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## Research Paper

# Applying Optimal Control Algorithm to Estimate Muscle Activation during the Gait Cycle

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**Abstract.** The effective procedure for activating muscles and improving movements in individuals who have a central nervous system injury is Functional electrical stimulation (FES). To have accurate movements of the limbs, appropriate stimulation patterns should be generated and exerted on the muscles. The challenging problem is the redundancy in the musculoskeletal system that makes it necessary to use optimal control to derive muscle activations. For doing this, the redundant musculoskeletal model of the ankle joint is derived in this paper, then an optimal control method for estimating muscle activation during the gait cycle is designed. The aim of optimal control theory is to minimize the cost function, including muscle activation as the control input and joint angle tracking error as the output of the model. Two optimal control algorithms are implemented in this research. These approaches present direct and indirect control methods to solve the optimization problem of ankle movement achieved by experimental research on two healthy males during the gait cycle. Based on the results, the direct method is smoother for nonlinear systems and estimates desired muscle activation more precisely and the cost function coefficients are chosen with logical and less values in comparison to the indirect method.

**Keywords:** Functional electrical stimulation, Optimal Control Algorithm, Human Muscle Activation, Musculoskeletal Model, Gait Cycle.

## 1. Introduction

### 1.1. Background

Neurological disorders occur in people with spinal cord injury, stroke and cause limited ability for doing specific tasks such as lifting the feet by muscle contractions in walking. This disability is known as foot drop [1]. A practical method that causes individuals with impaired motor function to move their body limbs is artificial muscle stimulation, known as functional electrical stimulation (FES). The primary goal of FES is to restore or improve functional movements and abilities that may be compromised due to conditions such as spinal cord injury, stroke, multiple sclerosis, or cerebral palsy [2]. By targeting the muscles involved in particular tasks, FES can help individuals regain functions such as walking, grasping objects, or controlling bladder and bowel movements. functional electrical stimulation holds promise as an effective tool in rehabilitation for individuals with neurological conditions to regain independence and improve their quality of life [3, 4].

### 1.2. Formulation of the problem of interest for this investigation

FES has been shown to improve post-stroke patient recovery in rehabilitation program and investigate the facilitation of vertical patient lifts in individuals with spinal cord injury [5] and also, FES is applied to control the knee joint musculoskeletal system in dynamic condition as the flatness technique [6]. Applying the FES method to different muscles of the body has some disadvantages, which prevents being widely used. Therefore, due to the complexity, redundancy, and nonlinearity of the musculoskeletal system [7], the FES approach is not used for various joints involved different muscles and it is suggested that FES should be used for distal joints and the rehabilitation devices should be used for proximal joints [8]. Tu et al. [8] has applied the Hybrid Rehabilitation and FES with sliding control algorithm to track the rehabilitation exercises. They have used the hybrid leg rehabilitation to reduce the cost the complexity of the of designing hip-knee-ankle exoskeleton. FES as an external stimulation can help to adjust or modulate weak muscle signals. Due to the nonlinearity of the musculoskeletal dynamics, Different nonlinear control strategies [9, 10] have been presented for FES therapy and also optimal control [11, 12] for dealing with model redundancy. Optimal control estimates the optimum muscle activation required for stimulating muscles for doing specific tasks based on various cost functions. Optimal control has been used to consider the effects of shoe materials on the energy cost function while running. For this purpose, a musculoskeletal model with seven body limbs has been derived. Then optimal control is applied to obtain the redundancies of the model [13]. Dorschky et al. [14] has proposed an optimal control to track accelerometer and gyroscope data measured by inertial



measurement units. They have used a planar musculoskeletal model with Hill-type muscle model. The muscular effort and tracking error were minimized to obtain the joints moments and compare with the IMU results.

### 1.3. Literature survey

An optimal control problem is solved by either explicit (indirect method) or implicit formulations (direct collocation method) of the musculoskeletal dynamic model [15-17]. Explicit formulation of the musculoskeletal model is applied in the indirect method. The ankle musculoskeletal model utilizes the Hill-type musculoskeletal model to calculate muscle forces and use instantaneous ground reaction forces as external forces during the gait cycle to compute the control signals by Hamiltonian methods [18]. In the indirect method, the optimality conditions are derived using state and co-state variables, and the optimal control problem is converted into two-point boundary values problem [19]. Indirect methods can be solved analytically in some cases, such as linear systems or quadratic cost functions [20]. Pikulinski et al. [21] proposed the adjoint method Hamiltonian framework with respect to the gradient approach and it has proved that the computational costs are reduced. They have employed the adjoint Hamiltonian methods for the optimization problems and generated the control signals successfully. On the other hand, the direct collocation approach does not require any formulation of the musculoskeletal model and solves the system equations implicitly by considering the cost function and inequality constraints. The direct collocation method is applied to the musculoskeletal model to simulate human gait and calculate muscle forces [22]. The direct collocation method is implemented in OpenSim to predict movements biomechanics. Two direct collocation and shooting methods were compared based on the convergence and computation time [23]. A whole-body neuromuscular model was presented, and direct collocation was performed to calculate muscle forces while ground reaction forces were measured for the human gait cycle [24].

