J. Appl. Comput. Mech., 11(4) (2025) 861-870
DOI: 10.22055/jacm.2024.46641.4562

ISSN: 2383-4536
jacm.scu.ac.ir

Shahid Chamran University of Ahvaz

Journal of Applied and Computational Mechanics

Research Paper

Heat Transfer Analysis of 2D Steady Laminar Mixed Convection CNTs Blood base Nanofluid Flow over a Stretching Surface: Analytical Simulation

Ali Rehman[1], Ma Chau Khun[1], Dolat Khan[2], Rashid Jan[3]

[1] Forensic Engineering Center Institute for Smart Infrastructure and Innovative Construction, Faculty of Civil Engineering, Universiti Teknologi Malaysia, Malaysia, Email: machaukhun@utm.my (M.C.K.)

[2] Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand, Email: dolat.ddk@gmail.com

[3] Institute of Energy Infrastructure (IEI), Department of Civil Engineering, College of Engineering, Universiti Tenaga Nasional (UNITEN), Putrajaya Campus, Jalan IKRAM-UNITEN, 43000 Kajang, Selangor, Malaysia

Received April 17 2024; Revised October 23 2024; Accepted for publication November 23 2024.
Corresponding authors: M.C. Khun (machaukhun@utm.my), D. Khan (dolat.ddk@gmail.com)

Abstract. With the significant advancements in nanofluids, the fields of fluid dynamics and thermal management have experienced further development. Due to their enhanced thermal properties and capacity for customization to specific applications, nanofluids present a viable option for improving the efficacy and efficiency of various technological and industrial processes. The objective of this research article is to investigate the heat transfer properties of a blood-based steady laminar mixed convection flow of nanofluid over perpendicular stretched sheets. The non-Newtonian nanofluid's unique thermal properties, attributable to carbon nanotubes (CNTs), render it a suitable medium for enhancing heat transfer in biomedical applications. The flow model equations, one for momentum and another for energy, are transformed into a set of nonlinear ordinary differential equations through the application of appropriate similarity transformations. The governing nonlinear equations are solved semi-numerically using the Homotopy Analysis Method. This method is effective for obtaining series solutions to highly nonlinear differential equations. The Homotopy Analysis Method, developed by Shijun Liao in the 1990s, has applications in numerous disciplines of science and engineering. Several significant factors, including Local Grashof number, couple stress parameter, nanoparticle volume fraction, thermal radiation parameter, temperature exponent, and Prandtl number, are investigated, and their impact on the temperature and velocity profiles is examined. The study also considers the Nusselt number and skin friction coefficient. The results demonstrate that the addition of CNTs significantly increases heat transfer efficiency, thereby enhancing cooling in biological domains. This study contributes to our understanding of heat transfer enhancement in blood-based nanofluids with carbon nanotubes and provides important new insights for the development and improvement of thermal control systems in biomedical engineering. Furthermore, the findings of this study may inform the development of novel strategies to improve the efficacy of heat exchangers and cooling devices in medical applications.

Keywords: Mathematica software, Stretching surface, Multi Wall Carbon Nanotube, Single Wall Carbon Nanotube, Homotopy Analysis Method.

1. Introduction

A base fluid, which can be either a liquid or a gas, is combined with solid particles, known as nanoparticles, which have an average size of less than 100 nanometers, to create a class of fluids known as nanofluids. These nanoparticles might be metallic (copper, silver, and gold), oxide-based (titania, alumina), or non-metallic (graphene oxide, carbon nanotubes). The base fluid can be ethylene glycol, water, oil, or another type of coolant. Nanofluids are colloidal suspensions of finely dispersed nanoparticles in a base fluid that increase the fluid's viscosity, specific heat capacity, and thermal conductivity. The high surface area to volume ratio of nanoparticles is primarily responsible for the increase in these characteristics. Enhancing the heat transfer characteristics of traditional heat transfer fluids is one of the main uses of nanofluids. The resulting nanofluid can have its thermal conductivity further enhanced by spreading nanoparticles throughout the base fluid. This feature is used by many industries, such as thermal solar systems, electronics cooling, automotive, and aerospace. Improved thermal conductivity makes nanofluids useful as coolants in a range of cooling systems. CNTs is the abbreviation for carbon nanotubes. These are cylindrical nanostructures, similar to graphite, made entirely of carbon atoms arranged in a hexagonal pattern. Carbon nanotubes offer exceptional mechanical, electrical, and thermal properties due to their unique structure. There are two types of them: multi-walled (MWCNTs) and single-walled (SWCNTs), which are composed of multiple layers of nested concentric tubes. SWCNTs are built of a single layer of carbon atoms wrapped into a seamless tube. Carbon nanotubes have remarkable electrical properties and are small and light, making them useful in a variety of electronic parts and gadgets like interconnects, sensors, displays, and transistors. Carbon nanotubes are being investigated by researchers as a potential means of improving the efficiency of energy storage and conversion in super capacitors, batteries, and fuel cells. Their huge surface area and electrical conductivity make them advantageous for electrode materials. Carbon nanotubes are being researched for a variety of biomedical applications, including imaging, bio sensing, tissue engineering, and drug administration, due to their unique properties, low toxicity, and biocompatibility. Ellahi et al. [1] studied a non-Newtonian nanofluid's magneto hydrodynamic flow through a pipe. They note that the temperature of the fluid is lower than the temperature of the pipe. They investigated the two models of temperature-dependent viscosity. Alshomrani et al. [2] studied the investigation of spray dispersal taking into account a thin coating of nanofluid past a cylinder's stretched and slick surface when visible to heat radiation. The spray rate distribution is recognized based on the thickness of the nan layer. With the inclusion of the viscous dissipation element, the applied temperature used during the spray phenomena has been taken as a reference temperature. Asadi et al. [3] examine the MgO–MWCNT/thermal oil hybrid nanofluid's heat transfer performance and pumping power. They investigate the samples' thermal conductivity using a KD2 Pro thermal analyzer. Based on the growing growth of the nanofluid's thermal conductivity with increasing temperature and mass concentration, they find that the maximum development in thermal conductivity was approximately 65%. It has been demonstrated that there is a very strong association between temperature and solid concentration when it comes to calculating the thermal conductivity of the nanofluid. Gul et al. [4] investigated the spraying a finite layer of Al_2O_3-H_2O and Al_2O_3-$C_2H_6O_2$ nanoliquid onto an elongating cylinder. They find that the thin film nanofluid stream is subjected to a constant reference temperature and a uniform magnetic field. Haq et al. [5] studied the flow of carbon nanotubes, both single- and multiple-wall, through the base fluids (water, engine oil, and ethylene glycol) over a stretched surface. The effects of viscous dissipation and magnetohydrodynamics were examined, as well as the thermal conductivity and viscosity of equal volume. Khan et al. [6] used the homogeneous flow model to study the flow and heat transfer of carbon nanotubes on a flat surface under the circumstances of homogenous heat flux and Navier slip. They combined single- and multi-wall CNTs with kerosene, or engine oil, as the basis fluid. Aman et al. [7] studied the heat transfer of SWCNTs and MWCNTs for mixed convection Poiseuille flow of a Casson fluid down a vertical channel is investigated analytically in this paper. Three distinct base fluid types water, kerosene, and engine oil are used to suspend these CNTs. For the effective thermal conductivity of CNTs. In order to increase the thermal conductivity of the nanofluid, a transverse magnetic field of uniform strength is added to the flow. Aman et al. [8] examined the boost up of heat transmission in free convection flow of carbon nanotube containing Maxwell nanofluids across a perpendicularly motionless surface with a constant wall temperature. They used four different base fluid kerosene oil, engine oil, water, and ethylene glycol which are suspended with two different forms of carbon nanotubes, single walls carbon nanotubes and multiple walls carbon nanotubes. Asadi et al. [9] examined the dynamic viscosity of the t MWCNT/MgO (20–80)–SAE50 hybrid nano-lubricant. This investigation was conducted at temperatures between 25°C to 50°C and solid contents between 0.25% to 2%. These results showed that at all investigated temperatures and solid concentrations, the nano-lubricant behaves in a Newtonian manner. Moreover, the experimental findings established that as temperature enhancing, dynamic viscosity reduced. Asadi et al. [10] investigated the hybrid nanofluid of MWCNT/ZnO engine oils with the dynamic viscosity. This investigation was conducted with solid concentrations ranging from 0.125% and 1% and at numerous temperatures between 5°C to 55°C. They used Brookfield cone and plate viscometer, the viscosity of the MWCNT/ZnO nanoparticles distributed in engine oil with a mean diameter of 30 nm was determined. Asadi et al. [11] examined the hybrid nanofluid Al_2O_3-MWCNT/thermal oil's heat transfer efficiency at numerous temperatures 25°C to 50°C and solid concentrations 0.125%–1.5%. They used Zeta potential analysis examine the stability of the nano-oil. Next, they study the nanofluid's dynamic viscosity and thermal conductivity. Asadi et al. [12] evaluated the $Mg(OH)_2$/MWCNT hybrid nano-lubricant's capacity for heat transfer. The impact of temperature and solid volume fraction on the dynamic viscosity and thermal conductivity of the $Mg(OH)_2$/MWCNT hybrid nano-lubricant engine oil were investigated experimentally for the first time. They used the range of temperatures (25, 30, 35, 40, 45, 50, 55, and 60 degrees Celsius) and solid concentrations (0.25%, 0.5%, 0.75%, 1%, 1.5%, and 2%) for their investigation. Asadi et al. [13] study the innovative impression of MHD 3D rotating nanofluid flow over a stretched surface through modelling and simulations. They used water as the base fluid and single wall carbon nanotubes as nano-sized materials. They discussed the phenomenon of heat exchange in relation to thermal radiation and the influence of nanoparticles.

