J. Appl. Comput. Mech., xx(x) (2024) 1-11
DOI: 10.22055/jacm.2024.46217.4482

ISSN: 2383-4536
jacm.scu.ac.ir

Shahid Chamran University of Ahvaz

Journal of Applied and Computational Mechanics

Research Paper

Dynamic Behavior of Shafts, Couplings and Working Body of the Machine under Torsional Impact Moment

Isa Ali Khalilov[1], Abdullah H. Sofiyev[2,3,4]

[1] Department of Machine Design and Industrial Technologies, Azerbaijan Technical University, H. Javid ave. 25, AZ1073, Baku, Azerbaijan, E-mail khalilov@aztu.edu.az

[2] Department of Mathematics, Istanbul Ticaret University, Beyoglu, 34445 Istanbul, Turkey, E-mail: aavey@ticaret.edu.tr

[3] Scientific Research Department of Azerbaijan University of Architecture and Construction, Baku 1073, Azerbaijan

[4] Scientific Research Centers for Composition Materials of UNEC Azerbaijan State Economic University, Baku 1001, Azerbaijan

Received February 25 2024; Revised May 01 2024; Accepted for publication May 19 2024.
Corresponding author: A.H. Sofiyev (aavey@ticaret.edu.tr)

Abstract. In this study, the influence of the torsional rigidity of the connected shafts, couplings and the working body of the machine, as well as the damping capacity of the coupling, on the torsional impact moment generated in the machine transmission is investigated. Unlike existing classical calculation models, the torsional stiffness of the connected shafts, the torsional damping ability of the coupling and the effects of the moment ratio are taken into consideration together. Under these conditions an analytical expression for the shock moment or resonance coefficient is obtained. The main novelties in obtaining of this expression are the ratio of the torsional stiffness of the connected shafts with the torsional stiffness of the coupling and the acceptance of the moment of resistance of the working body of the machine depending on the torsional stiffness. It has been found that the considered factors have a significant effect on the resonance zone. Finally, different and overlapping conditions are determined when determining the value of the resonance coefficient characterizing the torque impact moment, calculated according to the classical and proposed models.

Keywords: Torsional stiffness, damping capacity, resonance coefficient, mass moment of inertia, moment of resistance.

1. Introduction

Each new generation of machines differs from its predecessors in speed, which leads to additional dynamic loads. The correct assessment of these loads and finding ways to reduce them is always an urgent task facing designers. One of the main issues during the design of the transmission of modern high-speed machines is the correct assessment of the torsional impact moment in the resonance zone and the implementation of constructive measures to reduce it. One of the main parameters characterizing the dynamic characteristics of the machine transmission is the resonance coefficient or, in other words, the function of the rise (increase) of the relative rotation angle of the connecting coupling [1]. The resonance coefficient depends on many factors such as the oscillatory movements of the machine due to the operation of the drive and working bodies, the torsional stiffness of the transmission system elements, the damping ability and so on [2-5].

In the existing classical reporting models, the dependence of the resonance coefficient on the relative damping coefficient of the connecting clutch, the rotation frequency of the transmission shaft and the forced oscillations of the driving torque [1]. At this time, the connecting shafts are considered absolutely rigid, as well as the moment of resistance of the working body of the machine was not taken into account. The rigidity and damping capabilities of the connected shafts, as well as the nature of the loading of the working body of the machine, significantly affect the resonance coefficient of the transmission, including the torque growth coefficient affecting the clutch [2]. These questions have not been widely covered in the existing literature. Thus, recent studies conducted in this area investigated the influence of inaccuracies, gaps, changes in stiffness over time, changes load during transmission at the moment of impact when starting and braking [6-8]. Chen et al. [9] investigated the effect of open shaft running clearance on torsional stiffness and evaluated its effect on impact torque. When studying the impact torque created in a car transmission, in most cases studied the influence of vibrations occurring in the engine and transmission elements [10-14]. Most of these studies were carried out using experimental or numerical methods. Kim [15] investigated the torque transmission characteristics of a press-fit coupling between an aluminium shaft and a fine-toothed steel ring. Sondkar and Kahraman [16] proposed a dynamic model of a two-cylinder planetary gear. Guan et al. [17] proposed a new dynamic model of light-weight spur gear transmission system considering the elasticity of the shaft and gear body. Rao et al. [18] studied the dynamic characteristics of a bidirectional functionally graded rotor shaft. Lin et al. [19] determined an indirect method for nonlinearly elastic shear stress-strain constitutive relationships for nonlinear torsional vibration of nonlinearly elastic shafts. Yang et al. [19] presented rigid-flexible coupled modelling of compound multistage gear system considering flexibility of shaft and gear elastic deformation. Bavi et al. [20,

Published online: May 28 2024

21] studied parametric resonance, bifurcation of and stability analysis of thin-walled composite shafts. In addition to these studies, conducted the dynamic behaviour of some shaft-shaped structural elements in dynamic torsion analysis [22-25] and studied nonlinear forced vibrations in primary resonance [26-28].

The literature review reveals that while analytically determining the resonance coefficient characterizing the torque pulse moment created in the transmission of the machine, the torsional stiffness of the connecting shafts and the change in the moment of resistance of the working body are not examined. In this study, when determining the impact moment affecting the transmission clutch of the machine, in other words, the resonance coefficient, in addition to the torsional stiffness and damping ability of the connecting coupling, as well as the torsional stiffness of the transmission shafts and the resistance moment of the working body of the machine, unlike the classic models, it is taken into account.

The structure of the paper is followed as: In Section 2, the formulation of the problem and assumptions is presented. In Section 3, the basic equations are derived and their solution method is presented. In Section 4, the parametric analysis is included and in Section 5, the conclusions are given.

2. Formulation of the Problem

Let's describe the reporting scheme as a two-mass oscillating system for the study of the resonance coefficient characterizing the torsional impact moment, taking into account the dynamic characteristics of the machine transmission, which uses a linear elastic compensating coupling to connect the shafts (Fig. 1).

The main difference of the proposed scheme from existing schemes is that here the connecting drive shafts are assumed to have a certain torsional stiffness rather than being necessarily rigid. It is also taken into account that the moment of resistance occurring in the working body of the machine is associated with a certain torsional stiffness.

