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Letter to the Editor

Comment on "A Radial Basis Function Collocation Method for Space-dependent Inverse Heat Problems, M. Nawaz Khan, I. Ahmad, H. Ahmad, J. Appl. Comput. Mech., 6, 2020, 1187-1199"

Saifudden Saif[1], Hossein Hosseinzadeh[2], Zeinab Sedaghatjoo[3]

[1] Department of Mathematics, Takhar University, Taleqan Takhar, Afghanistan, Email: saifuddinsaif@takhar.gov.af, saifuddinsaif97@gmail.com
[2] Department of Mathematics, Persian Gulf University, Bushehr, Iran, Email: h_hosseinzadeh@aut.ac.ir, hosseinzadeh@pgu.ac.ir
[3] Department of Mathematics, Persian Gulf University, Bushehr, Iran, Email: z.sedaqatjoo@aut.ac.ir, zeinab.sedaghatjoo@gmail.com

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Corresponding author: H. Hosseinzadeh (h_hosseinzadeh@aut.ac.ir)

Abstract. The purpose of this note is to address an error in the paper titled "A Radial Basis Function Collocation Method for Space-dependent Inverse Heat Problems," published in the Journal of Applied and Computational Mechanics, 6, 2020, 1187-1199, which affects the numerical results presented therein. Consequently, we propose a numerical scheme to solve the inverse heat problem using the Radial Basis Functions Collocation Method. Then the paper is modified by this note.

Keywords: Meshless method, Radial basis function, Inverse heat source problem, Non-uniform nodes.

1. Introduction

In [1], Inverse Heat Conduction Problems (IHCP) were studied numerically. The paper involves determining unknown space-dependent heat sources $f(x)$ presented in equation:

$$w_t(x,t) = \Delta w(x,t) + f(x), \quad x \in \Omega, t > 0. \tag{1}$$

The equation was equipped with initial and boundary conditions:

$$w(x,0) = w_0(x), \quad x \in \Omega, \tag{2}$$

$$w(x,t) = g(t), \quad x \in \partial\Omega, \tag{3}$$

where $\partial\Omega$ is the boundary of Ω. An extra final condition:

$$w(x,T) = w_T(x), \quad x \in \Omega, \tag{4}$$

was also imposed in [1] at final time T to find unknown functions w and f at $(x,\ t) \in \Omega \times [0,\ T]$. The authors differentiated from both sides of the equation, Eq. (1), respect to t, and then function f was removed from the equation. Consequently, the initial equation was replaced by:

$$u_t(x,\ t) = \Delta u(x,\ t), \ \ x \in \Omega, \ \ t > 0, \tag{5}$$

where $u(x,\ t) = w_t(x,\ t)$. The authors of [1] assumed Eq. (11) with false initial condition:

$$u(x,0) = 0. \tag{6}$$

The false condition is presented in Eq. (10) of [1]. One can see that Eq. (2) does not essentially lead to Eq. (6). For example, the function:

$$w(x,t) = \sin(2\pi x) + x^2 + 2tx, \quad f(x) = 4\pi^2 \sin(2\pi x) + 2x - 2, \tag{7}$$

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presented in [1], satisfies:

$$u(x,0) = w_t(x,\ t)|_{t=0} = 2x, \tag{8}$$

which is in contrast with Eq. (6). It is important to note that the erroneous condition (6), referenced in [1], arose because the authors assumed:

$$u(x,0) = \partial w(x,0)/\partial t, \tag{9}$$

instead of using Eq. (8). Since the numerical method presented in [1] is based on this false assumption, the numerical results are not trustable. An alternative RBFCM is presented in Section 2 to solve the heat source problem and find w and f at $(x,\ t) \in \Omega \times [0,\ T]$. Finally, the proposed method is applied to some numerical examples in Sections 3 and 4 to verify its efficiency.