Nasir et al. [14] investigated the metallic tactile covering of nanoparticles forming a shiny thin film on the surface of a rotating disc. They used gold and silver particles with base fluid, ethanol, methanol, ethylene-glycol, and water. They using a focus on comparative multi-nanofluid analysis, a sound determination of which fluid was most appropriate for the metallic sparkling process of spin coating. Ellahi et al. [15] studied a Casson fluid's erratic MHD free flow over an oscillating perpendicular plate surface with a fixed wall temperature. The fluid is moving through a permeable material and conducting electricity. Asma et al. [16] examined how the heat and mass transfer with slip affect the Couette and generalized Couette flow in a third-grade non-Newtonian viscous fluid that is homogenous and thermodynamically compatible. They find the exact solutions for temperature and velocity for Couette flow problem. Ellahi et al. [17] examined that CNT nanofluids have higher heat transfer ratios than other nanofluids, which is important in the engineering and industrial sectors. They investigate the time dependent stagnation point flow of a CNT nanofluid across a stretching surface with analytically approach. In order to examine the different physical constraints, single and multi-walled carbon nanotubes were employed. Rehman et al. [18] studied how Marangoni convection affects heat transport and magnetohydrodynamic viscous dissipation on hybrid nanofluids in a rotating system between two surfaces. Rehman et al. [19] studied the consequence of the magnetic field on the graphene oxide-water and graphene oxide-ethylene glycol nanofluids' stagnation point flow. Using the similarity transformation, they convert the partial differential equation to a nonlinear ordinary differential equation. Rehman et al. [20] examined the unsteady squeezing flow of water-based carbon nanotubes (CNTs) in the presence of a magnetic field and variable thermal conductivity, they used an analytical solution for both MWCNT and SWCNT. They used the similarity transformation, to convert the partial differential equation into a nonlinear ordinary differential equation. Rehman et al. [21] studied the nanoliquid film's time-dependent flow over a flexible surface. Water was employed as the basis fluid, and they also used ethylene glycol and graphene oxide. By using a magnetic field and viscous dissipation, they compared the two kinds of nanoliquids and their uses for enhancing heat transmission. Rehman et al. [22] studied the idea of heat transfer, which is found in contemporary technology. They determine the analytical solution to the hybrid nanofluid's time-dependent MHD flow across a spinning sphere. Rehman, et al. [23] studied the effects of the couple stress Casson nanofluid-flow on a stretching surface. Using the mentioned similarity transformation, they transform the basic governing PDE into two linked, non-linear ODEs. The semi numerical solution for the flow problem that was simulated was found using the approximate analytical approach. In a fluid flow system, mixed convection flow is the simultaneous occurrence of forced convection and spontaneous (or free) convection. This kind of flow occurs when external forces, such as pressure or fans, combine with buoyant forces, which are caused by temperature variations. Natural Convection: Occurs when buoyancy forces and temperature fluctuations in the fluid induce variations in density. Warmer liquids rise, whereas colder fluids descend. Forced Convection: When a fan, pump, or pressure gradient is present, it induces fluid motion. Sadighi et al. [31] studied the MHD micropolar fluid flow and mixed convection features, as well as the entropy production analysis of an inclined permeable stretched surface. They take into account the effects of buoyant forces, Lorentz, heat source/sink, and thermal radiation. By using appropriate similarity variables, they convert the PDE system into a system of highly non-linearity coupled ODEs, which are then solved analytically. Jamrus et al. [32] investigated the dynamics of fluid flow and heat transfer in the stagnation zone of a mixed convection scenario involving a thermally stratified ternary hybrid nanofluid are numerically. The investigation takes into account velocity slip and the effects of a magneto hydrodynamic in addition to a permeable sheet that has the ability to expand and contract. Shah et al. [33] studied the magneto hydrodynamics, 2D incompressible steady-state stagnation point flow, and heat transport of a viscous fluid with mixed convection past a linearly diminishing permeable sheet. The controlling non-linear partial differential equations are transformed into ordinary differential equations by using the pertinent similarity variables. They used MATLAB's bvp4c to numerically solve the resulting ODEs. Zainodin et al. [34] studied the impact on stagnation point over an exponentially stretching/shrinking surface of magneto hydro dynamics, velocity slip, heat source/sink, mixed convection flow related to Fe_3O_4-$CoFe_2O_4$/water hybrid ferrofluids. The recommended system of differential equations is reduced with the use of MATLAB to solve the similarity transformation method. Two solutions are gained for several values of the governing parameters; the stability analysis shows that the first solution is stable. Sadighi et al. [35] investigated the influence of the permeable stretched cylinder, magnetic field, curvature, penetrable media, radiation, Joule heating, viscous dissipation, and heat generation/absorption. After solving high non-linear coupled PDEs, high non-linear coupled ODEs are obtained. The energy equation is solved using the confluent hypergeometric function and the PHF (prescribed heat flux) boundary condition. Sadighi et al. [36] studied the impact of a constant, laminar, two-dimensional (2D) nanofluid flow under the impact of an angled magnetic field, curvature, permeable medium, thermal radiation, electrical resistance heating, viscous dissipation, heat source/sink, chemical reaction, and concentration power-law exponent. Two-dimensional (2D) fluid flow is an idealized notion in fluid dynamics where the third dimension is either uniform or ignored and the flow parameters (velocity, pressure, etc.) fluctuate in just two spatial dimensions. In practical situations, there exist several physical explanations or rationales for assuming a two-dimensional fluid flow. The fluctuation of flow parameters in the thin (third) dimension is frequently smaller than the changes in the plane of flow when a fluid flow takes place in a thin layer, such as boundary layers close to surfaces. Some examples are as follows. Flow over boundary layers, or level surfaces. Flow (also known as Poiseuille flow or plane Couette flow) between closely spaced parallel plates. Thin-film flows, in which the thickness of the film is minimal in relation to the other dimensions. Mbogba et al. [38] investigated constant incompressible laminar flow through a semi-porous circular pipe as a function of an external constant pressure gradient. Gazambeti et al. [39] investigated continuous laminar flow of an incompressible fluid under pressure via a porous tube with parallel impermeable walls. Kalibe et al. [40] investigated the effects of laminar flows in semi-porous channels with a rectangular cross section under porous-wall acceleration. Makon et al. [41] investigated regarding the presence and identification of the incompressible laminar flow situated in the polar plane of a porous annular conduit. Highly nonlinear PDEs are transformed into highly nonlinear ODEs via similarity transformations. Motivated by the previously discussed study literature, numerous researchers have investigated a wide range of nanofluid types to uncover their intriguing thermal characteristics.

