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Research Paper

Moving Loads on Thermo-viscoelastic Micropolar Solid Medium with Voids and Two Temperature

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Abstract. This work involves the study of the effects of moving loads in an isotropic, homogeneous, micropolar, porous thermo-viscoelastic solid material with two temperatures. The problem is solved in the context of Green-Naghdi theory (G-N II and G-N III). The analytical expressions of physical quantities in the physical domain are obtained by Normal modal analysis. These expressions are numerically evaluated for a given material and shown graphically by comparing the G-N II and G-N III theories with and without moving initial stresses.

Keywords: Thermo-viscoelasticity; Voids; Green-Naghdi; Two-temperatures; Moving loads.

1. Introduction

The investigation of porous media exhibiting both viscoelastic and thermal characteristics, along with their response to diverse stimuli, continues to be a critical field of research, several works [1–7] explored employing mechanical models to represent linear viscoelastic behavior and proposed solutions to linear boundary value problems for viscoelastic material with temperature fluctuations in the dynamic and quasi-static cases. For this scientific field, several researchers [8–10] related the solutions of the linear problems of visco-elastic to the corresponding elastic one and proposing an approximation technique for these solutions. In these books [11–13], some boundary value problems of thermo-viscoelasticity and coupling problems of continuum mechanics are expressed, referring to experimental results for shedding light on the mechanical properties of viscoelastic materials. Here are some works [14–16] in which were discussed some problems of viscoelasticity. Regarding the elastic micropolar material theory and the description of the properties of these materials, Eringen [17] proposed it with the continuum micropolar mechanics theory, which considers the microstructure of the material. As for the heat conduction equations and the thermal aspect, there are several theories, that introduce generalized thermoelasticity, which presents the coupled relation between the thermal and mechanical effects on the elastic bodies, including the three models proposed by Green and Naghdi [18–20] (GN-I, II, and III). The linearized version of model I corresponds to the classical thermoelasticity theory. In model II, the internal entropy production rate is assumed to be equal to zero, which means that there is no thermal energy loss. Model III combines Model I and Model II in one equation with a parameter for energy dissipation. In these works [21–27], some problems of thermoelasticity and viscosity are discussed in the context of some different thermal conduction theories. The theory of elastic materials that contain voids focuses on elastic materials riddled with small, distributed cavities, known as voids, whose volume is one of the fundamental kinematic variables. For this theory, Nunziato and Cowin [28] demonstrated the connection between changes in void volume and an internal energy dissipation phenomenon, which in turn gives rise to the material's relaxation properties. In contrast, the linear theory of elastic materials with voids, pioneered by Cowin and Nunziato [29], treats the void volume fraction as a separate and independent kinematic variable. Several investigators [30–33] have conducted studies on this theory on many sides, including plane wave behavior in the material, the influence zone theorem, the heat flow-dependent void thermoelasticity theory, and the viscoelastic behavior of the porous mediums. Iesan [34] provided a development of the mentioned linear theory based on the work of Cowin and Nunziato [29]. In other work, Iesan [35] presented two models for linear and non-linear theories of viscous thermo-elastic materials containing voids. Some different issues of thermo-viscoelasticity with voids are discussed here [36–40]. There is a theory of heat conduction presented in the works [41–43] that depends on two different temperatures: the conduction temperature and the thermodynamic temperature. Boley and Tolins [44] found that these two temperatures, and strain, are represented in the form of traveling waves plus reactions that occur instantaneously throughout the body. The element that distinguishes the two-temperature thermoelasticity (2TT) from the classical theory (CTE) is a constant (depending on the material properties) called the temperature difference. In these works [45–47], some different problems of thermoelasticity with two temperatures were discussed.

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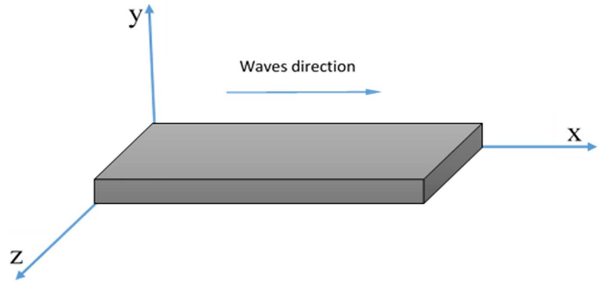

Fig. 1. Schematic configuration of the half-space.

2. Formulation of the Problem

The problem is formulated in the 2-dimensional Cartesian coordinates in half-space $z \geq 0$, where $\mathbf{u} = (u, 0, w)$ is assumed to be a function of (t,x,z) as depicted in Fig. 1. Iesan [37] and Green and Naghdi [22] formulated the basic linear equations which describe the mechanical state for a viscoelastic, homogeneous, micropolar, thermally conducting and isotropic medium containing voids with two temperatures under moving loads in the absence of body forces:

$$(\mu^* + \kappa^*)\nabla^2\mathbf{u} + (\mu^* + \lambda^*)\nabla(\nabla.\mathbf{u}) - \nu^*\nabla T + \kappa^*(\nabla\times\varphi) + b^*\nabla\phi = \rho\ddot{\mathbf{u}}, \tag{1}$$

$$(\alpha^* + \beta^* + \gamma^*)\nabla(\nabla.\varphi) - \gamma^*\nabla\times(\nabla\times\varphi) + \kappa^*(\nabla\times\mathbf{u}) - 2\kappa^*\varphi = J\rho\ddot{\varphi}, \tag{2}$$

$$A^*\nabla^2\phi - \xi_1\phi - \xi_2\dot{\phi} - B^*(\nabla.\mathbf{u}) + (\tau\nabla^2 + m)T = \rho\chi\ddot{\phi}, \tag{3}$$

$$\rho C_e\ddot{T} + \nu^* T_0\ddot{e} + (mT_0 - \varsigma\nabla^2)\dot{\phi} = K\nabla^2\theta + K^*\nabla^2\dot{\theta}. \tag{4}$$