Since the analytical solution of the problem under consideration is associated with some difficulties, the following assumptions are taken into account:

1) It is assumed that the driving torque varies according to the law of sinus as follows: $M_M = M_n + M_1 \sin(\omega t)$, where M_n is the nominal torque, M_1 is the amplitude of variable sum of torque, ω is the forced sum of variable torque the frequency of their dances and t is the time;

2) The moment of resistance of the working body of the machine (M_{res}) is proportional to the resistance stiffness (k_w) in any torsion, namely, $M_{res} \sim k_w$;

3) The torsional damping capacity of shafts made of steel is not considered because it is extremely small $(c_1 = c_2 = 0)$.

3. Basic Equations and Solution Method

According to the considered reporting scheme, the differential equations of motion of the driving and driven shafts can be expressed as follows:

$$\frac{d^2\varphi_1}{dt^2} = \frac{1}{J_1}(M_n + M_1 \sin\omega t) - \frac{k_1}{J_1}\varphi_1 - \frac{1}{J_1}\left[k_M(\varphi_1 - \varphi_2) + c_M\left(\frac{d\varphi_1}{dt} - \frac{d\varphi_2}{dt}\right)\right] \tag{1a}$$

$$\frac{d^2\varphi_2}{dt^2} = -\frac{k_2}{J_2}\varphi_2 + \frac{1}{J_2}\left[k_M(\varphi_1 - \varphi_2) + c_M\left(\frac{d\varphi_1}{dt} - \frac{d\varphi_2}{dt}\right)\right] - \varphi_2\frac{k_w}{J_2} \tag{1b}$$

where φ_1 and φ_2 are the rotation angles of the leading and driven half couplings, respectively; J_1 and J_2 are the induced mass moments of inertia of the rotating masses in the leading and driven arms of the transmission, respectively; k_1 and k_2 are the torsional stiffness of the leading and driven shafts, respectively; k_M and c_M are the torsional stiffness and torsional damping coefficient of the elastic coupling, respectively.

The initial conditions are as follows:

$$\varphi_1 = \varphi_2 = 0, \frac{d\varphi_1}{dt} = \frac{d\varphi_2}{dt} = 0 \text{ at } t = 0, \tag{2}$$

where t is the time.

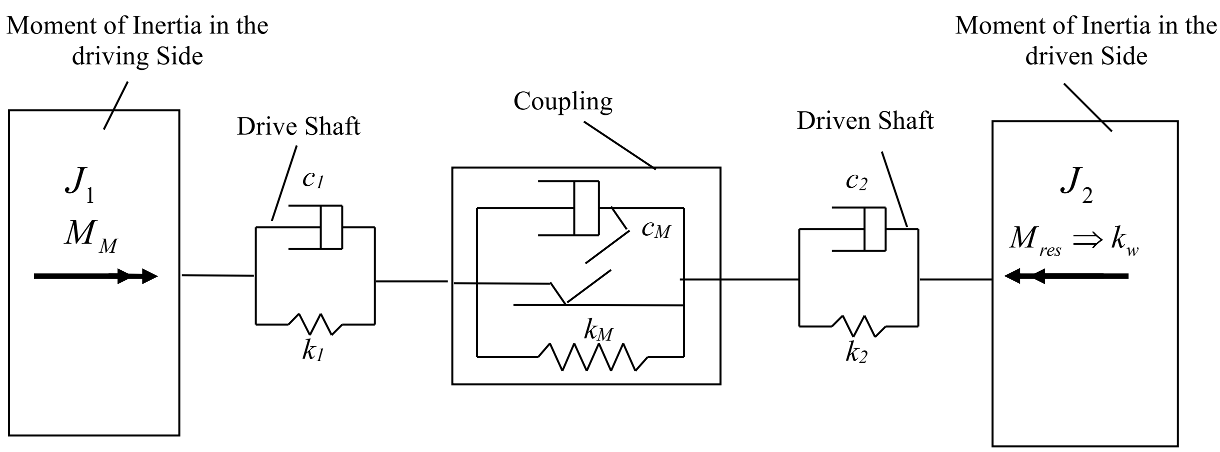

Fig. 1. Illustration of a machine transmission with an elastic compensating coupling as an oscillating system.

To analyze the influence of the clutch on the dynamic characteristics of the transmission, let's write an expression for the purpose of determining the difference $\varphi=\varphi_1-\varphi_2$, since the resonance coefficient (V), which characterizes the increase in the amplitude of the relative rotation angle of the clutch, is used. For this, let's subtract the second expression of the system of equations (1) from the first expression. From here we get that:

$$\frac{d^2\varphi}{dt^2}=m_0+m_1\sin\omega t-2K\frac{d\varphi}{dt}+\varphi_1\left(-\frac{k_1}{J_1}-\frac{1}{J_1}k_M-\frac{1}{J_2}k_M\right)+\varphi_2\left(\frac{1}{J_1}k_M+\frac{k_2}{J_2}+\frac{k_M}{J_2}+\frac{k_w}{J_2}\right) \tag{3}$$

where the following definitions are applied:

$$m_0=\frac{M_n}{J_1},\ m_1=\frac{M_1}{J_1},\ 2K=\frac{c_M}{J_1}+\frac{c_M}{J_2} \tag{4}$$

To write this expression in terms of φ_1, we assume that:

$$\frac{k_1}{J_1}=\frac{k_2}{J_2}+\frac{k_w}{J_2} \tag{5}$$

where, $k_w=J_2(k_1/J_1-k_2/J_2)=k_2(k\cdot\overline{J}-1)>0,\ k>1/J,\ \ k=k_1/k_2$ is the dimensionless coefficient of the ratio of stiffness of the leading and driven shafts, $\overline{J}=J_2/J_1$ is the dimensionless coefficient of the ratio of the moments of inertia of the rotating masses.

Taking into account the above transformations, one gets:

$$\frac{d^2\varphi}{dt^2}+2K\frac{d\varphi}{dt}+S\varphi=m_0+m_1\sin\omega t \tag{6}$$

where

$$S=\frac{1}{J_1}(k_1+k_M)+\frac{k_M}{J_2} \tag{7}$$

The initial conditions are as follows:

$$\varphi=\frac{d\varphi}{dt}=0 \text{ as the } t=0 \tag{8}$$

Thus, the solution of Eq. (6) is as follows:

$$\varphi(t)=e^{-Kt}\left(B_1\sin\beta t+B_2\cos\beta t\right)+\frac{m_0}{S}-\frac{m_1}{A_0}\left[\left(\omega^2-S\right)\sin\omega t+2K\omega\cos\omega t\right] \tag{9}$$

where $\beta=\sqrt{S-K^2}$ $(S>K^2)$; $A_0=4K^2\omega^2+\left(\omega^2-S\right)^2$ and $B_i(i=1,2)$ are determined from the initial conditions (8) as follows:

$$B_1=\frac{1}{\beta}\left(2K\omega\frac{m_1}{A_0}-\frac{m_0}{S}\right)K+\frac{m_1}{\beta A_0}\omega\left(\omega^2-S\right),\ B_2=\frac{-m_0}{S}+\frac{m_1}{A_0}2K\omega \tag{10}$$

Equation (9) can be easily transformed into the following form:

$$\varphi(t)=e^{-Kt}\cdot\sqrt{B_1^2+B_2^2}\cdot\sin\left(\beta t+\alpha_1\right)+\frac{m_0}{S}-\frac{m_1}{A_0}\sqrt{\left(\omega^2-S\right)^2+4K^2\omega^2}\cdot\sin\left(\omega t+\alpha_2\right) \tag{11}$$

where, α_1 and α_2 are the phase shifts of free and forced oscillations, respectively.