Nanofluid applications have been the subject of extensive research utilizing a wide range of mathematical models and physical applications; however, to the authors' knowledge, no studies have employed modern semi-numerical approaches on blood-based CNT nanofluids using this model. To address this research gap, the objective of this work is to examine the semi-numerical analysis of 2D steady, laminar mixed convection, CNT blood-based nanofluid along a perpendicular, stretching sheet with heat transfer analysis. To create two distinct nanofluids, MWCNT and SWCNT nanoparticles were suspended in blood. The governing equations in the form of PDEs are reduced to a nonlinear coupled system of ODEs by applying suitable similarity transformations. Through semi-numerical investigation of the flow problem, the behavior of skin friction and the Nusselt number with respect to various physical parameters, as well as the variation of the temperature and velocity fields, have been explored and are presented as tables and graphical data. A comparison between the numerical solutions and HAM is presented. It is anticipated that the current research work will stimulate the necessary experimental investigations.

2. Problem Formulation

In situations where the effect of the third dimension is negligible or where symmetry or thin geometries greatly simplify the problem, 2D flow is frequently justified. This approximation preserves the key flow properties while lowering computational and analytical complexity. Consider a 2D, time independent incompressible laminar boundary layer flow of CNTs blood base nanofluid, including, thermal radiation, using a couple stress model over a stretching surface at a constant temperature T_∞. Temperature and velocity on the continuous stretching surface are expected to follow a power-law curve variations, are $u_w = Bx^m$ and $T_w = T_\infty + Ax^n$, A and B are constants where m and n are the exponents. Assuming that the stretching sheet is warmer than the surrounding fluid, A is a positive constant. With the slot serving as the origin, the positive x coordinate is measured along the direction of motion, while the positive y coordinate is measured normal to the sheet and outward towards the fluid. The governing equations for the problem under discussion can be expressed as follows using the Boussinesq approximation and boundary layer assumptions.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_{nf}\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \gamma_0\frac{\partial^4 u}{\partial y^4}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y}, \tag{3}$$

where are velocity components in x and y-directions. According to Rosseland's approximation for radiation radiative heat flux $q_r = -(4\sigma_s / 3k_1)(\partial T^4 / \partial y)$. T^4 can be expanded with Taylor's series, the temperature variation may be about T_∞ as $T^4 = 4T_\infty^3 T - 3T_\infty^4$, with the uses of this expansion the radiative heat flux becomes $q_r = -(16\sigma_s T_\infty^3 / 3k_1)(\partial T / \partial y)$. Thus, Eq. (3) gets the form:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma_s T_\infty^3}{3k_1\rho c_p}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

The following are appropriate boundary conditions for the consider flow problem:

$$\begin{aligned} &u=Bx^m,\quad v=0,\quad T=T_\infty+Ax^n \quad \text{at}\ \ y=0,\\ &u\to 0,\quad T\to T_\infty \quad \text{as}\ \ y\to\infty, \end{aligned} \tag{5}$$

Thermophysical properties of nanofluid, according to [24], are:

$$\begin{aligned} &\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}},\\ &(\rho C_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_{MWCNT},\\ &\frac{k_{nf}}{k_f}=\frac{[(k_{MWCNT}+2k_f)-2\phi(k_f-k_{MWCNT})]}{[(k_{MWCNT}+2k_f)+\phi(k_f-k_{MWCNT})]},\\ &\rho_{nf}=(1-\phi)\rho_f+\phi\rho_{MWCNT},\\ &\frac{\sigma_{nf}}{\sigma_f}=\frac{[(\sigma_{MWCNT}+2\sigma_f)-2\phi(\sigma_f-\sigma_{MWCNT})]}{[(\sigma_{MWCNT}+2\sigma_f)+\phi(\sigma_f-\sigma_{MWCNT})]}, \end{aligned} \tag{6}$$

Next, we apply the similarity transformation by assuming that the velocity and temperature filed is independent of the streamwise coordinate, meaning that all parameters are dimensionless. The fluid is blown or suctioned uniformly over the whole stretching surface. The dimensionless variables are used as a similarity transformation to transform the established partial differential equations (2) to (4) into nonlinear ordinary differential equations together with boundary conditions (5) as:

$$\psi=\sqrt{\nu x u_w}\,f(\eta),\quad T=T_\infty+(T_w-T_\infty)\theta(\eta), \tag{7}$$

where ψ is known as stream function, T is temperature, η is similarity parameter defined as $\eta=y\sqrt{u_w/\nu x}$. Using definition of stream function, we have. Using Eqs. (5) and (6), Eqs. (2) and (4) are reduced as the following ordinary differential equations:

$$\frac{1}{(1-\phi)^{2.5}\left[\left\{\left(1-\phi+\phi\frac{\rho_{SWCNT}}{\rho_f}\right)\right\}+\phi\frac{\rho_{MWCNT}}{\rho_f}\right]}f'''+\frac{\rho_{nf}\mu_{bf}}{\mu_{nf}\rho_{bf}}\left(\frac{m+1}{2}\right)ff''-mf'^2+G^*\theta-\gamma f^{v}=0, \tag{8}$$

$$(3R+4)\frac{1}{(1-\phi)^{2.5}\left[\left\{\left(1-\phi+\phi\frac{(\rho C_p)_{SWCNT}}{(\rho C_p)_f}\right)\right\}+\phi\frac{(\rho C_p)_{MWCNT}}{(\rho C_p)_f}\right]}\theta''+\frac{n+1}{2}\frac{(\rho C_p)_{nf}}{(\rho C_p)_{bf}}\frac{k_{bf}}{k_{nf}}f\theta'-(2n-1)f'\theta=0, \tag{9}$$

and the boundary conditions (5) get the form:

$$\begin{aligned} &f'(0)=1,\quad f(0)=0,\theta(0)=1,\\ &f'(\infty)=0,\quad \theta(\infty)=0. \end{aligned} \tag{10}$$

$G^*=Gr_x/\mathrm{Re}_x^2$, shows the buoyancy parameter. $Gr_x=g\beta x^3(T_w-T_\infty)/\nu^2$, shows the local Grashof number, $P_r=\mu c_p/k$, shows the Prandtl number, $R=kk_1/4\sigma_s T_\infty^3$, shows the thermal radiation parameter, $\gamma=\gamma_0 a/\nu^2{}_{bf}\rho_{bf}$, is couple stress parameter, n is the temperature exponent parameter, and m is the velocity exponent magnetic parameter.