The relations between stresses and displacements are:

$$\sigma_{ij} = \mu^* u_{i,j} + (\mu^* + \kappa^*)u_{j,i} - \kappa^*\varepsilon_{ijk}\varphi_k + [\lambda^* u_{k,k} - \nu^* T + b^*\phi]\delta_{ij}, \tag{5}$$

$$m_{ij} = \alpha^*\varphi_{k,k}\delta_{ij} + \beta^*\varphi_{i,j} + \gamma^*\varphi_{j,i}, \tag{6}$$

$$T = (1 - a\nabla^2)\theta. \tag{7}$$

The parameters $\lambda^*, \mu^*, \nu^*, b^*, A^*, B^*, \kappa^*, \alpha^*, \beta^*$ and γ^* are defined as:

$$\begin{gathered}
\lambda^* = \lambda(1 + \alpha_0\partial_{,t}),\ \mu^* = \mu(1 + \alpha_1\partial_{,t}),\ \nu^* = \nu(1 + \nu_0\partial_{,t}),\ b^* = b(1 + \alpha_2\partial_{,t}),\ A^* = A(1 + \alpha_3\partial_{,t}), \\
B^* = B(1 + \alpha_4\partial_{,t}),\ \alpha^* = \alpha(1 + \alpha_5\partial_{,t}),\ \beta^* = \beta(1 + \alpha_6\partial_{,t}),\ \gamma^* = \gamma(1 + \alpha_7\partial_{,t}), \\
\kappa^* = \kappa(1 + \alpha_8\partial_{,t}),\quad \nu_0 = \frac{1}{\nu}(3\lambda\alpha_0 + 2\mu\alpha_1 + \kappa\alpha_8)\alpha_t,\quad \nu = (3\lambda + 2\mu + \kappa)\alpha_t,
\end{gathered} \tag{8}$$

The dot notation is used to denote time differentiation. Equation (1) can be expressed in the $xz-$plane by two equations below:

$$[\mu(1 + \alpha_1\partial_{,t}) + \kappa(1 + \alpha_8\partial_{,t})]\nabla^2 u + [\lambda(1 + \alpha_0\partial_{,t}) + \mu(1 + \alpha_1\partial_{,t})]e_{,x} - \nu(1 + \nu_0\partial_{,t})(1 - a\nabla^2)\theta_{,x} - \kappa(1 + \alpha_8\partial_{,t})\varphi_{2,x} + b(1 + \alpha_2\partial_{,t})\phi_{,x} = \rho u_{,tt}, \tag{9}$$

$$[\mu(1 + \alpha_1\partial_{,t}) + \kappa(1 + \alpha_8\partial_{,t})]\nabla^2 w + [\lambda(1 + \alpha_0\partial_{,t}) + \mu(1 + \alpha_1\partial_{,t})]e_{,z} - \nu(1 + \nu_0\partial_{,t})(1 - a\nabla^2)\theta_{,z} + \kappa(1 + \alpha_8\partial_{,t})\varphi_{2,z} + b(1 + \alpha_2\partial_{,t})\phi_{,z} = \rho w_{,tt}, \tag{10}$$

where $e = u_{,x} + w_{,z}$. For simplifications, the following dimensionless quantities are used:

$$\begin{gathered}
x_i' = \frac{\tilde{\omega}}{c_1}x_i,\ u_i' = \frac{\rho c_1\tilde{\omega}}{\nu T_0}u_i,\ \{T', \theta'\} = \frac{1}{T_0}\{T, \theta\},\ \phi' = \frac{\rho c_1^2}{\nu T_0}\phi,\ \varphi_i' = \frac{\rho c_1^2}{\nu T_0}\varphi_i,\ t' = \tilde{\omega}t, \\
\sigma_{ij}' = \frac{1}{\nu T_0}\sigma_{ij},\ m_{ij}' = \frac{1}{\nu T_0 c_1}m_{ij},\ a' = \frac{\tilde{\omega}}{c_1^2}a,\ \tilde{\omega} = \frac{\rho C_e c_1^2}{K},\ \varepsilon = \frac{\nu^2 T_0}{\rho C_e(\lambda + 2\mu + \kappa)}, \\
\{\alpha_0', \alpha_1', \alpha_2', \alpha_3', \alpha_4', \alpha_5', \alpha_6', \alpha_7', \alpha_8'\} = \tilde{\omega}\{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8\}, \\
c_1^2 = \frac{(\lambda + 2\mu + \kappa)}{\rho},\ c_2^2 = \frac{\mu}{\rho},\ c_3^2 = \frac{\kappa}{\rho},\ \delta^2 = \frac{c_2^2}{c_1^2},\ \delta_1^2 = \frac{c_3^2}{c_1^2}.
\end{gathered} \tag{11}$$

In the non-dimensional variables (11), Eqs. (9) to (10) and (2) to (4) become, respectively (after dropping the dashed for convenience):

$$[\delta^2(1+\alpha_1\partial_{,t})+\delta_1^2(1+\alpha_8\partial_{,t})]\nabla^2 u+[(1-2\delta^2-\delta_1^2)(1+\alpha_0\partial_{,t})+\delta^2(1+\alpha_1\partial_{,t})]e_{,x} \\ -\delta_1^2(1+\alpha_8\partial_{,t})\varphi_{2,x}-(1+\nu_0\partial_{,t})(1-a\nabla^2)\theta_{,x}+a_1(1+\alpha_2\partial_{,t})\phi_{,x}=u_{,tt}, \tag{12}$$

$$[\delta^2(1+\alpha_1\partial_{,t})+\delta_1^2(1+\alpha_8\partial_{,t})]\nabla^2 w+[(1-2\delta^2-\delta_1^2)(1+\alpha_0\partial_{,t})+\delta^2(1+\alpha_1\partial_{,t})]e_{,z} \\ +\delta_1^2(1+\alpha_8\partial_{,t})\varphi_{2,z}-(1+\nu_0\partial_{,t})(1-a\nabla^2)\theta_{,z}+a_1(1+\alpha_2\partial_{,t})\phi_{,z}=w_{,tt}, \tag{13}$$

$$(1+\alpha_7\partial_{,t})\nabla^2\varphi_2+a_2(1+\alpha_8\partial_{,t})(u_{,z}-w_{,x})-2a_2(1+\alpha_8\partial_{,t})\varphi_2=a_3\varphi_{2,tt}, \tag{14}$$

$$(1+\alpha_3\partial_{,t})\nabla^2\phi-a_4(1+\xi\,\partial_{,t})\phi-a_5(1+\alpha_4\partial_{,t})e+(a_6\nabla^2+a_7)(1-a\nabla^2)\theta=a_8\phi_{,tt}, \tag{15}$$

$$(1-a\nabla^2)\theta_{,tt}+\varepsilon(1+\nu_0\,\partial_{,t})e_{,tt}+(a_9-a_{10}\nabla^2)\phi_{,t}=\nabla^2(\varepsilon_2\theta+\varepsilon_3\theta_{,t}). \tag{16}$$