### 1.4. Scope and contribution of this study

In this study, a musculoskeletal model of the ankle joint, including Soleus and Tibialis anterior muscles, is derived during the gait cycle. Ground reaction forces during the stance phase, from heel contact to toe-off, are measured by the force plate. Also, the trajectory of the foot center of pressure (COP) during the stance phase is measured to calculate the moment arms of the reaction forces. The experiments are performed in the Motion Analysis Lab of the Shiraz University of Medical Sciences. Muscles are modeled based on the Hill-type muscle-tendon model, and the moment arms of muscles are computed empirically. Then, the optimal control algorithm for solving the redundancies of the human musculoskeletal ankle model is applied. In optimal control approach, two types of direct and indirect collocation methods are presented. Optimal methods minimize the cost function subjected to discrete sets of inequality constraints and dynamic equations of the system in some procedures. The trade-off between muscle activation and joint angle error must be occurred to determine the contribution of muscles activation of the ankle joint during the gait cycle. The aim of this study is to compare direct collocation and indirect optimization approach for solving optimal control problem to minimize the cost function during the human gait cycle. The results show that the direct approach estimates muscle activation more precisely that is used for the functional electrical stimulations (FES). The schematic configuration of the proposed system for FES rehabilitation therapy during the gait cycle is illustrated in Fig. 1.

The main contributions of the current research with respect to previous studies are as follows:

- To present the musculoskeletal model with major TA and Sol muscles based on the Hill type muscles model.
- Propose the direct collocation optimal control to estimate muscles activation which is applicable for FES.
- To determine the best coefficients to track the desired ankle angle by two optimal control methods.

### 1.5. organization of the paper

The organization of this article is as follows: the ankle joint musculoskeletal model with Hill type muscle model simulation is presented in section 2. in the next section, the theory of the optimal control strategies based on the two direct and indirect collocation approach is explained. in section 4, the process of measuring the kinematic and the kinetic data of the experiments is described. In section 5, the results of the direct and indirect collocation approaches are compared.

## 2. Materials and Methods

### 2.1. Ankle joint musculoskeletal dynamics

The ankle joint musculoskeletal model, which has one degree of freedom, is presented in this section. The external forces were then obtained using an experimental setup, and the muscle forces were computed using a Hill-type muscle-tendon model. As a result, the musculoskeletal dynamic model and the muscle force model are discussed in the sections below.

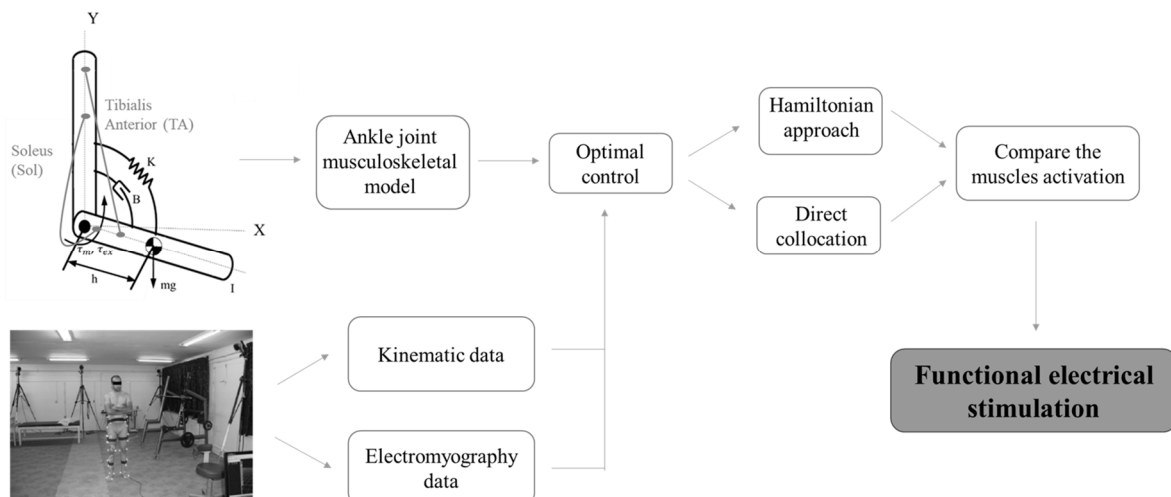


Fig. 1. The schematic configuration of the proposed system for task-specific FES rehabilitation therapy.