For the given problem, the physical quantities of the flow and wall heat transfer are also examined as:

$$C_f=\frac{2\tau_w}{\rho(u_w)^2},\qquad \text{where}\quad \tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{11}$$

$$Nu=\frac{xq_w}{k(T_w-T_\infty)},\qquad \text{where}\quad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{12}$$

Equations (11) and (12), in the dimensionless form, under similarity (6) are:

$$C_f\mathrm{Re}_x^{\frac{1}{2}}=2f''(0), \tag{13}$$

$$Nu\,\mathrm{Re}_x^{-\frac{1}{2}}=-\theta'(0), \tag{14}$$

where $\mathrm{Re}_x=u_w/\nu$, represents the local Reynold's number, which is the primary controlling parameter of viscous fluid flow.

3. Solution by HAM

The series solution is one of the helpful procedures for solving the nonlinear differential equations. Non-linear difficulties are frequently found in the scientific and engineering areas. A very new and useful method for solving coupled and non-linear differential equations is known as HAM. BVPh 1.0 and BVPh 2.0 are the latest HAM programs that boost up the convergence of the proposed problems. The BVPh 2.0 package is difficult to apply up to the 100th iteration, while these packages are helpful for fast convergence. We employ an approximate analytical process known as the homotopy analysis method to handle the nonlinear ordinary differential equation system represented by equations (8–9) and with the provided transform boundary condition (10). The significant factor in assuring the convergence of the approximate analytical solution is the introduction of an auxiliary parameter. The following is how the initial solution is applied:

$$F_0(\eta) = 1 - e^{-\eta},\ \theta_0(\eta) = e^{-\eta}\ , \tag{15}$$

The given problem's linear operators are defined as L_F and L_θ :

$$L_F(F) = F''',\ L_\theta(\theta) = \theta'', \tag{16}$$

with the following characteristics:

$$L_F(D_1 + D_2\eta + D_3\eta^2) = 0, \ \text{and}\ \ L_\theta(D_4 + D_5\eta) = 0, \tag{17}$$

They make up the constants while thinking about solving a problem overall. N_F and N_θ are the operators that are not linear provided as:

$$\frac{d^3F(\eta,l)}{d\eta^3} + \left(\frac{m+1}{2}\right)F(\eta,l)\frac{d^2F(\eta,l)}{d\eta^2} - m\left(\frac{dF(\eta,l)}{d\eta}\right)^2 + G^{*}\theta - \gamma\frac{d^5F(\eta,l)}{d\eta^5} = 0 \tag{18}$$

$$(3R+4)\frac{d^2\theta(\eta,l)}{d\eta^2} + \frac{m+1}{2}\left(F(\eta,l)\frac{d\theta(\eta,l)}{d\eta}\right) - (2m-1)\left(\theta(\eta,l)\frac{dF(\eta,l)}{d\eta}\right) = 0 \tag{19}$$

As described by [26–30], the basic understanding of HAM for Eqs. (18–19) is:

$$\{1-l\}L_F\left[F(\eta,l) - F_0(l)\right] = l h_F N_F\left\{F(\eta,l)\right\}, \tag{20}$$

$$\{1-l\}L_\theta\left[\theta(\eta,l) - \theta_0(l)\right] = l h_\theta N_\theta\left[F(\eta,l), \theta(\eta,l)\right], \tag{21}$$

where $l \in [0,1]$, is inserting factor, when $l = 0$ and $l = 1$, we get:

$$F(\eta,1) = F(\eta)\ \ \text{and}\ \ \theta(\eta,1) = \theta(\eta), \tag{22}$$

Expanding $F(\eta,l)$ and $\theta(\eta,l)$ in Taylor's series about $l = 0$, gives:

$$\begin{aligned} F(\eta,l) &= F_0(\eta) + \sum_{e=1}^{\infty} F_e(\eta) l^e, \\ \theta(\eta,l) &= \theta_0(\eta) + \sum_{e=1}^{\infty} \theta_e(\eta) l^e, \end{aligned} \tag{23}$$

4. Results and Discussion

The graphical results of our problem are investigated for different effects of the related parameters in this part. Equations (8–9)'s governing parameters were calculated over a large range of values, and the Mathematica function bvp 2 was used to determine semi-numerical solution subject to the transform boundary condition (10), by means of a well semi numerical method (HAM). Enhancing the rate of heat transfer is the main objective of the proposed model. The key findings derived from the data are presented both mathematically and graphically. Blood was used as the foundation fluid, and SWCNT and MWCNT nanoparticles were combined to create two different nanofluids. Single wall carbon nanotubes are referred as SWCMT and multi wall carbon nanotubes as MWCNT. The geometry of the flow problem is shown in Fig. 1.

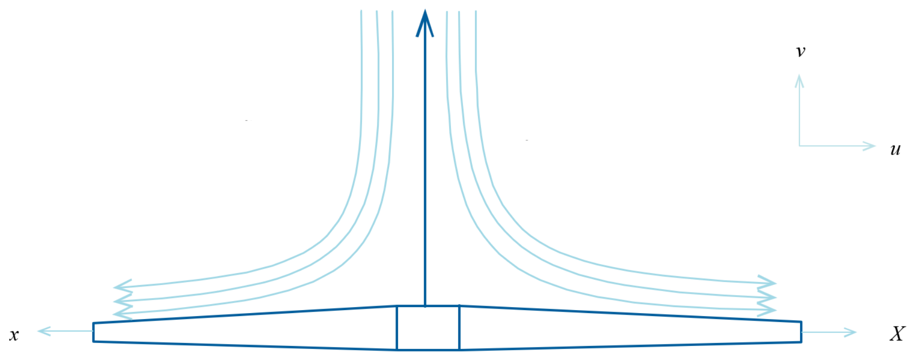

Fig. 1. Geometry of the flow problem.

