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Research Paper

A Study of Thermoelastic Double Porosity Rod under a Thermal Shock

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Abstract. This research article deals with the one-dimensional study of a thermoelastic double porosity rod of length l. A thermal shock is applied at the end $x = l$ of the rod. The equations of motion are solved analytically to obtain the expressions of displacement, stress, temperature distribution, and pressure in macro and micro pores. The analytical results are then complemented with a numerical example to show the variation of these quantities with horizontal distance. The effect of thermal-pores inertia on the quantities is also shown through these graphical representations.

Keywords: Double porosity, Thermal shock, Macro and micro pores, Thermal pores inertia.

1. Introduction

Expansion and contraction of a medium is a natural process whenever the body is heated or cooled. The expansion and contraction take place in the pores present in the medium. This verifies that every material that exists in nature is porous. It was more than fifty years ago that the initial problems of elastic solids with double porosity were discussed. The particles try to migrate back to their original form and play a vital role in contraction and expansion. The study of multiple pores and the temperature of the medium plays an important role in many industrial problems, which include petroleum engineering, chemical engineering, earthquake engineering, etc. Lord and Shulman [1] were the first to introduce thermal relaxation time to the theory of coupled thermoelasticity [2]. Later, many generalizations [3-11] were proposed by researchers in the field of thermoelasticity. A lot of work has been done in the field of thermoelasticity. Some prominent works in the early twentieth century have been listed [12-25].

The thermal-pores inertia has a significant effect on displacement, stress, temperature distribution, and pressure in the rod. The presence of pores can cause uneven thermal expansion and temperature gradient may be steeper in porous regions. These pores may also be responsible for disrupting stress distribution which can cause increased localized stress. The pressure distribution in the rod is affected due to the sudden heating of the rod. The initial work on double porosity was studied by Barenblatt and Zheltov [26] and Wilson and Aifantis [27]. Double porosity in thermoelastic medium with applications in seismic attenuation was studied by Armstrong [28] and Pride et al. [29]. Kana et al. [30] and Zhang et al. [31] explored double porosity in geothermal exploration. Practically, there arise many situations where deformation may be induced by changing thermal effects. This is true in the case of elastic bodies with double porosity where cracks appear due to thermal stresses [32-38]. Some recent work on thermoelastic double porous media include Chirila and Marin [39]; Hobiny and Abbas [40]; Chirita and Arusoaie [41]; Mahato and Biswas [42]; Kalkal et al. [43]; Sheokand et al. [44]; Kumar et al. [45]. The application of isotropic and homogeneous material has created a lot of interest for the researchers [46-49] in their study.

This paper aims to study one-dimensional deformation of a thermoelastic double porosity rod of length l due to a sudden shock applied at one end of the rod. The expressions of displacement, mechanical stress, temperature field and pressure in macro and micro pores are obtained by analytical method. The values of these expressions are then obtained numerically by generating code in MATLAB software.

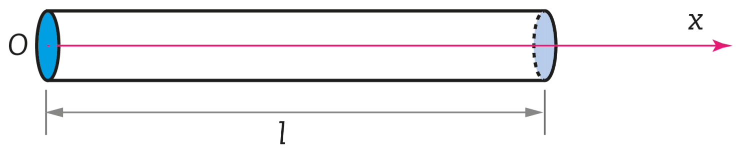

Fig. 1. Geometry of the Problem.

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These values are then plotted against horizontal distance to show the effect of time-lapse on the quantities. The problem discussed in the paper finds applications in manufacturing processes: glass and ceramic industry, Aerospace: Rods and other components in spacecraft and satellites, Energy Sector: components like turbine blades in power plants.

2. Problem Definition

The thermoelastic double porosity rod of length l occupying the region $x \geq 0$ is considered (Fig. 1). A rectangular Cartesian coordinate system (x,y,z) with x-axis pointing horizontally is taken. The displacement vector in a thermoelastic double porosity medium may be taken as $\vec{u}=(u_1,0,0)$ where $u_1=u(x,z,t)$. The field equations and constitutive relations in absence of body forces are given by [50-54]:

$$\mu u_{r,mm}+(\lambda+\mu)u_{m,mr}-\xi p_{,r}-\chi q_{,r}-\varsigma\theta_{,r}=\rho\ddot{u_r}, \tag{1}$$

