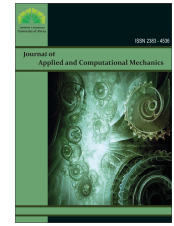




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## Research Paper

# Magnetohydrodynamic Mixed Convection of a Non-Newtonian Nanofluid over a Vertical Cylinder: Slip Effects, Heat Transfer Enhancement, and Entropy Generation

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**Abstract.** Mixed-convection magnetohydrodynamic (MHD) flows of non-Newtonian nanofluids are of considerable importance in advanced thermal management systems, energy conversion devices, and industrial cooling applications due to their enhanced heat-transfer capabilities. The primary objective of this study is to investigate the effects of magnetic field strength, velocity slip, thermal slip, and Brownian motion on the flow behavior, heat transfer characteristics, nanoparticle concentration distribution, and entropy generation in a non-Newtonian nanofluid flowing over a vertical cylinder. The governing nonlinear partial differential equations are transformed into a system of nonlinear ordinary differential equations using suitable similarity transformations and solved analytically using the Homotopy Analysis Method (HAM). The results reveal that increasing the magnetic parameter significantly suppresses the fluid velocity due to the Lorentz force while enhancing the heat transfer rate by approximately 20%. An increase in the thermal slip parameter reduces the thermal boundary layer thickness and lowers the fluid temperature distribution near the cylinder surface. Furthermore, stronger Brownian motion substantially modifies nanoparticle concentration profiles and influences heat transport within the boundary layer. Entropy generation increases with both the magnetic parameter and the temperature gradient parameter, indicating greater thermodynamic irreversibility. The present findings provide valuable guidelines for the design and optimization of nanofluid-based thermal systems, including heat exchangers, electronic cooling devices, energy transport units, and other heat transfer enhancement technologies.

**Keywords:** Magnetohydrodynamics (MHD), Mixed convection flow, non-Newtonian nanofluids, Multiple slip effects, Entropy generation, Vertical cylinder, Heat and mass transfer, Thermodynamic efficiency, Nanoparticle concentration.

## 1. Introduction

When nanoparticles are suspended in a base fluid, the resulting fluid is known as a nanofluid. The suspended nanoparticles' sizes are in the nanometer range. The nanoparticles are made of oxides, metals, and carbides, whereas the base fluid is usually oil, water, or glycol. Nanofluid applications attract many researchers. Nanofluids boost the efficiency of the thermal system by enhancing the heat transfer rate. Nanofluids have vast applications in many fields, including medical and engineering. In the medical field, they are used to decrease the thermal resistance of equipment. Routbort [1] states that nanofluids are very effective for industrial cooling. It can also be used on computers to reduce microchips' temperature. Sheikholeslami et al. [2] studied natural convection in thermal radiation. Al-Odat and Ghamdi [3] also numerically investigated natural convection flow over a vertical plate and analyzed the effects of various parameters. Rudra Deka and Neog [4] examined the unsteady natural convection flow passing over the vertical plate. Asimoni et al. [5] studied the magnetohydrodynamic convective flow passing over a vertical plate. They concluded that increasing the magnetic field increases temperature and decreases velocity. Daniel et al. [6] studied the unsteady convective flow passing over a vertical plate.

Hang Xu et al. [7] employed the Buongiorno model to investigate the flow of mixed-convection nanofluid in a vertical channel. They found that, with a correctly selected nanofluid, heat transfer improves. Liu and Lo [8] numerically examined the entropy generation number in a mixed-convection magnetohydrodynamic flow through a vertical channel. They found that the entropy generation and the Bejan number are, respectively, minimum and maximum at the center of the channel. Nazar et al. [9] analyzed the mixed-convection nanofluid flow over a horizontal cylinder embedded in a porous medium filled with nanofluid. Rana et al. [10] examined the mixed-convection flow of alumina-water nanofluid over the hollow cylinder. Grosan and Pop [11] numerically studied the axisymmetric mixed convection past a vertical cylinder in a nanofluid. Sheikholeslami and Ganji [12] studied the MHD and ferrohydrodynamics effects on convective heat transfer and Ferrofluid flow.

