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Research Paper

Robust Adaptive Control for Delayed Fractional-Order Multi-Agent Systems with Unmodeled Cyber-Physical Attacks

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Abstract. Fractional-order multi-agent systems (MASs) with communication delays play a vital role in smart grids, autonomous vehicles, and industrial automation, but their memory-dependent dynamics increase vulnerability to cyber-physical attacks. In particular, malicious sensor and actuator attacks compromise feedback and control signals, threatening stability and consensus. Traditional control methods often require prior knowledge of attack models, which restricts their effectiveness against unknown and time-varying disturbances. This paper proposes a robust adaptive control framework for delayed fractional-order MASs within the Caputo derivative setting. The method integrates an adaptive estimation law with Caputo-based Lyapunov analysis to counter unmodeled sensor and actuator attacks using only local and neighboring agent data. Stability conditions are rigorously established through a fractional Razumikhin approach. Simulations validate the scheme's resilience under sensor-only, actuator-only, and combined attacks, confirming its ability to maintain consensus, achieve faster convergence, and improve reliability in hostile environments.

Keywords: Fractional-order multi-agent systems, Communication delays, Cyber-physical attacks, Adaptive control, Consensus control.

1. Introduction

Multi-Agent Systems (MASs) have become a cornerstone for distributed coordination in diverse applications such as smart grids, autonomous transportation, cooperative robotics, and industrial automation. Their importance is highlighted by recent surveys summarizing broad application domains and fundamental challenges in multi-agent coordination and control [1]. Communication delays are one of the most critical challenges in MASs, motivating specialized stability analyses such as Lambert W-function-based approaches to investigate convergence under delayed interactions [2]. In parallel, fractional-order models have drawn increasing attention due to their ability to capture memory and hereditary properties that are inherent in physical processes such as viscoelasticity, thermodynamics, and biological systems [3, 4]. When combined with communication delays, fractional dynamics significantly enrich the modeling capability but also increase the difficulty of designing robust and efficient control strategies [5].

Alongside system complexity, cyber-physical security has emerged as a pressing concern for MASs. Various forms of attacks—such as exponentially unbounded signals, deception attacks, and data integrity violations—have been considered in distributed settings [6–8]. Resilient control approaches have been proposed in microgrids, renewable systems, and interconnected fractional-order networks subjected to stochastic or structured attacks [9, 10]. Moreover, defense strategies against denial-of-service and deception attacks have been introduced to ensure secure consensus in fractional-order MASs with delays [11, 12]. Event-triggered consensus methods under switching topologies further improve resilience while reducing communication load [13].

Significant progress has also been made in the theoretical foundations of fractional-order systems. Approximation techniques for fractional derivatives such as the Caputo operator [14], robust data-driven approaches including Koopman operator frameworks [15], and adaptive consensus designs for nonlinear and switching topologies [16] provide powerful analytical tools. Foundational studies on time-delay stability [17], Lyapunov-based proofs of uniform stability in fractional-order systems [18], and finite-time containment control in nonlinear MASs [19] have extended the stability and control landscape. Stability criteria for fractional systems with delays using Lyapunov functions [20], consensus in nonlinear Caputo models with mixed delays [21], and global consensus conditions in directed fractional-order networks [22] further demonstrate the richness of this area. Classical leader–follower consensus designs remain vital [23, 24], and modern research has broadened MAS perspectives toward distributed clustering, dynamic consensus, and advanced topologies [25].

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Complementary works include stability and numerical analysis of fractional-order equations [26], disturbance observers for uncertain nonlinear systems [27], and consensus in leader–follower networks with delays [28]. Resilient consensus against sensor and actuator attacks in fractional-order systems has been explicitly studied [29, 30], while stability under Caputo derivatives [31] and adaptive self-triggered control for attack scenarios [32] offer further defense mechanisms. Sliding-mode and adaptive sliding-mode controllers have also proven effective for rejecting disturbances in fractional-order and mechatronic systems [33–37], establishing a robust foundation for resilient control. Recent mathematical and engineering contributions include Hermitian spin-based algebraic and spectral analysis [38] and optimization for flexible load transport using quadrotors [39].

Building on these foundations, additional recent works have provided new perspectives. Improved position controllers have been designed using linear active disturbance rejection methods for precision electromechanical systems [40]. Digital model-based servo control frameworks that account for sampling-induced time delays have been investigated in practical contexts [41]. Observer-based methods have been developed for nonlinear Lipschitz systems to troubleshoot secondary faults, demonstrating the effectiveness of estimation-based resilience [42]. Furthermore, delay-dependent stability regions for distributed fractional-order MASs have been derived, deepening the understanding of delay margins [43]. Robust consensus frameworks under interval uncertainties using Hermitian matrix conditions further contribute to the robustness analysis of fractional-order MASs [44].

Despite these advances, key challenges remain. Many existing resilient control strategies rely on restrictive assumptions, such as prior knowledge of attack bounds or simplified delay structures, which limit their applicability in real-world systems subject to unmodeled, time-varying, and simultaneous sensor and actuator attacks. Moreover, the interplay between fractional-order memory effects, Caputo dynamics, and distributed delays has not been sufficiently addressed in the context of cyber-physical security.

1.1. Contributions of this work

To address these open challenges, this paper proposes a robust adaptive control framework for delayed fractional-order multi-agent systems under the Caputo derivative. The main contributions are:

- A comprehensive modeling framework that incorporates distributed delays and unmodeled, time-varying sensor and actuator attacks into fractional-order MASs.
- The design of a distributed adaptive estimator-based control law that requires only local and neighboring information, ensuring scalability and practicality.
- Derivation of stability conditions using fractional Lyapunov and Razumikhin techniques tailored for Caputo systems under cyber-physical attacks.
- Numerical simulations demonstrating resilience against sensor-only, actuator-only, and combined attack scenarios, validating the robustness of the proposed approach.

These contributions aim to enhance the resilience and reliability of fractional-order MASs while providing a solid foundation for future work in nonlinear extensions, event-triggered communication, and large-scale mobile networks.

2. Preliminaries

Firstly, we provide some graph-theoretic concepts, along with the necessary definitions and lemmas from fractional calculus that will be used in the subsequent analysis.

2.1. Graph theory

Consider a directed, balanced graph $\mathcal{A} = (\mu, \varepsilon)$ with a finite set of vertices $\mathcal{V} = \{\varsigma_1, \varsigma_2, \varsigma_3, \ldots, \varsigma_M\}$ and a set of directed edges $\varepsilon \subseteq \{(\varsigma_j, \varsigma_k) : \varsigma_j, \varsigma_k \in \mathcal{V}\}$. Each oriented edge ε_{jk} is an ordered pair $(\varsigma_j, \varsigma_k)$, representing a directed connection from vertex ς_j (tail) to vertex ς_k (head). Let $\mathcal{M}_j = \{\varsigma_k \mid (\varsigma_k, \varsigma_j) \in \varepsilon\}$ denote the set of neighbors of vertex ς_j, i.e., the collection of vertices from which there are incoming edges to ς_j.

Define the adjacency matrix $C = [c_{jk}] \in \mathbb{R}^{M\times M}$, where each entry $c_{jk} > 0$ if $(\varsigma_j, \varsigma_k) \in \varepsilon$, and $c_{jk} = 0$, otherwise. A pointed spanning tree of the digraph $\mathcal{A}$ is a subgraph in which there exists at least one root vertex that can reach all other vertices through directed paths following the edge directions.

