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Research Paper

Some Models in Unmagnetized Plasma Involving Kaniadakis Distributed Electrons and Temperature Ratio: Dust Ion Acoustic Solitary Waves

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Abstract. The current paper studies the influence of the temperature ratio of ion-to-electron α, dust concentration μ, and κ-deformed parameter on dust ion acoustic solitary waves in an unmagnetized plasma with Kaniadakis distributed electrons. More precisely, the reductive perturbation technique is utilized to extract the Korteweg-de Vries and modified Korteweg-de Vries equations. Both compressive and rarefactive Korteweg-de Vries solitons are found to exist in the ranges $0 < \mu \le 0.677$ and $0.677 < \mu < 1$, respectively, and only compressive modified Korteweg-de Vries solitons in the range $0 < \mu \le 0.11$. In an unmagnetized plasma with Kaniadakis distributed electrons, the influence of the ion-to-electron temperature ratio on dust ion acoustic solitary waves can have several fascinating applications and consequences in plasma physics and astrophysics.

Keywords: Unmagnetized plasma; Kaniadakis distributed; KdV; mKdV; Compressive and rarefactive solitons.

1. Introduction

For several decades, ion-acoustic waves (IAWs) in a variety of plasma systems, including many astrophysical space plasmas and laboratory produced plasma, have been researched. Both theoretical and experimental studies of IAWs have been conducted [1-3]. Theoretical investigations based on the Sagdeev quasi-potentials or the reductive perturbation technique (RPT) have been developed, resulting in the derivation of nonlinear Schrodinger-type equations or the Korteweg-deVries (KdV) equations that characterize dynamical systems. In a nonplanar spherical geometry, the nonlinear characteristics of IAWs in an electron-ion quantum plasma with the effects of quantum corrections have been explored [4].

Since dusty plasma exists in actual systems of charged particles, such as laboratory plasmas and space plasmas, researchers are becoming more and more interested in the subject. This interest is also a result of the theoretical calculation and subsequent experimental validation of two original plasma modes, specifically the dust ion-acoustic wave (DIAW) and the dust acoustic wave (DAW), as well as the application of unique physics in its description. The thorough explanation of the above-mentioned novel acoustic modes described in [5-18] is invaluable for understanding astrophysical phenomena. Meanwhile, low-frequency waves known as dust ion-acoustic solitary waves (DIASWs) are crucial in the nonlinear dusty plasma system that is present in most astrophysical and space environments, including cometary tails, planetary atmospheres, asteroid zones, molecular clouds, interstellar clouds, and circumstellar, as well as the Earth's ionosphere [19-21].

In the last few decades, other entropic forms that generalize the well-known Boltzmann–Gibbs–Shannon have received a great amount of attention Renyi entropy [22], Havrda–Charvat entropy [23], Tsallis entropy [24], etc. As an illustration, the Tsallis nonextensive theory demonstrated its efficacy in handling particularly complicated systems and showed an unexpectedly high degree of agreement with experimental data [25]. Within the theoretical framework of the κ-statistics arising from the Kaniadakis entropy [26], blackbody radiation [27], and quantum entanglement [28] were revisited. The alleged κ-deformed distributions resulting from the Kaniadakis entropy have been studied in relation to cosmic rays [29], the generation of quark-gluon plasma [30], the kinetics of interacting photons and atoms [31], and nonlinear kinetics [32]. Unmagnetized plasma with a