In fluid dynamics, the no-slip condition is the boundary condition applied at a solid boundary. Here, the fluid velocity is zero relative to the boundary. However, no-slip conditions do not hold in nanofluid flow and non-Newtonian fluids. Slip conditions are



needed to investigate such flows and obtain precise results. Yoshimura et al. [13] analyzed the partial slip conditions and concluded that, at the first derivative, velocity and stress have non-zero finite values at the boundary. Hayat et al. [14] examined the impact of homogeneous and heterogeneous reactions in the MHD flow of nanofluid related to second-order slip velocity. Gul et al. [15] investigated heat transfer in MHD mixed-convection flow of a Ferrofluid along a vertical channel.

Anderson [16] investigated magnetohydrodynamic slip flow on a stretching surface and solved the Navier-Stokes equation. Zheng et al. [17] studied radiative convection nanofluid passing through a sheet to analyze the effects of hydrodynamic and thermal slip. With Buongiorno's model, Dhanai et al. [18] studied the magnetohydrodynamic mixed-convection critical value parameter for the existence of dual solutions that pass through an inclined cylinder with thermal slip effects and first-order velocity slip. Dhanai et al. [19] have employed Lie group analysis to study the effects of slip on magnetohydrodynamic bioconvective flow past an inclined sheet. The energy in thermal systems vanishes due to irreversibility, including viscous dissipation, diffusion, and other such effects. This leads to entropy generation in such thermal systems. Bejan et al. [20] derived the entropy generation equation for forced convective heat transfer from a circular cylinder and a plate. The effects of Reynolds number and magnetic field on entropy generation have been investigated by Aiboud et al. [21]. With the optimal HAM method, Abolbashari et al. [22] investigated viscoelastic nanofluid flow. Nogrehabadi et al. [23] also analyzed this phenomenon with a first-order velocity-slip effect. Recent advances in thermal-fluid systems have significantly improved the understanding of coupled heat and momentum transport in complex engineering configurations, particularly in microscale and nanoscale applications. For example, Jagadha et al. [24] investigated the influence of higher-order chemical reactions and Joule heating on radiative magnetohydrodynamic Casson nanofluid flow, highlighting the importance of nonlinear transport phenomena in thermal systems. Modern investigations continue to emphasize enhanced modeling strategies for improving thermal efficiency and flow control.

Microchannel and nanomaterial-based heat transfer systems have attracted considerable attention due to their compact geometry and superior heat removal capability. Kumar et al. [25] examined the effects of time-varying nanofluid flow and particle aggregation on homogeneous and heterogeneous reaction kinetics in a squeezing porous slider, demonstrating the critical role of nanoparticle transport mechanisms in thermal-fluid applications.

Due to their enhanced thermophysical properties, nanofluids have emerged as effective heat-transfer media. The significance of nanoparticle dynamics and transport processes has been demonstrated by Gowda et al. [26], who analyzed magnetic and thermal characteristics using a Touchard polynomial collocation technique. Such studies indicate that Brownian motion, thermophoresis, and magnetic interactions strongly influence heat and mass transfer behavior. Entropy generation and thermal optimization have become important considerations in modern thermal systems. Kumar and Sharma [27] employed machine-learning-assisted optimization techniques to investigate surface-drag phenomena in coaxial disk systems, emphasizing the importance of minimizing energy losses and improving system efficiency. In thermal engineering applications, significant efforts have been devoted to improving heat transfer performance. Kumar et al. [28] conducted a computational analysis of a wet triangular spine fin and demonstrated the effectiveness of advanced numerical approaches in enhancing thermal performance. Similarly, Bankar et al. [29] investigated time-varying fluid mechanisms using an airfoil-polynomial-based collocation method, providing valuable insights into coupled heat transfer and fluid-flow behavior. Transport phenomena in complex fluids have received increasing attention due to their nonlinear rheological characteristics and practical engineering relevance. Varun Kumar et al. [30] analyzed the thermal performance of perforated fin structures with internal heat generation and highlighted the strong coupling between fluid flow and thermal transport mechanisms. Furthermore, Punith Gowda et al. [31] studied nanofluid flow in rough rotating disk systems under different magnetic field configurations, demonstrating the significant influence of magnetic effects on flow and thermal characteristics. Chemical engineering applications frequently involve coupled momentum, heat, and mass transfer processes. Banakar et al. [32] investigated particle transport and accumulation in chemically reactive fluids near off-centered stagnation points, while Kattimani et al. [33] analyzed temperature-induced and heat-driven diffusion effects in stagnation-point flows. These studies further confirmed the importance of species diffusion and thermal gradients in determining overall system behavior. Despite these significant advancements, limited attention has been devoted to the combined effects of magnetohydrodynamic mixed convection, nonlinear slip conditions, and entropy generation in non-Newtonian nanofluid flow over cylindrical geometries. In particular, the simulation interactions between Brownian motion, thermophoresis, and dual-slip effects remain insufficiently explored.