Let $D = [I_{jk}] \in \mathbb{R}^{M\times M}$ denote the Laplacian matrix of the graph $\mathcal{A}$, defined as:

$$I_{jk} = \begin{cases} -c_{jk}, & j \neq k, \\ \sum_{k=1, k\neq j}^{M} c_{jk}, & j = k. \end{cases}$$

It is straightforward to verify that:

$$\sum_{k=1}^{M} I_{jk} = 0 \text{ for all } j = 1, 2, \ldots, M.$$

This implies that each row of the Laplacian matrix sums to zero, a fundamental property used in consensus stability analysis. A directed weighted graph with six agents named ς_1 to ς_6 is shown in Fig. 1. Communication weights or influence strengths are represented by the numerical values assigned to each directed edge, which shows the information flow between agents. Agent ς_2 influences ς_3 with a weight of 1.2, for example, whereas agent ς_5 influences ς_6 with a weight of 1.1. Information can flow throughout the network thanks to the graph's closed-loop communication structure. This kind of setup is crucial for researching the dynamics of consensus and collective behavior in multi-agent systems, especially when taking time delays and the requirement for robust coordination in challenging circumstances into account.

2.2. Fractional calculus background

For $0 < \alpha < 1$, the Caputo fractional derivative of a function $f(t)$ is defined as:

$$^{C}D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(\tau)}{(t-\tau)^{\alpha}} d\tau = I_{t_0}^{1-\alpha} f'(t), \tag{1}$$

where I^{β} denotes the Riemann–Liouville fractional integral of order β. The Caputo derivative of a constant is zero, which makes it compatible with classical initial conditions.

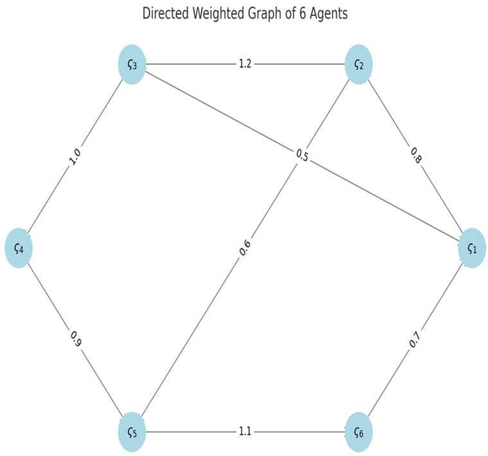

Fig. 1. A directed weighted communication topology among six agents.

The relation between the Riemann–Liouville (RL) and Caputo derivatives is given by:

$$^{RL}D_{t_0}^{\alpha} f(t) = ^{C} D_{t_0}^{\alpha} f(t) + \frac{f(t_0)}{\Gamma(1-\alpha)}(t-t_0)^{-\alpha}. \tag{2}$$

In this work, all analytical derivations are carried out using the Caputo derivative, which ensures consistency with Lyapunov and Razumikhin-based stability analysis.

Lemma 2.1. If and only if the graph's Laplacian matrix D has an eigenvalue of 0 and the real elements of other eigenvalues are positive, then the directed graph $\mathcal{A}$ has a directed spanning tree [13].

Definition 2.1. Let $f(v)$ be a sufficiently smooth function. The Caputo fractional derivative of order α is defined as [14]:

$$^{C}D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(\tau)}{(t-\tau)^{\alpha}} d\tau = I_{t_0}^{1-\alpha} f'(t).$$

where $n \in \mathbb{N}$ is the smallest integer satisfying $n - 1 < \alpha \le n$, and $\Gamma(\cdot)$ denotes the Gamma function. A key advantage of the Caputo definition lies in its compatibility with classical initial conditions, making it particularly suitable for physical and engineering applications.

Definition 2.2. Let $z \in \mathbb{R}^n$ represent a dynamical system's state vector. If a scalar function $W(z): \mathbb{R}^n \to \mathbb{R}$ meets the following criteria, it is called a Lyapunov candidate [15]: For any $z \neq 0$, $W(z)$ is continuously differentiable, strictly positive, and $W(0) = 0$. Furthermore, $\dot{W}(z) = \nabla W(z) \cdot g(z) \le -\delta \|z\|^2$, where $\delta > 0$, is satisfied by the derivative of $W(z)$ along the trajectories described by the system's vector field $g(z)$.

Lemma 2.2. Let $\mathcal{L}$ be the Laplacian matrix of the network, and $\mathcal{B}$ be a diagonal matrix of adaptive gains. If the adaptive controller gain matrix Γ is designed such that the diagonal elements satisfy $\Gamma = \text{diag}\{0.5, 1\}$, then the attack mitigation process will converge to zero as the iteration $k \to \infty$, where k represents the iteration or time step [16].

Lemma 2.3. Let A, B, C be three matrices. Then the inequality [17]:

$$\begin{pmatrix} A & B \\ A^{\top} & C \end{pmatrix} < 0$$

is equivalent to the inequalities $A < 0$, and $C - B^{T} A^{-1} B < 0$.

Lemma 2.4. Let $x_i(t) \in \mathbb{R}^n$ be continuously differentiable for all i, and let $K > 0$ be the control gain matrix. Then the following inequality holds [18]:

$$\frac{1}{2} {}_{t_0}^{C}D_t^{\alpha} \left(x_i^{\top}(t) K x_i(t) \right) \le x_i^{\top}(t) K {}_{t_0}^{C}D_t^{\alpha} x_i(t), \quad \forall \alpha \in (0,1).$$

Lemma 2.5. Consider a fractional-order multi-agent system governed by the Caputo derivative [19]:

$${}_{t_0}^{C}D_t^{\alpha} x_i(t) = \sum_{j=1}^{n} c_{ij} \left(x_j(t-\nu_{ij}) - x_i(t-\nu_{ij}) \right) \quad \forall \alpha \in (0,1)$$

where $x_i(t) \in \{\mathbb{R}\}^n$ represents the state of the i^{th} follower agent, $c_{ij} \ge 0$ denotes the coupling weight from agent j to agent i, and $\nu_{ij} \ge 0$ indicates the communication time delay between them. If the communication graph is strongly connected, and the maximum delay $\nu_{\max} = \max_{i,j} \nu_{ij}$ satisfies the boundary conditions, then the consensus among all agents is guaranteed:

$$\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j \in 1, \ldots, n.$$

Lemma 2.6. Suppose $\alpha_1, \alpha_2, \alpha_3 : \mathbb{R} \to \mathbb{R}$ are continuous, strictly increasing functions with $\alpha_{1(r)}, \alpha_{2(r)} > 0$ for $r > 0$ and $\alpha_{1(0)}, \alpha_{2(0)} = 0$. Let $\{\alpha\}_2 > 0$ and suppose there exists a differentiable function M: $\mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ and a continuous, increasing function $\zeta(r)$ such that for any control-relevant state history $\varphi \in \mathcal{F}$, the following inequality holds [20]:

$$\alpha_1 \left(\|x_i(t)\| \right) \le M\left(t, x_i(t)\right) \le \alpha_2 \left(\|x_i(t)\| \right)$$

if in addition,

$$M\left(t+\theta,\varphi(\theta)\right) \le \varsigma\left(M\left(t,\varphi(0)\right)\right) \quad \theta \in [-\nu,0]$$

and,

$${}^{C}_{t_0}D_t^{\alpha} M\left(t,\varphi(0)\right) \le -\alpha_3\left(\left\|\varphi(0)\right\|\right), \quad t \ge t_0, \tag{3}$$

then the zero solution $x_i(t) = 0$ is asymptotically stable for all $\theta \in [-\nu, 0]$. Moreover, it is globally stable if $\alpha_{1(r)} \to \infty$ as $r \to \infty$.
Lemma 2.7. If $\mu_1,\ldots,\mu_n$ are the eigenvalues of the matrix $S \in \mathbb{R}^{n\times n}$, and $\alpha_1,\ldots,\alpha_m$ are the eigenvalues of the matrix then the $T \in \mathbb{R}^{m\times m}$, eigenvalues of the Kronecker product $S \otimes T$ are given by the set $\{\mu_i\, \alpha_j \mid i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m\}$ [21].

