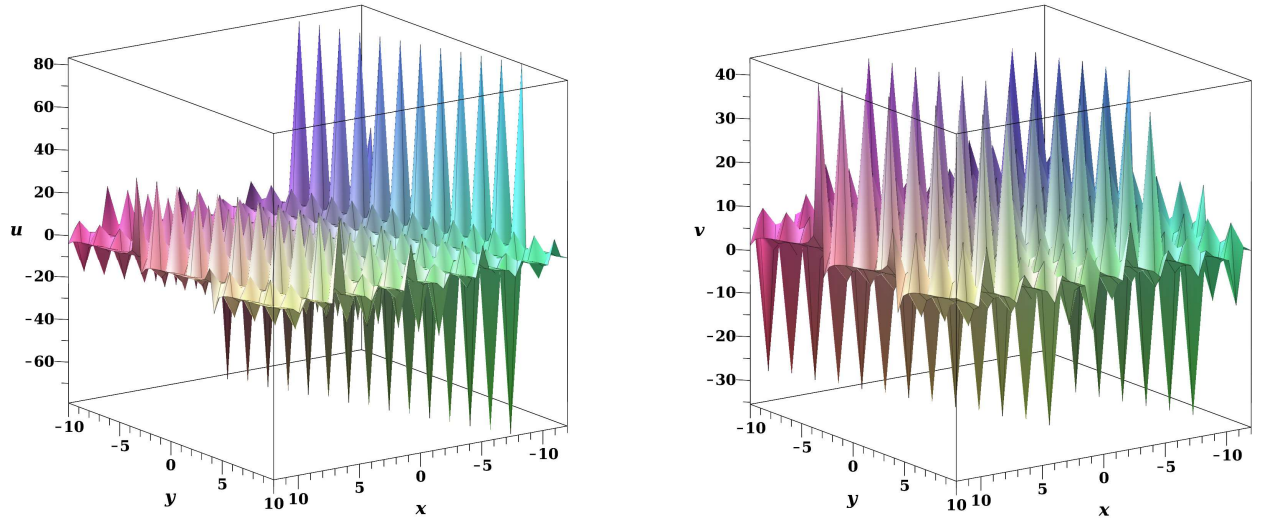
(a) (b)

Fig. 3. (a) The solitary wave for the longitudinal motion, (b) the anti-wave for the transversal motion at $\alpha = 0.5$, $B = 15$, $A = 2$, $D = 10$, $k = 0.5$, $l = 0.2$, $c = 1$, $t = 0.5$, $a = 0.2$ and $b = 5$.



(a) (b)

Fig. 4. (a) The periodic wave for the longitudinal motion, (b) the transversal motion at $\alpha = 0.4$, $B = 5$, $A = 10$, $D = 10$, $k = 0.5$, $l = 0.2$, $c = 1$, $C_1 = 0.1$, $t = 0.5$, $a = 1$ and $b = -3$.



(a) (b)

Fig. 5. (a) The multi-soliton wave for the longitudinal motion, (b) the transversal motion at $\alpha = 0.4$, $B = 5$, $A = 10$, $D = 10$, $k = 0.5$, $l = 0.2$, $c = 1$, $C_1 = 0.1$, $t = 0.5$, $a = 1$ and $b = 5$.

$$u_3 = \frac{-B}{3A} + \sqrt{\frac{3AD - B^2}{3A^2}} \tan_\alpha \left(\sqrt{\frac{3AD - B^2}{6A\left(c^{2\alpha} - C_1^2 k^{2\alpha} - C_1^2 l^{2\alpha}\right)}} (kx + ly + ct) \right), 3AD - B^2 > 0, A \neq 0. \tag{40}$$

$$v_3 = a\left(\frac{-B}{3A} + \sqrt{\frac{3AD - B^2}{3A^2}} \tan_\alpha \left(\sqrt{\frac{3AD - B^2}{6A\left(c^{2\alpha} - C_1^2 k^{2\alpha} - C_1^2 l^{2\alpha}\right)}} (kx + ly + ct) \right) \right) + b,\ 3AD - B^2 > 0, A \neq 0. \tag{41}$$

These waves are presented in Fig. 4, showing the periodic wave for the longitudinal and transversal motions at the following arbitrary constants $\alpha = 0.4$, $B = 5$, $A = 10$, $D = 10$, $k = 0.5$, $l = 0.2$, $c = 1$, $C_1 = 0.1$, $t = 0.5$, $a = 1$ and $b = -3$.