The study of magnetohydrodynamic (MHD) mixed convection in non-Newtonian nanofluid flows is critical for advancing thermal management technologies and optimizing energy-efficient systems across various industrial applications. Incorporating multiple slip effects provides a more realistic representation of boundary conditions encountered in micro- and nanofluidic devices, enhancing the applicability of the findings to modern engineering challenges. Furthermore, understanding the interplay of heat transfer and entropy generation in such flows is vital for designing sustainable thermal systems with minimal energy dissipation. Despite significant progress in MHD nanofluid research, limited studies have addressed the combined effects of non-Newtonian fluid properties, multiple slips, and entropy generation over complex geometries such as vertical cylinders. This research seeks to bridge these gaps, offer novel insights into the thermofluidic behavior of nanofluids under magnetic fields, and pave the way for innovative solutions for heat transfer optimization and energy conservation. The subsequent problems are highly nonlinear in fluid mechanics. Most of the difficulty comes from solving such problems. It is almost impossible to obtain an analytically convergent series solution to such problems. Many methods have been applied to solve such problems numerically and analytically. The most important and widely used method in fluid mechanics is the homotopy analysis method. It is a very powerful technique for obtaining the solution of coupled, non-coupled, and non-homogenous nonlinear equations. We are free to choose the appropriate guesses and auxiliary linear operators. Many investigators have effectively used this technique. The main aim of this research is to analyze the thermal effects and the effect of order-two velocity slip as the fluid passes over the vertical cylinder. The technique used to solve subsequent problems is the HAM method.

## 2. Mathematical Modeling

Consider a Newtonian, consistent, and incompressible fluid in the horizontal and vertical axes. The  $r$  - axis and  $z$  - axis referred to as horizontal and vertical. The fluid is passing over the vertical cylinder. The magnetic field, as shown in Fig. 1, is applied in the direction of  $r$  - axis, showing the fluid's electrical conductivity. The velocity that is used to stretch the fluid is  $W_w = W_0 z$ . The temperature of the cylinder surface and surroundings is assumed to be  $T_w$  and  $T_\infty$ , and it is considered that the value of  $T_w$  is larger than the value of  $T_\infty$ . Due to the temperature difference, the buoyancy force is obtained on the vertical axis. On the surface of the cylinder, the concentration has been controlled by the condition given in Eq. (1):

$$D_B \frac{\partial C}{\partial r} + \frac{D_T}{T_\infty} \frac{\partial C}{\partial r} = 0. \quad (1)$$



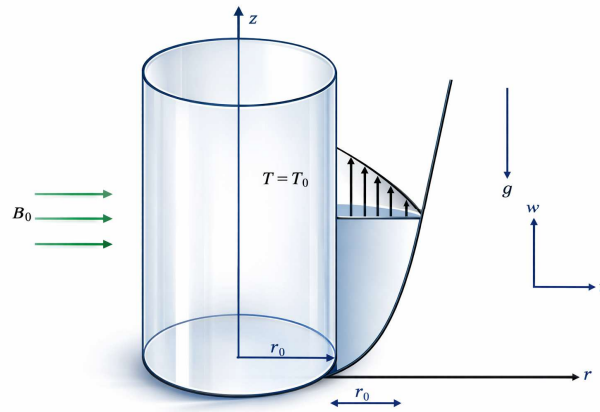


Fig. 1. Geometry of the problem.