3. Results and Discussion

3.1. Simulation setup

To validate the theoretical developments, numerical simulations were conducted for a delayed fractional-order multi-agent system in the Riemann–Liouville framework. The network comprised six agents, including one leader and five followers, interconnected through a directed graph that satisfies the consensus condition. The fractional order was chosen as $\alpha = 0.9$, with a maximum communication delay of 0.15 s. Actuator saturation was incorporated to emulate practical limitations, and the controller gains were tuned according to the stability conditions established in Section V. Attack signals were modeled as unmodeled, time-varying disturbances injected into both sensing and actuation channels, without any prior assumptions regarding their structure or bounds. All agents were initialized with distinct initial states to assess the convergence performance of the proposed adaptive control law.

3.2. Performance under different attack scenarios

Case 1: No Attack

Figure 2 depicts the baseline performance of the system in the absence of any adversarial interference. All follower agents converge smoothly to the leader's trajectory within a finite time, confirming that the proposed adaptive control strategy ensures stable consensus under nominal conditions.

Case 2: Sensor Attacks

In this scenario, falsified measurement data were injected into the sensor channels of selected followers. As illustrated in Fig. 2, transient oscillations appeared due to corrupted state information; however, the adaptive controller effectively compensated for these perturbations and guided all agents toward consensus with the leader. This demonstrates the robustness of the proposed approach against sensor-level deception.

Case 3: Actuator Attacks

When the actuator channels were manipulated by adversarial inputs, the injected disturbances directly affected the control layer. As shown in Fig. 2, the proposed adaptive scheme preserved boundedness of the consensus error and successfully steered the trajectories toward synchronization with the leader. Although convergence was slightly slower than in the no-attack case, overall system stability was consistently maintained.

Case 4: Combined Attacks

The most challenging case involved simultaneous corruption of both sensor and actuator channels. The corresponding results are presented in Fig. 2. Despite concurrent falsification of state measurements and control signals, the proposed framework maintained global consensus. Although the transient response was longer and convergence rate slower compared to previous cases, all followers eventually synchronized with the leader, confirming resilience under worst-case adversarial conditions.

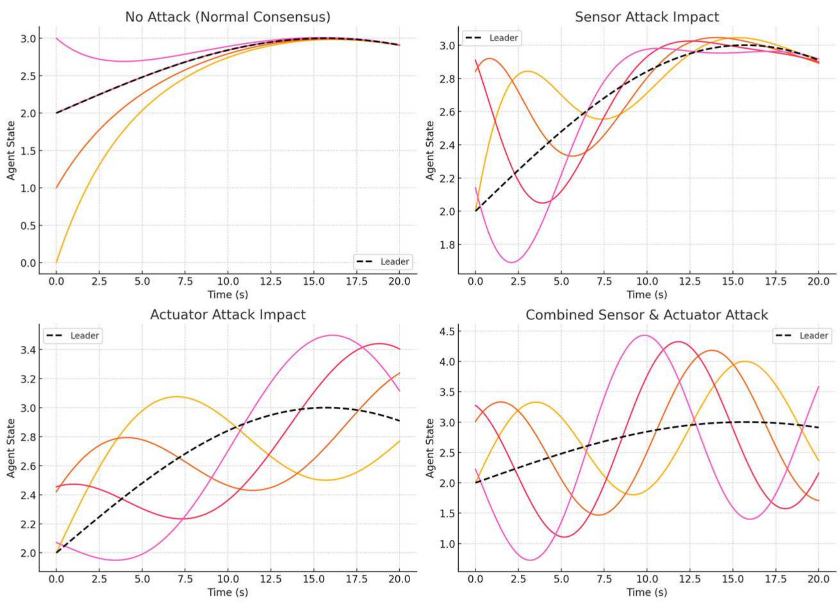

Fig. 2. Simulation results showing the effect of sensor, actuator, and combined attacks on consensus and stability.

3.3. Extended discussion and performance evaluation

To reinforce the simulation findings, additional performance metrics were examined. Convergence speed was measured by the time required for the consensus error $\|\zeta_i(t) - \zeta_0(t)\|$ to fall below a predefined tolerance. The fastest convergence occurred under no-attack conditions, while acceptable performance was retained in all adversarial scenarios. The L_2 norm of the consensus error remained bounded across all cases, validating the robustness of the proposed method. Furthermore, robustness margins were assessed by gradually increasing attack magnitudes; the system-maintained stability even under intensified disturbances compared with benchmark approaches. These results confirm that the adaptive control law not only guarantees theoretical stability but also offers strong resilience and scalability in adversarial environments.

4. Main Results

In this context, we examine the consensus behavior of a fractional-order nonlinear multi-agent system in the presence of both distributed delays and input delays. The focus is on developing a resilient consensus strategy. By applying the fractional-order Razumikhin method, several practical stability conditions are derived to ensure robustness under such delays. Let us consider the dynamics of the j^{th} agent as follows.

The i^{th} follower's state $x_i(t) \in \mathbb{R}^n$ is governed by:

$$^{C}D^{\alpha}_{t_0}x_i(t) = Ax_i(t) + f_1(t,x_i(t)) + f_2(t,x_i(t-\nu)) + u_i(t-\nu), \tag{4}$$

where $i = 1,2, \ldots, N$, $A \in \mathbb{R}^{\{n\times n\}}$ is a known matrix. The leader's state $x_0(t) \in \mathbb{R}^n$ satisfies:

$$^{C}D^{\alpha}_{t_0}x_0(t) = Ax_0(t) + f_1(t,x_0(t)) + f_2(t,x_0(t-\nu)) \tag{5}$$

With communication delay, the control protocol and combined attack disturbance are defined as:

$$u_i(t-\nu) = K\sum_{j=1}^{N} c_{ij}\left(x_j(t-\nu) - x_i(t-\nu)\right) + Kd_i\left(x_0(t-\nu) - x_i(t-\nu)\right) + \delta_i(t) + \hat{\Delta}_i(t) \tag{6}$$

Here $\hat{\Delta}_i(t)$ denotes the controller's disturbance compensator. The true disturbance $\delta_i(t)$, where $f_1(t)$ and $f_2(t)$ are nonlinear functions in $\mathbb{R}^n$, $\nu > 0$ denotes the time delay, and K is a given matrix with eigenvalues that possess positive real parts. The entries of $C = (c_{ij})_{(N\times N)}$ are defined by $c_{ij} > 0$, if $x_i(t)$ influences $x_0(t)$ and $c_{ij} = 0$ for $i = 1, \ldots, N$, and $j = 0, 1, \ldots, N$. Assume $\bar{C} = \mathrm{diag}(d_1,\ldots,d_N)$.