2. A Correction

By differentiating both sides of Eq. (1) with respect to t, we obtain the partial differential equation (PDE):

$$w_{tt}(x,t) = \Delta w_t(x,t), \quad x \in \Omega,\ 0 < t < T, \tag{10}$$

which it can be simplified to the following PDE:

$$w_{tt}(\bar{x}) = \Delta w_t(\bar{x}), \quad \bar{x} = (x,\ t) \in \Omega \times (0,T). \tag{11}$$

Radial basis functions:

$$\varphi_i(\bar{x}) = \varphi(\|\bar{x} - \bar{x}_i\|), \tag{12}$$

can be applied to approximate function w as:

$$w(\bar{x}) = \sum_{j=1}^{N} \alpha_j \varphi_j(\bar{x}). \tag{13}$$

Here, α_j is a constant number which must be determined by solving a system of linear equations [2]. This system is derived by PDE (11) along with initial, boundary and extra conditions (2), (3) and (4) as follows:

$$A\alpha = b, \tag{14}$$

where $\alpha = [\alpha_1,\ \alpha_2,\ \ldots,\ \alpha_N]^T$. Matrix A and vector b are evaluated as:

$$A(i,j) = \varphi_{jtt}|_{\bar{x}=\bar{x}_i} - \Delta\varphi_{jt}|_{\bar{x}=\bar{x}_i}, \quad b(i) = 0, \tag{15}$$

When $\bar{x}$ is in $\bar{\Omega} = \Omega \times (0,T)$ and:

$$A(i,j) = \varphi_j(\bar{x}_i), \quad b(i) = w(\bar{x}_i), \tag{16}$$

when $\bar{x}$ is on $\partial\bar{\Omega} = (\partial\Omega \times [0,\ T]) \cup (\Omega \times \{0,\ T\})$.

After solving system (14) and finding α, function w can be approximated using the RBFs, as presented in Eq. (13). Consequently, heat source f can be obtained from Eq. (1) as follows:

$$f(\bar{x}) = w_t(\bar{x}) - \Delta w(\bar{x}). \tag{17}$$

Note that, the function f obtained through this equation is a function of $\bar{x} = (x,t)$, and one can fix t (*i.g.* $t = T/2$) to express it as a function of x.

3. A One-dimensional Numerical Example

This section is dedicated to a numerical experiment using Gaussian and Inverse Multiquadric (IMQ) Radial Basis Functions with the following formulations:

$$\varphi_i(\bar{x}) = \varphi_i(x,\ t) = e^{-\varepsilon^2\left((x-x_i)^2+(t-t_i)^2\right)}, \tag{18}$$

$$\varphi_i(\bar{x}) = \sqrt{\varepsilon^2 + (x-x_i)^2 + (t-t_i)^2}. \tag{19}$$

Functions w and f are obtained numerically using Eqs. (13) and (17) for the exact solutions presented in Eq. (7). In this example $x \in [0,1]$ and $t \in [0,1]$, then $\bar{\Omega} = [0,1] \times [0,1]$ is the computational domain. Three types of collocation points are considered within $\bar{\Omega}$: regular points, Gauss-Lobatto points and random points. These points are illustrated in Fig. 1.

The root mean square (RMS) error for w is calculated as follows:

$$RMS = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(w_{ex}(x_i) - w_{nu}(x_i)\right)^2}, \tag{20}$$

where w_{ex} and w_{nu} represent the exact and numerical values of w, respectively. The same formula applies to function f as well. In this example, the number of collocation points, N, equals 21 × 21. The RMS error of f for the RBFs is reported in Table 1 when the optimal values of the shape parameter ε, are used. Analytical and numerical values of functions w and f at the uniform points are shown in Fig. 2. From Table 1, it is evident that the RBFs can accurately approximate the exact solution. The experimental results indicate that the accuracy of the RBFCM improves with the addition of more computational points.

Table 1. RMS error for function *f*.