$$\alpha_1\dot{p}+\alpha_3\dot{q}+\gamma_1\dot{\theta}=k_1p_{,rr}-\gamma(p-q)-\xi\dot{u_{r,r}}, \tag{2}$$

$$\alpha_3\dot{p}+\alpha_2\dot{q}+\gamma_2\dot{\theta}=k_2q_{,rr}+\gamma(p-q)-\chi\dot{u_{r,r}}, \tag{3}$$

$$\gamma_1\dot{p}+\gamma_2\dot{q}+a\dot{\theta}=k_3\theta_{,rr}-\varsigma\dot{u_{r,r}}, \tag{4}$$

$$t_{rs}=2\mu e_{rs}+\lambda e_{mm}\delta_{rs}-(\xi p+\chi q+\varsigma\theta)\delta_{rs}, \tag{5}$$

The parameters k_3, a and ς are given by [55, 56]:

$$k_3=\frac{k}{\theta_0},\qquad a=\frac{\rho C_e}{\theta_0},\quad \varsigma=-3K\beta,$$

The field equations and constitutive relations for a thermoelastic double porosity rod in one dimension reduce to:

$$(\lambda+2\mu)\frac{\partial^2u}{\partial x^2}-\xi\frac{\partial p}{\partial x}-\chi\frac{\partial q}{\partial x}-\varsigma\frac{\partial\theta}{\partial x}=\rho\frac{\partial^2u}{\partial t^2}, \tag{6}$$

$$\alpha_1\frac{\partial p}{\partial t}+\alpha_3\frac{\partial q}{\partial t}+\gamma_1\frac{\partial\theta}{\partial t}=k_1\frac{\partial^2p}{\partial x^2}-\gamma(p-q)-\xi\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right), \tag{7}$$

$$\alpha_3\frac{\partial p}{\partial t}+\alpha_2\frac{\partial q}{\partial t}+\gamma_2\frac{\partial\theta}{\partial t}=k_2\frac{\partial^2q}{\partial x^2}+\gamma(p-q)-\chi\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right), \tag{8}$$

$$\gamma_1\frac{\partial p}{\partial t}+\gamma_2\frac{\partial q}{\partial t}+a\frac{\partial\theta}{\partial t}=k_3\frac{\partial^2\theta}{\partial x^2}-\varsigma\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right). \tag{9}$$

3. Problem Solution

For the sake of numerical computations, the equations are made dimensionless using the below mentioned variables:

$$x'=\frac{\omega^*}{c_1}x,\quad u'=\frac{\rho\omega^*c_1}{\varsigma\theta_0},\quad t'=\omega^*t,\quad \theta'=\frac{\theta}{\theta_0},\quad t'_{ij}=\frac{t_{ij}}{\varsigma\theta_0},\quad \{p',q'\}=\frac{\{p,q\}}{\varsigma\theta_0}, \tag{10}$$

where,

$$\omega^*=\frac{\rho C_e c_1^2}{k_3},\quad c_1^2=\frac{\lambda+2\mu}{\rho}.$$

Using Eq. (10) in equations (6)-(9), the equations may be recast in dimensionless form as:

$$\frac{\partial^2u}{\partial x^2}-\xi\frac{\partial p}{\partial x}-\chi\frac{\partial q}{\partial x}-\frac{\partial\theta}{\partial x}=\frac{\partial^2u}{\partial t^2}, \tag{11}$$

$$\alpha_1\frac{\partial p}{\partial t}+\alpha_3\frac{\partial q}{\partial t}+\gamma_1\frac{\partial\theta}{\partial t}=\frac{k_1\omega^*}{c_1^2}\frac{\partial^2p}{\partial x^2}-\frac{\gamma}{\omega^*}(p-q)-\frac{\xi}{\rho c_1^2}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right), \tag{12}$$

$$\alpha_3\frac{\partial p}{\partial t}+\alpha_2\frac{\partial q}{\partial t}+\gamma_2\frac{\partial\theta}{\partial t}=\frac{k_2\omega^*}{c_1^2}\frac{\partial^2q}{\partial x^2}+\frac{\gamma}{\omega^*}(p-q)-\frac{\chi}{\rho c_1^2}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right), \tag{13}$$

$$\gamma_1\frac{\partial p}{\partial t}+\gamma_2\frac{\partial q}{\partial t}+\frac{a}{\varsigma}\frac{\partial\theta}{\partial t}=\frac{k_2\omega^*}{\varsigma c_1^2}\frac{\partial^2\theta}{\partial x^2}-\frac{\varsigma}{\rho c_1^2}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right). \tag{14}$$

Assuming the solution of the variables in the form of normal modes as:

$$[u, p, q, \theta, t_{ij}] = [\bar{u}, \bar{p}, \bar{q}, \bar{\theta}, \overline{t_{ij}}]e^{\omega t}, \tag{15}$$

where ω is complex frequency.