$$u_4 = \frac{-B}{3A} - \sqrt{\frac{3AD - B^2}{3A^2}} \cot_\alpha \left(\sqrt{\frac{3AD - B^2}{6A\left(c^{2\alpha} - C_1^2 k^{2\alpha} - C_1^2 l^{2\alpha}\right)}} (kx + ly + ct) \right), 3AD - B^2 > 0, A \neq 0. \tag{42}$$

$$v_4 = a\left(\frac{-B}{3A} - \sqrt{\frac{3AD - B^2}{3A^2}} \cot_\alpha \left(\sqrt{\frac{3AD - B^2}{6A\left(c^{2\alpha} - C_1^2 k^{2\alpha} - C_1^2 l^{2\alpha}\right)}} (kx + ly + ct) \right) \right) + b,\ 3AD - B^2 > 0, A \neq 0. \tag{43}$$

The multi-soliton waves u_4 and v_4 for the longitudinal and transversal motions are presented in Fig. 5, for the following arbitrary constants $\alpha = 0.4$, $B = 5$, $A = 10$, $D = 10$, $k = 0.5$, $l = 0.2$, $c = 1$, $C_1 = 0.1$, $t = 0.5$, $a = 1$ and $b = 5$.

Finally, we have:

$$u_5 = \frac{-B}{3A} - \sqrt{\frac{2}{A}\left(c^{2\alpha} - C_1^2 k^{2\alpha} - C_1^2 l^{2\alpha}\right)} \frac{\Gamma(1+\alpha)}{(kx + ly + ct)^\alpha + \omega},\ \omega = \text{constant}, \theta = 0, A \neq 0. \tag{44}$$

$$v_5 = a\left(\frac{-B}{3A} - \sqrt{\frac{2}{A}\left(c^{2\alpha} - C_1^2 k^{2\alpha} - C_1^2 l^{2\alpha}\right)} \frac{\Gamma(1+\alpha)}{(kx + ly + ct)^\alpha + \omega} \right) + b,\ \omega = \text{constant}, \theta = 0, A \neq 0. \tag{45}$$

On the other hand, when we think about Eq. (24), we found that it is similar to the space time fractional generalized reaction duffing equation [53], with some constants renamed:

$$ {}_0^A D_t^{2\alpha} u + E\, {}_0^A D_x^{2\alpha} u + F\, {}_0^A D_y^{2\alpha} u + qu^3 + ru^2 + su = 0. \tag{46}$$

The importance of the space time fractional generalized reaction duffing equation (46) is that it can be reduced to many well-known nonlinear fractional wave equations such as the following:

- Fractional Klein-Gordon equation:

$$ {}_0^A D_t^{2\alpha} u - {}_0^A D_x^{2\alpha} u - d_1 u - d_2 u^3 = 0,\ t > 0\ , 0 < \alpha < 1. \tag{47}$$

- Fractional Landau-Ginzburg-Higgs equation:

$$ {}_0^A D_t^{2\alpha} u - {}_0^A D_x^{2\alpha} u - m^2 u + g^2 u^3 = 0,\ t > 0\ , 0 < \alpha < 1. \tag{48}$$

- Fractional φ^4 equation:

$$ {}_0^A D_t^{2\alpha} u - {}_0^A D_x^{2\alpha} u + u - u^3 = 0,\ t > 0\ , 0 < \alpha < 1. \tag{49}$$

- Fractional duffing equation:

$$ {}_0^A D_t^{2\alpha} u + d_1 u + d_2 u^3 = 0,\ t > 0\ , 0 < \alpha < 1. \tag{50}$$

- Fractional Sine-Gordon equation:

$$ {}_0^A D_t^{2\alpha} u - {}_0^A D_x^{2\alpha} u + u - \frac{1}{6} u^3 = 0,\ t > 0\ , 0 < \alpha < 1. \tag{51}$$

The importance of the space time fractional generalized reaction duffing equation enables us to spread the solution of the DNA system.

5. Conclusions

The genetic instructions for every living thing on Earth are stored in a molecule of nucleic acid called deoxyribonucleic acid (DNA). The DNA double-chain fractional-order dynamical system was solved analytically in two dimensions. The major contribution of this study was the investigation of the fractional differential order of double-chain DNA dynamical system, considering Atangana's conformable fractional derivative. The developed novel solutions also provided a visual simulation of the DNA's behavior. Both longitudinal and transverse solitary wave solutions were discussed, and their visualizations were shown. The applied method has the potential to uncover a wealth of fascinating wave solutions for the double-chain DNA dynamical system. The information obtained is vital for genetic studies of the Coronavirus and the development of vaccines.

Author Contributions

S.M. Mabrouk and A.S. Rashed wrote the main manuscript and prepared the figures. A-M. Wazwaz revised the whole manuscript linguistically and scientifically.

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Conflict of Interest

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ORCID iD

Samah M. Mabrouk https://orcid.org/0000-0003-1877-3944
Abdul-Majid Wazwaz https://orcid.org/0000-0002-8325-7500
Ahmed S. Rashed https://orcid.org/0000-0001-7800-2768

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