RBF	Mesh	Optimal ε	RMS
Gaussian	Uniform points	1	3×10^{-2}
Gaussian	Gaus-Lobottow points	2	3×10^{-2}
Gaussian	Random points	1.5	4×10^{-2}
IMQ	Uniform points	0.7	7×10^{-2}
IMQ	Gaus-Lobottow points	0.7	5×10^{-2}
IMQ	Random points	0.5	1×10^{-1}

4. A Two-dimensional Numerical Example

Three dimensional Gaussian Radial Basis Functions with formulation:

$$\varphi_i(\bar{x}) = \varphi_i(x,y,t) = e^{-\varepsilon^2\left((x-x_i)^2+(y-y_i)^2+(t-t_i)^2\right)}, \tag{21}$$

are applied here to obtain unknown functions w and f by Eqs. (13) and (17) for exact solutions:

$$w(x,y,t) = (\cos(2x)+\cos(2y))(1-e^{-4t}), \tag{22}$$

$$f(x,y) = 4(\cos(2x)+\cos(2y)). \tag{23}$$

In this example, $x\in[0,1]$, $y\in[0,1]$ and $t\in[0,1]$. Then $\bar{\Omega}=[0,1]^2\times[0,1]$ is the computational domain. Error functions E_u and E_f can be calculated as follows:

$$E_u(x_i,\ y_i,\ t_i) = |w_{ex}(x_i,\ y_i,\ t_i) - w_{nu}(x_i,\ y_i,\ t_i)|, \tag{24}$$

$$E_f(x_i,\ y_i) = |f_{ex}(x_i,\ y_i) - f_{nu}(x_i,\ y_i)|, \tag{25}$$

for $N = 11 \times 11 \times 11$ points selected in the computational domain, regularly. The error functions are shown in Fig. 3 at some points. From Fig. 3, the functions are vanished at internal points but they increase significantly at some boundary points. This fact has happened because the approximation power of the RBFs decreases at boundary points.

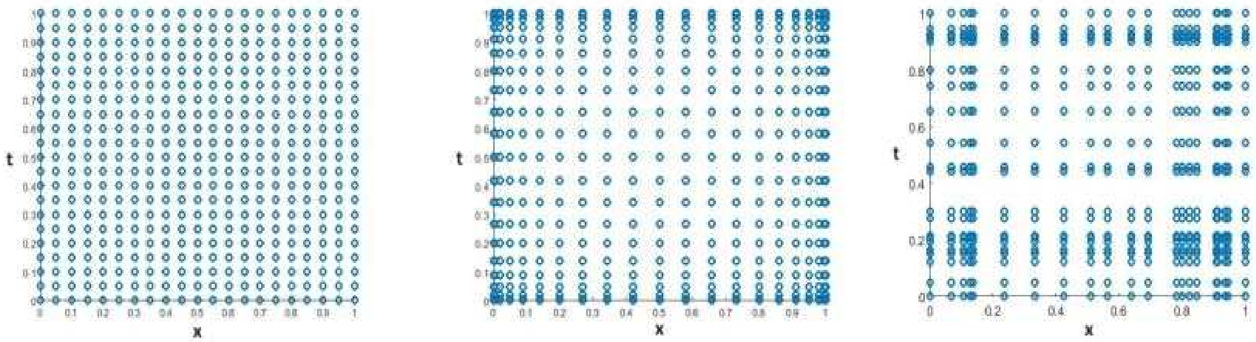

Fig. 1. From left to right: 21×21 uniform points, Gauss-Lobotto points and random points selected in $\bar{\Omega}=[0,1]\times[0,1]$ as collocation points.