The constitutive relations (5) to (7) in non-dimensional versions take the form:

$$\sigma_{ij}=\delta^2(1+\alpha_1\partial_{,t})u_{i,j}+[\delta^2(1+\alpha_1\partial_{,t})+\delta_1^2(1+\alpha_8\partial_{,t})]u_{j,i}-\delta_1^2(1+\alpha_8\partial_{,t})\varepsilon_{ijk}\varphi_k \\ +[(1-2\delta^2-\delta_1^2)(1+\alpha_0\partial_{,t})u_{k,k}-(1+\nu_0\partial_{,t})(1-a\nabla^2)\theta+a_1(1+\alpha_2\partial_{,t})\phi]\delta_{ij}, \tag{17}$$

$$m_{ij}=a_{13}(1+\alpha_5\partial_{,t})\varphi_{k,k}\delta_{ij}+a_{12}(1+\alpha_6\partial_{,t})\varphi_{i,j}+a_{11}(1+\alpha_7\partial_{,t})\varphi_{j,i}, \tag{18}$$

$$T=(1-a\nabla^2)\theta. \tag{19}$$

where $a_i, i=1,\ldots,12,\ \xi,\varepsilon_2,\varepsilon_3$ are defined in Appendix A. According to the Helmholtz theorem, the displacement components $u(x,z,t),\ w(x,z,t),$ can be decomposed into potential Φ and vortex parts Ψ:

$$u=\Phi_{,x}+\Psi_{,z},\quad w=\Phi_{,z}-\Psi_{,x}. \tag{20}$$

Using the Eqs. (20), the system of Eqs. (12) to (16) tends to:

$$(1+\delta_0\partial_{,t})\nabla^2\Phi-(1+\nu_0\partial_{,t})(1-a\nabla^2)\theta+a_1(1+\alpha_2\partial_{,t})\phi=\Phi_{,tt}, \tag{21}$$

$$[\delta^2(1+\alpha_1\partial_{,t})+\delta_1^2(1+\alpha_8\partial_{,t})]\nabla^2\Psi-\delta_1^2(1+\alpha_8\partial_{,t})\varphi_2=\Psi_{,tt}, \tag{22}$$

$$(1+\alpha_7\partial_{,t})\nabla^2\varphi_2+a_2(1+\alpha_8\partial_{,t})\nabla^2\Psi-2a_2(1+\alpha_8\partial_{,t})\varphi_2=a_3\varphi_{2,tt}, \tag{23}$$

$$(1+\alpha_3\partial_{,t})\nabla^2\phi-a_4(1+\xi\partial_{,t})\phi-a_5(1+\alpha_4\partial_{,t})\nabla^2\Phi+(a_3\nabla^2+a_7)(1-a\nabla^2)\theta=a_8\phi_{,tt}, \tag{24}$$

$$(1-a\nabla^2)\theta_{,tt}+\varepsilon(1+\nu_0\partial_{,t})\nabla^2\Phi_{,tt}+a_9\phi_{,t}-a_{10}\nabla^2\phi_{,t}=\nabla^2\left(\varepsilon_2\theta+\varepsilon_3\theta_{,t}\right), \tag{25}$$

where $\delta_0=\alpha_0+2(\alpha_1-\alpha_0)\delta^2+(\alpha_8-\alpha_0)\delta_1^2$.

3. Normal Mode Analysis

The considered physical quantities can be decomposed as the following form:

$$\{\Phi,\Psi,\theta,\varphi_2,\phi\}(x,z,t)=\{\bar\Phi,\bar\Psi,\bar\theta,\bar\varphi_2,\bar\phi\}(z)e^{(\omega t+inx)}. \tag{26}$$

Equations (21) to (25) with the aid of Eq. (26) become, respectively:

$$(b_1D^2-b_2)\bar\Phi+(b_3D^2-b_4)\bar\theta+b_5\bar\phi=0, \tag{27}$$

$$(b_6D^2-b_7)\bar\Psi-b_8\bar\varphi_2=0, \tag{28}$$

$$(b_9D^2-b_{10})\bar\Psi+(b_{11}D^2-b_{12})\bar\varphi_2=0, \tag{29}$$

$$(b_{13}D^2-b_{14})\bar\Phi+(b_{15}D^4+b_{16}D^2+b_{17})\bar\theta-(b_{18}D^2-b_{19})\bar\phi=0, \tag{30}$$

$$(b_{20}D^2-b_{21})\bar\Phi+(b_{22}D^2-b_{23})\bar\theta-(b_{24}D^2-b_{25})\bar\phi=0, \tag{31}$$

where, $D = d/dz$, and $b_j,\ j = 1,...,25$, are defined in Appendix B. Eliminating $\bar{\Psi}$ and $\bar{\varphi}_2$ between Eqs. (28) and (29), and also eliminating $\bar{\Phi}, \bar{\phi}$ and $\bar{\theta}$ between Eqs. (27), (30) and (31), lead to the following ODEs:

$$(D^4 - d_1 D^2 + d_2)\{\bar{\Psi}, \bar{\varphi}_2\} = 0, \tag{32}$$

$$(D^8 - d_3 D^6 + d_4 D^4 - d_5 D^2 + d_6)\{\bar{\Phi}, \bar{\theta}, \bar{\phi}\} = 0, \tag{33}$$

where all the constants are defined in Appendix A. The Eqs. (32) and (33), can be factored respectively as follows:

$$(D^2 - k_1^2)(D^2 + k_2^2)\{\bar{\Psi}, \bar{\varphi}_2\} = 0, \tag{34}$$

$$(D^2 - k_3^2)(D^2 - k_4^2)(D^2 - k_5^2)(D^2 - k_6^2)\{\bar{\Phi}, \bar{\theta}, \bar{\phi}\} = 0, \tag{35}$$

where, k_1^2, k_2^2 and $k_3^2, k_4^2, k_5^2, k_6^2$ are roots of characteristic equations (34) and (35), respectively, so, one has:

$$k^4 - d_1 k^2 + d_2 = 0, \tag{36}$$

$$k^8 - d_3 k^6 + d_4 k^4 - d_5 k^2 + d_6 = 0. \tag{37}$$

The bounded solutions at $z \to \infty$ for the two equations (34) and (35) take the form:

$$\{\bar{\Psi}, \bar{\varphi}_2\} = \sum_{i=1}^{2} \{1, L_{1i}\} R_i e^{-k_i z}, \tag{38}$$

$$\{\bar{\Phi}, \bar{\theta}, \bar{\phi}\} = \sum_{i=3}^{6} \{1, L_{1j}, L_{2j}\} R_i e^{-k_j z}, \tag{39}$$

where, L_{1i}, L_{1j} and L_{2j} are defined in Appendix B. Using Eqs. (38) and (39) with Eq. (26), gives the following:

$$\{\Psi, \varphi_2\} = \sum_{i=1}^{2} \{1, L_{1i}\} R_i\, e^{(-k_i z + \omega t + inx)}, \tag{40}$$

$$\{\Phi, \theta, \phi\} = \sum_{j=3}^{6} \{1, L_{1j}, L_{2j}\} R_i e^{(-k_j z + \omega t + inx)}. \tag{41}$$