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κ-deformed Kaniadakis electron distribution has been studied [27] for arbitrary amplitude electron-acoustic waves. By using the bifurcation theory of planar dynamical systems, the bifurcations of nonlinear DIASWs in a magnetized dusty plasma have been characterized [33]. A nonperturbative technique has used to examine nonlinear solitary and periodic waves [34, 35] in unmagnetized plasmas containing non-Maxwellian positrons and electrons. The Sagdeev technique have utilized [36] to describe the characteristics of IASWs in KD electron plasma. They demonstrated how the structural properties of IASWs are only very minimally altered by the κ-deformed parameter. In a four-component plasma with a κ-deformed electron and hot positron distribution, Saha and Tamang [37] investigated positron acoustic waves. The investigation [38] of the application of the κ-deformed KD in the case of arbitrary amplitude IASWs in a magnetized plasma (consisting of non-Maxwellian electrons and cool fluid ions), where the electrons follow the κ-deformed KD has made. The mobility of the DIASWs has been investigated [39] with non-Maxwellian electrons. With the use of the RPT, the KdV equation and mKdV equation were generated, and respective solitary wave solutions were examined. The interested reader is referred to see [40-49] for additional studies on various wave shapes of nonlinear evolution equations. The uniqueness of the findings likely results from the interaction of fluid ions, immobile dust particles with negative charges, temperature, and nonthermal electron elements, and it may help us learn more about the physics of plasmas in non-equilibrium situations and how solitary waves behave in these environments. Additionally, it might have consequences for real-world applications in fields like controlled fusion, space plasmas, or astrophysics. The qualitative investigation of DIASWs in unmagnetized dusty plasmas with κ-deformed KD electrons is undertaken in this paper. The KdV equation for such a system is determined first, followed by the mKdV equation in the case when the KdV equation fails. The study of the influence of various physical characteristics on dust ion acoustic solitary waves of the KdV and mKdV equations, such as the dust concentration μ, real parameter κ, and temperature ratio α is provided. The current paper is an extension of the reference [39].

The paper is made up of the following: In Section 2, the basic equations are presented. The KdV and mKdV equations are derived in Sections 3 and 4 using the RPT, respectively. Section 5 presents the criteria for the presence of DIASWs, and Section 6 provides the results and discussion.

2. Basic Equations

An unmagnetized dusty plasma made up of warm fluid ions, immobile dust particles with negative charges and nonthermal electrons is considered. The set of normalized fluid equations used to study DIAWs is as follows [39]:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=0, \quad \text{Equation for continuity} \tag{1}$$

$$\left(\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x}\right)v_i+\frac{\alpha}{n_i}\frac{\partial n_i}{\partial x}+\frac{\partial \varphi}{\partial x}=0, \quad \text{Equation for momentum} \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2}=(1-\mu)n_e-n_i+\mu. \quad \text{Poisson equation} \tag{3}$$

The number densities of ion (n_i) and electron (n_e) are normalized by their equilibrium equivalents, n_{i0} and n_{e0}, respectively. The velocity v_i is normalized by IA speed $C_s=\sqrt{T_e/m}$ and $\varphi=(e\varphi)/T_e$ is the normalized electrostatic wave potential. The time and space variables have been normalized by $\omega_{pi}^{-1}=\sqrt{m/(4\pi n_0 e^2)}$ and Debye length $\lambda_D=\sqrt{T_e/(4\pi n_0 e^2)}$, respectively, with m (ion mass) and T_e (electron temperature). Furthermore, $\mu=n_{d0}/n_{i0}$ represents the dust concentration ratio, n_{d0} denotes the equilibrium number density of dust grains and α indicates ion to electron temperature ratio.

The electrons follow κ-deformed KD [26], which is expressed as:

$$f_e^{(\kappa)}(v)=A_\kappa \exp_\kappa\left(-\frac{m_e v^2/2-e\varphi}{T_e}\right), \tag{4}$$

with

$$\exp_\kappa(x)=\left(\sqrt{1+\kappa^2x^2}+\kappa x\right)^{\frac{1}{\kappa}}, \tag{5}$$

where A_k is the normalized constant indicated by:

$$A_\kappa=n_{e0}\left(\frac{m_e|\kappa|}{\pi T_e}\right)^{\frac{3}{2}}\frac{\Gamma\left(\frac{1}{2|\kappa|}+\frac{3}{4}\right)}{\Gamma\left(\frac{1}{2|\kappa|}-\frac{3}{4}\right)}\left(1+\frac{3}{2}|\kappa|\right). \tag{6}$$

The following standard integration was applied for calculating A_κ:

$$\int_0^\infty x^{r-1}\exp_\kappa(-x)dx=\frac{[1+(r-2)|\kappa|]|2\kappa|^{-r}\,\Gamma\left(\frac{1}{2|\kappa|}-\frac{r}{2}\right)}{[1-(r-1)|\kappa|]^2-\kappa^2\Gamma\left(\frac{1}{2|\kappa|}+\frac{r}{2}\right)}\Gamma(r). \tag{7}$$