The advantage of normal mode analysis is that it gives exact solutions in the elastic medium without any assumed restrictions on the actual physical quantities appearing in the governing equations. Normal mode analysis complements traditional approaches with computational efficiency and applicability to large systems that are beyond the reach of older methods.

Using Eq. (15) in Eqs. (11) to (14), we get:

$$\left(\frac{d^2}{dx^2} - \omega^2\right)\bar{u} - \xi\frac{d\bar{p}}{dx} - \chi\frac{d\bar{q}}{dx} - \frac{d\bar{\theta}}{dx} = 0, \tag{16}$$

$$\eta_1\frac{d\bar{u}}{dx} + \left(\eta_2\frac{d^2}{dx^2} - \eta_3\right)\bar{p} + \eta_4\bar{q} - \eta_5\bar{\theta} = 0, \tag{17}$$

$$\eta_6\frac{d\bar{u}}{dx} + \eta_4\bar{p} + (\eta_8\frac{d^2}{dx^2} - \eta_9)\bar{q} - \eta_{10}\bar{\theta} = 0, \tag{18}$$

$$\eta_1\frac{d\bar{u}}{dx} - \eta_5\bar{p} - \eta_{10}\bar{q} + (\eta_{11}\frac{d^2}{dx^2} - \eta_{12})\bar{\theta} = 0, \tag{19}$$

where,

$$\eta_1 = -\frac{\varsigma\omega}{\rho c_1^2}, \quad \eta_2 = \frac{k_1\omega^*}{c_1^2}, \quad \eta_3 = \frac{\gamma}{\omega^*} + \alpha_1\omega, \quad \eta_4 = \frac{\gamma}{\omega^*} - \alpha_3\omega,$$

$$\eta_5 = \gamma_1\omega, \quad \eta_6 = -\frac{\chi}{\rho c_1^2}, \quad \eta_8 = \frac{k_2\omega^*}{c_1^2}, \quad \eta_9 = \frac{\gamma}{\omega^*} + \alpha_2\omega, \tag{20}$$

$$\eta_{10} = \gamma_2\omega, \quad \eta_{11} = \tfrac{k_3\omega^*}{\varsigma c_1^2}, \quad \eta_{12} = -\tfrac{a}{\varsigma}.$$

On solving equations (16)-(19), we get an eighth order differential equation given by:

$$[\Delta^8 + A_1\Delta^6 + A_2\Delta^4 + A_3\Delta^2 + A_4](\bar{u}, \bar{p}, \bar{q}, \bar{\theta}) = 0, \tag{21}$$

where,

$$A_1 = \frac{g_1g_{10} + g_2g_9 - g_4g_6}{g_1g_9}, \quad A_2 = \frac{g_1g_{11} + g_2g_{10} + g_3g_9 - g_5g_6 - g_4g_7}{g_1g_9}, \tag{22a}$$

$$A_3 = \frac{g_2g_{11} + g_3g_{10} - g_5g_7 - g_4g_8}{g_1g_9}, \quad A_4 = \frac{g_3g_{11} - g_5g_8}{g_1g_9}, \tag{22b}$$

$$g_1 = -f_4f_6, \quad g_2 = f_1f_9 + f_5f_6 + f_4f_7, \quad g_3 = -f_5f_7, \quad g_4 = f_2 + \eta_2, \tag{22c}$$

$$\mu g_5 = f_8 - f_3, \quad g_6 = f_6f_{14} - f_9\eta_{11}, \quad g_7 = -(f_6f_{15} + f_7f_{14} + f_9f_{10}), \tag{22d}$$

$$g_8 = f_7f_{15} - f_9f_{11}, \quad g_9 = -\eta_2f_{14}, \quad g_{10} = f_9f_{12} + \eta_2f_{15} - f_8f_{14}, \tag{22e}$$

$$g_{11} = f_8f_{15} - f_9f_{13}, \quad f_1 = \eta_1\eta_{10} - \eta_5\eta_6, \quad f_2 = \eta_2\eta_{10}, \tag{22f}$$