Inserting Eqs. (40) and (41) into Eq. (20), gives the components u and w, that are bounded at $z \to \infty$, in the form:

$$u = [-\sum_{i=1}^{2} k_i R_i\, e^{-k_i z} + \sum_{j=3}^{6} in R_j e^{-k_j z}] e^{(\omega t + inx)}, \tag{42}$$

$$w = -[in \sum_{i=1}^{2} R_i\, e^{-k_i z} + \sum_{j=3}^{6} k_j R_j\, e^{-k_j z}] e^{(\omega t + inx)}. \tag{43}$$

The stress components, the micro-stress and the temperature distributions can be obtained using Eqs. (40) to (42) with Eqs. (17) to (19) as follows:

$$\begin{aligned} \sigma_{xx} &= in[(1 - 2\delta^2 - \delta_1^2)(1 + \alpha_0 \omega) - b_1] \sum_{i=1}^{2} k_i R_i e^{(-k_i z + \omega t + inx)} \\ &+ \sum_{j=3}^{6} [(1 - 2\delta^2 - \delta_1^2)(1 + \alpha_0 \omega) k_j^2 - b_1 n^2 + \{b_3 (k_j^2 - n^2) - (1 + \nu_0 \omega)\} L_{1j} + b_5 L_{2j}] R_j e^{(-k_j z + \omega t + inx)}, \end{aligned} \tag{44}$$

$$\begin{aligned} \sigma_{zz} &= in[b_1 - (1 - 2\delta^2 - \delta_1^2)(1 + \alpha_0 \omega)] \sum_{i=1}^{2} k_i R_i e^{(-k_i z + \omega t + inx)} \\ &+ \sum_{j=3}^{6} [b_1 k_j^2 - (1 - 2\delta^2 - \delta_1^2)(1 + \alpha_0 \omega) n^2 + \{b_3 (k_j^2 - n^2) - (1 + \nu_0 \omega)\} L_{1j} + b_5 L_{2j}] R_j e^{(-k_j z + \omega t + inx)}, \end{aligned} \tag{45}$$

$$\sigma_{xz} = \sum_{i=1}^{2} [b_6 n^2 + (b_6 - b_8) k_i^2 + b_8 L_{1i})] R_i e^{(-k_i z + \omega t + inx)} - in(2b_6 - b_8) \sum_{j=3}^{6} k_j R_j e^{(-k_j z + \omega t + inx)}, \tag{46}$$

$$m_{zy} = -a_{11} b_{11} \sum_{i=1}^{2} k_i L_{1i} R_i e^{(-k_i z + \omega t + inx)}, \tag{47}$$

$$\theta = \sum_{j=3}^{6} [1 - a(k_j^2 - n^2)] L_{1j} R_j e^{(-k_j z + \omega t + inx)}. \tag{48}$$

4. The Boundary Conditions

For determining R_i $(i=1,2)$, R_j $(j=3,4,5,6)$, the boundary conditions at $z=0$ are taken to be:

$$\sigma_{zz}=-p_1 N(x,t),\quad \sigma_{xx}=\sigma_{xz}=\frac{\partial\phi}{\partial z}=m_{zy}=0,\quad \theta=p_2 M(x,t). \tag{49}$$

where $N(x,t), M(x,t)$ are known functions, a load with velocity ν_0 is chosen to be acting on the surface $z=0$ of the medium in the normal direction, so $p_1=p_0(1+\nu_0)$, where, p_0 is the magnitude of the mechanical force. Using the boundary conditions (49), leads to the following equations:

$$\sum_{i=1}^{2} h_{1i}R_i+\sum_{j=3}^{6} h_{1j}R_j=-p_1 N_0, \tag{50}$$

$$\sum_{i=1}^{2} h_{2i}R_i+\sum_{j=3}^{6} h_{2j}R_j=0, \tag{51}$$

$$\sum_{i=1}^{2} h_{3i}R_i+\sum_{j=3}^{6} h_{3j}R_j=0, \tag{52}$$

$$\sum_{j=3}^{6} h_{4j}R_j=0, \tag{53}$$

$$\sum_{i=1}^{2} h_{4i}R_i=0, \tag{54}$$

$$\sum_{j=3}^{6} h_{5j}R_j=p_2. \tag{55}$$

where,

$$h_{1i}=in[b_1-(1-2\delta^2-\delta_1^2)(1+\alpha_0\omega)]k_i,\quad h_{2i}=in[(1-2\delta^2-\delta_1^2)(1+\alpha_0\omega)-b_1]k_i, \tag{56}$$

$$h_{3i}=b_6 n^2+(b_6-b_8)k_i^2+b_8 L_{1i},\quad h_{4i}=-a_{11}b_{11}L_{1i}k_i, \tag{57}$$

$$h_{1j}=b_1k_j^2-(1-2\delta^2-\delta_1^2)(1+\alpha_0\omega)n^2+[b_3(k_j^2-n^2)-(1+\nu_0\omega)]L_{1j}+b_5L_{2j}, \tag{58}$$

$$h_{2j}=(1-2\delta^2-\delta_1^2)(1+\alpha_0\omega)k_j^2-b_1n^2+[b_3(k_j^2-n^2)-(1+\nu_0\omega)]L_{1j}+b_5L_{2j}, \tag{59}$$

$$h_{3j}=-in(2b_6-b_8)k_j,\quad h_{4j}=-L_{2j}k_j,\quad h_{5j}=[1-a(k_j^2-n^2)]L_{1j}. \tag{60}$$

Solving Eqs. (50) to (55) for R_i $(i=1,2)$, R_j $(j=3,4,5,6)$, by using the inverse of matrix method as follows:

$$X=A^{-1}B. \tag{61}$$

where,

$$X=\begin{pmatrix}R_1\\R_2\\R_3\\R_4\\R_5\\R_6\end{pmatrix},\quad A=\begin{pmatrix}h_{11}&h_{12}&h_{13}&h_{14}&h_{15}&h_{16}\\h_{21}&h_{22}&h_{23}&h_{24}&h_{25}&h_{26}\\h_{31}&h_{32}&h_{33}&h_{34}&h_{35}&h_{36}\\0&0&h_{43}&h_{44}&h_{45}&h_{46}\\h_{41}&h_{42}&0&0&0&0\\0&0&h_{53}&h_{54}&h_{55}&h_{56}\end{pmatrix},\quad B=\begin{pmatrix}-p_1N_0\\0\\0\\0\\0\\p_2\end{pmatrix}. \tag{62}$$