Here, Γ stands for the universal gamma function and κ is a real parameter that indicates the degree of deformation. The inequality $-1<\kappa<1$ must hold for the real parameter's value. Additionally, the quantity κ measures the dispersion from the Maxwellian distribution; hence, when $\kappa\to 0$, the KD function is transformed into the Maxwell–Boltzmann distribution as follows:

$$\lim_{\kappa\to 0} f_e^{(\kappa)}(v) = n_{e0}\left(\frac{m_e}{2\pi T_e}\right)^{\frac{3}{2}} \exp\left(-\frac{(m_e v^2/2)-e\varphi}{T_e}\right), \tag{8}$$

with $\lim_{\kappa\to 0}\exp_\kappa(x) \equiv \exp(x)$.

Prior to continuing, it is crucial to limit the acceptable range of κ. Calculating the mean square speed $\langle v^2\rangle$ requires:

$$\langle v^2\rangle = \frac{\iiint v^2 f_e^{(\kappa)}(v) d^3v}{\iiint f_e^{(\kappa)}(v) d^3v} = \frac{2T_e/m_e}{|2\kappa|^{\frac{5}{2}}\left(1+\frac{5}{2}|\kappa|\right)} \frac{\Gamma\left(\frac{1}{2|\kappa|}-\frac{5}{4}\right)}{\Gamma\left(\frac{1}{2|\kappa|}+\frac{5}{4}\right)}. \tag{9}$$

In order to maintain the physical meaning of $\langle v^2\rangle$, which requires that $\langle v^2\rangle$ be finite and whose value diverges at $|\kappa|\to 0.4$. Moreover, the divergence from the Maxwellian behaviour is also represented by the κ-deformed parameter. It is observed that the condition $|\kappa|\le 0.4$ must be satisfied by the value of the real parameter κ. This range is set in order to preserve the framework's stability and well-behaved mathematical features, preventing divergent or unphysical behaviour that would result from exceedingly large or tiny values of κ. Additionally, an increase in κ signifies a rise in state energy, suggesting that the κ-deformation parameter influences the system's energy levels and needs to be limited to an acceptable range to preserve physical interpretability. It should be noted that this restriction has been considered when calculating A_κ and the standard KE (kinetic energy) of the particles, $m\langle v^2\rangle/2$, and that the interacting particles have been disregarded, i.e., $\varphi = 0$.

The electron density n_e expresses how many electrons there are in a specific plasma volume. It is an essential parameter that defines the plasma and is vital in establishing its characteristics and behaviour. The normalized number density for the electrons in these equations is provided by [39], which is generated by integrating the κ-deformed Kaniadakis distribution over the velocity space and is given by:

$$n_e = \exp_k(\varphi) = \left(\sqrt{1+\kappa^2\varphi^2}+\kappa\varphi\right)^{\frac{1}{\kappa}}. \tag{10}$$

Expansion of Eq. (10) up to the third-order for $\varphi << 1$ is:

$$n_e = 1+\varphi+\frac{1}{2}\varphi^2+\frac{(1-\kappa^2)}{6}\varphi^3+\cdots. \tag{11}$$

3. KdV Equation and Its Extraction

The dependent variables are expanded to derive the KdV equation from Eqs. (1) to (3) for understanding the spread of IAWs as follows:

$$\begin{pmatrix} n_i \\ v_i \\ \varphi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \varepsilon \begin{pmatrix} n_{i1} \\ v_{i1} \\ \varphi_1 \end{pmatrix} + \varepsilon^2 \begin{pmatrix} n_{i2} \\ v_{i2} \\ \varphi_2 \end{pmatrix} + \varepsilon^3 \begin{pmatrix} n_{i3} \\ v_{i3} \\ \varphi_3 \end{pmatrix} + \cdots. \tag{12}$$

By taking the stretched variables:

$$\xi = \varepsilon^{\frac{1}{2}}(x-Ut), \quad \tau = \varepsilon^{\frac{3}{2}}t, \tag{13}$$

such that:

$$\frac{\partial}{\partial x} \equiv \varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial t} \equiv \varepsilon^{\frac{3}{2}}\frac{\partial}{\partial \tau} - U\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi},$$

where U is the perturbation mode's phase speed and ε is a smallness parameter indicating the dispersion's weakness.