Figure 2 shows how the couple stress parameter is influenced by fluid velocity for both nanofluid when $\gamma = 0.50, 0.70, 0.90$ $\phi = 0.02, Gr = 0.50, R = 1, n = 0.30$. The behavior of nanofluids is significantly influenced by the couple stress parameter, particularly in terms of how it affects the velocity profile. Usually speaking, a thicker boundary layer is formed by enhancing the couple stress parameter. This is a result of the fluid near the boundary slowing down due to the improved resistance to angular distortion. The velocity gradient near to the boundary decreasing as the couple stress parameter rises. An increased couple stress parameter results in a rise in the overall flow resistance. This may lower the flow velocity, particularly in areas that have seen a large amount of angular distortion. By flexible the boundary layer properties, the couple stress parameter in heat exchangers can be optimized to increase the velocity profile and heat transfer efficiency. The couple stress parameter can be adjusted to fine-tune the velocity distribution and confirm exact fluid manipulation in systems where exact control over fluid flow is crucial. By enhancing the effective viscosity and shear stress, a greater coupling stress parameter can improve the load-carrying capacity for lubrication using nanofluids. The velocity profile of nanofluids is greatly influenced by the couple stress parameter, which modifies the boundary layer thickness, velocity gradient, shear stress, flow resistance, and velocity distribution. The performance of many systems using nanofluids can be improved by comprehending and managing this parameter. The inspiration of SWCNT and MWCNT nanoparticle volume fractions on fluid velocity are depicted in Fig. 3, when $\gamma = 0.70, \phi = 0, 0.01, 0.02, Gr = 0.20,$ $R = 0.80, n = 0.50$. The velocity filed of nanofluids are strongly influenced by the volume percentage of nanoparticles in the nanofluid. The actual viscosity of the nanofluid usually increases as the volume percentage of nanoparticles enhancing. Additional flow resistance from this increased viscosity can decreases nanofluid velocity, mostly when the pressure gradient is the same. The total density of the nanofluid rises with greater nanoparticle volume fractions. The higher density can amplify the buoyant forces in buoyancy-driven flows (like natural convection), which may increase velocity in certain areas while decreasing in others. The thermal conductivity of the nanofluid is improved by greater volume fractions of nanoparticles. As a result, temperature gradients within the fluid may be reduced and heat transfer may be more effective. The direct consequence on velocity, on the other hand, is more complicated and usually depends on the particular application and flow circumstances. Certain nanofluids show shear thinning, a non-Newtonian behavior, with larger volume fractions. Higher shear rates may result in greater velocities for shear-thinning fluids (whose viscosity decreases with increased shear rate) and decreased velocities for shear-thickening fluids (whose viscosity increases with increased shear rate). Increasing the volume proportion of nanoparticles in heat exchangers can improve heat transfer rates, but the increased viscosity may necessitate more pumping power. Optimizing the nanoparticle volume fraction in electronic cooling systems is essential to striking a compromise between effective heat removal and controllable flow rates. By modifying the viscosity, density, thermal conductivity, and flow stability of nanofluids, the nanoparticle volume fraction is a crucial factor that affects their velocity. Lower volume fractions have the opposite effect, while higher volume fractions typically increase viscosity and density, possibly limiting fluid velocity.

The effects of local Grashof number on the velocity profiles of both nanofluids is depicted in Fig. 4, when $\gamma = 0.30, \phi = 0.01,$ $Gr = 0.10, 0.50, 0.90, R = 0.50, n = 0.10$. The dimensionless parameter called the local Grashof number defines how significant buoyant forces are in relation to viscous forces in a fluid flow. It is particularly important for the study of nanofluid dynamics and plays a key role in natural convection. More buoyant forces in relation to viscous forces are specified by a greater Grashof number. This frequently leads to greater fluid velocity in the case of nanofluids because of stronger natural convection flows.

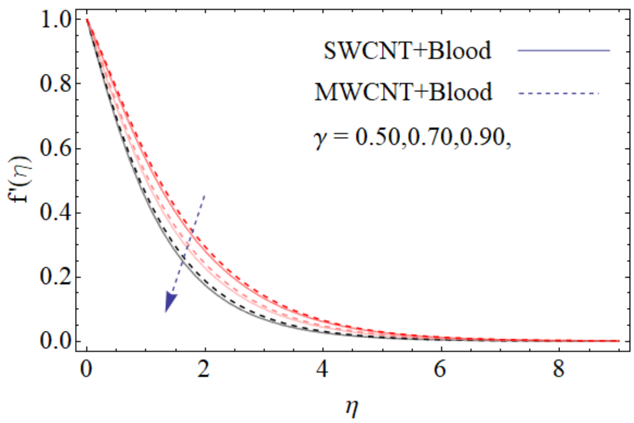

Fig. 2. Couple stress parameter impact on nanofluid velocity.

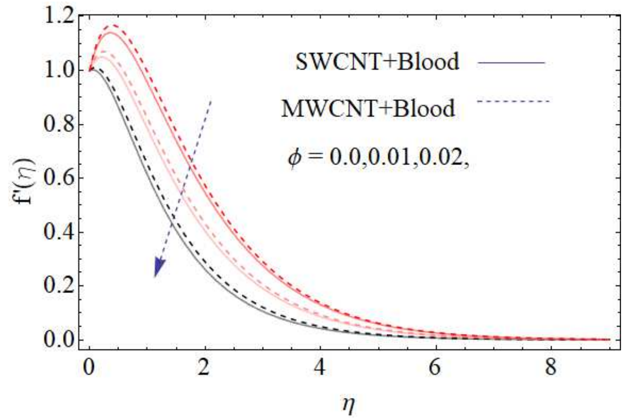

Fig. 3. Nanoparticle's volume friction impact on nanofluid velocity.

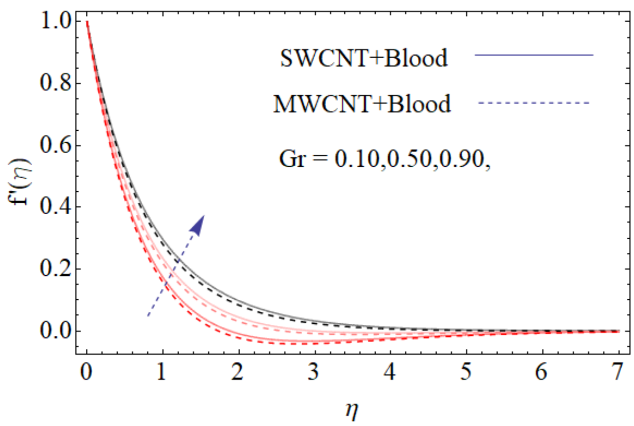

Fig. 4. Local Grashof number impact on nanofluid velocity.

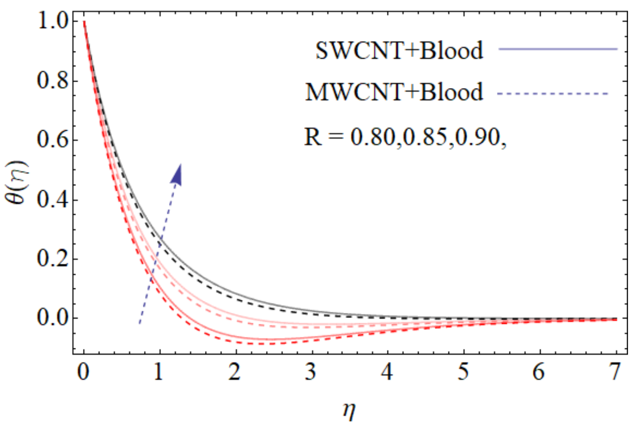

Fig. 5. Thermal radiation parameter impact on nanofluid temperature.

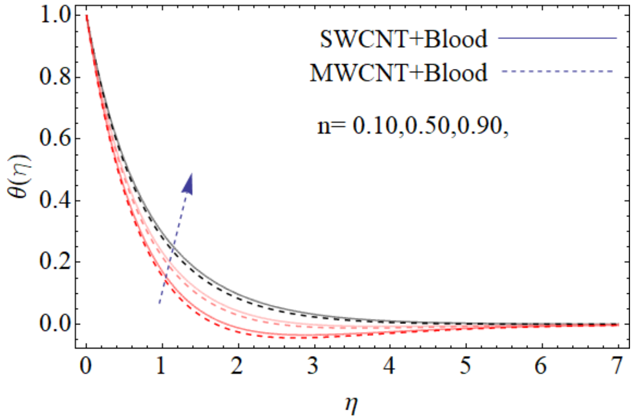

Fig. 6. Temperature exponent parameter impact on nanofluid temperature.