5. Numerical Results and Discussions

The magnesium crystal-like thermoelastic micropolar material was chosen for the purpose of calculating some numerical results, where its physical data are given from [46] in SI units:

$$T_0=298\ K^\circ,\ \{\lambda,\mu\}=\{9.4,4\}\times10^{10}\,kg\,m^{-1}s^{-2},\ \rho=1.74\times10^3\ kg/m^3, \tag{63}$$

$$\alpha_t=7.4033\times10^{-7}K^{-1},\quad C_e=1.04\times10^3\ kg.m^{-3},\quad K=386\ W\,m^{-1}K^{-1},\quad a=0.15\times10^{-2}. \tag{64}$$

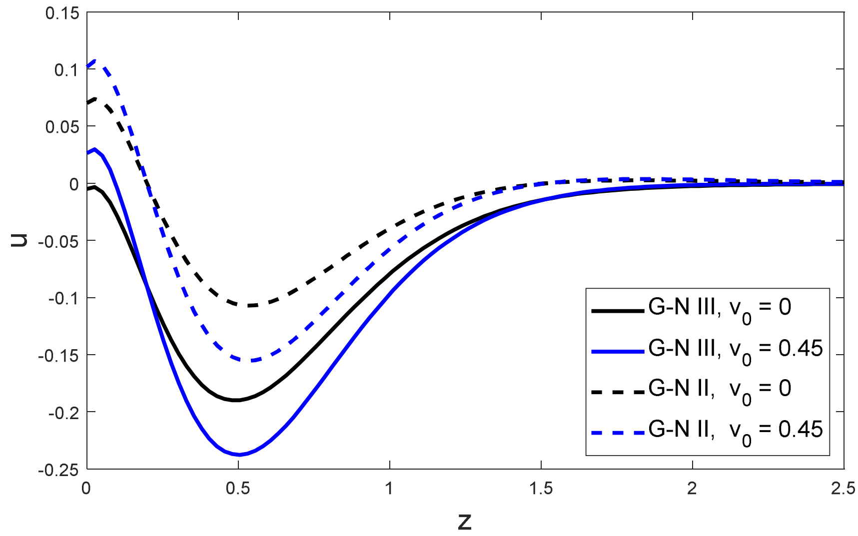

Fig. 2. Distribution of u in z direction.

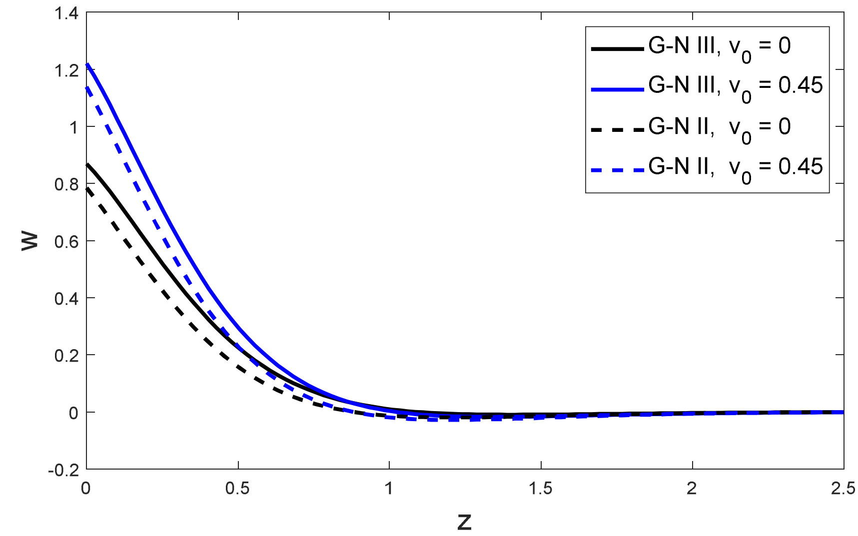

Fig. 3. Distribution of w in z direction.

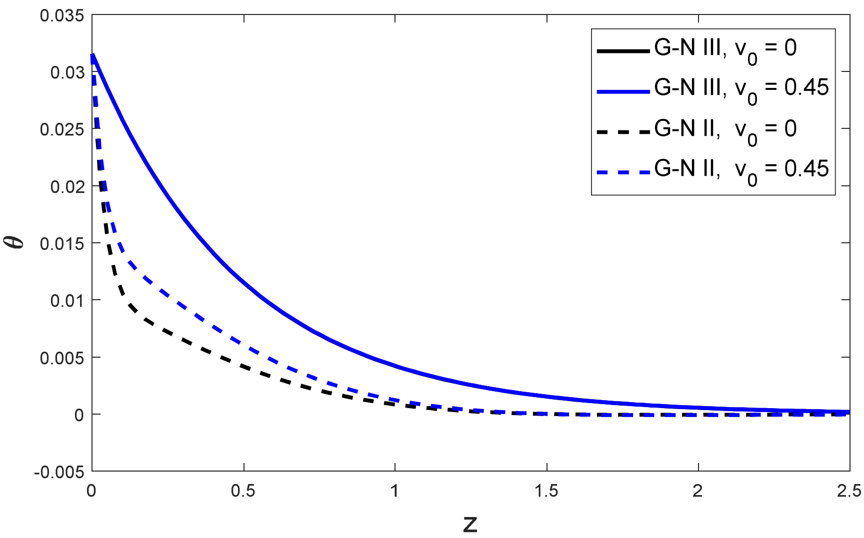

Fig. 4. Distribution of θ in z direction.