The stretching variables ξ and τ include the small perturbative parameter ε, which describes the typical IA mode of weakly nonlinear plasma waves with small amplitudes and long wave lengths. The stretched coordinate system is based on the Galilean frame of reference. However, time variation in the stationary wave frame exists because of a slight but finite dispersive impact in the dispersion relation; for this reason, the coordinate system is referred to as stretching. Slow variation of time with respect to space, in contrast to fast change of space scale, is dictated by the self-consistent derivation of the stretching coordinate system.

We get from ε order equations, the following expressions:

$$n_{i1} = \frac{\varphi_1}{U^2-\alpha}, \quad v_{i1} = \frac{U\varphi_1}{U^2-\alpha}, \quad (1-\mu)\varphi_1 - n_{i1} = 0, \tag{14}$$

under the boundary conditions $n_{i1}=0$, $v_{i1}=0$, $\varphi_1=0$ at $|\xi|\to\infty$, by using Eqs. (12) and (13) in the normalized set of Eqs. (1) to (3) are obtained.

Putting the value of n_{i1} in the third expression of Eq. (14), the equation for U can be obtained as:

$$U = \sqrt{\alpha+\frac{1}{1-\mu}}. \tag{15}$$

This gives the phase velocity of Khalid et al. [39] for $\alpha = 0$.

The KdV equation can be determined by entering Eq. (14) in the set of ε^2 – order equations that comes out from Eqs. (1) to (3) as:

$$\frac{\partial\varphi_1}{\partial\tau}+p\varphi_1\frac{\partial\varphi_1}{\partial\xi}+q\frac{\partial^3\varphi_1}{\partial\xi^3}=0, \tag{16}$$

where $p=A/B$ and $q=1/B$ with:

$$A=\frac{(3U^2-\alpha)-(1-\mu)(U^2-\alpha)^3}{(U^2-\alpha)^3},\quad B=\frac{2U}{(U^2-\alpha)^2}.$$

4. mKdV Equation and Its Extraction

Equation (16) reveals that the nonlinear coefficient p depends on the physical parameters μ and α. For $p>0$, we discover a compressive solitary wave; for $p<0$, we find a rarefactive solitary wave. According to Fig. 1(a), for a given value of $\mu(\mu_c)$, the nonlinear coefficient p vanishes. At that crucial point, nonlinearity disappears, and as a result, the amplitude of the IASW solution has an infinite divergence. We take into account higher-order nonlinearity to examine the IASW in the critical point as well as in adjacent zones of the critical point.

Therefore, by setting $p=0$, we get:

$$\mu_c=1-\frac{3U^2-\alpha}{(U^2-\alpha)^3}. \tag{17}$$

It can be concentrated on identifying mKdV solitons for different values of the critical density ratio $\mu_c=n_{d0}/n_{i0}$ given by Eq. (17). Different stretched variables are provided for higher-order non-linearity as:

$$\xi=\varepsilon(x-Ut),\qquad \tau=\varepsilon^3 t. \tag{18}$$

Equation (15) for the phase velocity U is produced by the expansion (12) in equations (1) through (3). The second-order equations in ε are subject to the boundary conditions: $n_{i2}=0$, $v_{i2}=0$, and $\varphi_2=0$, at $|\xi|\to\infty$.

Integrating yields the results shown below:

$$n_{i2}=\frac{3U^2-\alpha}{2(U^2-3\alpha)^3}\varphi_1^2+\frac{1}{U^2-\alpha}\varphi_2,$$

$$v_{i2}=\frac{2U^3}{2(U^2-3\alpha)^3}\varphi_1^2+\frac{U}{U^2-\alpha}\varphi_2.$$