## 2.2. Ankle joint dynamic model

Figure 2 shows the schematic of the ankle joint dynamic model and the equation of motion of one degree musculoskeletal model is as follows [25]:

$$I\ddot{q} + B\dot{q} + Kq = \tau_m + \tau_{ex} - mgh \cos(q) \quad (1)$$

where  $\tau_m$  denotes the joint torque applied by plantar and dorsiflexor muscles (Soleus and Tibialis Anterior) during walking; To identify the joint torques ( $\tau_m$ ) from induced muscle contraction, Hill-type muscle model is utilized [26].  $\tau_{ex}$  is the interaction torque with the peripheral environment due to the ground reaction forces;  $q$  is the rotation angle of ankle,  $I$ ,  $B$  and  $K$  denote the moment of inertia, damping, and stiffness of ankle joint, respectively. Also,  $m$  and  $h$ , respectively, denote the mass of the foot and the distance between the center of the mass to the ankle joint.

During the gait cycle, the two primary muscles of the ankle joint are the Soleus (Sol) and Tibialis Anterior (TA) (Fig. 3). Each muscle's moment arm versus the ankle joint is calculated empirically [27].

## 2.3. Hill-type muscle model

### A) Activation dynamics

The activity of each muscle is a value between 0 and 1, indicating the muscle's ability to produce force. Isometric muscle force ( $F_{mx}$ ) will generate at the optimal fiber length. In this study, the activation dynamic of each muscle as the controller input is presented by  $a(t)$ . By adjusting  $a(t)$  for each muscle, the variance between the desired and calculated ankle angle is minimized during the gait cycle.

### B) Contraction dynamics

Each muscle is represented by a three-element Hill model (Fig. 4). The passive properties of muscle fibers and soft tissues are represented by a contractile element (CE) and a pair of parallel elastic (spring and damper) elements, respectively. The tendons which are in series with the muscle fibers, are represented by the series elastic (spring) elements [28].

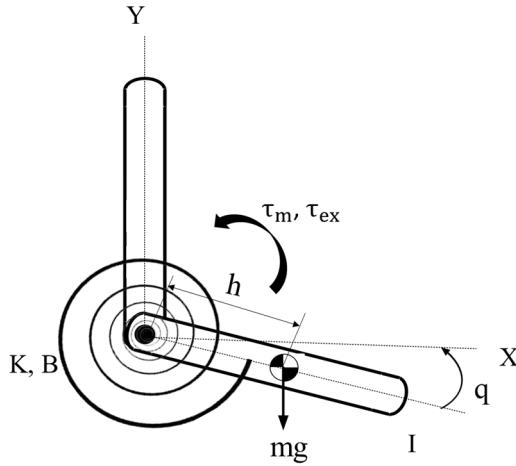


Fig. 2. Dynamic model of human ankle joint.

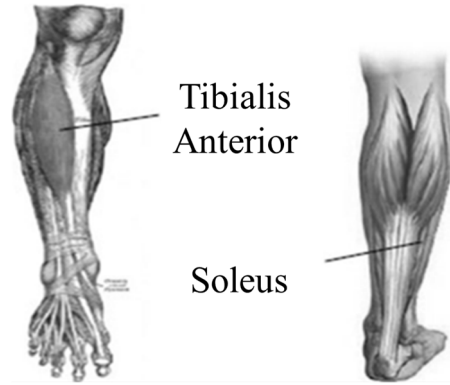


Fig. 3. Soleus and tibialis anterior muscles.

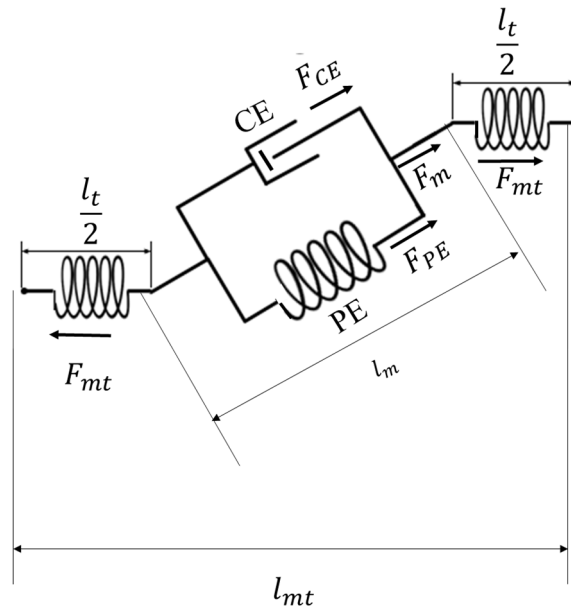


Fig. 4. Schematic of Hill-based muscle-tendon model [27].



As shown in Fig. 4, the contractile and passive muscle fibers are denoted by  $F_{ce}$  and  $F_{pe}$ , respectively, to generate active and passive force.  $F_{mt}$  indicates the muscle-tendon unit force, which is expressed as:

$$F_{mt} = F_m \cos \psi = (F_{ce} + F_{pe}) \cos \psi = F_t \quad (2)$$

where  $F_m$ ,  $F_{ce}$ ,  $F_{pe}$  and  $F_t$  designate the forces generated by the muscle fibers, contractile element, passive element, and tendon, respectively and  $\psi$  is the pennation angle.