$$f_3 = \eta_3\eta_{10} + \eta_4\eta_5, \quad f_4 = \eta_4\eta_8, \quad f_5 = \eta_5\eta_9 + \eta_4\eta_{10}, \quad f_6 = \eta_5 - \eta_1, \tag{22g}$$

$$f_7 = \omega^2\eta_5, \quad f_8 = \xi\eta_5 - \eta_3, \quad f_9 = \chi\eta_5 + \eta_4, \tag{22h}$$

$$f_{10} = \eta_1 - \eta_{12} - \omega^2\eta_{11}, \quad f_{11} = \omega^2\eta_{12}, \quad f_{12} = \xi\eta_{11}, \tag{22i}$$

$$f_{13} = \xi\eta_{12} - \eta_5, \quad f_{14} = \chi\eta_{11}, \quad f_{15} = \chi\eta_{12} - \eta_{10}, \quad \Delta \equiv \frac{d}{dx}. \tag{22j}$$

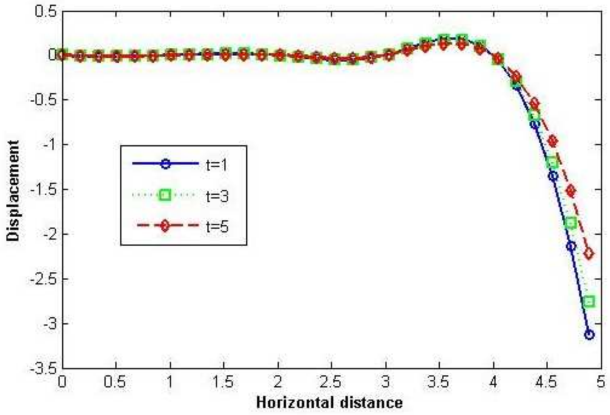

Fig. 2. Variation of displacement.

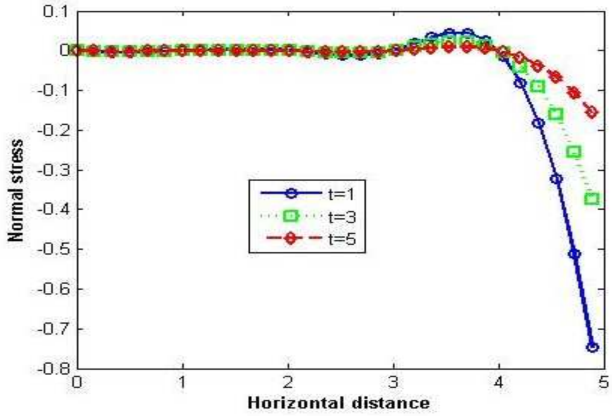

Fig. 3. Variation of stress.

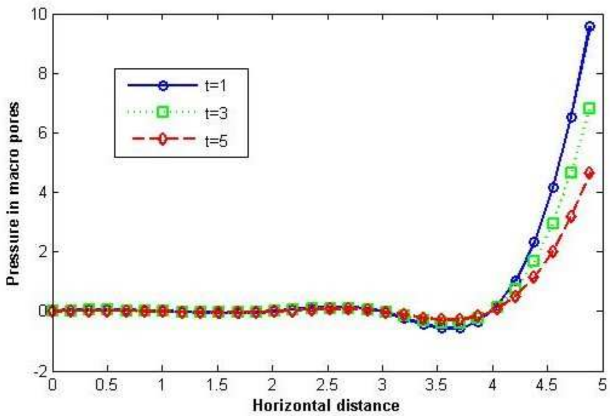

Fig. 4. Variation of pressure in macro pores.

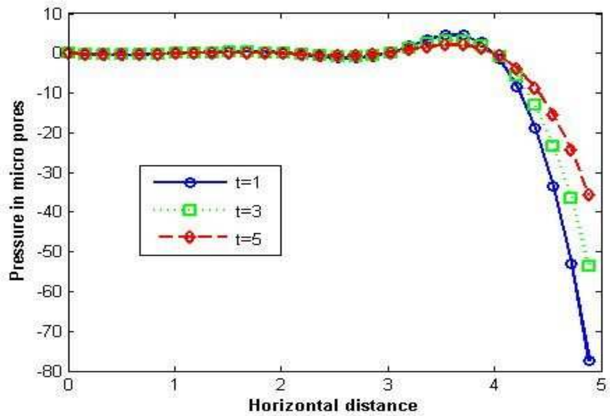

Fig. 5. Variation of pressure in micro pores.