By putting the value of n_{i2} in the relevant second-order Poisson equation:

$$(1-\mu_c)\left(\frac{1}{2}\varphi_1^2+\varphi_2\right)-n_{i2}=0,$$

the following expression can be obtained:

$$1-\mu_c-\frac{3U^2-\alpha}{(U^2-\alpha)^3}=0. \tag{19}$$

The equations for the third-order perturbations in ε are as follows:

$$\frac{\partial n_{i1}}{\partial\tau}-U\frac{\partial n_{i3}}{\partial\xi}+\frac{\partial v_{i3}}{\partial\xi}+\frac{\partial}{\partial\xi}(n_{i1}v_{i2})+\frac{\partial}{\partial\xi}(n_{i2}v_{i1})=0,$$

$$\frac{\partial v_{i1}}{\partial\tau}-U\frac{\partial v_{i3}}{\partial\xi}-Un_{i1}\frac{\partial v_{i2}}{\partial\xi}+v_{i1}\frac{\partial v_{i2}}{\partial\xi}+n_{i1}v_{i1}\frac{\partial v_{i1}}{\partial\xi}+\alpha\frac{\partial n_{i3}}{\partial\xi}+\frac{\partial\varphi_3}{\partial\xi}+n_{i1}\frac{\partial\varphi_2}{\partial\xi}+n_{i2}\frac{\partial\varphi_1}{\partial\xi}=0, \tag{20}$$

$$\frac{\partial^2\varphi_1}{\partial\xi^2}=(1-\mu_c)\left\{\varphi_3+\varphi_1\varphi_2+\frac{(1-\kappa^2)}{6}\varphi_1^3\right\}-n_{i3}.$$

When the last equation of (20) is differentiated, the following equation is obtained:

$$\frac{\partial^3\varphi_1}{\partial\xi^3}=(1-\mu_c)\left(\frac{\partial\varphi_3}{\partial\xi}+\frac{\partial}{\partial\xi}(\varphi_1\varphi_2)+\frac{(1-\kappa^2)}{6}\frac{\partial\varphi_1^3}{\partial\xi}\right)-\frac{\partial n_{i3}}{\partial\xi}. \tag{21}$$

Elimination of $\partial n_{i3}/\partial\xi$ from Eq. (21) using Eqs. (15), (19) and (14), yields the following mKdV equation:

$$\frac{\partial\varphi_1'}{\partial\tau}+p'(\varphi_1')^2\frac{\partial\varphi_1'}{\partial\xi}+q'\frac{\partial^3\varphi_1'}{\partial\xi^3}=0,\quad \varphi_1'=\varphi_1, \tag{22}$$

from Eqs. (20) by using Eq. (21) where $p' = A'/4B'$ and $q' = 1/2B'$ with:

$$A' = \frac{3\left(U^4 - 3U^2\alpha + \alpha^2\right)}{\left(U^2-\alpha\right)^5} - (1-\kappa^2)(1-\mu_c),$$

$$B' = \frac{U}{\left(U^2-\alpha\right)^2}.$$

5. The Required Condition for the Presence of Solitons

Equation (15) yields:

$$\alpha + \frac{1}{1-\mu} > 0.$$

Additionally, for the compressive soliton, $p > 0$ i.e. $A > 0$. This implies:

$$1 > \mu > 1 - \frac{3U^2 - \alpha}{\left(U^2-\alpha\right)^3},$$

while for the rarefactive soliton, $p < 0$ i.e. $A < 0$. This leads to:

$$\mu < 1 - \frac{3U^2 - \alpha}{\left(U^2-\alpha\right)^3}.$$

Using the transformation $\eta = \xi - V\tau$, Eq. (16) can be integrated under the boundary conditions $\varphi_1 = 0$ and $\left(\partial^2\varphi_1\right)/\left(\partial\eta^2\right) = 0$ as $\eta \to \pm\infty$ to give:

$$-V\varphi_1 + \frac{1}{2}p\varphi_1^2 + q\frac{\partial^2\varphi_1}{\partial\eta^2} = 0.$$

Now, the solitary wave solution can be obtained by using the ansatz method [50] as:

$$\varphi_1 = \frac{3V}{p}\operatorname{sech}^2\left(\sqrt{\frac{V}{4q}}\eta\right).$$