The force  $F_{ce}$  is related to muscle activation input  $a(t)$  and maximum isometric muscle force  $F_{mx}$  as:

$$F_{ce} = f(l)f(v)F_{mx}a(t) \quad (3)$$

where  $f(l)$  and  $f(v)$  are the active force-length relationship and force-velocity relationship as defined by Eq. (4), respectively. The dimensionless muscle fiber length,  $l$ , which is the ratio of the current muscle fiber length  $l_m$  to the optimum fiber length  $l_{m0}$  [29]:

$$\begin{aligned} f(l) &= \sin(-1.317l^2 - 0.403l + 2.454) \\ f(v) &= 1 \end{aligned} \quad (4)$$

The passive element force  $F_{pe}$  can be defined as:

$$F_{pe} = f_p(l)F_{mx} \quad (5)$$

where  $f_p(l)$  is the passive elastic force-length relationship as defined by [30]:

$$f_p(l) = \exp(10l - 15) \quad (6)$$

The pennation angle is the angle formed between the tendon and the muscle fibers, and it is defined as:

$$\psi(t) = \sin^{-1}\left(\frac{l_{m0}\psi_o}{l_m}\right) \quad (7)$$

where  $l_m$  is the current muscle fiber length and  $\psi_o$  denotes the optimal pennation angle.

During the gait cycle, the length of the ankle joint muscle-tendon complex ( $l_{mt}$ ) is a function of the corresponding joint angle ( $q$ ). A first-order polynomial can be used to describe muscle length:

$$l_{mt} = l_t + l_m \cos(\psi) = l_{mt0} - d \times q \quad (8)$$

where  $l_{mt0}$  is the length of the muscle-tendon unit when all joint angles are zero and  $d$  is the moment arm of the muscle.

According to Eq. (8), the physiological parameters  $l_t$ ,  $l_{m0}$ ,  $\psi_o$  are considered to be constant when calculating the muscle length ( $l_m$ ). So, the muscle force can be calculated by knowing the value of the parameter  $l_m$ . The tibialis anterior (TA) and soleus (S) moment arms are determined empirically.

### 3. Control Strategy

Optimal control theory is used to solve the redundancies of the musculoskeletal dynamic system over the human gait cycle by compromising between muscle activities and joint angles data for computing the controller inputs by direct collocation and indirect methods as explained in the following. The cost function, including muscle activation as the control input and joint angle tracking error as the output of the model, is represented:

$$J = \int_{t_0}^{t_f} W_1(q - q_d)^2 + W_2(a_{TA} - a_{dTA})^2 + W_3(a_S - a_{dS})^2 \quad (9)$$

This cost function is used to minimize the tracking error in relation to human muscle activation and joint angle data by applying optimal control laws and designing parameters. Weight coefficients of the cost function are defined as  $W_1$ ,  $W_2$  and  $W_3$ . These positive parameter values show the significance of state and control input. The coefficient value  $W_1$  shows the importance of the state tracking, and the coefficient values  $W_2$  and  $W_3$  show the importance of muscle activation tracking error. By changing the coefficient values, the cost function value will change.  $q$  and  $q_d$  are the state and the desired value one.  $a_S$  and  $a_{TA}$  are tibialis anterior (TA) and soleus (S) activation,  $a_{dTA}$  and  $a_{dS}$  are desired ones, respectively. Each muscle activation was defined in Eq. (9), must be within a specified bound:

$$0 < |a| \leq 1 \quad (10)$$

where the value of muscle activation applied to the system is normalized. Also, the system state is subjected to the physiological limits of the joints, which must be satisfied as follows:

$$q_{\min} \leq q \leq q_{\max} \quad (11)$$

#### 3.1. Indirect collocation method

Indirect methods derive analytical expressions based on the calculus of variations. Indirect optimization methods based on the gradient require the twice continuously differentiable cost and constraints functions. The indirect method converts optimal control problems into two-point boundary value problems.

Hamiltonian of the optimal system is defined as:

$$H = J + p_i \dot{x}_i \quad i = 1, 2 \quad (12)$$

where  $\dot{x}_i$  ( $i = 1, 2$ ) are the states of system equations.



For optimality, the below necessary conditions must be satisfied [31]:

$$\begin{aligned}\dot{x} &= \frac{\partial H}{\partial p}(x^*(t), u^*(t), p^*(t), t) \\ \dot{p} &= -\frac{\partial H}{\partial x}(x^*(t), u^*(t), p^*(t), t) \\ 0 &= \frac{\partial H}{\partial u}(x^*(t), u^*(t), p^*(t), t)\end{aligned}\quad (13)$$

where  $u$  is the control signal,  $x$  is the state space and  $p$  is the coefficients of the system equations states. In this study, the initial and final conditions of muscle activation and joint angle are measured by experimental data.