Here,  $C_\infty$  is the ambient concentration. The effects of external forces and the pressure gradient are neglected. The resulting equations are given below:

$$\frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0. \quad (2)$$

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = \frac{v}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w}{\partial r} \right) - \frac{\sigma B_0^2}{\rho} w + \left[ (1 - C_\infty)(T - T_\infty)\beta - \frac{(\rho_p - \rho)(C - C_\infty)}{\rho} \right] g - \lambda_1 \left[ w^2 \frac{\partial^2 w}{\partial z^2} + u^2 \frac{\partial^2 w}{\partial r^2} + 2uw \frac{\partial^2 w}{\partial r \partial z} \right] = 0. \quad (3)$$

$$(\rho c_f) \left( u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = \frac{k}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + (\rho c_p) \left[ D_B \frac{\partial C}{\partial r} \frac{\partial T}{\partial r} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial r} \right)^2 \right] + \mu \left( \frac{\partial w}{\partial r} \right)^2. \quad (4)$$

$$u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} = \frac{D_B}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) + \frac{D_T}{T_\infty} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right). \quad (5)$$

Here, the velocity components are  $u$  and  $w$ , along  $(r, z)$  axis. The dynamic viscosity and kinematic viscosity are represented by  $\nu$  and  $\mu$ , respectively. The thermal conductivity and volumetric thermal expansion coefficient of the nanofluid are represented by  $k$  and  $\beta$ , respectively. The diffusion coefficients for thermophoresis and Brownian motion are represented by  $D_T$  and  $D_B$ , respectively. The base fluid and nanoparticles' heat capacities are denoted with  $\rho c_f$  and  $\rho c_p$ . The densities of the fluid and nanoparticles are represented with  $\rho$  and  $\rho_p$ . The gravitational constant is  $g$ , the electrical conductivity of the nanofluid is denoted with  $\sigma$ , and the concentration of nanoparticles and temperature of the nanofluid are represented with  $C$  and  $T$ .

The boundary conditions as described by Dhanai et al. [18, 19] are stated below:

At  $r = r_0$ :

$$u_w = u_w(z), \quad w(zr) = W_w + W_s, \quad T = T_w + N \frac{\partial T}{\partial r}, \quad D_B \frac{\partial C}{\partial r} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial r} = 0, \quad (6)$$

As  $r \rightarrow \infty$ :

$$w(zr) = 0, \quad T = T_\infty, \quad C = C_\infty, \quad (7)$$

The velocity slip  $W_s$  is represented as:

$$W_s = \frac{2}{3} \left( \frac{3 - \alpha_m l^3}{\alpha_m} - \frac{3}{2} \frac{1 - l^2}{K_n} \right) \lambda \frac{\partial w}{\partial r} - \frac{1}{4} \left( l^4 + \frac{2}{K_n^2} (1 - l^2) \right) \lambda^2 \frac{\partial^2 w}{\partial r^2} = A \frac{\partial w}{\partial r} + B \frac{\partial^2 w}{\partial r^2}, \quad (8)$$

The following similarity transformations are used to transform the Eqs. (2) to (5) and the boundary conditions (6) to (8) as described by Dhanai et al [18, 19]:

$$\eta = \frac{r^2 - r_0^2}{2r_0} \sqrt{\frac{W_0}{v}}, \quad \psi = \sqrt{W_0} v z r_0 f(\eta), \quad u = -\frac{r_0}{r} \sqrt{W_0} v f(\eta), \quad (9)$$

$$w = W_0 z f'(\eta), \quad \phi = \frac{C - C_\infty}{C_\infty}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}.$$