Since m_0 is a coefficient characterizing the constant sum of the torque affecting the machine movement and its effect on V is very small compared to m_1, that is not taken into account in the subsequent calculations. When the expression (10) is replaced in (9) and considering that $m_0=0$, it takes the following form:

$$\varphi(t)=e^{-Kt}\cdot\sin\left(\beta t+\alpha_1\right)\cdot\frac{m_1\omega}{\beta}\cdot\frac{1}{\sqrt{A_0}}-\frac{m_1}{\sqrt{A_0}}\cdot\sin\left(\omega t+\alpha_2\right)=\frac{m_1}{\sqrt{A_0}}\left[e^{-Kt}\cdot\sin\left(\beta t+\alpha_1\right)\cdot\frac{\omega}{\beta}-\sin\left(\omega t+\alpha_2\right)\right] \tag{12}$$

The amplitude of the relative rotation angle of the semi-coupling is characterized as follows [1]:

$$\max\varphi=\pm\frac{M_1}{J_1\omega_0^2}V_\varphi \tag{13}$$

From expressions (12) and (13), we obtain:

$$V_\varphi=\frac{\omega_0^2}{\sqrt{A_0}}=\left[4K^2\frac{\overline{\omega}^2}{\omega_0^2}+\left(\overline{\omega}^2-1\right)^2\right]^{-\frac{1}{2}} \tag{14}$$

where $\overline{\omega}=\omega/\omega_0$ is the dimensionless coefficient of the oscillation frequency of the system, the quantity A_0 is expressed as follows:

$$\begin{aligned}A_0&=4K^2\omega^2+\left(\omega^2-S\right)^2=4K^2\omega^2+\left(\omega^2-\omega_0^2\right)^2=4K^2\omega^2+\omega_0^4\left(\overline{\omega}^2-1\right)^2=\\&=4K^2\omega_0^2\overline{\omega}^2+\omega_0^4\left(\overline{\omega}^2-1\right)^2=\omega_0^4\left[4K^2\frac{\overline{\omega}^2}{\omega_0^2}+\left(\overline{\omega}^2-1\right)^2\right]\end{aligned} \tag{15}$$

in which $S=\omega_0^2$ here ω_0 is the characteristic frequency of the oscillation of the system.

From Eq. (15), the following expression for the torque growth function (resonance coefficient) is obtained:

$$V=\sqrt{\frac{1+4K^2\dfrac{\varpi^2}{\omega_0^2}}{\left(1-\varpi^2\right)^2+4K^2\dfrac{\varpi^2}{\omega_0^2}}} \tag{16}$$

On the other hand, in classic studies, the torque growth factor is determined as follows [16, 18]:

$$V_{cl}=\sqrt{\frac{1+\dfrac{\psi^2}{4\pi^2}}{\left(1-\varpi_{cl}^2\right)^2+\dfrac{\psi^2}{4\pi^2}}} \tag{17}$$

where, $\varpi_{cl}=\omega/\omega_{0cl}$ is the dimensionless coefficient of the oscillation frequency of the classical system. It is obvious that the characteristic frequency of the classical system (ω_{0cl}) will differ from the characteristic frequency of the system we are considering (ω_0). This difference is explained by the fact that in classical reporting schemes, the connected shafts are considered absolutely rigid, as well as the moment of resistance of the working body of the machine is not taken into account.

The relative damping coefficient ψ is defined as the ratio of the work done during coupling damping (A_D) and elastic deformation (A_{el}) [1]:

$$\psi=\frac{A_D}{A_{el}}=\frac{4\pi K}{S}\beta \tag{18}$$

where

$$\beta^2=S-K^2=\omega_0^2\left(1-\frac{K^2}{\omega_0^2}\right),\qquad \frac{S^2}{\omega^2}+S-K^2=\omega_0^2\left(1-\frac{K^2}{\omega_0^2}+(\varpi)^{-2}\right) \tag{19}$$

Thus, the torsional moment growth factor (16) for current case can be expressed as follows:

$$V=\sqrt{\frac{1+\dfrac{\psi^2}{4\pi^2}\cdot\dfrac{\omega_0^2\varpi^2}{\beta^2}}{\left(1-\varpi^2\right)^2+\dfrac{\psi^2}{4\pi^2}\cdot\dfrac{\omega_0^2\varpi^2}{\beta^2}}} \tag{20}$$

According to our adopted reporting scheme, the relative damping factor ψ will be determined as follows:

$$\psi=\frac{4\pi K}{S}\beta=\frac{4\pi K}{S}\sqrt{S-K^2};\quad S>K^2 \tag{21}$$

To simplify the analytical expression (20), we make the following transformations:

$$\frac{\psi}{2\pi}=2\frac{K}{\omega_0}\sqrt{1-\left(\frac{K}{\omega_0}\right)^2} \tag{22}$$

Due to the physical meaning of the coefficients K and ψ, we consider from (22) only the root with a negative sign:

$$\frac{K}{\omega_0}=\sqrt{\frac{1-\sqrt{1-\dfrac{\psi^2}{4\pi^2}}}{2}} \tag{23}$$

Substituting expression (23) into (20), it takes the following form:

$$V=\sqrt{\frac{1+\dfrac{\psi^2}{4\pi^2}\cdot\dfrac{\varpi^2}{1-(K/\omega_0)^2}}{\left(1-\varpi^2\right)^2+\dfrac{\psi^2}{4\pi^2}\varpi^2\left(\dfrac{1}{1-(K/\omega_0)^2}\right)}} \tag{24}$$