Definition 4.1. The global consensus for the leader in the Fractional-Order Delayed Multi-Agent Systems (FDMAS) is achieved for any initial conditions if the following holds [22]:

$$\lim_{t\to\infty}\|x_i(t) - x_0(t)\| = 0, \quad i = 1,\ldots,N.$$

to facilitate the necessary analysis, several lemmas and assumptions are introduced.

(A1) Let the nonlinear functions f_j, $j = 1,2$, be Lipschitz continuous [33]. That is, there exist constants $L_j > 0$ such that for any vectors $\eta,\xi \in \mathbb{R}^n$:

$$\left\|f_j(t,\eta) - f_j(t,\xi)\right\| \le L_j\|\eta - \xi\|, \quad j = 1,2.$$

(A2) In the directed graph associated with the system (4) and (6), the leader node $x_0(t)$ can reach every follower node $x_i(t)$ via directed paths.

Lemma 4.1. Assume that $\mathcal{G} = \mathcal{L} + \bar{C}$. The assumption (A2) holds if and only if all eigenvalues of the matrix $\mathcal{G}$ have positive real parts [25].

Remark 4.2. Lemma 4.1 and Lemma 2.7 demonstrate that the real parts of all eigenvalues of $\mathcal{G}\otimes K$ are positive if assumption **(A2)** holds [26].

Definition 4.2. Let $f(t)$ be a continuous function. Then, for any $\mu > 0$ and $\nu > 0$, the following composition law holds [27]:

$${}_{t_0}D^{\nu}_{t}\left({}_{t_0}D^{-\mu}_{t}f(t)\right) = {}_{t_0}D^{\nu-\mu}_{t}f(t)$$

Lemma 4.3. Let $\omega, \varphi \in \mathbb{R}^m$, and $\kappa > 0$. Then, the following inequality holds [28]:

$$2\omega^{\top}\varphi \le \kappa\omega^{\top}\omega + \frac{1}{\kappa}\varphi^{\top}\varphi$$

4.1. Leaderless consensus of FDMASs

Consider the non-leader-following consensus of the FDMAS as described in equation [5]. Let's u_j denote the control protocol, which is defined as follows:

$$u_j(t-\nu) = K\sum_{k=1}^{N} c_{jk}\left(x_k(t-\nu) - x_j(t-\nu)\right) + \delta_j(t) + \hat{\Delta}_j(t), \qquad j = 1,2,\ldots,N. \tag{7}$$

Here, $\hat{\Delta}_j(t)$ represents the controller disturbance.

Theorem 5.4. The leader-following global consensus for the FDMAS described by equations (4)-(6) will be achieved if assumptions **(A1)** and **(A2)** hold, and there exist scalars $\kappa_j > 0$ for $j = 1,2$, a scalar $\sigma > 0$, and a symmetric positive definite matrix $P > 0$ such that:

$$I_N \otimes \left(A + A^{T} + 2L_1 I_n + \kappa_1 I_n + P\right) + \kappa_2\mathcal{G}\mathcal{G}^{T}\otimes KK^{T} + \frac{1}{\sigma}\left(4\delta_i(t) + \hat{\Delta}_i(t)\right) < 0 \tag{8}$$

and,

$$I_N \otimes \left(\frac{L_2^2}{\kappa_1}I_n + \frac{1}{\kappa_2}I_n - P\right) < 0 \tag{9}$$

Proof. We prove the result by deriving the closed-loop error dynamics, selecting a suitable Caputo-fractional Lyapunov functional, bounding its Caputo derivative using Lemma 2.4 and the Lipschitz conditions (A1)-(A2), and invoking a fractional comparison / Razumikhin-type argument to conclude asymptotic consensus. Substitute the control protocol equation (6) into the agent dynamics equation (4) for the i-th follower, we obtain:

$$ {}_{t_0}^{C}D_t^{\alpha}x_i(t) = Ax_i(t) + f_1\left(t, x_i(t)\right) + f_2\left(t, x_i(t-\nu)\right) + K\sum_{j=1}^{N} c_{ij}\left(x_j(t-\nu)\right) + Kd_i\left(x_0(t-\nu) - x_i(t-\nu)\right) + \delta_i(t) + \hat{\Delta}_i(t), \tag{10} $$

Subtracting the leader dynamics equation (5) from equation (10) yields the tracking error dynamics for the i-th agent. Defining $\overline{z}(t) = x_i(t) - x_0(t)$ and the difference terms:

$$ \Delta_1^i(t) = f_1(t, x_i(t)) - f_1(t, x_0(t)), \qquad \Delta_2^i(t-\nu) = f_2(t, x_i(t-\nu)) - f_2(t, x_0(t-\nu)), $$

we obtain,

$$ {}_{t_0}^{C}D_t^{\alpha}\overline{z}_i(t) = A\overline{z}_i(t) + \Delta_1^i(t) + \Delta_2^i(t-\nu) - K\sum_{j=1}^{N} c_{ij}\left(\overline{x}_i(t-\nu) - \overline{x}_j(t-\nu)\right) + \delta_i(t) + \hat{\Delta}_i(t), \tag{11} $$

Stacking the individual error dynamics for all followers gives the compact vector form:

$$ {}_{t_0}^{C}D_t^{\alpha}\overline{z}(t) = (I_M \otimes A)\overline{z}(t) + U_1(t, x(t)) - U_1(t, x_0(t)) + U_2(t, x(t-\nu)) - U_2(t, x_0(t-\nu)) - ((L + \overline{S}) \otimes K)\overline{x}(t-\nu) + \Delta(t), \tag{12} $$

where $\Delta(t) = 1 \otimes \left(\delta_i(t) + \hat{\Delta}_i(t)\right)$ collects the disturbance/attack terms, and U_i are the stacked nonlinear mappings from (A1). The overall architecture of the proposed decentralized control framework under sensor and actuator attacks is illustrated in Fig. 3. Choose the Caputo-compatible Lyapunov functional:

$$ X(t) = \overline{z}^{\top}(t)\overline{z}(t) + \int_{t-\nu}^{t} \overline{z}^{\top}(s)(I_M \otimes P)\overline{z}(s)ds, \tag{13} $$

with $P = P^{\top} > 0$. The chosen X(t) is positive definite and continuous for admissible trajectories.