The solution of equation (21) is given by:

$$\bar{u} = D_1\cosh(m_1x) + D_2\sinh(m_1x) + D_3\cosh(m_2x) + D_4\sinh(m_2x) + D_5\cosh(m_3x) + D_6\sinh(m_3x) + D_7\cosh(m_4x) + D_8\sinh(m_4x), \tag{23}$$

$$\bar{p} = D_1^{(1)}\cosh(m_1x) + D_2^{(1)}\sinh(m_1x) + D_3^{(1)}\cosh(m_2x) + D_4^{(1)}\sinh(m_2x) + D_5^{(1)}\cosh(m_3x) + D_6^{(1)}\sinh(m_3x) + D_7^{(1)}\cosh(m_4x) + D_8^{(1)}\sinh(m_4x), \tag{24}$$

$$\bar{q} = D_1^{(2)}\cosh(m_1x) + D_2^{(2)}\sinh(m_1x) + D_3^{(2)}\cosh(m_2x) + D_4^{(2)}\sinh(m_2x) + D_5^{(2)}\cosh(m_3x) + D_6^{(2)}\sinh(m_3x) + D_7^{(2)}\cosh(m_4x) + D_8^{(2)}\sinh(m_4x), \tag{25}$$

$$\bar{\theta} = D_1^{(3)}\cosh(m_1x) + D_2^{(3)}\sinh(m_1x) + D_3^{(3)}\cosh(m_2x) + D_4^{(3)}\sinh(m_2x) + D_5^{(3)}\cosh(m_3x) + D_6^{(3)}\sinh(m_3x) + D_7^{(3)}\cosh(m_4x) + D_8^{(3)}\sinh(m_4x), \tag{26}$$

where the constants $D_m^{(n)}(m = 1,2,\ldots 8; n = 1,2,3)$ are given by:

$$\left[D_j^{(s)}, D_{j+1}^{(s)}\right] = [L_{si}D_j, -L_{si}D_{j+1}], \quad (s = 1,2,3; i = 1,2,3,4; j = 1,3,5,7)$$

$$L_{1i} = \frac{g_1m_i^4 + g_2m_i^2 + g_3}{m_i(g_4m_i^2 + g_5)}, \qquad L_{2i} = -\frac{[f_1m_i + L_{1i}(f_2m_i^2 - f_3)]}{f_4m_i^2 - f_5}, \tag{27}$$

$$L_{3i} = \frac{L_{1i}(\eta_2m_i^2 - \eta_3) + L_{2i}\eta_4 - \eta_1m_i}{\eta_5}.$$

4. Boundary Conditions

We raise one end of the rod by a known function to a prescribed temperature, while the other end is fixed and maintained at the initial temperature such that the initial conditions are assumed to be:

$$(i)\ u(x,0) = 0, \quad 0 \le x \le l; \qquad (ii)\ \frac{\partial u(x,0)}{\partial t} = 0, \quad 0 \le x \le l; \quad (iii)\ t_{xx} = 0, 0 \le x \le l\ .$$

The rod has a uniform temperature θ_0. We assume that the rod is subjected to heating of a general function and is traction free, so that the boundary conditions take the following forms:

The displacement condition along the end points of the rod: $(i)\ u(x,t) = 0 \ \ at\ x = 0, l$.

The pressure in the macro and micro pores vanishes at the end points of the rod: $(ii)\ \ p(x,t) = 0 \ \ at\ x = 0, l$. $(iii)\ q(x,t) = 0 \ \ at\ x = 0, l$.

The boundary of the rod is isothermal: $(iv)\ T(0,t) = 0$.

We consider the finite rod $0 \le x \le l$ at a uniform temperature θ_0 with its boundary $x = 0$ subjected to thermal shock:

$$(iv)\ T(l,t) = P_0 \exp(\omega t). \tag{28}$$

where P_0 is a constant representing the strength of the shock on the boundary.

Using Eqs. (15), (23)-(26) in the boundary conditions (28), we obtain a non-homogeneous system of eight equations. These equations may be solved numerically with the help of MATLAB software and the values of unknown parameters $D_v (v = 1,2 \ldots .8)$ are obtained. These values are then used to evaluate the values of displacement, stress, temperature distribution, and pressure in macro and micro pores.