### 3.2. Direct collocation method

Direct collocation methods do not require any analytical expressions and involve both the control and state variables as the constrained equations. This method converts the optimal control problems into a nonlinear programming problem. The process of this method is parameterizing continuous variables. The parameterization of variables is discretizing of the continuous variables such as time domain and also muscle activation and joint angle data [32].

Discretization of time domain:

$$t_1 < t_2 < \dots < t_N \quad (14)$$

Discretization of muscle activation as control inputs and joint angle data:

$$\begin{aligned}q(t) &\rightarrow \{q_1, q_2, \dots, q_N\} \\ a(t) &\rightarrow \{a_1, a_2, \dots, a_N\}\end{aligned}\quad (15)$$

So, the vector of variables is defined as:

$$X = \{t_i, q_i, a_i\} \quad i = 1, 2, \dots, N \quad (16)$$

In this approach, the cost function will be minimized subject to the implicit dynamic equation and linear inequality constraints. The nonlinear programming controller is solved by the “fmincon” function in MATLAB to estimate muscle activation. “fmincon” command in MATLAB applies the cost function with inequality constraint.

## 4. Experimental Results

Experiments have been performed to validate the proposed model and investigate the actual human movement while walking. The trajectories of the bony landmarks were measured using the eight cameras (ProReflex, QualisysR Ltd., Sweden) at 100 Hz sampling frequency to track the markers fixed to the skin. The joint kinematics were extracted based on the recommendations of International Society of Biomechanics [33]. To measure the electromyography data of the muscles, all participants were instrumented with bipolar Ag-AgCl surface electrodes as shown in Fig. 5. All experiments were done at the Motion Analysis Lab under supervision of the Research Ethics Committee, School of Rehabilitation Sciences, Shiraz University of Medical Sciences. All the subjects involved in the experiments gave informed consent to the work [34]. EMG data were collected based on the European recommendations for surface electromyography [35]. During each experiment, the positions of the ankle joint, the values of ground reaction forces and the electromyography data at each time during the gait cycle are recorded using marker-based visual analysis.

During the gait cycle, Figs. 6 (a), (b) show the Ground Reaction Force (GRF) exerted on the foot, and Figs. 6 (c), (d) show the Center of Pressure (COP) at each moment in the X and Y directions, respectively. The ground reaction forces are multiplied by the foot center of pressure during the gait cycle to determine the external torque with the environment ( $\tau_{ex}$ ).

## 5. Results

In this section, simulations of applying optimal control are performed to measure the required muscle activation to reach the desired ankle angle during the gait cycle obtained by experimental setups. So, the results of direct collocation and indirect optimization methods are shown in the following. By the experimental results, the initial and final conditions are known for both optimal control methods. In the indirect collocation method, the Hamiltonian function and derivatives are applied to solve the optimal controller by two-point boundary values. The direct method uses the “fmincon” function with inequality constraints are used to estimate muscle activation. The parameters that are required for the calculation are illustrated in Table 1 and the relation between each limb length, mass and inertia and the body mass and the body height is extracted from the study of Winter [36]. The damping and stiffness of the joint is extracted from the literature [37] and also, the gait cycle time is the time of simulation.

### 5.1. Indirect optimization approach

An optimal control algorithm based on the Indirect method is constructed and simulated in this section, and the results of compromising between muscle activation and joint angle data are shown. The governing equation of the Hamiltonian approach is solved through the boundary value problem (BVP). The approximated muscles activation due to tracking the desired ankle angle are shown in Figs. 7 (a), (b) and (c). Choosing the appropriate coefficients  $W_1$ ,  $W_2$  and  $W_3$  effect on the accuracy of the tracking to minimize the cost function. By increasing the coefficient of angle tracking ( $W_1$ ) with respect to muscle activation coefficients ( $W_2$ ,  $W_3$ ), the role of angle tracking will be more significant. So, choosing the appropriate coefficients for the optimal control problem is very vital.

### 5.2. Direct collocation approach

In this section, the Direct collocation approach is used to reduce the optimal control problem to a nonlinear programming problem. In this optimal control approach, the cost function will be minimized subject to the implicit dynamic equation and linear inequality constraints by the “fmincon” function in MATLAB. By Choosing the appropriate coefficients  $W_1$ ,  $W_2$  and  $W_3$ , the results of this approach are shown in Figs. 8 (a), (b) and (c). As Table 1 showed, the indirect approach needs higher coefficients to track the desired angle and it will be difficult to be controlled in reality.







Fig. 5. The experimental setup [34].