The stream function  $\psi$  is defined as  $u = -\frac{1}{r} \frac{\partial \psi}{\partial r}$  and  $w = \frac{1}{r} \frac{\partial \psi}{\partial z}$ , and  $\eta$  is the similarity variable. The continuity equation (2) has been satisfied with the stream function, whereas Eqs. (3) to (5) have been transformed into the following ordinary differential equations of Momentum, energy, and concentration with the help of similarity transformation:

$$(1 + 2\eta\gamma) f'''' + 2\gamma f'' - M^2 f' + f f'' - f'^2 + Ri(\theta - Nr\phi) - \lambda_1 W_0 \left[ \left( \frac{\gamma}{1 + 2\eta\gamma} \right) f^2 f'' + f^2 f''' - 2f f' f'' \right] = 0, \quad (10)$$

$$\frac{[(1 + 2\eta\gamma)(\theta'' + Nb\phi'\theta' + Nt\theta'^2) + 2\gamma\theta']}{Pr} + (1 + 2\eta\gamma) E_C f'^2 + f\theta' = 0, \quad (11)$$

$$(1 + 2\eta\gamma)\phi'' + 2\gamma\phi' + 2 \left( \frac{Nt}{Nb} \right) \gamma\theta' + (1 + 2\eta\gamma) \left( \frac{Nt}{Nb} \right) \theta'' + Scf\phi' = 0, \quad (12)$$



The transformed boundary conditions are:

When  $\eta = 0$ ,

$$\left. \begin{aligned} f(0) &= 0, & f'(0) &= 1 + \lambda_1 f''(0) + \lambda_2 f'''(0), \\ \theta &= 1 + \delta \theta'(0), & Nb\phi'(0) + Nt\theta'(0) &= 0 \end{aligned} \right\} \quad (13)$$

and as  $\eta \rightarrow \infty$ ,

$$\phi(\eta) = 0, \quad \theta(\eta) = 0, \quad f'(\eta) = 0, \quad (14)$$

### 3. Second Law Entropy Generation Analysis

Bejan et al. [20] derived the following expression for entropy generation:

$$S_{gen} = \frac{k}{T_\infty^2} \left( \frac{\partial T}{\partial r} \right)^2 + \frac{\mu}{T_\infty} \left( \frac{\partial w}{\partial r} \right)^2 + \frac{RD}{C_\infty} \left( \frac{\partial C}{\partial r} \right)^2 + \frac{RD}{T_\infty} \left( \frac{\partial T}{\partial r} \right) \left( \frac{\partial C}{\partial r} \right) + \frac{\sigma B_0^2}{T_\infty} w^2. \quad (15)$$

The above equation shows the effects that generate entropy. Due to heat transfer, heat-transfer irreversibility is the first effect of entropy generation. Next is the fluid-friction irreversibility, which is caused by viscous dissipation. The third effect is diffusion irreversibility, caused by the transfer of mass over a finite concentration difference. The last effect is due to the magnetic field.

The characteristic entropy is represented as:

$$S_c = \frac{k(\Delta T)^2}{z^2 T_\infty^2}. \quad (16)$$

The entropy number is expressed as:

$$N_S = \frac{S_{gen}}{S_c}. \quad (17)$$

Using the similarity transformation, the entropy generation number is expressed as:

$$N_S = (1 + 2\eta\gamma)Re \left[ \theta'^2 + \frac{PrEc}{\Omega} f''^2 + \frac{\chi}{\Omega^2} \phi'^2 + \frac{\chi}{\Omega} \theta' \phi' \right] + \frac{M^2 Re}{\Omega} PrEc f'^2. \quad (18)$$

where  $\chi = \frac{RDC_\infty}{k}$  is the diffusion constant parameter and  $\Omega = \frac{\Delta T}{T_\infty}$  is the dimensionless temperature.

### 4. Analytical Solution via HAM

The HAM method is applied to get the analytical power solution. For this, the base function used is  $\{\eta^m e^{-c\eta} | m \geq 0, c \geq 0\}$ . As described by Van Gorder [34], we have chosen the initial assumed values and linear operators as:

$$F_0(\eta) = \frac{1}{1 + \lambda_1 - \lambda_2} (1 - \exp(-\eta)), \theta_0(\eta) = \frac{1}{1 + \delta} \exp(-\eta), \quad (19)$$

$$\phi_0(\eta) = -\frac{Nt}{Nb} \exp(-\eta), 1 + \lambda_1 - \lambda_2 \neq 0, 1 + \delta \neq 0, \quad (20)$$