5. Numerical Results

The material constants of a double porosity thermoelastic solid are considered for Berea sandstone. The values are given by [50, 56, 57]:

$$\begin{gathered}
\mu = 4.8 \times 10^9 \frac{N}{m^2}, \quad \lambda = 2.6 \times 10^9 \frac{N}{m^2}, \quad \rho = 2115 \frac{Kg}{m^3}, \quad k_2 = 6.18 \times 10^{-12}\ m^2/(\sec Pa), \\
k_1 = 2.47 \times 10^{-11}\ m^2/(\sec Pa), \quad \alpha_1 = 5.4 \times 10^{-6}\ Pa^{-1}, \quad \alpha_2 = 1.35 \times 10^{-6}\ Pa^{-1}, \\
\xi = 0.11875, \quad \varsigma = -3K\beta, \quad \chi = 0.95, \quad a = \frac{\rho C_e}{\theta_0}, \quad k_3 = \frac{k}{\theta_0}, \quad k = 2.34\ W/(m℃) \\
\rho C_e = 1.76 \times 10^6\ J/(m^3(℃)), \quad \beta = 1.5 \times 10^{-6}\ 1/℃, \quad \theta_0 = 273^0 K, \quad \gamma = 0.5\ Pa^{-1} sec^{-1}.
\end{gathered} \tag{29}$$

The computations are carried out at three different times i.e $t = 1.0, 3.0, 5.0$ and for a wide range of $x = 0.0$ up to 5.0. The magnitude of thermal shock is assumed to be $P_0 = 1.0$ and the length of rod is taken as $l = 1.0$, with $\omega = \omega_0 + i\omega_1$, where $\omega_0 = -0.2$ and $\omega_1 = 0.1$.

6. Discussions

The values of displacement and normal stress are less in magnitude in the range $0 < x \le 4.0$. However, these values decrease sharply in the range $4.0 < x \le 5.0$. The magnitude of values of normal stress is less as compared to the values of displacement.

The variations of displacement and mechanical stress, being similar in nature are shown in Figs. 2 and 3, respectively. The variations of pressure in macro and micro pores are opposite in nature in the range $4.0 < x \le 5.0$ but these values, which are again less in magnitude in the range $0 < x \le 4.0$ observe a similar behaviour. At a particular instant, the pressure in macro pores is more in comparison to the pressure in micro pores. The variations of pressure in macro and micro pores are shown in Figs. 4 and 5, respectively. The variation of temperature field is similar in nature to the variation of pressure in macro pores with difference in magnitude of the values. The variation of temperature field may be observed in Fig. 6. It is also interesting to see from the figures that effect of time lapse is significant in the range $4.0 < x \le 5.0$. Also, the variations of displacement, normal stress, pressure in the pores and temperature along the boundary $0 \le x \le 1.0,\ 0 \le y \le 1.0$ in three dimensions are shown in Figs. 7 to 11, respectively.

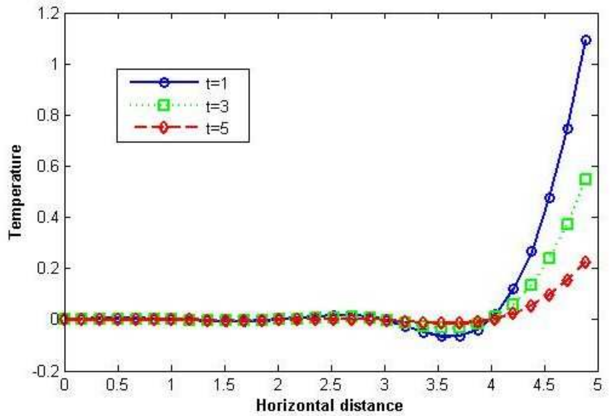

Fig. 6. Variation of temperature field.

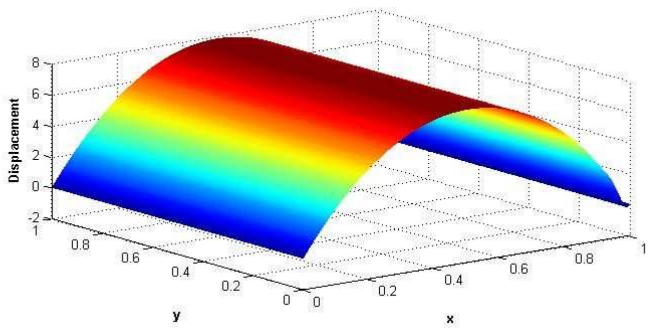

Fig. 7. Variation of displacement in 3D.