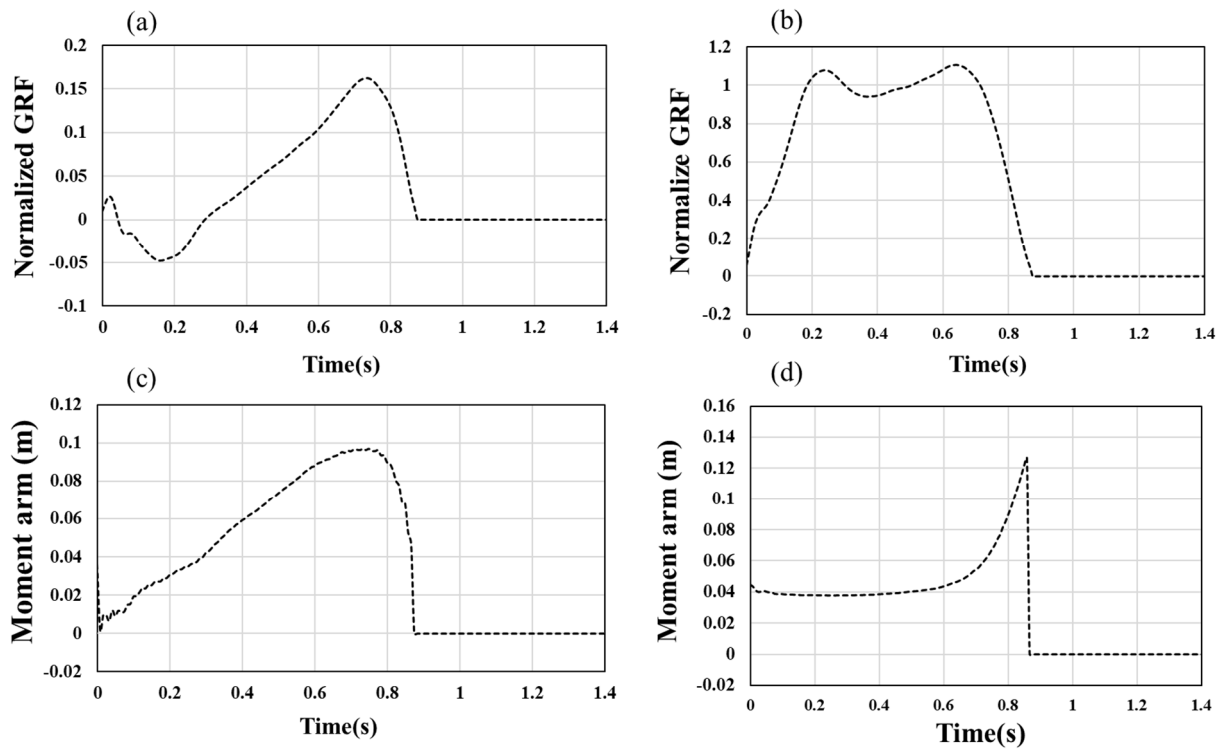


Fig. 6. Experimental results; (a) the Ground Reaction force per body weight in X Direction, (b) The Ground Reaction force per body weight in Y Direction, (c) Moment arm of the Reaction Force in X Direction, (d) Moment arm of the Reaction Force in Y Direction.

Table 1. The parameters of model.

Parameter	Value	Parameter	Value
Body Height	1.80 m	Foot Inertia	0.1887 kg.m <sup>2</sup>
Body Mass	75 kg	Damping (B)	0.1 Nms/rad
Foot Mass	1.0875 kg	Stiffness (K)	5000 N.m/rad
Foot Length	0.2736 m	h (Center of Foot to Ankle)	0.137 m
Gravity acc.	9.81 N/kg	Gait Cycle Time	1.40 s
W <sub>1</sub> (indirect)	1000	W <sub>2</sub> (indirect)	500
W <sub>3</sub> (indirect)	500	W <sub>1</sub> (direct)	100
W <sub>2</sub> (direct)	50	W <sub>3</sub> (direct)	50