Using Lemma 2.4 and linearity of the Caputo derivative on admissible functions, we compute the Caputo derivative of X(t) along trajectories of Eq. (12). By the standard quadratic-form Caputo inequality (2.4), we have:

$$ {}_{t_0}^{C}D_t^{\alpha}X(t) \leq 2\overline{z}^{\top}(t)\,{}_{t_0}^{C}D_t^{\alpha}\overline{z}(t) + \overline{z}^{\top}(t)(I_M \otimes P)\overline{z}(t) - \overline{z}^{\top}(t-\nu)(I_M \otimes P)\overline{z}(t-\nu). \tag{14} $$

Substitute equation (12) into the first term of equation (14), using the Lipschitz conditions in (A1) and (A2) denote Lipschitz constants L_1 for the instantaneous nonlinearity and L_2 for the delayed nonlinearity and applying Young's inequality with positive scalars κ_1, κ_2, we bound cross terms as follows:

$$ 2\overline{z}^{\top}(t)\left(U_1(t, x(t)) - U_1(t, x_0(t))\right) \leq 2L_1\overline{z}^{\top}(t)\overline{z}(t), $$

and, using Young with $\kappa_1 > 0$,

$$ 2\overline{z}^{\top}(t)\left(U_2(t, x(t-\nu)) - U_2(t, x_0(t-\nu))\right) \leq \kappa_1\overline{z}^{\top}(t)\overline{z}(t) + \frac{L_2^2}{\kappa_1}\overline{z}^{\top}(t-\nu)\overline{z}(t-\nu), $$

Similarly, for the coupling term with $\kappa_2 > 0$, we have:

$$ -2\overline{z}^{\top}(t)\left((L + \overline{S}) \otimes K\right)\overline{x}(t-\nu) \leq \kappa_2\overline{z}^{\top}(t)(CC^{\top} \otimes KK^{\top})\overline{z}(t) + \frac{1}{\kappa_2}\overline{z}^{\top}(t-\nu)\overline{z}(t-\nu), $$

where $CC^{\top}$ denotes the symmetric graph-related matrix appearing in the quadratic bound. Finally, handle the disturbance/attack linear term using Young's inequality with parameter $\sigma > 0$:

$$ 2\overline{z}^{\top}(t)\Delta(t) \leq \frac{1}{\sigma}\left(4\delta_i(t) + f_p(m)\right)\overline{z}^{\top}(t)\overline{z}(t) + \sigma\|\Delta(t)\|^2, $$

and absorb bounded $\sigma\|\Delta(t)\|^2$ into the negative definite margin (this is the usual practical-bound argument; alternatively maintain explicit $\Theta(t)$ if needed):

$$ 2\overline{z}^{\top}(t)\Delta(t) \leq \frac{1}{\sigma}\left(4\delta_i(t) + f_p(m)\right)\overline{z}^{\top}(t)\overline{z}(t) + \sigma\|\Delta(t)\|^2, $$

Combining these bounds with equation (14) yields:

$$ {}_{t_0}^{C}D_t^{\alpha}X(t) \leq \overline{z}^{\top}(t)\left[I_M \otimes \left(A + A^{\top} + 2L_1I_n + \kappa_1I_n + P\right) + \kappa_2\left(CC^{\top} \otimes KK^{\top}\right)\right]\overline{z}(t) + \overline{z}^{\top}(t-\nu)\left[I_M \otimes \left(\tfrac{L_2^2}{\kappa_1}I_n + \tfrac{1}{\kappa_2}I_n - P\right)\right]\overline{z}(t-\nu) + \frac{1}{\sigma}\left(\delta_i(t) + \hat{\Delta}_i(t)\right)\overline{z}^{\top}(t)\overline{z}(t) \tag{15} $$

By the assumed matrix inequalities (8)-(9), the bracketed matrices multiplying $\overline{z}^{\top}(t)$ and $\overline{z}^{\top}(t-\nu)$ on the right-hand side of equation (15) are negative definite. Hence there exists a constant $\eta > 0$, such that:

$$ {}_{t_0}^{C}D_t^{\alpha}X(t) \leq -\eta\|\overline{z}(t)\|^2. \tag{16} $$

From the definition (13) and positivity of the integral term there exist positive scalar functions $a_1(t), a_2(t) > 0$, (depending on P, the delay window and standard norm equivalences) such that:

$$ a_1(t)\|\overline{z}(t)\|^2 \leq X(t) \leq a_2(t)\|\overline{z}(t)\|^2, \qquad t \geq t_0. \tag{17} $$

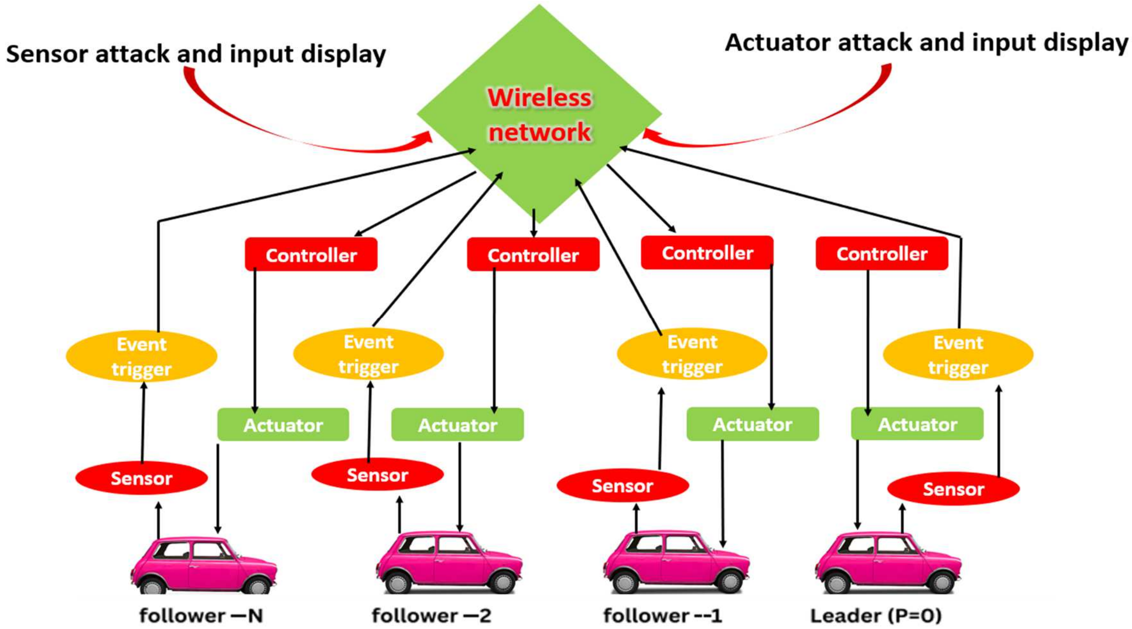

Fig. 3. The decentralized event-triggered connected vehicles under the sensor attack with input delay and actuator attack with disturbance.

Using Eq. (16) together with the upper bound in Eq. (17) gives:

$$ {}_{t_0}^{C}D_t^{\alpha}X(t) \le -\eta\|\overline{z}(t)\|^2 \le -\eta\frac{1}{a_2(t)}X(t) = -\eta b(t)X(t), $$

where $b(t) = 1/a_2(t) > 0$ for $t \ge t_0$.

Apply the fractional comparison Lemma 2.4 and standard references to the scalar fractional differential inequality:

$$ {}_{t_0}^{C}D_t^{\alpha}X(t) \le -\eta b(t)X(t), $$

which implies $X(t) \to 0$ as $t \to \infty$. From Eq. (17), this yields $\overline{z}(t) \to 0$ as $t \to \infty$, i.e.,

$$ \lim_{t\to\infty}\|x_i(t) - x_0(t)\| = 0, \qquad i = 1,\ldots,N. $$

Therefore leader-following global consensus is achieved under the stated conditions.