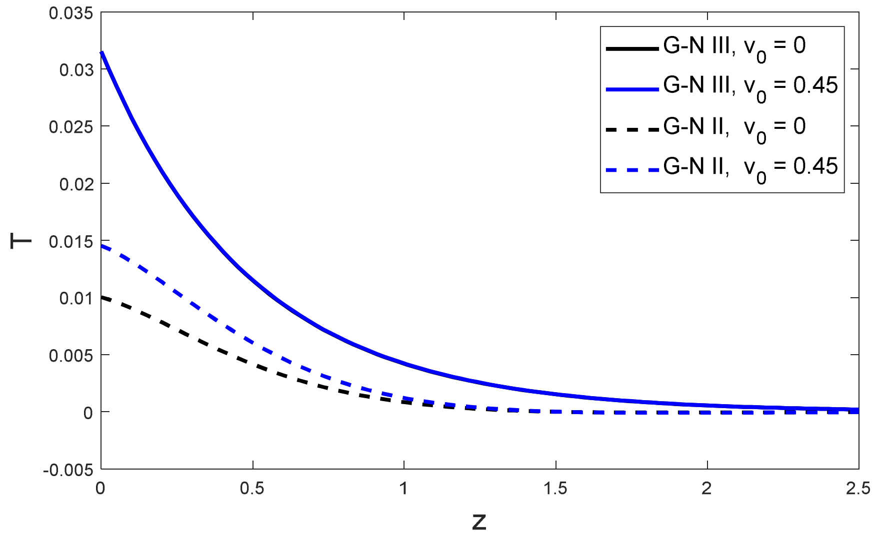

Fig. 5. Distribution of T in z direction.

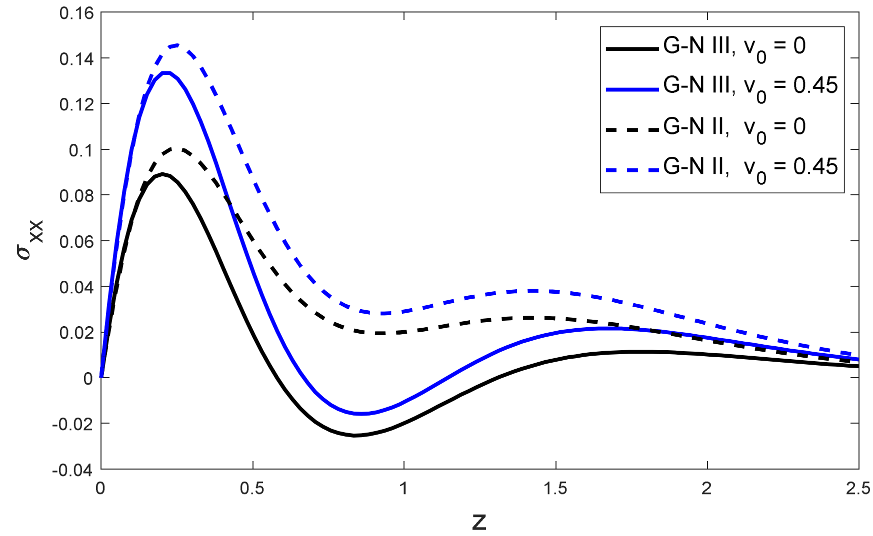

Fig. 6. Distribution of σ_{xx} in z direction.

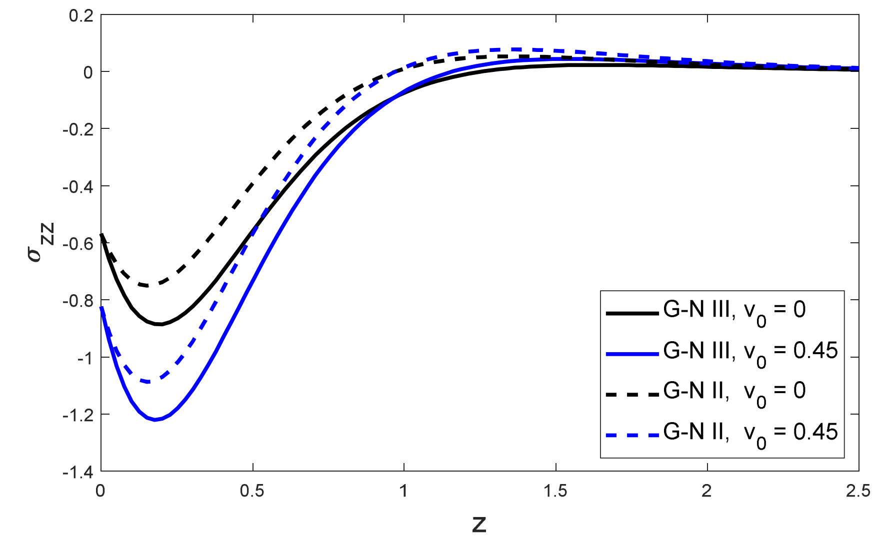

Fig. 7. Distribution of σ_{zz} in z direction.

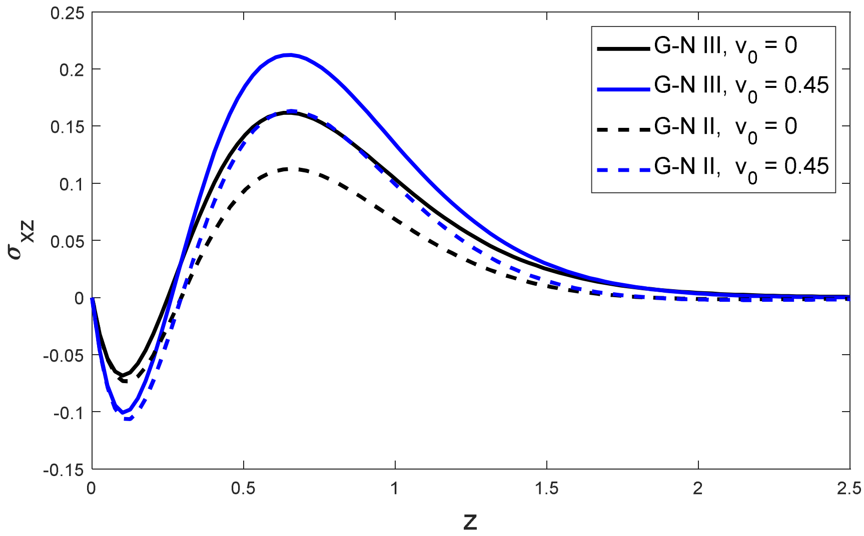

Fig. 8. Distribution of σ_{xz} in z direction.

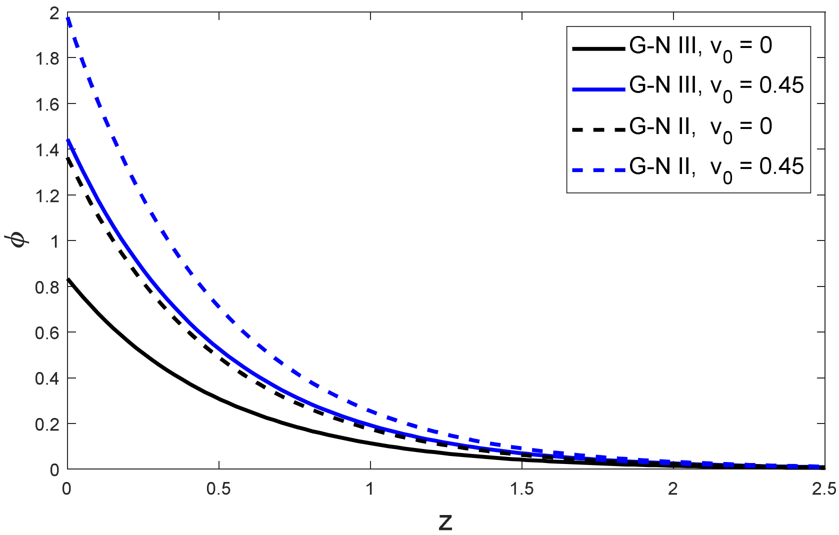

Fig. 9. Distribution of ϕ in z direction.