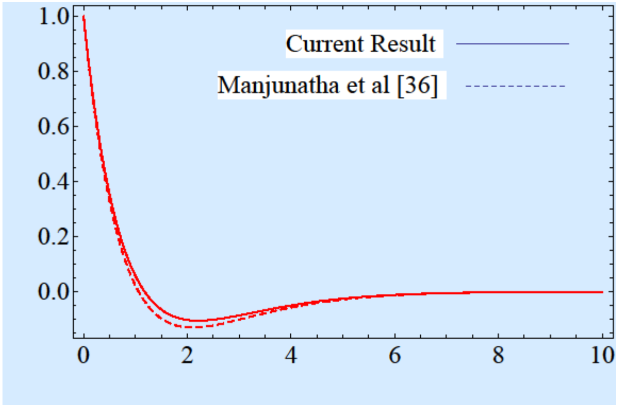

Fig. 7. Validation of the current code with Manjunatha et al. [36].

Temperature deviations drive the fluid motion additional, creating a more forceful flow. Convective heat transfer is enhanced with a superior Grashof number due to the improved velocity and decreased thickness of the boundary layer. This is valuable in situations when effective heat elimination is required, such cooling systems. When designing systems that use nanofluids for energy harvesting, heating, or cooling, the Grashof number can play a crucial role. Increased concentrations of nanoparticles can improve thermal conductivity, which in turn affects the velocity profile and effective Grashof number. The local Grashof number affects the equilibrium between buoyancy and viscous forces, which has a substantial effect on the velocity of nanofluids. Stronger natural convection currents, thinner boundary layers, and possible flow instabilities all contribute to higher Grashof numbers, which increase fluid velocity and improve heat transfer. Slower, more stable flows with larger boundary layers are produced by lower Grashof numbers. In many engineering applications, maximizing the Grashof number is crucial to realizing the chosen fluid dynamics and thermal performance. Figure 5 shows the relationship between the fluid temperature and the thermal radiation parameter when $\gamma = 0.90, \phi = 0.04, Gr = 0.90, R = 0.80, 0.85, 0.90, n = 0.30.$ The temperature filed of the nanofluid flow is observed to improve as the values of the thermal radiation parameter rise. One of the most significant parameters affecting the temperature distribution in a nanofluid is the thermal radiation parameter. When the thermal radiation parameter increases, it advises that thermal radiation is more significant than conduction. As radiation tends to equilibrium out temperature gradients, this results in extra uniform distribution of temperature within the nanofluid. As of the combined effects of radiation and conduction, the temperature at the surface of a heated object within the nanofluid can increase noticeably when the thermal radiation parameter is greater. As a result, the fluid's overall rate of heat transfer from the surface is improved. A thinner thermal boundary layer may result from a superior thermal radiation parameter. Radiation's improved heat transmission lowers the temperature differential across the boundary layer, influencing the layer's thickness. The total heat transfer rate increases with a high thermal radiation value. This is especially useful in applications where effective heating or cooling is required since the nanofluid has a superior capacity to absorb or release heat. Productivity gains in heat exchangers can be attained by optimizing the thermal radiation parameter. Elevated radiation parameters have the potential to increase heat transfer rates and decrease the surface area needed for a given heat transfer job. The temperature profile and properties of heat transport in nanofluids are strongly influenced by the thermal radiation parameter. Improved heat transfer rates, thinner thermal boundary layers, and more uniform temperature distributions are all caused by higher thermal radiation parameters. Lower thermal radiation parameters, on the other hand, lead to thicker thermal boundary layers, steeper temperature gradients, and slower heat transfer rates. An analysis of the impact of the temperature exponent parameter on temperature distribution is shown in Fig. 6, when $\gamma = 0.80, \phi = 0.03,$ $Gr = 0.50, R = 0.90, n = 0.10, 0.50, 0.90.$ It is discovered that the temperature field increases together with the temperature exponent parameter values. When describing how a property of the nanofluid changes with temperature, the temperature exponent parameter is frequently employed in empirical or semi-empirical correlations. The property is highly dependent on temperature when the temperature exponent value is higher. The temperature profiles inside the nanofluid may become more nonlinear as a result. For instance, areas of the fluid at higher temperatures will transfer heat more efficiently if thermal conductivity increases noticeably with temperature. This could result in temperature gradients that are steeper. The thermal conductivity of the nanofluid rises quickly with temperature when the temperature exponent value is high. This causes the fluid's temperature to equilibrate more quickly by improving heat transfer in areas with higher temperatures. Significant changes in fluid properties, such as reduced viscosity and improved thermal conductivity at higher temperatures, which promote enhanced convective currents and more efficient heat transfer, can be brought about by high temperature exponent parameter values. Thermal behavior and temperature distribution of nanofluids are significantly influenced by the temperature exponent parameter. More nonlinear temperature profiles, more convection, and thinner thermal boundary layers are the outcomes of higher temperature exponent parameter values, which cause more noticeable changes in characteristics like thermal conductivity and viscosity with temperature. More linear temperature dependencies, which produce smoother temperature gradients and more stable fluid properties, are produced by lower values of the temperature exponent parameter. For systems that use nanofluids to function better and be more efficient, the temperature exponent parameter must be optimized. Under similar situations, a systematic comparison for the validation the existing computational code with Manjunatha et al. [36] is presented in Fig. 7. Its use for more research and applications in heat transfer and nanofluid dynamics is made possible by the successful validation, which increases trust in the code's performance and correctness. The thermophysical characteristics of CNTs and blood base fluid are displayed in Table 1. The skin friction values that were calculated for each of the coupling stress, nanoparticle volume friction, and velocity exponent parameter are displayed in Table 2. Enhancements to multiple elements at both the top and lower walls were observed to elevate the skin friction value. Both nanofluids had an improvement in skin friction. The Nusselt numbers for the temperature exponent parameter, the volume friction of the nanoparticles, and the thermal radiation for both nanofluids are displayed numerically in Table 3. The results are shown in Table 3, which shows that when the temperature exponent parameter, volume friction of nanoparticles, and thermal radiation increase, so does the rate of heat transmission. Table 4 shows the resudle error via the number of itration.

Table 1. The base fluid Blood and CNTs thermophysical properties as reported in [24, 25].

	$\rho\left(\frac{kg}{m^3}\right)$	$k\left(\frac{W}{mK}\right)$	$\sigma\left(\frac{s}{m}\right)$	$c_p\left(\frac{j}{kgK}\right)$
Blood	4250	0.492	-	3594
SWCNT	2600	6600	10^6 - 10^7	425
MWCNT	1600	3000	1.9×10^{-4}	796

Table 2. Skin friction via m,γ,ϕ.

m	γ	ϕ	C_f
0.70			0.2143
0.80			0.1703
0.90			0.1412
	0.10		0.3319
	0.20		0.3691
	0.30		0.3907
		0.01	0.4289
		0.02	0.4416
		0.03	0.4733

Table 3. Nusselt number via n,R,ϕ.