The amplitude (φ_0) and width (Δ_0) are described by:

$$\varphi_0 = \frac{3V}{p},$$

$$\Delta_0 = 2\sqrt{\frac{q}{V}}.$$

For the mKdV equation (22), the solitary wave solution can be deduced using the ansatz method [50] as:

$$\varphi_1' = \sqrt{\frac{6V}{p'}}\operatorname{sech}\left(\sqrt{\frac{V}{q'}}\eta\right),$$

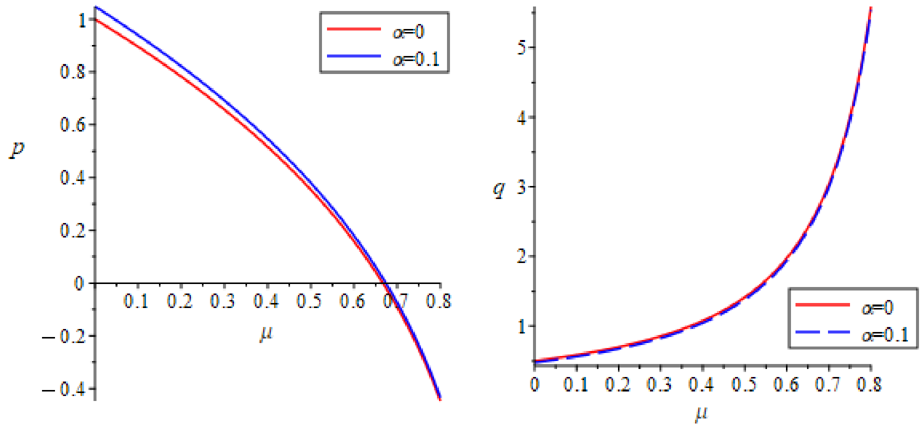
where the amplitude and width are given by:

$$\varphi_0' = \sqrt{\frac{6V}{p'}},$$

$$\Delta_0' = \sqrt{\frac{q'}{V}}.$$

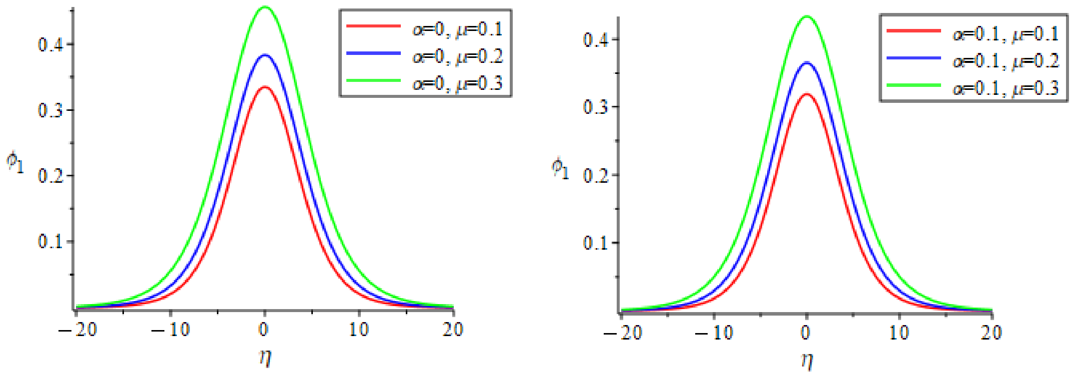
6. Results and Discussion

In the context of Kaniadakis distributed κ-deformed electrons, DIASW properties have been studied in the present manuscript. The effect of various parameters including the temperature (α) and the deformation parameter (κ) on DIASWs is analyzed. The dust concentration μ and temperature ratio α have a considerable influence on the coefficients p and q in the KdV equation. It should be stated at the outset that all of Khalid et al. [39] findings obtain if the ion's thermal influence $(\alpha = 0)$ is eliminated. Figure 1 may be comparable with Fig. 1 of Khalid et al. [39]. Figure 1 depicts a graph showing how p and q change for two scenarios, i.e. $\alpha = 0$ and $\alpha = 0.1$, as μ rises from 0 to 0.8. It can be observed that the nonlinear coefficient p in Fig. 1(a) can take both positive and negative values whereas the dispersion coefficient q in Fig. 1(b) is always positive. The existence of DIASWs is defined as having both positive and negative potential. Thus, both rarefactive and compressive solitary structures can be found in the KdV model of plasma.