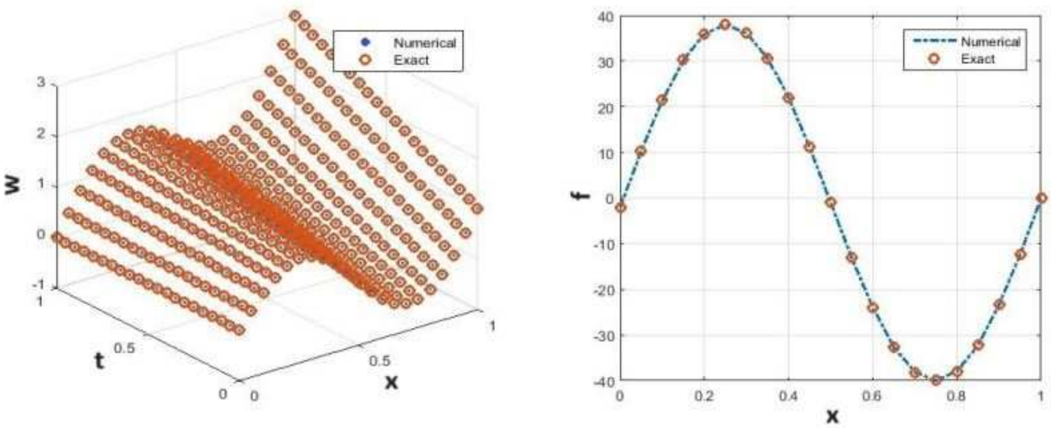

Fig. 2. Numerical and exact solutions of *w* and *f* at the uniform collocation points.

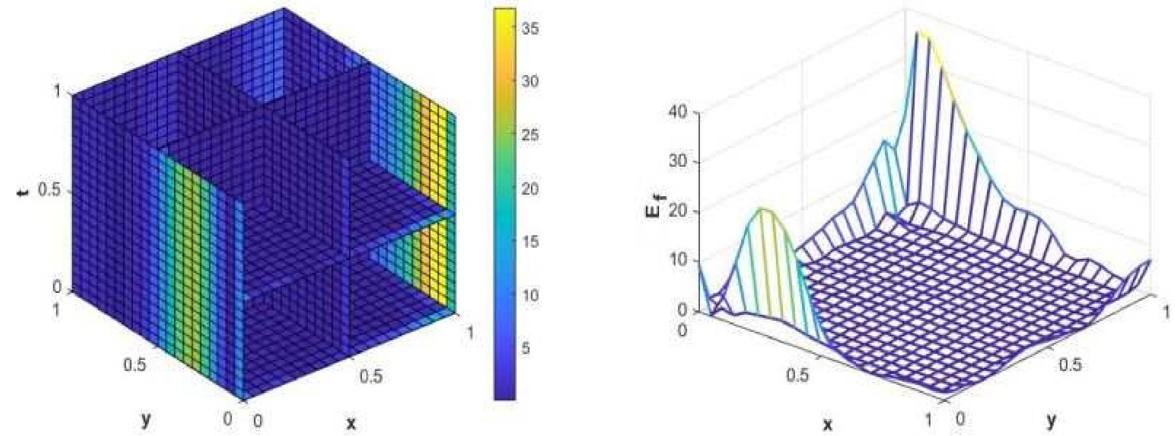

Fig. 3. Error functions E_u (left) and E_f (right) at some collocation points. The functions are vanished at internal points but they increase significantly at some boundary points.

Author Contributions

S. Saif designed the scheme, initiated the project, and drafted the initial format of the manuscript. H. Hosseinzadeh revised the manuscript, conducted the experiments, and analyzed the numerical results. Z. Sedaghatjoo developed and analyzed the numerical results for two-dimensional problems. The manuscript was collaboratively written with contributions from all authors. All authors discussed the results, reviewed the manuscript, and approved the final version.

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Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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ORCID iD

Saifudden Saif https://orcid.org/0009-0002-0292-7427
Hossein Hosseinzadeh https://orcid.org/0000-0002-0480-5717
Zeinab Sedaghatjoo https://orcid.org/0000-0001-8175-061X

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