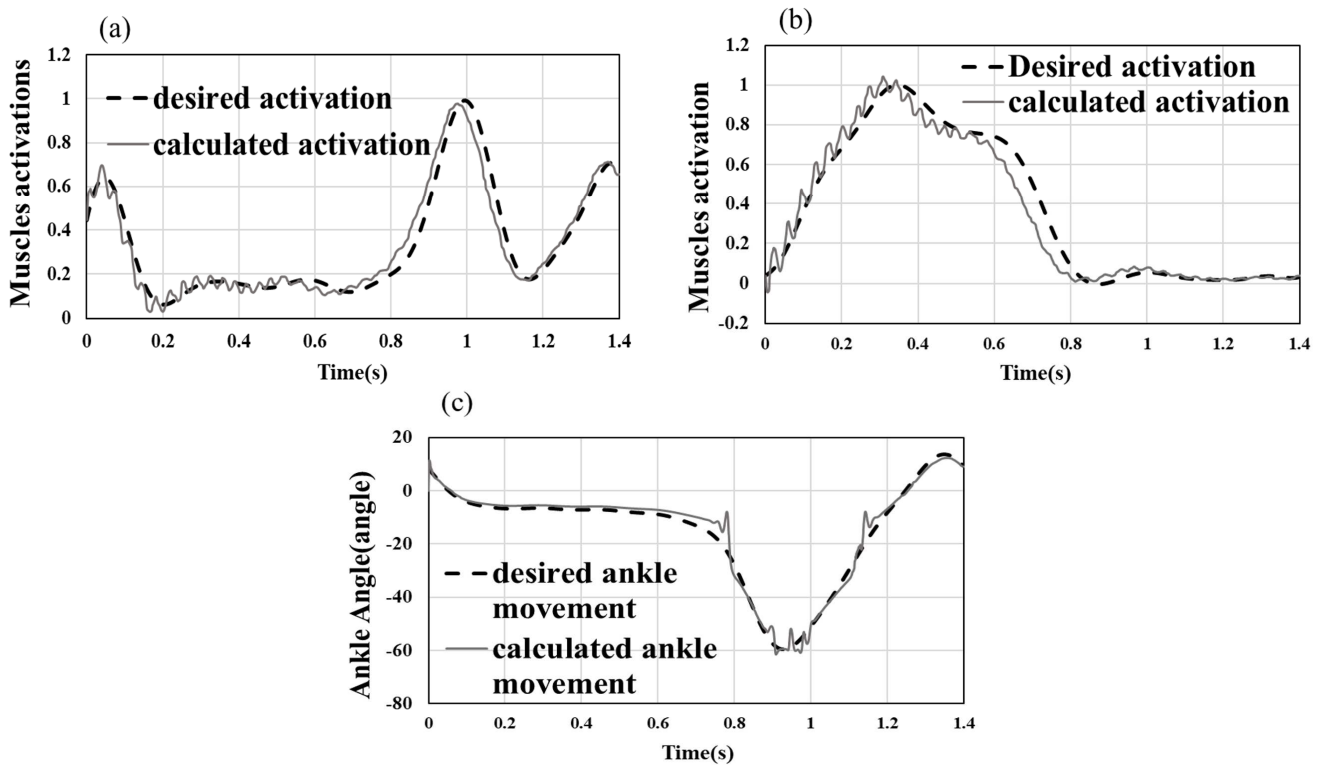


Fig. 7. Indirect optimization approach, (a) Desired and calculated TA activation, (b) Desired and calculated Sol activation, (c) Desired and calculated ankle movement.