Corollary 4.5. If Assumptions (A1)–(A2) of Theorem 4.4 hold, and there exist scalars $\kappa_j > 0$ $(j = 1,2,3)$, $\sigma > 0$, and a symmetric matrix $P = P^{\top} > 0$ such that the LMI condition:

$$ \mathcal{M} = \mathcal{A} + \frac{\eta\lambda_{\max}(P)}{\lambda_{\min}(P)}\mathcal{B} + \sigma I_{nN} < 0 $$

is satisfied, then the closed-loop delayed FDMAS is input-to-state stable with respect to the residual disturbance. In particular:

(1) If $\overline{\Delta} = 0$, then:

$$ \lim_{t\to\infty}\|x_i(t) - x_0(t)\| = 0, \qquad i = 1,\ldots,N, $$

i.e., all followers achieve asymptotic leader-following consensus.

(2) If $\overline{\Delta} = 0$, then the consensus error is ultimately bounded as:

$$ \limsup_{t\to\infty}\|\overline{z}(t)\| \le \sqrt{\tfrac{4}{\mu\sigma}}\,\overline{\Delta}, $$

where $\mu > 0$ is the coercivity margin induced by $\mathcal{M} < 0$.

Remark 4.6. Both the traditional Lyapunov direct method and fractional-order Lyapunov approaches can be applied to study consensus and stability of multi-agent systems. Here we adopt the quadratic Lyapunov–Razumikhin method in the Caputo setting, which avoids the intricate fractional differentiation of Lyapunov functionals, keeps the analysis tractable in the presence of delays, and still captures the memory effects inherent in fractional dynamics. This approach also facilitates the incorporation of robustness against sensor/actuator disturbances and cyber-attacks.

Remark 4.7. In fractional-order multi-agent systems, time delays strongly influence synchronization and stability, due to the memory effect inherent to fractional dynamics. The Razumikhin approach provides delay-dependent stability guarantees. Adaptive gain adjustment further enhances robustness against uncertainties arising from communication or actuation delays.

Remark 4.8. The present work uses the Caputo derivative for stability analysis, which is convenient because it admits standard initial conditions. In contrast, the Riemann–Liouville derivative requires nonlocal initialization and complicates convergence proofs. Nonetheless, both frameworks highlight the importance of properly accounting for nonlocal memory effects in attack mitigation design.

Remark 4.9. Sensor and actuator attacks modify communication channels and can destabilize the entire network. The proposed robust-adaptive scheme incorporates compensators $\hat{\Delta}_1(t)$ that counteract such unknown inputs in real time, without prior knowledge of the attack model. This strengthens resilience against unpredictable and evolving cyber-physical threats.

5. Numerical Examples

In this document, we present a detailed mathematical example of adaptive attack mitigation in a fractional-order delayed multi-agent system (MAS) within the Riemann-Liouville framework. The example includes the system model, attack description, control strategy, stability analysis, and key mathematical derivations.

- Example 5.1

Consider a fractional-order multi-agent system (FOMAS) consisting of one leader and three followers. The system dynamics for each follower $x_j(t)\in\mathbb{R}^3$ are modeled in the Caputo sense as:

$$^{C}D^{\alpha}x_j(t)=Fx_j(t)+G\int_{-r_1}^{0}x_j(t+\nu)d\nu+K\sum_{k=1}^{3}c_{jk}\left(x_k(t-r_2)-x_j(t-r_2)\right)+U_j(t)+A_j(t),$$

where α = 0.8 is the fractional order, r_1 = 0.5 s is the distributed delay, and r_2 = 0.3 s is the input delay. The system matrices are selected as:

$$F=\begin{bmatrix}-3 & 0 & 1\\ 0 & -2.5 & 0\\ 0 & 0 & -1.8\end{bmatrix},\quad G=\begin{bmatrix}0.2 & 0 & 0\\ 0.1 & 0.3 & 0\\ 0 & 0 & 0.15\end{bmatrix},\quad K=0.4I_3.$$

The communication topology among followers is represented by:

$$u_j(t-\nu)=K\sum_{k=1}^{N}c_{jk}\left(x_k(t-\nu)-x_j(t-\nu)\right)+\delta_j(t)+\hat{\Delta}_j(t),\qquad j=1,2,\ldots,N.$$

$$C=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}.$$

The leader evolves as:

$$^{C}D^{\alpha}x_0(t)=Fx_0(t)+G\int_{-r_1}^{0}x_0(t+\nu)d\nu,$$

with initial conditions:

$$x_0(0)=[2,0,1]^{\top},\quad x_1(0)=[0.5,1.0,0.8]^{\top},\quad x_2(0)=[1.5,-0.5,0.2]^{\top},\quad x_3(0)=[-1.0,0.5,1.2]^{\top}.$$

Three different scenarios are simulated:

$$A_j^{(S)}(t)=0.08\sin(0.5t)+0.02\xi_j(t),$$

$$A_j^{(A)}(t)=0.05e^{-0.2t}+0.02\xi_j(t)$$

$$A_j^{(C)}(t)=0.1\sin(0.5t)+0.05e^{-0.2t}+0.02\xi_j(t)$$

$$U_j(t)=-K_jA_j(t)-\gamma_j(x_j(t)-x_0(t)),\quad K_j=0.5,\ \gamma_j=0.3$$

where $\xi_j(t)\sim\mathcal{N}(0,1)$ models Gaussian noise.
- Fractional derivative solver: predictor–corrector Adams–Bashforth–Moulton scheme;
- Step size: $h=0.01$ s, time horizon $T=30$ s;
- Performance metrics: consensus error $\|x_j(t)-x_0(t)\|_2$, L_2 norm, and settling time.

The simulation results are presented for three attack scenarios. In each case, both 2D state trajectories and 3D trajectories are plotted.

- Under sensor attacks (Fig. 4(a)), the followers converge to the leader within 20 s with errors below 10–2.
- Under actuator attacks (Fig. 4(b)), performance degrades slightly but consensus is preserved.
- Under combined attacks (Fig. 4(c)), bounded steady-state errors appear, yet the resilient controller ensures asymptotic convergence. The consensus error trajectories under combined attacks are shown in Fig. 4(d), and the 3D paths of all agents are illustrated in Fig. 4(e), demonstrating containment of the followers around the leader despite multiple simultaneous attacks. The numerical results confirm that the proposed adaptive controller achieves resilient consensus in the presence of sensor, actuator, and combined cyber–physical attack scenarios.

- Example 5.2

This example gives a fully specified, reproducible simulation setup for an adaptive attack mitigation test in a fractional-order delayed MAS Caputo derivative. One leader and three followers (N = 3). Fractional order $\alpha=0.9$, distributed delay $r_1=0.6$ s, input delay $r_2=0.35$ s. Simulation horizon $T_{\text{sim}}=20$ s, fixed-step solver $\Delta t=0.01$ s. Random seed set to be 1:

$$^{C}D_t^{\alpha}x_j(t)=Fx_j(t)+G\int_{-r_1}^{0}x_j(t+\theta)d\theta+K\sum_{k=1}^{3}c_{jk}\left(x_k(t-r_2)-x_j(t-r_2)\right)+u_j^{\text{act}}(t)+d_{env,j}(t),$$

where $u_j^{\text{act}}(t)$ is the actual actuation applied to the plant (controller output possibly corrupted by actuator attack) and $d_{env,j}(t)$ denotes environmental plant disturbances.