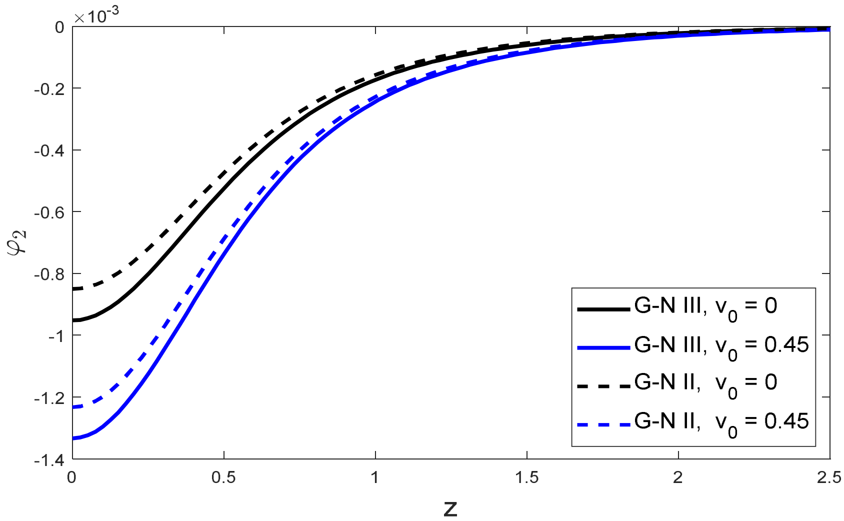

Fig. 10. Distribution of φ_2 in z direction.

The voids parameters are:

$$A = 3.668 \times 10^{-4} kg\, m s^{-2},\ b = 1.13849 \times 10^{11} kg\, m^{-1} s^{-2},\ m = 2.0\ kg s^{-2} K^{-1}, \tag{65}$$

$$\chi = 1.753 \times 10^{-15} m^2,\ \xi_1 = 1.475 \times 10^{11} kg\, m^{-1} s^{-2},\ \xi_2 = 0.0787 \times 10^{-2} kg\, m^{-1} s^{-3}, \tag{66}$$

$$\tau = 0.2 \times 10^{-8} kg\, m s^{-2} K^{-1}, \quad \varsigma = 0.1 \times 10^{-7} kg\, m s^{-2}. \tag{67}$$

The micropolar parameters are:

$$\kappa = 10^{-10} N\, m^{-2}, \quad \gamma = 7.779 \times 10^{-7} N, \quad J = 2 \times 10^{-20} m^2. \tag{68}$$

The comparisons were carried out for:

$$p_1 = 0.45,\ p_2 = 0.025,\ \nu_0 = 0.45,\ t = 0.45,\ x = 0.06,\ \omega = 2.05 + 2.05i,\ n = 2.05, \tag{69}$$

$$\alpha_0 = 0.2544 \times 10^{-4},\ \alpha_1 = 3.9053 \times 10^{-4},\ \alpha_2 = 6.5088 \times 10^{-4},\ \alpha_3 = 0.18 \times 10^4, \tag{70}$$

$$\alpha_4 = 1.9527 \times 10^{-4},\ \alpha_7 = 0.5088 \times 10^3,\ \alpha_8 = 0.2018 \times 10^{-4},\ 0 \le z \le 2.5. \tag{71}$$

The numerical calculation was used for presenting the distribution of the real parts of all the physical quantities $(u, w, \theta, T, \sigma_{xx}, \sigma_{zz}, \sigma_{xz}, \varphi_2, \phi)$ with the distance Z in the context of G-N II and G-N III with and without moving load effect. All distributions are shown graphically in Figs. 2 to 10. At $\nu_0 = 0$, the black solid lines represent the solution in the context of the G-N II and the black dashed lines represent the solution for the G-N III. In the case of $\nu_0 = 0.45$, the blue solid lines represent the solution in the context of the (G-N II) and the blue dashed lines represents the solution for the (G-N III). All physical quantities indicate that all curves converge to zero, and initial stress effects play an important role in these quantities.

6. Conclusion

The normal modal analysis was used to analyse all the physical quantities mentioned due to moving loads in a micropolar, porous thermo-viscoelastic solid material with two temperatures. According to the mathematical analysis of this problem, we can conclude that:

- The employed solving method provides exact solutions without requiring assumptions about the actual physical quantities involved in the problem under consideration.
- The values of all physical quantities mentioned converge to zero with increasing distance and are continuous.
- It was found that moving loads also play an important role in all physical quantities considered, since the magnitude of these physical quantities' changes (increases or decreases) with size.
- It was finally concluded that the deformation of the material considered depends on the type of forces, the influence of moving loads, and the type of boundary conditions.

Author Contributions

The authors contributed equally to this work in terms of planning the scheme, initiating the project, validating the system of equations for the considered problem, developing the mathematical modelling, analysing the results, writing the manuscript, and approving it for publication.

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Conflicts of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

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Data Availability Statements

The datasets generated and/or analysed during the current study are available from the corresponding author on reasonable request.