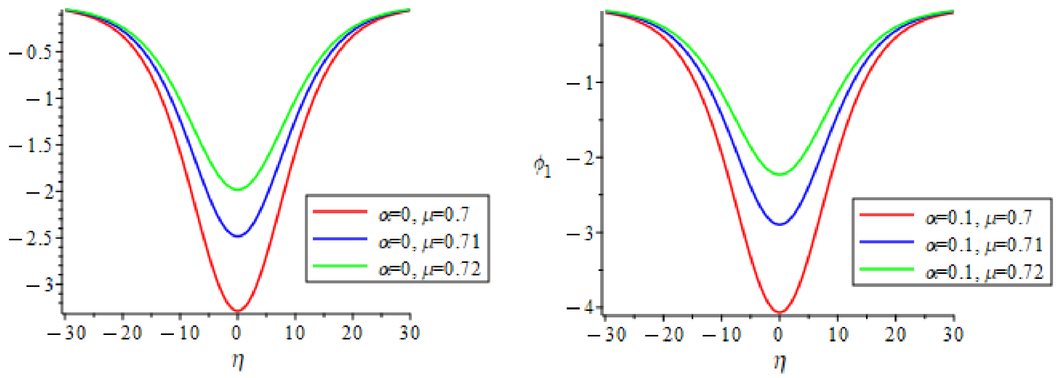
(a) (b)

Fig. 1. (a) Variation of p with μ for $\alpha = 0$ and $\alpha = 0.1$; (b) variation of q with μ for $\alpha = 0$ and $\alpha = 0.1$.



(a) (b)

Fig. 2. Variation of amplitude and width of the compressive KdV soliton for $V = 0.1$ when (a) $\alpha = 0$ and $\mu = 0.1, 0.2, 0.3$; (b) $\alpha = 0.1$ and $\mu = 0.1, 0.2, 0.3$.



(a) (b)

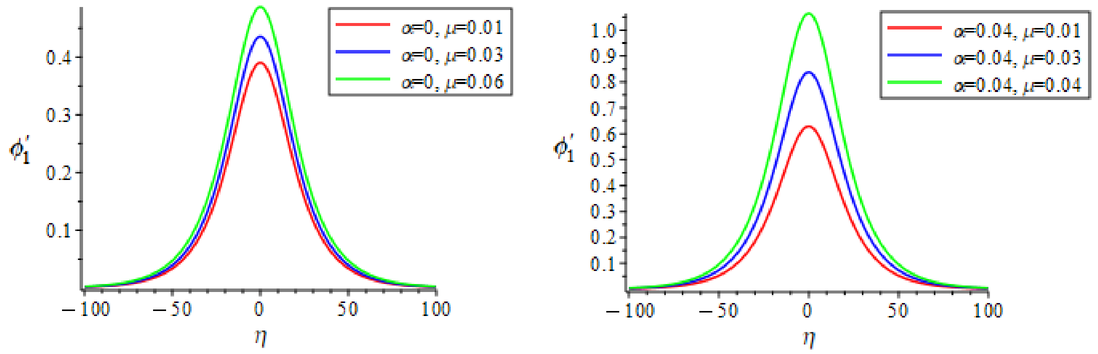
Fig. 3. Variation of depth and width of the rarefactive KdV soliton for $V = 0.1$ when (a) $\alpha = 0$ and $\mu = 0.7, 0.71, 0.72$; (b) $\alpha = 0.1$ and $\mu = 0.7, 0.71, 0.72$.

Figure 2 illustrates how DIASWs behave under various values of α and μ. It is evident from Fig. 2 that when μ rises, the amplitude and width of the positive potential DIASWs increase. The authors examine the effect of the ion-to-electron temperature ratio α for $\alpha = 0$ in Fig. 2(a) and for $\alpha = 0.1$ in Fig. 2(b). It is seen that the magnitude of the amplitude is higher for $\alpha = 0$ than $\alpha = 0.1$.

With rising values of μ for the temperatures $\alpha = 0$ and $\alpha = 0.1$, Figs. 3(a) and 3(b) show how negative potential DIASWs behave differently than positive ones. It is observed when μ increases, the depth and width of the negative potential DIASWs decrease. Additionally, it is clear that the magnitude of the amplitude is higher for $\alpha = 0.1$ than $\alpha = 0$.