ϕ	n	R	Nu
0.01			0.6725
0.05			0.6900
0.09			0.7319
	0.10		0.7527
	0.30		0.7749
	0.50		0.8041
		0.90	0.8315
		1.00	0.8517
		1.10	0.8810

Table 4. The method's convergence for the temperature equation and velocity.

m	$\theta(\eta)$	$f'(\eta)$
5	0.09747	0.07483
10	0.08671	0.06351
15	0.00673	0.00825
20	0.00093	0.00089
25	0.00043	0.00054

5. Conclusion

The current work examined heat transfer analysis of two dimensional steady, laminar mixed convection, CNTs blood base non-Newtonian nanofluid over a stretching sheet with the influence of thermal radiation. The effects of the local Grashof number, thermal radiation, temperature exponent, and nanoparticle volume friction were all included. (SWCNT + Blood) and (MWCNT + Blood) demonstrated two different nanofluids. The analytical results were executed using the HAM method, and the validation was performed with a numerical approach. The behavior and characteristics of nanofluids can be studied to further the fields of nanotechnology and materials research. This entails comprehending the stability, dispersion, and interactions of nanoparticles in various fluid matrices. The following are the key conclusions of the current analysis:

1. The nanofluid velocity field exhibits a negative correlation with the couple stress parameter and the volume fraction of nanoparticles.
2. An increase in the Local Grashof number enhances the nanofluid velocity field.
3. Increasing the temperature exponent and thermal radiation parameters also enhances the temperature field of the nanofluid.
4. Increasing the temperature exponent parameter, nanoparticle volume fraction, and thermal radiation also enhances the Nusselt number.
5. Increasing the velocity exponent parameter, couple stress, and nanoparticle volume enhances the skin friction.
6. As the number of iterations increases, the residual error decreases.

Author Contributions

A. Rehman and M.C. Khun planned the scheme, initiated the project, and suggested the experiments; D. Khan conducted and analyzed the numerical results; R. Jan developed the mathematical modeling and examined the theory validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

Acknowledgments

Ali Rehman is a postdoctoral fellow of University Technology Malaysia under the postdoc fellowship scheme for the project; Q.J130000.21A2.07E1107E11.

Conflict of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

Funding

The authors received no financial support for the research, authorship, and publication of this article.

Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

Symbol	Description	Symbol	Description
u_w	Stretching velocity	Gr	Local Grashof number
u and v	x and y components of velocity	x,y	Plane coordinate axis
T_w	Surface temperature	C_{fx}	Coefficient of skin friction
T_∞	Ambient temperature	β	Thermal expansion coefficient
ψ	Stream function	γ	Couple stress parameter
A, B	Constants	Nu_x	Nusselt number
k	Thermal conductivity	m and n	Exponents
Re_x	Reynold number	α	Thermal diffusivity
ρ_{nf}	Nanofluid density	μ	Dynamic viscosity
w	Surface condition	f'	Velocity without dimension
σ	Electrical conductivity of the base fluid	∞	Ambient condition
$(\rho c_p)_{nf}$	Capacity of heat in Nanofluid	ν	Kinematic viscosity
q_r	Stefan-Boltzmann constant	θ	Temperature without dimension
g	Gravitation acceleration	Pr	Prandtl number
η	Similarity variable	τ_w	Share stress of wall surface
ϕ	Volume fraction of nanoparticles	k_1	Absorption coefficient
nf	Nanofluid	q_w	Wall heat flux
f	Base fluid	MWCNT	Multi wall carbon nanotube
PDEs	Partial differential equations	ODEs	Ordinary differential equations
HAM	Homotopy analysis method	SWCNT	Single wall carbon nanotube
G^*	Buoyancy parameter	R	Thermal radiation parameter

References

[1] Ellahi, R., The effects of MHD and temperature dependent viscosity on the flow of a non-Newtonian nanofluid in a pipe, Analytical solution, *Applied Mathematical Modelling*, 37, 2013, 1451–1457.
[2] Alshomrani, A. S., Gul, T., A convective study of Al2O3-H2O and Cu-H2O nano-liquid films sprayed over a stretching cylinder with viscous dissipation, *The European Physical Journal Plus*, 132, 2017, 495–512.
[3] Asadi, M., Asadi, A., Aberoumand, S., An experimental and theoretical investigation on the effects of adding hybrid nanoparticles on heat transfer efficiency and pumping power of an oil-based nanofluid as a coolant fluid, *International Journal of Refrigeration*, 89, 2018, 83–92.
[4] Gul, T., Nasir, S., Islam, S., Shah, Z., Khan, M. A., Effective Prandtl number model influences on the Al2O3-H2O and Al2O3-C2H6O2 nanofluids spray along a stretching cylinder, *Arabian Journal for Science and Engineering*, 44, 2019, 1601–1616.
[5] Haq, R. U., Nadeem, S., Khan, Z. H., Noor, N. F. M., Convective heat transfer in magnetohydrodynamic slip flow over a stretching surface in the presence of carbon nanotubes, *Physica B: Condensed Matter*, 457, 2015, 40–47.
[6] Khan, W. A., Khan, Z. H., Rahi, M., Fluid flow and heat transfer of carbon nanotubes along a flat plate with Navier slip boundary, *Applied Nanoscience*, 4, 2014, 633–641.
[7] Aman, S., Khan, I., Ismail, Z., Salleh, M. Z., Alshomrani, A. S., Magnetic field effect on Poiseuille flow and heat transfer of carbon nanotubes along a vertical channel filled with Casson fluid, *AIP Advances*, 7, 2017, 1–18.
[8] Aman, S., Khan, I., Ismail, Z., Salleh, M. Z., Al-Mdallal, Q. M., Heat transfer enhancement in free convection flow of CNTs Maxwell nanofluids with four different types of molecular liquids, *Scientific Reports*, 7, 2017, 1–13.
[9] Asadi, A., Asadi, M., Rezaei, M., Siahmargoi, M., Asadi, F., The effect of temperature and solid concentration on dynamic viscosity of MWCNT/MgO (20–80)–SAE50 hybrid nano-lubricant and proposing a new correlation: An experimental study, *International Communications in Heat and Mass Transfer*, 78, 2016, 48–53.
[10] Asadi, M., Asadi, A., Dynamic viscosity of MWCNT/ZnO-engine oil hybrid nanofluid: An experimental investigation and new correlation in different temperatures and solid concentrations, *International Communications in Heat and Mass Transfer*, 76, 2016, 41–45.
[11] Asadi, A., Asadi, M., Rezaniakolaei, A., Rosendahl, L. A., Afrand, M., Heat transfer efficiency of Al2O3-MWCNT/thermal oil hybrid nanofluid as a cooling fluid in thermal and energy management applications, *International Journal of Heat and Mass Transfer*, 117, 2018, 474–486.
[12] Asadi, A., Asadi, M., Rezaniakolaei, A., Rosendahl, L. A., Wongwises, S., An experimental and theoretical investigation on heat transfer capability of $Mg(OH)_2$/MWCNT-engine oil hybrid nano-lubricant adopted as a coolant and lubricant fluid, *Applied Thermal Engineering*, 129, 2018, 577–586.
[13] Nasir, S., Islam, S., Gul, T., Shah, Z., Khan, M. A., Khan, W., Khan, A. Z., Khan, S., Three-dimensional rotating flow of magnetohydrodynamic single-wall carbon nanotubes over a stretching sheet in presence of thermal radiation, *Applied Nanoscience*, 8, 2018, 1361–1378.
[14] Ellahi, R., Zeeshan, A., Hussain, F., Abbas, T., Study of shiny film coating on multi-fluid flows of a rotating disk suspended with nano-sized silver and gold particles: A comparative analysis, *Coatings*, 8, 2018, 422.
[15] Asma, K., Khan, I., Arshad, K., Sharidan, S., Unsteady magnetohydrodynamic free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium, *Engineering Science and Technology, an International Journal*, 18, 2015, 309–317.
[16] Ellahi, R., Shivanian, E., Abbasbandy, S., Rahman, S. U., Hayat, T., Analysis of steady flows in viscous fluid with heat/mass transfer and slip effects, *International Journal of Heat and Mass Transfer*, 55, 2012, 6384–6390.
[17] Rehman, A., Saeed, A., Salleh, Z., Jan, R., Kumam, P., Analytical investigation of the time-dependent stagnation point flow of a carbon nanotube nanofluid over a stretching surface, *Nanomaterials*, 12, 2022, 1108.
[18] Rehman, A., Salleh, Z., Influence of Marangoni convection on magnetohydrodynamic viscous dissipation and heat transfer on hybrid nanofluids in a rotating system among two surfaces, *Mathematics*, 9, 2021, 2242.
[19] Rehman, A., Salleh, Z., Analytical investigation of magnetic field on unsteady boundary layer stagnation point flow of water-based graphene oxide-water and graphene oxide-ethylene glycol nanofluid over a stretching surface, *Mathematical Problems in Engineering*, 2021, 1–15.
[20] Rehman, A., Salleh, Z., Gul, T., Analytical study of unsteady squeezed flow of water-based carbon nanotube nanofluid with magnetic field and variable thermal conductivity over a stretching surface, *Frontiers in Heat and Mass Transfer*, 14, 2020, 1–10.
[21] Rehman, A., Salleh, Z., Gul, T., The impact of the magnetic field and viscous dissipation on the thin film unsteady flow of GO-EG/GO-W nanofluids, *Journal of Physics: Conference Series*, 1366, 2019, 012031.
[22] Rehman, A., He, Z. Y., Wang, M. K., Almaghrabi, O. A., Alsallami, S. A., Khan, W., Analytical study of time-dependent magnetohydrodynamic flow of hybrid nanofluid around a rotating sphere, *Waves in Random and Complex Media*, 2022, 1–18.
[23] Rehman, A., Jan, R., Elamin, A. E. A., Abdel-Khalek, S., Inc, M., Analytical study of magnetohydrodynamic couple stress Casson nanofluid flow over a stretching surface, *Thermal Science*, 26, 2022, 397–403.
[24] Saeed, A., Khan, N., Gul, T., Kumam, W., Alghamdi, W., Kumam, P., The flow of blood-based hybrid nanofluids with couple stresses by the convergent