Finally, the expression (24) for the torsional moment growth factor can also be written as follows:

$$V=\sqrt{\frac{1+\dfrac{\psi^2}{4\pi^2}\cdot\dfrac{2\varpi^2}{1+\sqrt{1-\psi^2/4\pi^2}}}{\left(1-\varpi^2\right)^2+\dfrac{\psi^2}{4\pi^2}\cdot\dfrac{2\varpi^2}{1+\sqrt{1-\psi^2/4\pi^2}}}} \tag{25}$$

where the following equality is taken into account:

$$\frac{1}{1-(K/\omega_0)^2}=\frac{2}{1+\sqrt{1-\dfrac{\psi^2}{4\pi^2}}} \tag{26}$$

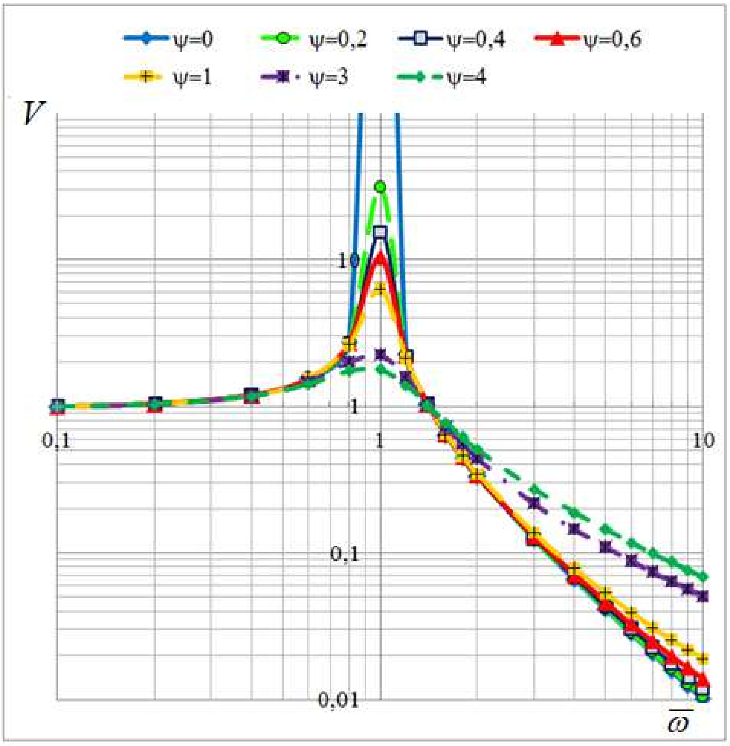

Fig. 2. Diagram of the dependence of the torque growth coefficient on the dimensionless coefficient of the system oscillation frequency for different values of the coupling relative damping coefficient (ψ).

4. Results and Discussion

In this section, the adequately of our proposed method and the influence of individual parameters of the mechanical system on the resonance coefficient of the twisting moment are presented. According to the expression (20), the curves of the dependence of the torque growth coefficient on the dimensionless coefficient of the system oscillation frequency for different values of the relative damping coefficient of the coupling are illustrated in Fig. 2. From Fig. 2, it can be seen that as the relative torsional damping coefficient of the coupling increases, the torque increase coefficient or impact torque decreases significantly. This factor is especially pronounced in the resonance zone. At small values of the dimensionless value of the oscillation frequency of the mechanical system $(\overline{\omega} \leq 1)$, the value of the relative torsional damping coefficient does not significantly affect the resonance coefficient. On the contrary, as the dimensionless value of the oscillation frequency $(\overline{\omega} \geq 1)$ increases, the increase in the relative torsional damping coefficient of the coupling increases the difference between the values of the resonance coefficients calculated by the classical and proposed methods.

The analysis of the charts shows the adequacy of the established reporting scheme. Thus, the growth of the torque in the form of an impact occurs in the resonance zone. As you move away from the resonance zone, the torque growth decreases. In addition, the relative damping coefficient of the coupling has a substantial effect on the value of the torque. For absolutely rigid couplings, the torsional impact moment grows to infinity at the resonance value of the frequency. With the increase of the relative damping coefficient, a significant part of the shock torque is absorbed due to the elastic element and the shock torque decreases.

In order to compare the results (26) obtained for the proposing scheme we have prepared with the results (17) obtained for the classical scheme, it is necessary to bring them to a single scale and establish relations between the relevant dimensionless quantities, especially between $\overline{\omega}$ and $\overline{\omega}_{cl}$:

$$\overline{\omega}_{cl} = \left(\frac{\omega}{\omega_{0cl}}\right) = \overline{\omega} \cdot \frac{\omega_0}{\omega_{0cl}} \tag{27}$$

where

$$\omega_0 = \sqrt{S} = \sqrt{\frac{1}{J_1}(k_1 + k_M) + \frac{k_M}{J_2}} \tag{28}$$

Then according to Ref. [1], one gets:

$$\omega_{0cl} = \sqrt{\frac{k_M}{J_1} + \frac{k_M}{J_2}}. \tag{29}$$

To determine the ratio ω_0/ω_{0cl}, let's perform the following transformation according to expressions (28) and (29):

$$\frac{\omega_0}{\omega_{0cl}} = \frac{\sqrt{\overline{J}\left(\overline{k}+1\right)+1}}{\sqrt{\overline{J}+1}} \tag{30}$$

where $\overline{k} = k_1 / k_M$ is a dimensionless coefficient of the ratio of torsional stiffness of the drive shaft and coupling.

Then expression (27) will take the following form:

$$\bar{\omega}_{cl} = \bar{\omega} \cdot \frac{\sqrt{\bar{J}\left(\bar{k}+1\right)+1}}{\sqrt{\bar{J}+1}} \tag{31}$$

Substituting (31) into (17), we obtain the following expression:

$$V_{cl} = \sqrt{\frac{1+\dfrac{\psi^2}{4\pi^2}}{\left(1-\left[\bar{\omega}\cdot\dfrac{\sqrt{\bar{J}\left(\bar{k}+1\right)+1}}{\sqrt{\bar{J}+1}}\right]^2\right)^2+\dfrac{\psi^2}{4\pi^2}}} \tag{32}$$

In order to evaluate the obtained results, according to the expressions (25) and (32), the curves of the dependence of the torque growth coefficient (V) on the dimensionless coefficients ($\bar{\omega}$) for different values of the relative damping coefficient of the coupling are plotted in Figs. 3 to 6. The analysis of the diagrams shows that the results obtained from the presented reporting scheme for rigid couplings completely coincide with the results obtained from the classical scheme. However, as a result of increasing the relative damping coefficient of the coupling, the difference between the obtained results increases significantly. This factor must be taken into account when designing the transmission of machines. This fact is one of the factors that confirm the correctness of the approach to solving the problem. This fact is one of the factors that confirm the correctness of the approach to solving the problem.