Nomenclature

$\boldsymbol{u}$	Vector of displacement	u, w	Components of displacement
x,z	Cartesian coordinates	t	Time
λ, μ	Lame's constants	σ_{ij}	Components of stress tensor
ϕ	Volume fraction field	$A^*, \xi_1, \xi_2, B, \tau, \varsigma, m, \chi$	Material constants due to presence of voids

T	Temperature of thermodynamic	θ	Conductive temperature
T_0	Reference temperature	K	Thermal conductivity
ρ	Density	C_e	Specific heat at constant strain
$\alpha_i\,(i=0..8)$	Viscoelasticity parameters	α_t	Coefficient of linear thermal expansion
δ_{ij}	Kronecker's delta	$\tilde{\omega}$	Characteristic frequency of the material
c_1, c_2	Longitudinal and shear wave velocities in the medium	n	The wave number in x-direction
ω	The frequency	ODE	Ordinary differential equation
$\partial_{,t} = \partial / \partial t,\ u_{,t} = \partial u / \partial t$		$i=\sqrt{-1}$	

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Appendix A

$$a_1=\frac{b}{\rho c_1^2},\ a_2=\frac{\kappa c_1^2}{\gamma\tilde{\omega}^2},\ a_3=\frac{J\rho c_1^2}{\gamma},\ a_4=\frac{\xi_1 c_1^2}{A\tilde{\omega}^2},\ a_5=\frac{b c_1^2}{A\tilde{\omega}^2},\ a_6=\frac{\tau\rho c_1^2}{A\nu},\ a_7=\frac{m\rho c_1^2}{A\nu\tilde{\omega}^2}, \tag{A.1}$$

$$a_8=\frac{\chi\rho c_1^2}{A},\ a_9=\frac{\nu m T_0}{\rho^2 C_e c_1^2\tilde{\omega}},\ a_{10}=\frac{\varsigma\nu\tilde{\omega}}{\rho^2 C_e c_1^4},\ a_{11}=\frac{\gamma\tilde{\omega}^2}{\rho c_1^4},\ a_{12}=\frac{\beta\tilde{\omega}^2}{\rho c_1^4},\ \xi=\frac{\xi_2\tilde{\omega}}{\xi_1}, \tag{A.2}$$

$$\varepsilon_2=\frac{K}{\rho C_e c_1^2},\ \varepsilon_3=\frac{K^*}{\rho C_e c_1^2}, \tag{A.3}$$

$$d_1=\frac{-f_1}{f_{01}},\ d_2=\frac{f_2}{f_{01}},\ d_3=\frac{-f_3}{f_{02}},\ d_4=\frac{f_4}{f_{02}},\ d_5=\frac{-f_5}{f_{02}},\ d_6=\frac{f_6}{f_{02}}, \tag{A.4}$$

$$f_{01}=b_6 b_{11},\ f_{02}=-b_1 b_{15} b_{24}, \tag{A.5}$$

$$f_1=b_8b_9-b_6b_{12}-b_7b_{11},\ f_2=b_7b_{12}-b_8b_{10},\ f_3=b_3b_{13}b_{24}-b_5b_{15}b_{20}+b_1b_{15}b_{25}-b_1b_{16}b_{24}-b_1b_{18}b_{22}+b_2b_{15}b_{24}-b_3b_{18}b_{20}, \tag{A.6}$$

$$\begin{aligned} f_4 &= b_5b_{15}b_{21}-b_3b_{13}b_{25}+b_3b_{14}b_{24}-b_4b_{13}b_{24}-b_5b_{13}b_{22}-b_5b_{16}b_{20}-b_1b_{16}b_{25}-b_3b_{17}b_{24}+b_1b_{18}b_{23} \\ &\quad +b_1b_{15}b_{25}+b_1b_{19}b_{224}-b_2b_{15}b_{25}+b_2b_{16}b_{24}+b_2b_{18}b_{22}+b_3b_{18}b_{21}+b_3b_{19}b_{20}+b_4b_{18}b_{20}, \end{aligned} \tag{A.7}$$

$$\begin{aligned} f_5 &= b_5b_{13}b_{23}+b_5b_{14}b_{22}+b_3b_{14}b_{25}+b_4b_{13}b_{25}+b_4b_{14}b_{24}+b_5b_{16}b_{21}-b_5b_{17}b_{20}+b_1b_{17}b_{25} \\ &\quad -b_1b_{19}b_{23}-b_2b_{16}b_{25}+b_2b_{17}b_{24}-b_2b_{18}b_{23}-b_2b_{19}b_{22}-b_3b_{19}b_{21}-b_4b_{18}b_{21}+b_4b_{19}b_{20}, \end{aligned} \tag{A.8}$$

$$f_6=b_5b_{17}b_{21}-b_4b_{14}b_{25}-b_5b_{14}b_{23}-b_2b_{17}b_{25}+b_2b_{19}b_{23}+b_4b_{19}b_{21}. \tag{A.9}$$

Appendix B

$$b_1=1+\omega\delta_0,\ b_2=b_1n^2+\omega^2,\ b_3=(1+\omega\nu_0)a,\ b_4=b_3n^2+(1+\omega\delta_0),\ b_5=(1+\omega\alpha_2)a_1, \tag{B.1}$$

$$b_6=\delta^2(1+\omega\alpha_1)+\delta_1^2(1+\omega\alpha_8),\ b_7=b_6n^2+\omega^2,\ b_8=\delta_1^2(1+\omega\alpha_8),\ b_9=a_2(1+\omega\alpha_8),\ b_{10}=b_9n^2, \tag{B.2}$$

$$b_{11}=(1+\omega\alpha_7),\ b_{12}=b_{11}n^2+2a_2(1+\omega\alpha_8)+a_3\omega^2,\ b_{13}=a_5(1+\omega\alpha_4),\ b_{14}=b_{13}n^2,\ b_{15}=a_6a, \tag{B.3}$$

$$b_{16}=a_7a-a_6-2b_{15}n^2,\ b_{17}=b_{15}n^4+(a_7a-a_6)n^2-a_7,\ b_{18}=(1+\omega\alpha_3),\ b_{19}=b_{18}n^2+a_2(1+\omega\xi)+a_8\omega^2,\ b_{20}=\varepsilon(1+\omega\nu_0)\omega^2, \tag{B.4}$$

$$b_{21}=b_{20}n^2,\ b_{22}=a\omega^2+\varepsilon_2+\varepsilon_3\omega,\ b_{23}=b_{22}n^2+\omega^2,\ b_{24}=a_{10}\omega,\ b_{25}=b_{24}n^2+a_9\omega, \tag{B.5}$$

$$L_{1i}=\frac{b_6k_i^2-b_7}{b_8},\quad L_{1j}=\frac{b_5(b_{20}k_j^2-b_{21})+(b_1k_j^2-b_2)(b_{24}k_j^2-b_{25})}{b_5(b_{22}k_j^2-b_{23})-(b_3k_j^2-b_4)(b_{24}k_j^2-b_{25})}, \tag{B.6}$$

$$L_{2j}=\frac{(b_{20}k_j^2-b_{21})(b_3k_j^2-b_4)+(b_1k_j^2-b_2)(b_{22}k_j^2-b_{23})}{(b_{24}k_j^2-b_{25})(b_3k_j^2-b_4)-b_5(b_{22}k_j^2-b_{23})}. \tag{B.7}$$

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