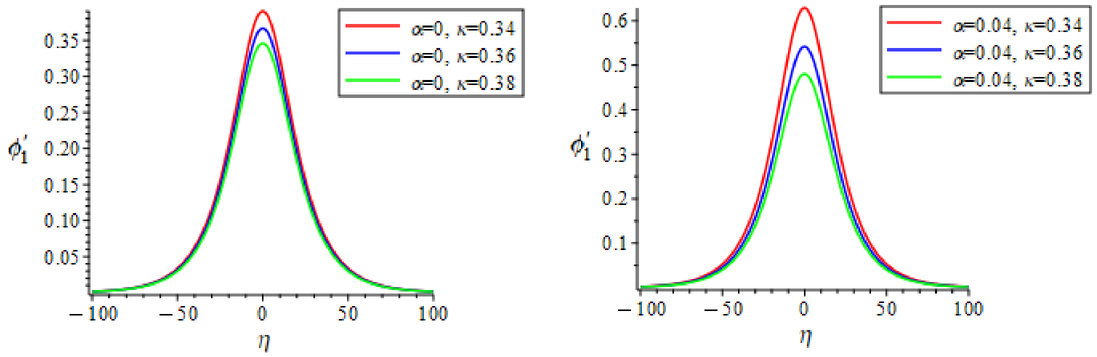
For various values of α and μ, the variation of the DIASW of the mKdV equation against η has been investigated in Fig. 4. The amplitude and width of the compressive DIASW increase by the increase of μ. Additionally, it is figured out that the magnitude of the amplitude is higher for $\alpha = 0.04$ than $\alpha = 0$. So, the temperature α has a significant effect on the dynamics of the DIASW of the mKdV equation.

The change of the DIASW of the mKdV equation versus η has been studied in Fig. 5 for different selections of α and κ. It is clear when κ rises the amplitude and width of the compressive DIASW decrease. In addition, it is revealed that the magnitude of the amplitude is higher for $\alpha = 0.04$ than $\alpha = 0$. The present work is different from the work of Khalid et al. [39], because in their investigation, the KdV and mKdV equations have been derived for solitary waves in nonthermal ions. In the present investigation, we have shown the effect of ion to electron temperature ratio which significantly modifies the soliton behaviour and propagation properties in this model of plasma.



(a) (b)

Fig. 4. Variation of amplitude and width of the compressive mKdV soliton for $V = 0.002$ and $\kappa = 0.34$ when (a) $\alpha = 0$ and $\mu = 0.01, 0.03, 0.06$; (b) $\alpha = 0.04$ and $\mu = 0.01, 0.03, 0.04$.



(a) (b)

Fig. 5. Variation of amplitude and width of the compressive mKdV soliton for $V = 0.002$ and $\mu = 0.01$ when (a) $\alpha = 0$ and $\kappa = 0.34, 0.36, 0.38$; (b) $\alpha = 0.04$ and $\kappa = 0.34, 0.36, 0.38$.

7. Conclusion

The present study investigated the effect of α, μ and κ on DIASWs in an unmagnetized plasma containing KD electrons. It has carefully explored how plasma properties affect the region where solitary waves can exist. It should be noted that the presence of a κ-deformed KD affects the DIASW's standard behavior. The characteristics of solitary waves were investigated using a parametric analysis, and the findings can be concluded as follows:

- The authors found both compressive and rarefactive KdV solitons and only compressive mKdV solitons in such models of plasma.
- In a compressive soliton, heavier ions have higher energies than lighter electrons. In contrast, it is observed in rarefactive solitons, the lighter electrons are more energetic and contribute more significantly to the plasma dynamics.
- It is clear that the magnitude of the amplitude is higher for $\alpha = 0.10$ than $\alpha = 0$.

Author Contributions

J. Kalita and R. Das wrote the original draft, K. Hosseini revised the original draft, and S. Salahshour and D. Baleanu reviewed the revised draft. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no conflict of interest.

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Data Availability Statements

Due to the nature of the research, supporting data is not available.

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