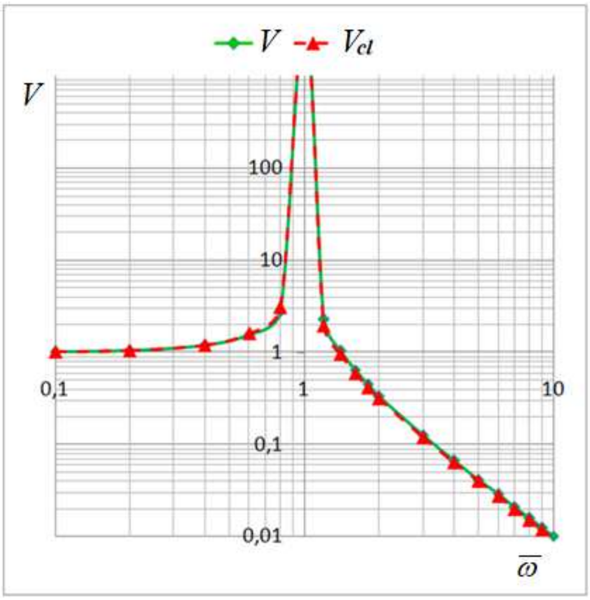

Fig. 3. Comparison of V and V_{cl} coefficients varying depending on frequency with $\psi = 0, \bar{J} = 1, \bar{k} = 0.1$.

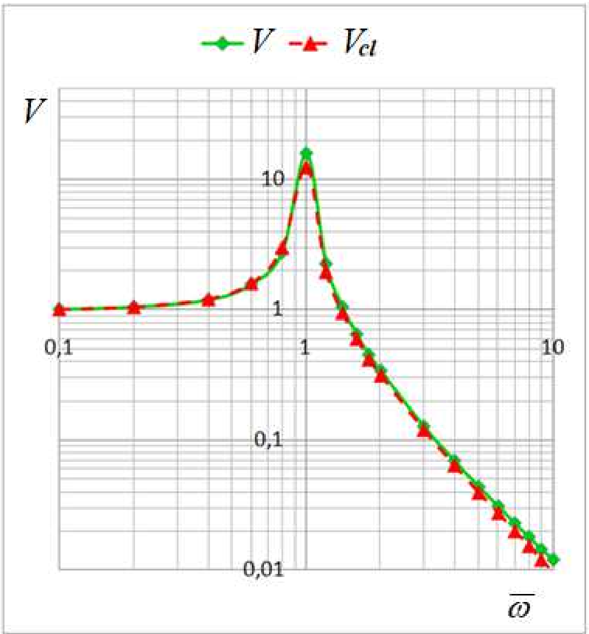

Fig. 4. Comparison of V and V_{cl} coefficients varying depending on frequency with $\psi = 0.4, \bar{J} = 1, \bar{k} = 0.1$.

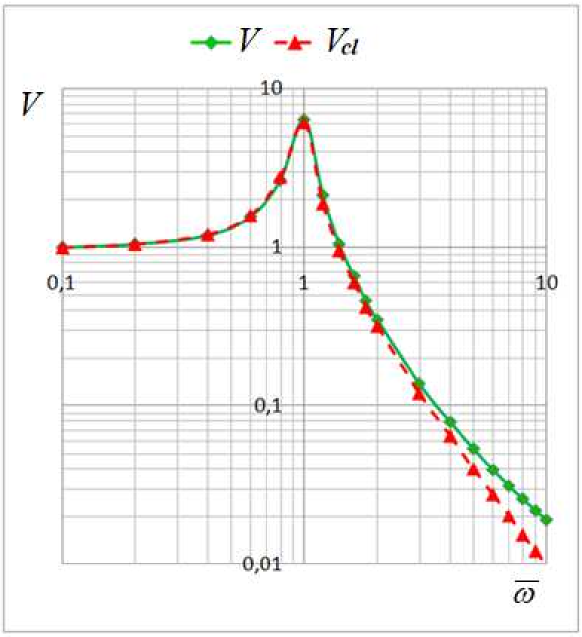

Fig. 5. Comparison of V and V_{cl} coefficients varying depending on frequency for $\psi = 1, \overline{J} = 1, \overline{k} = 0.1$.

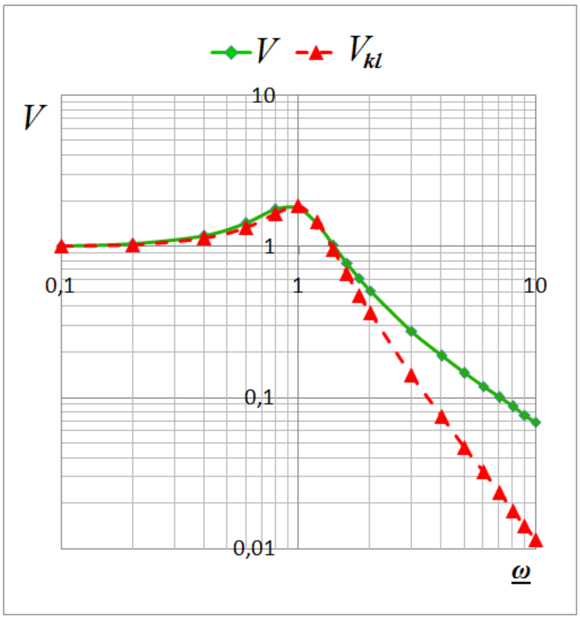

Fig. 6. Comparison of V and V_{cl} coefficients varying depending on frequency for $\psi = 4, \overline{J} = 1, \overline{k} = 0.1$.

It is clear that the ratio of the mass moments of inertia of the leading and driven branch will also have a significant effect on the twisting impact moment generated in the machine transmission. In order to investigate this issue, dependence curves $(V \sim \psi)$ of the relative damping coefficient (ψ) of the coupling in the resonance zone on the torque growth coefficient (V and V_{cl}) acting on the coupling are illustrated (Figs. 7 and 8). A comparison of the diagrams shows that the difference between the results obtained from the presented scheme and the results obtained from the classical scheme is large at small values of the relative damping coefficient of the coupling (when the stiffness of the coupling is high). As the value of the ratio of the mass moments of inertia of the leading and driven branch of the transmission increases, this difference increases, that is, the torsional impact moment increases. For large values of the relative damping coefficient (especially values greater than 1), the results obtained by the proposed scheme and the classical scheme coincide.