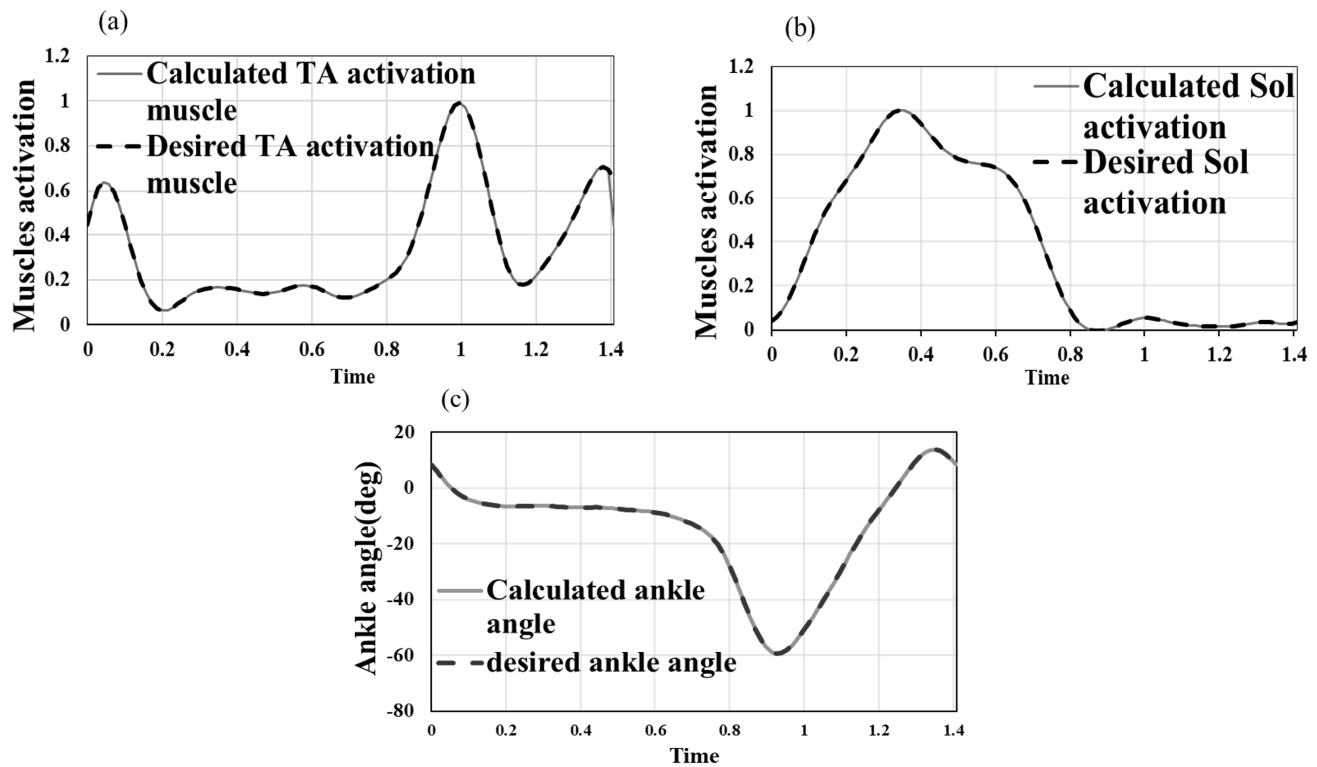


Fig. 8. Direct collocation approach, (a) Desired and calculated TA activation, (b) Desired and calculated Sol activation, (c) Desired and calculated ankle movement.

## Discussion

The aim of this study is to compare direct and indirect collocation approaches for solving optimal control problem in human gait for the application of FES. The electromyography and the kinematic data of participants were extracted to be the desired values of the experiments. Then, the musculoskeletal model with major TA and Sol muscles based on the Hill type muscles models was generated and the muscles parameters were chosen according to the literature review. To Track the desired values, the optimal control was applied to extract the muscles activations and angular magnitudes. The direct and indirect collocation were explained to optimize the cost function. In Indirect method, the derivatives of the Hamiltonian function were obtained subjected to the states



and co-states. However, the direct optimal method does not require any explicit dynamic system equation and is based on discretizing the cost function and parameterizing the control and state variables. Both approaches were performed on the ankle musculoskeletal model to compute the compromised muscle activation and joint angle for a human gait cycle. As shown in Fig. 8, direct collocation approach tracks the desired motion and estimates muscle activation much more precisely than the indirect approach. Based on results, the gradient-based solver in the indirect method has some errors to predict the muscles activation for tracking trajectory and desired motion due to the derivatives of the Hamiltonian function to calculate optimal control solution. So, it is realized that using derivatives of cost function and system dynamic equation cause errors around the desired values. The advantage of the direct collocation method is to solve optimal control problem numerically and by discretizing variables and it solves the problem of nonlinearity of the musculoskeletal model. The other point about the direct and indirect method is to choose the appropriate cost function coefficient to estimate muscle activation compromising with joint angle data. To achieve the desired result, the coefficients of the indirect method must be chosen to be higher than the direct method. Hence, performing the indirect method in experiments may face other challenges for the considered system. The briefly achievements of this research with respect to previous studies are as follows:

- To present the musculoskeletal model with major TA and Sol muscles based on the Hill type muscles model.
- To propose the direct collocation optimal control and compare it with indirect collocation to estimate muscles activation that is applicable for FES
- The appropriate coefficients to track the desired ankle angle are determined by two optimal control methods.

## 7. Conclusions

In this paper, the musculoskeletal model of the ankle joint with one degree of freedom and two major muscles was considered. The electromyography and the kinematic data were measured to compute the muscles activation and the ankle angle as the desired values. Since the number of muscles involved around the ankle joint is greater than the DOF, the optimization problem is necessary to be solved to compute the muscles activations. Also, the electromyography and the kinematic data were measured to compute the desired values of muscles activation and the ankle angle. Therefore, the optimal control was used to estimate muscles activation due to trajectory tracking and the differences of the measured and modelling muscles activation and the ankle trajectory must be minimized. The objective function with linear inequality constraints was defined to be minimized by optimal control laws. Two direct and indirect collocation methods with the goal of minimizing the cost function were presented to determine the portion of each muscle activation during the gait cycle and tracking the desired trajectory. Based on the results, the direct collocation approach estimates muscle activation and trajectory tracking much more precisely than the indirect method. Due to the nonlinearity of the system, conversion of the optimal control into a nonlinear programming problem, using discrete sets of variables and not using any derivatives of the Hamiltonian function when solving the optimal control problem. For future work, the direct collocation optimal controller is able to estimate dynamic activation of the multi-DOF musculoskeletal model such as the human shoulder.

## Author Contributions

M. Farrokhnia and S.A. Haghpanah planned the scheme, initiated the project, suggested the experiments, conducted the experiments and analyzed the empirical results, developed the mathematical modeling and examined the theory validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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## Conflict of Interest

The author(s) declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

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## Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

## Nomenclature

TA	Tibialis anterior	$F_{ce}$	Force generated by contractile element
Sol	Soleus	$F_{pe}$	Force generated by passive elements
FES	Functional electrical stimulation	$F_m$	Force generated by muscles
BVP	Boundary value problem	$F_t$	Force generated by tendon
COP	Center of pressure	$F_{mx}$	Maximum Isometric Force
GRF	Ground reaction forces	$F_{mt}$	Muscle-tendon unit force
$\psi$	Pennation angle	$f(l)$	Force-length relationship
$f(v)$	Force-velocity relationship	$l_m$	Elastic force-length relationship







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