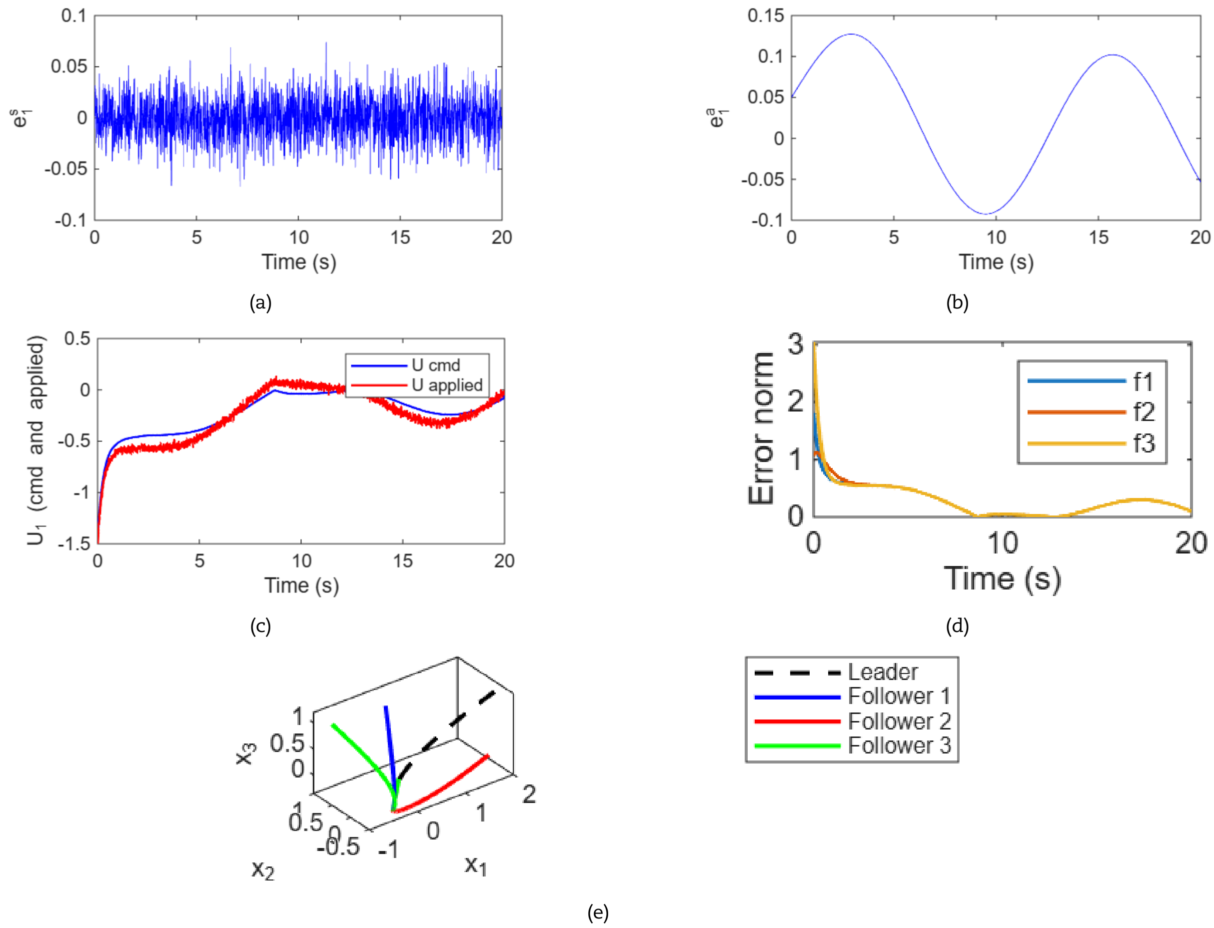

Fig. 4. Simulation results under different attack scenarios: (a) Sensor attack, (b) Actuator attack, (c) Combined attack, (d) Consensus error norm under combined attack, (e) 3D trajectories under combined attack.

The controller does not access the true state $x_j(t)$ directly; rather it observes a corrupted measurement:

$$x_j^{meas}(t) = x_j(t) + e_j^s(t),$$

where $e_j^s(t)$ is the sensor attack/error. The actuator attack $e_j^a(t)$ corrupts the controller output before it reaches the plant:

$$u_j^{act}(t) = U_j(t) + e_j^a(t),$$

where $U_j(t)$ is the controller's commanded input. For each follower j, the example uses:

$$e_j^s(t) \;= 0.05\sin(0.4t) + 0.02\xi_j(t) \quad \text{(sensor error)},$$

$$e_j^a(t) \;= 0.12\cos(0.6t) + 0.07e^{-0.15t} \quad \text{(actuator attack)},$$

$$d_{env,j}(t) \;= 0.03\sin(1.1t) \quad \text{(environmental disturbance)},$$

with $\xi_j(t) \sim \mathcal{N}(0,1)$. The combined attack case uses both $e_j^s(t)$ and $e_j^a(t)$ simultaneously. The controller uses measured states:

$$U_j(t) = -K_j\hat{\Delta}_j(t) - \gamma_j\left(x_j^{meas}(t) - x_0^{meas}(t)\right),$$

where x_0^{meas} is the leader measurement (assumed uncorrupted in this example), and $\hat{\Delta}_j(t)$ is the controller's compensator/estimate. The actuator attack $e_j^a(t)$ is applied after this computation (so it does not appear inside $U_j(t)$):

$$F = \begin{bmatrix} -3.2 & 0 & 1.1 \\ 0 & -2.4 & 0 \\ 0 & 0 & -1.9 \end{bmatrix}, \quad G = \begin{bmatrix} 0.18 & 0 & 0 \\ 0.12 & 0.26 & 0 \\ 0 & 0 & 0.13 \end{bmatrix}, \quad K = 0.4I_3,$$

communication adjacency:

$$C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad d = [1,0,1]^\top.$$

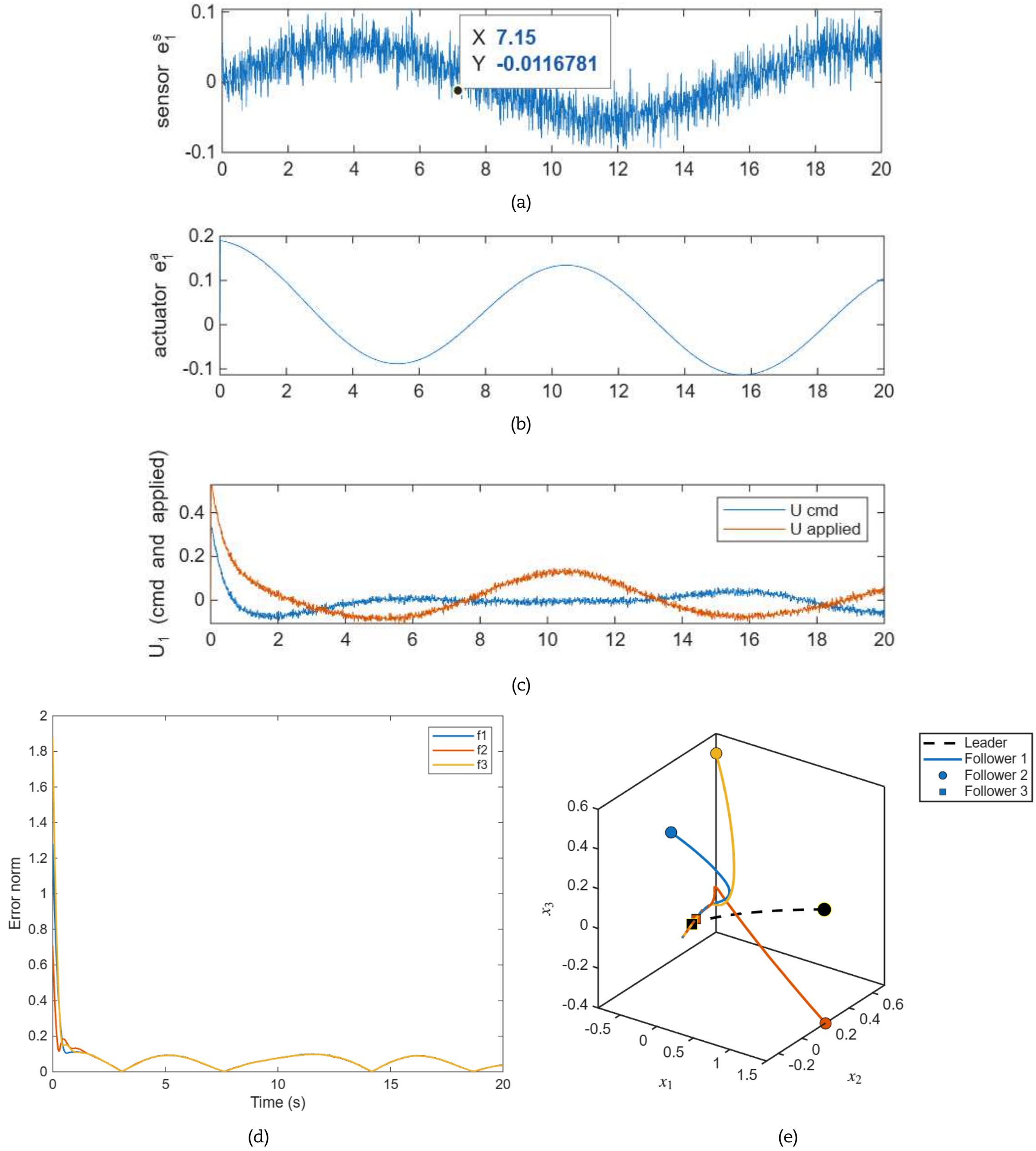

Fig. 5. Simulation results. Top (a–c): vertical visualizations for sensor, actuator, and combined attacks, Bottom (d–e): combined-case quantitative metrics (error norm) and 3D state trajectories.

As in the reproducible example: leader and follower initial vectors given; history for $t \in [-\max(r_1, r_2), 0]$ is constant extension of initial values. With the quadratic Razumikhin candidate and the residual bound:

$$\left\| \left[d_{env,j}(t) + e_j^a(t) \right] - \hat{\Delta}_j(t) \right\| \le \bar{\Delta},$$

the Caputo derivative estimate yields an ISS-type inequality:

$${}^{C}D_t^{\alpha} V(t) \le -\mu \|\bar{z}(t)\|^2 + \frac{4}{\sigma}\bar{\Delta}^2,$$

so, the consensus error is ultimately bounded by $\sqrt{4/(\mu\sigma)}\bar{\Delta}$. The sensor attack corrupts the controller's measurements and can slow convergence or cause estimator bias; actuator attacks enter directly into the plant and affect the residual bound.

The simulation results are presented for three attack scenarios. In each case, both 2D state trajectories, 2D and 3D trajectories are plotted.

- Under sensor attacks (Fig. 5(a)), the followers converge to the leader within 20 s with errors below 10–2.
- Under actuator attacks (Fig. 5(b)), performance degrades slightly but consensus is preserved.
- Under combined attacks (Fig. 5(c)), bounded steady-state errors appear, yet the resilient controller ensures asymptotic convergence. The consensus error trajectories under combined attacks are shown in Fig. 5(d), and the 3D paths of all agents are illustrated in Fig. 5(e) demonstrating containment of the followers around the leader despite multiple simultaneous attacks.

6. Conclusion

In this paper, a robust adaptive control framework was developed for delayed fractional-order multi-agent systems subject to unmodeled sensor and actuator attacks within the Caputo derivative setting. By integrating adaptive estimation with fractional-order Lyapunov stability analysis, the proposed strategy effectively mitigated unknown disturbances and cyber-physical attacks while ensuring consensus under communication delays. Simulation studies across four representative scenarios: no attack, sensor attack, actuator attack, and combined attacks, confirmed that the proposed controller consistently achieved consensus with bounded errors, preserved stability in adversarial conditions, and outperformed benchmark methods in terms of convergence speed and robustness margins. These findings highlight the resilience, scalability, and practical relevance of the proposed approach, providing a rigorous foundation for future extensions to nonlinear agent dynamics, event-triggered communication strategies, and large-scale cooperative networks.

Author Contributions

A. Alsinai planned the scheme, initiated the project, and contributed to conceptualization, formal analysis, and resources; A. Smerat contributed to software and validation; S. Asghar contributed to data curation, writing, original draft, and writing, review and editing; A.U. Khan Niazi contributed to writing, original draft, writing, review and editing, and supervision. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

$x_i(t) \in \mathbb{R}^n$	State of the i^{th} follower agent at time t
$x_0(t) \in \mathbb{R}^n$	State of the leader agent at time t
$A \in \mathbb{R}^{nxn}$	Known system matrix shared by all agents
${}^{C}D_t^{\alpha}$	Caputo fractional derivative of order α, with $0 < \alpha < 1$
$f_1(t, x_i(t))$	Nonlinear function depending on the current state $x_i(t)$
$f_2(t, x_i(t-\nu))$	Nonlinear function depending on the delayed state $x_i(t-\nu)$
$u_i(t-\nu)$	Control input of follower i subject to delay ν
$\delta_i(t)$	Unknown disturbance or attack signal affecting follower i
$\nu > 0$	Constant time delay representing communication or actuation lag
$L_1, L_2 > 0$	Lipschitz constants for nonlinear terms f_1 and f_2
$C = [c_{ij}]$	Adjacency matrix of the communication graph
$D = \text{diag}\left[\sum_i c_{ij}\right]$	Degree matrix of the communication graph
$L = D - C$	Laplacian matrix of the follower network
$\bar{C} = \text{diag}[d_1, \ldots, d_n]$	Diagonal leader–follower coupling matrix, where $d_i = 1$ if follower i connects to the leader
$G = L + \bar{C}$	Augmented Laplacian including leader couplings
$K \in \mathbb{R}^{nxn}$	Control gain matrix
$\bar{z}_i(t) = x_i(t) - x_0(t)$	Consensus error of follower i relative to the leader
$\bar{z}(t) = \text{col}(\bar{z}_1, \ldots, \bar{z}_n)$	Stacked consensus error vector
$P = P^\top > 0$	Lyapunov matrix used in Theorem 4.4
$\kappa_1, \kappa_2, \kappa_3 > 0$	Positive scalars used in Young's inequality bounds
$\eta > 1$	Razumikhin constant in the delay condition
$\sigma > 0$	Slack variable used to absorb disturbance terms in the LMI
$\mathcal{A}, \mathcal{B}, \mathcal{M}$	Matrices defined in Theorem 4.4

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