Figure 9 shows the change of torque increase coefficients for the near-resonance region $(\overline{\omega}_{cl} = 0.9)$, calculated by the classical and presented methods, depending on the dimensionless coefficient characterizing the torsional rigidity of the shaft, for given values of the relative damping coefficient of the coupling $\psi = 0,4$ and the ratio of the moments of inertia of the masses in the driving and driven branches drive $\overline{J} = 0.2$. Analysis of the graphs shows that the resonance coefficient calculated by the classical method does not depend on the torsional rigidity of the shaft. However, the resonance coefficient V calculated by the presented method varies depending on the dimensionless coefficient $\overline{k}$. As the ratio of the torsional stiffness of the drive shaft to the torsional stiffness of the coupling increases, the resonance coefficient, that is, the torque shock moment, decreases. This factor proves the correctness of the approach to the issue.

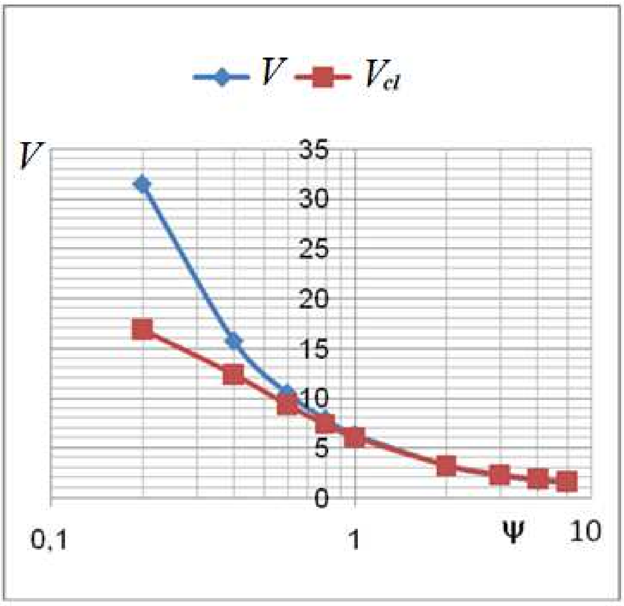

Fig. 7. Diagrams of the dependence of the torsional moment growth factor acting on the coupling ($\bar{\omega} \to 1$) on the relative damping coefficient (ψ) for the resonance zone with $\bar{J} = 1$ and $\bar{k} = 0{,}1$.

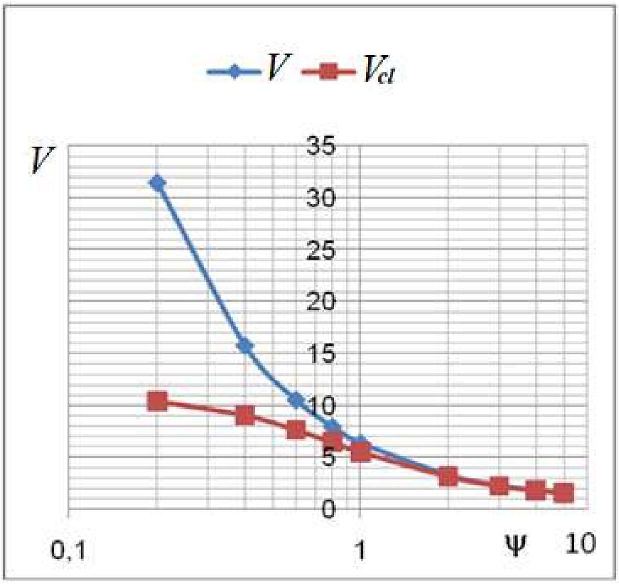

Fig. 8. Diagrams of the dependence of the torsional moment growth factor acting on the coupling ($\bar{\omega} \to 1$) on the relative damping coefficient (ψ) for the resonance zone with $\bar{J} = 10$ and $\bar{k} = 0{,}1$.

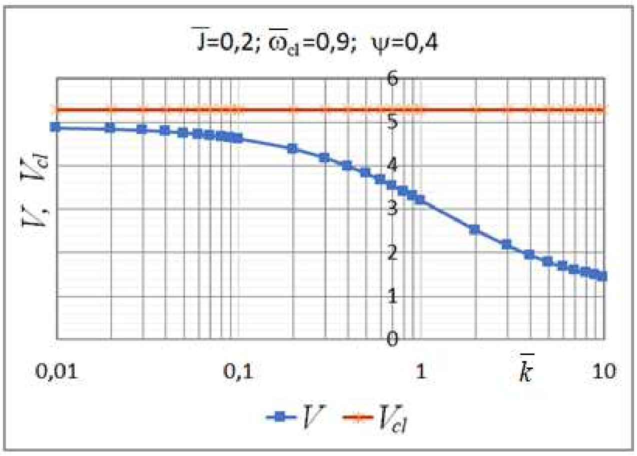

Fig. 9. Dependence of the torque increase coefficient on the torsional rigidity of the shaft.

A numerical assessment of the influence of the shaft torsional stiffness on the torsional shock moment over a wider range of vibrations is presented in Fig. 10. Analysis of the graphs shows that a significant difference between the values of the resonance coefficients for different values of the shaft torsional stiffness occurs predominantly around the resonance zone $(0.9 \le \bar{\omega}_{cl} \le 1.1)$. It is shown that the change in the torsional stiffness of the shaft does not significantly affect the torque increase as it moves away from the resonance zone. In addition, the difference between the values of the resonance coefficient is obvious at large values of the dimensionless coefficient $\bar{k}$. When the dimensionless coefficient $\bar{k}$ decreases, this difference is not noticeable. These observations confirm the adequacy of the adopted calculation scheme.

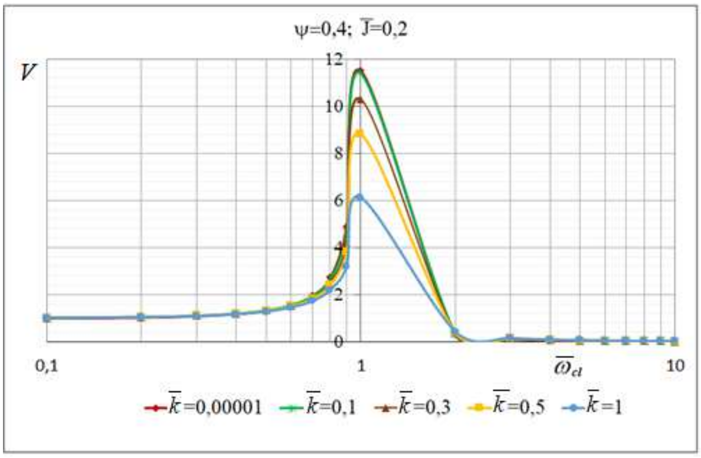

Fig. 10. Dependence of the torque increase coefficient on the oscillation frequency of the mechanical system for various values of shaft torsional rigidity.