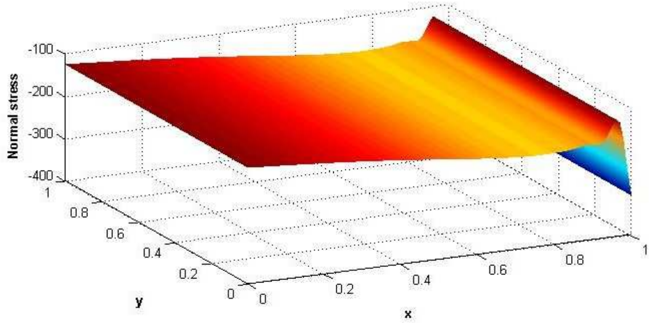

Fig. 8. Variation of stress in 3D.

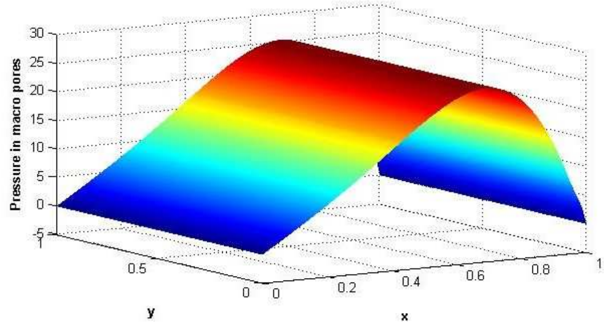

Fig. 9. Variation of pressure in macro pores in 3D.

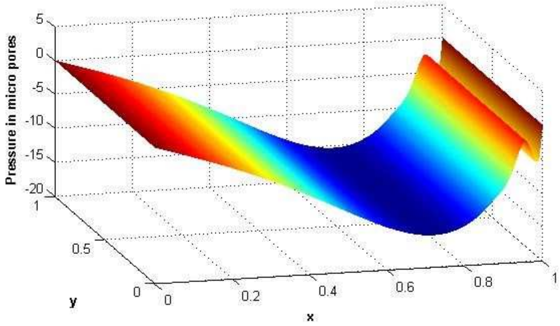

Fig. 10. Variation of pressure in micro pores in 3D.

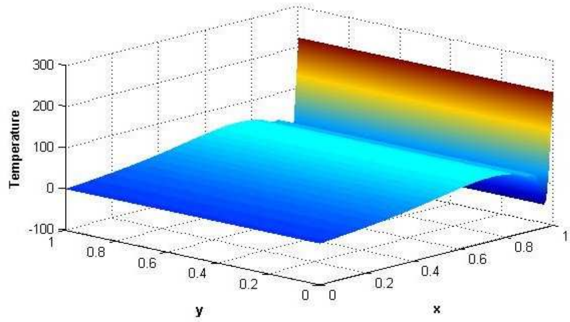

Fig. 11. Variation of temperature field in 3D.

7. Conclusion

The problem discussed in the paper and the results obtained concluded that:

1. There are four waves propagating in the medium.
2. The values of the physical quantities are almost similar in the initial range evaluated at different times. The effect of time is significant in the range $4.0 < x \leq 5.0$.
3. The variation of pressure in the pores (macro pores and micro pores) is opposite.
4. The behaviour of displacement and normal stress are similar.

Author Contributions

P. Ailawalia devised the problem and its proposed solution and finalized the graphical results. M. Marin developed the codes in MATLAB software for numerical results. J. Kaur performed the mathematical calculations and draft of the paper. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

ρ	Mass density
λ	Modulus of elasticity
μ	Shear modulus
$\gamma > 0$	Leakage parameter
$\alpha_1, \alpha_2, \alpha_3$	Coefficients describing the cross-coupling compressibility for fluid flow
γ_1, γ_2	Thermal-pores inertia defining the compressibility of the pores
k_1, k_2	Parameters describing the macroscopic intrinsic permeability associated with double pore system
δ_{rs}	Kronecker delta
k	Thermal conductivity coefficient
C_e	Specific heat at constant volume
β	Thermal expansion coefficient
$K = \lambda + 2\mu/3$	Bulk modulus
θ_0	Reference temperature

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