and divergent channel for the applications of drug delivery, *Molecules*, 26, 2021, 6330.
[25] Alsagri, A. S., Nasir, S., Gul, T., Islam, S., Nisar, K. S., Shah, Z., Khan, I., Magnetohydrodynamic thin film flow and thermal analysis of blood with carbon nanotube nanofluid, *Coatings*, 9, 2019, 175.
[26] Liao, S. J., *Homotopy analysis method in nonlinear differential equations*, Springer and Higher Education Press, 2012, 1451–1457.
[27] Liao, S. J., *Beyond Perturbation: Introduction to the Homotopy Analysis Method*, Chapman & Hall/CRC, 2003, 1–30.
[28] Liao, S. J., An optimal homotopy-analysis approach for strongly nonlinear differential equations, *Communications in Nonlinear Science and Numerical Simulation*, 15, 2010, 2003–2016.
[29] Liao, S. J., On the homotopy analysis method for nonlinear problems, *Applied Mathematics and Computation*, 147, 2004, 499–513.
[30] Liao, S. J., *Homotopy analysis method in nonlinear differential equations*, Beijing: Higher Education Press, 2012, 153–165.
[31] Sadighi, S., Afshar, H., Jabbari, M., Ashtiani, H. A. D., An analytical approach to entropy production in magnetohydrodynamic mixed convection micropolar fluid flow over an inclined porous stretching sheet, *Frontiers in Mechanical Engineering*, 8, 2022, 900316.
[32] Jamrus, F. N., Waini, I., Khan, U., Ishak, A., Effects of magnetohydrodynamics and velocity slip on mixed convective flow of thermally stratified ternary hybrid nanofluid over a stretching/shrinking sheet, *Case Studies in Thermal Engineering*, 104161, 2024, 1–15.
[33] Shah, S. H. A. M., Suleman, M., Khan, U., Dual solution of magnetohydrodynamic mixed convection flow and heat transfer over a shrinking sheet subject to thermal radiation, *Partial Differential Equations in Applied Mathematics*, 6, 2022, 100412.
[34] Zainodin, S., Jamaludin, A., Nazar, R., Pop, I., Magnetohydrodynamic mixed convection of hybrid ferrofluid flow over an exponentially stretching/shrinking surface with heat source/sink and velocity slip, *Mathematics*, 10, 2022, 4400.
[35] Sadighi, S., Afshar, H., Ashtiani, H. A. D., Jabbari, M., Magnetohydrodynamic flow and conductive heat transfer on a permeable stretching cylinder: Benchmark solutions, *Case Studies in Thermal Engineering*, 44, 2023, 102886.
[36] Sadighi, S., Jabbari, M., Afshar, H., Ashtiani, H. A. D., Magnetohydrodynamic heat and mass transfer nanofluid flow on a porous cylinder with chemical reaction and viscous dissipation effects: Benchmark solutions, *Case Studies in Thermal Engineering*, 40, 2022, 102443.
[37] Manjunatha, S., Puneeth, V., Gireesha, B. J., Chamkha, A., Theoretical study of convective heat transfer in ternary nanofluid flowing past a stretching sheet, *Journal of Applied and Computational Mechanics*, 8, 2022, 1279–1286.
[38] Mbogba, G. L., Ngo Nyobe, E., Lamara, M., Mbono Samba, Y. C., Effects of an external constant pressure gradient on a steady incompressible laminar flow through a semi-porous annular pipe, *Zeitschrift für Naturforschung A*, 77, 2022, 131–141.
[39] Gazambeti, Y., Ngo Nyobe, E., Lamara, M., Pemha, E., Pressure-exerted steady laminar flow of an incompressible fluid along a porous parallel-walled channel with an impermeable wall, *Zeitschrift für Naturforschung A*, 77, 2022, 675–687.
[40] Kalibe, B., Lamara, M., Nyobe, E. N., Pemha, E., Effects of porous-wall acceleration on laminar flows in semi-porous channels with a rectangular cross section, *Chinese Journal of Physics*, 89, 2024, 834–858.
[41] Makon, N. R., Nyobe, E. N., Lamara, M., Samba, Y. C. M., Pemha, E., On the existence and determination of the incompressible laminar flow located in the polar plane of a porous annular pipe, *Alexandria Engineering Journal*, 61, 2022, 2637–2650.
[42] Mbam Mbam, S. B., Lamara, M., Makon, N. R., Ngo Nyobe, E., Pemha, E., Polar-plane flow in porous annular ducts with accelerated rotating walls: A region of stagnation inside the fluid, *Zeitschrift für Naturforschung A*, 79, 2024, 229–251.

ORCID iD

Ali Rehman https://orcid.org/0009-0005-3956-5994
Ma Chau Khun https://orcid.org/0009-0008-4874-9690
Dolat Khan https://orcid.org/0000-0002-3723-0796
Rashid Jan https://orcid.org/0000-0001-9709-7045

How to cite this article: Rehman A., Khun M.C., Khan D., Jan R. Heat Transfer Analysis of 2D Steady Laminar Mixed Convection CNTs Blood base Nanofluid Flow together with Stretching Surface Analytical Simulation, *J. Appl. Comput. Mech.*, 11(4), 2025, 861-870. https://doi.org/10.22055/jacm.2024.46641.4562