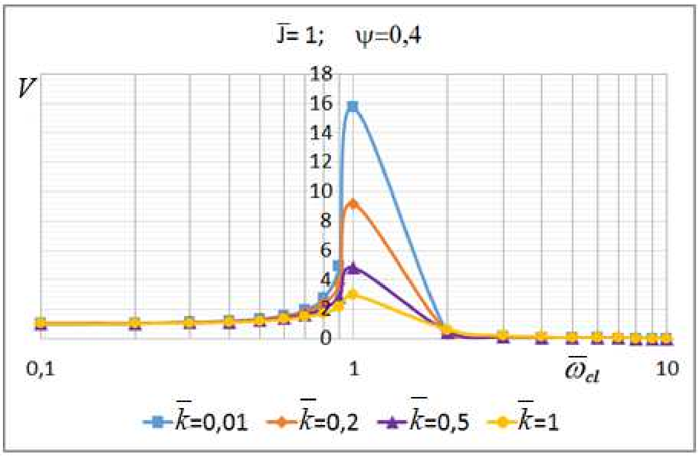

Fig. 11. Dependence of the torque increase coefficient on the oscillation frequency of the mechanical system with the $\bar{J} = 1$ and at different values of shaft torsional stiffness.

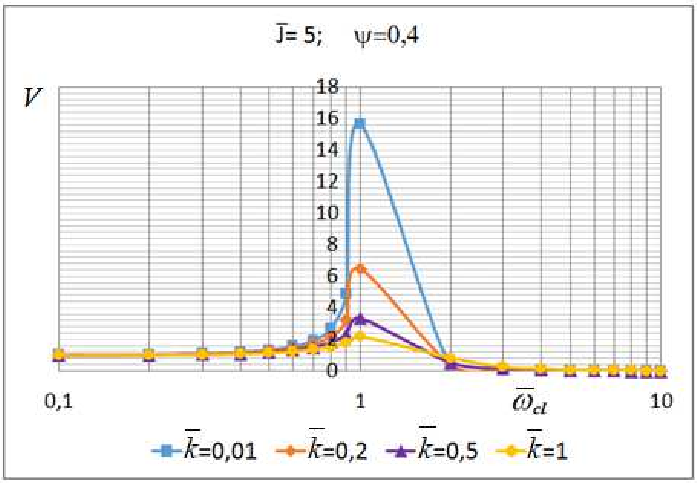

Fig. 12. Dependence of the torque increase coefficient on the frequency of the mechanical system with the $\bar{J} = 5$ and at different values of shaft torsional stiffness.

When analyzing the dependence of the torque increasing function on the torsional rigidity of the shaft, it is also necessary to take into account the ratios of the moments of mass inertia $(\bar{J} = J_2 / J_1)$ in the drive branches before and after the coupling. For this purpose, the variation of the resonance coefficient depending on the dimensionless oscillation frequency coefficient $(\bar{\omega}_{cl})$ of the mechanical system for various ratios of mass moments of inertia in the driving and driven branches of the drive and the same values of the torsional stiffness of the shaft is presented in Figs. 11 and 12. A comparative analysis shows that with the same values of the torsional stiffness of the shafts and the relative torsional damping coefficient of the coupling, the resonance coefficient will be lower when the ratio of the mass moments of inertia is high. This factor is especially manifested in large values of the coefficient characterizing the torsional rigidity of the shaft.

6. Conclusions

Taking into account the torsional stiffness of the connecting shafts and the change in the moment of resistance of the working body of the machine, a new analytical expression was obtained for determining the resonance coefficient characterizing the torsional impact moment created in the transmission of the machine.

The analysis results are generalized as follows:

- The difference between the increase in the relative damping coefficient of the coupling and the values of the torque growth coefficients, calculated according to the accepted scheme and according to the classical scheme, increases in the non-resonance zone, and practically coincides in the resonance zone.
- As the stiffness of the shafts increases, the V value obtained in the current study and the V_{cl} value approach each other, confirming the adequacy of the adopted reporting plan.
- At small values of the relative damping coefficient of the coupling around the resonance zone, the increase in the ratio of moments of inertia in the driven and driving branch of the system significantly affects the growth coefficient of the torsional impact moment.
- At large values of the stiffness of the connected shafts, as well as at large values of the relative damping coefficient of the coupling, the effect of the change in the ratio of moments of inertia in the driven and driving branch of the system on the torque growth coefficient is very insignificant.
- As the ratio of the torsional stiffness of the drive shaft to the torsional stiffness of the coupling increases, the resonance coefficient increases.
- In the resonance zone there is a significant difference between the values of the resonance coefficients for different values of the torsional stiffness of the shaft.
- Any change in the torsional stiffness of the shaft does not significantly affect the increase in torque as it moves away from the resonance zone.
- At large values of the stiffness of the connected shafts, as well as at large values of the relative damping coefficient of the coupling, the effect of the change in the ratio of moments of inertia in the driven and driving branch of the system on the torque growth coefficient is very insignificant.

The results obtained allow us to correctly design the transmission as a whole depending on the operating characteristics of the machine, that is, the nature of the moment of resistance, and to effectively select the torsional stiffness characteristics of its individual elements to reduce torsional shock.The proposed method can be successfully used to solve optimization problems to minimize the impact torque in machine transmissions.

Author Contributions

I.A. Khalilov: Conceptualization, methodology, software, validation, resources; I.A. Khalilov and A.H. Sofiyev: investigation, writing—original draft preparation, review and editing. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

Acknowledgments

Not Applicable.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

Funding

The authors received no financial support for the research, authorship and publication of this article.

Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

References

[1] Peeken, H., Troeder, Ch., *Elastische Kupplungen, Ausführungen, Eigenschaften, Berechnungen*, Springer-Verlag, 1986.
[2] Khalilov, I.A., Studies of the dynamic properties of geometrically closed mechanical clutches, *Mechanical Engineering*, 2, 2009, 25-31.
[3] Khalilov, I.A., Influence of the gap in clutches on the dynamic properties of drives, *Russian Engineering Research*, 30, 2010, 206-212.
[4] Khalilov, I.A., Temperature variation of elastic elements in clutches on damping, *Russian Engineering Research*, 32, 2012, 322-325.
[5] Khalilov, I.A., The importance of considering the temperature factors while choosing the elastic couplings, *IFAC International Federation of Automatic Control*, 18th IFAC Conference on International Stability, Technology and Culture 13-15 September 2018, Baku, Azerbaijan, 816-820, 2018.
[6] Jin, G., Ren, W., Zhu, R., Influence of torsional stiffness on load sharing coefficient of a power split drive system, *MATEC Web of Conferences*, 211, 2008, 17002.
[7] Liu, Q., Tang, C., Liu, X., Dynamic analysis and experimental study of confluence planetary row under transient torque shock, *9th International Symposium on Test Automat, & Instrument (ISTAI 2022)*, Online Conference, Beijing, China, 213-219, 2022.

[8] Zha, L., Lin, Y., Li, Z., Huang, C., Optimal starting control of zerosynchronous shock AMT based on torque compensation, *Science Progress*, 103(4), 2020, 36850420968675.
[9] Chen, Y., Zhu, R., Jin, G., Xiong, Y., Influence of shaft torsional stiffness on dynamic response of four-stage main transmission system, *Mathematical Problems in Engineering*, 2018, 2018, 6141035.
[10] Costas, A.M., Athanasios, N.S., Dynamic and vibration analysis of a multimotor dc drive system with elastic shafts driving a tissue paper machine, *IEEE Transactions on Industrial Electronics*, 54(4), 2007, 2033-2046.
[11] Krot P., Transient torsional vibrations control in the geared drive trains of the hot rolling mills, *IEEE Conference on Control Technology and Applications (CCTA)*, St. Petersburg, Russia, 1368-1373, 2009.
[12] Meeus, H., Verrelst, B., Moens, D., Guillaume, P., Lefeber, D., Experimental study of the shaft penetration factor on the torsional dynamic response of a drive train, *Machines*, 6, 2018, 31.
[13] Morimura, H., A Study of vehicle acceleration shock: analysis of driveability model and transient excitation torque rise, *Transactions of the Japan Society of Mechanical Engineers*, 69, 2003, 3228-3235.
[14] Kim, H.S., torque transmission characteristics of the press fit joint between the aluminum shaft and steel ring with small teeth, *Mechanics Based Design of Structures and Machines*, 39 (1), 2011, 100-117.
[15] Sondkar, P., Kahraman, A., A dynamic model of a double-helical planetary gear set, *Mechanizm and Machine Theory*, 70, 2013, 157–174.
[16] Guan, X.L., Tang, J.Y., Hu, Z.H., Wang, Q.S., Kong, X.N., A new dynamic model of light-weight spur gear transmission system considering the elasticity of the shaft and gear body, *Mechanism and Machine Theory*, 170, 2021, 104689.
[17] Rao, D.K., Swain, A., Roy, T., Dynamic responses of bidirectional functionally graded rotor shaft, *Mechanics Based Design of Structures and Machines*, 50(1), 2022, 302-330.
[18] Lin, F., Peng, J.S., Xu, S.F., Yang, J., An indirect method to determine nonlinearly elastic shear stress-strain constitutive relationships for nonlinear torsional vibration of nonlinearly elastic shafts, *Multidiscipline Modeling in Materials and Structures*, 18(4), 2022, 582-605.
[19] Yang, X., Lei, Y.G., Liu, H., Yang, B., Li, X., Li, N., Rigid-flexible coupled modeling of compound multistage gear system considering flexibility of shaft and gear elastic deformation, *Mechanical Systems and Signal Processing*, 200, 2023, 11063.
[20] Bavi, R., Mohammad-Sedighi, H., Hajnayeb, A., Shishesaz, M., Parametric resonance and bifurcation analysis of thin-walled asymmetric gyroscopic composite shafts: An asymptotic study, *Thin-Walled Structures*, 184, 2023, 110508.
[21] Bavi, R., Hajnayeb, A., Sedighi, H.M., Shishesaz, M., Simultaneous resonance and stability analysis of unbalanced asymmetric thin-walled composite shafts, *International Journal of Mechanical Sciences*, 217, 2022, 107047.
[22] Sofiyev, A.H., Schnack, E., The stability of functionally graded cylindrical shells under linearly increasing dynamic torsional loading, *Engineering Structures*, 26, 2004, 1321-1331.
[23] Sofiyev, A.H., Kadioglu, F., Khalilov, I.A., Sedighi, H.M., Vergul, T., Yenialp, R., On the torsıonal bucklıng moment of cylındrıcal shells consıstıng of functıonally graded materials resting on the Pasternak-type soıl, *SOCAR Proceedings*, 1, 2022, 16-22.
[24] Ramezannejad Azarboni, H., Heidari, H., Nonlinear primary frequency response analysis of self- sustaining nanobeam considering surface elasticity, *Journal of Applied and Computational Mechanics*, 8(4), 2022, 1196-207.
[25] Zhang, J.G., Song, Q.R., Zhang, J.Q., Wang, F., Application of he's frequency formula to nonlinear oscillators with generalized initial conditions, *Facta Universitatis, Series: Mechanical Engineering*, 21(4), 2023, 701-12.
[26] Avey, M., Sofiyev, A.H., Fantuzzi, N., Kuruoglu, N., Primary resonance of double-curved nanocomposite systems using improved nonlinear theory and multi-scales method: modeling and analytical solution, *International Journal of Nonlinear Mechanics*, 137, 2021, 103816.
[27] Sofiyev, A.H., Avey, M., Kuruoglu, N., An approach to the solution of nonlinear forced vibration problem of structural systems reinforced with advanced materials in the presence of viscous damping, *Mechanical Systems and Signal Processing*, 161, 2021, 107991.
[28] Ipek, C., Vibration analysis of shear deformable cylindrical shells made of heterogeneous anisotropic material with clamped edges, *Journal of Applied and Computational Mechanics*, 9(3), 2023, 861-869.

ORCID iD

Isa Ali Khalilov https://orcid.org/0000-0001-5026-5742
Abdullah H. Sofiyev https://orcid.org/0000-0001-7678-6351

How to cite this article: Khalilov I.A., Sofiyev A.H. Dynamic Behavior of Shafts, Couplings and Working Body of the Machine under Torsional Impact Moment, *J. Appl. Comput. Mech.*, xx(x), 2024, 1–11. https://doi.org/ 10.22055/jacm.2024.46217.4482