$$L_F(F) = \frac{\partial^3 F}{\partial \eta^3} + \frac{\partial^2 F}{\partial \eta^2}, L_\theta(\theta) = \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial \theta}{\partial \eta}, L_\phi(\phi) = \frac{\partial^2 \phi}{\partial \eta^2} + \frac{\partial \phi}{\partial \eta}, \quad (21)$$

where  $L_F, L_\theta$ , and  $L_\phi$  are satisfied, the conditions are:

$$L_F(C_1 + C_2\eta + C_3e^{-\eta}) = 0, L_\theta(C_4 + C_5e^{-\eta}) = 0, L_\phi(C_6 + C_7e^{-\eta}) = 0, \quad (22)$$

And the following are the deformation equations:

$$(1 - q)L_F[\psi_F(\eta, q) - F_0(\eta)] = qh_F H_F(\eta) N_F, \quad (23)$$

$$(1 - q)L_\theta[\psi_\theta(\eta, q) - \theta_0(\eta)] = qh_\theta H_\theta(\eta) N_\theta, \quad (24)$$

$$(1 - q)L_\phi[\psi_\phi(\eta, q) - \phi_0(\eta)] = qh_\phi H_\phi(\eta) N_\phi, \quad (25)$$

With boundary conditions:

$$\psi_F(0, q) = 0, \psi_F'(0, q) = 1 + \lambda_1 \psi_F''(0, q) + \lambda_2 \psi_F'''(0, q), \psi_F'(\infty, q) = 0, \quad (26)$$

$$\psi_\theta(0, q) = 1 + \delta \psi_\theta'(0, q), \psi_\theta'(\infty, q) = 0, \quad (27)$$

$$Nb\psi_\phi'(0, q) + Nt\psi_\phi'(0, q) = 0, \psi_\phi'(\infty, q) = 0, \quad (28)$$

Here, the auxiliary parameters and auxiliary functions are  $h_F, h_\theta, h_\phi, H_F, H_\theta$  and  $H_\phi$ . The embedding parameter  $q$  belongs to  $[0, 1]$ . The nonlinear operators  $N_F, N_\theta$ , and  $N_\phi$  are represented as:

$$N_F = \frac{(1 + 2\eta\gamma)\partial^3 \psi_F}{\partial \eta^3} + 2\gamma \frac{\partial^2 \psi_F}{\partial \eta^2} - \left( \frac{\partial \psi_F}{\partial \eta} \right)^2 - Nr\psi_\phi + \psi \frac{\partial^2 \psi_F}{\partial \eta^2} - M^2 \frac{\partial \psi_F}{\partial \eta} + Ri\psi_\theta, \quad (29)$$



$$N_\theta = \frac{1}{Pr} \left[ (1 + 2\eta\gamma) \left( \frac{\partial^2 \psi_\theta}{\partial \eta^2} + Nb \frac{\partial \psi_\theta}{\partial \eta} \frac{\partial \psi_\phi}{\partial \eta} \right) + 2\eta \frac{\partial \psi_\theta}{\partial \eta} \right] + (1 + 2\eta\gamma) \left( Ec \left( \frac{\partial^2 \psi_F}{\partial \eta^2} \right)^2 + \frac{1}{Pr} Nt \left( \frac{\partial \psi_\theta}{\partial \eta} \right)^2 \right) + \psi_F \frac{\partial \psi_\theta}{\partial \eta}, \quad (30)$$

$$N_\phi = (1 + 2\eta\gamma) \left( \frac{\partial^2 \psi_\phi}{\partial \eta^2} + \left( \frac{Nt}{Nb} \right) \frac{\partial^2 \psi_\theta}{\partial \eta^2} \right) + 2\gamma \frac{\partial \psi_\phi}{\partial \eta} + 2 \left( \frac{Nt}{Nb} \right) \gamma \frac{\partial \psi_\theta}{\partial \eta} + Sc \psi_F \frac{\partial \psi_\phi}{\partial \eta}, \quad (31)$$

The following are the expansions of  $F(\eta, q)$ ,  $\theta(\eta, q)$  and  $\phi(\eta, q)$  w.r.t to  $q$ , by using Taylor's series:

$$F(\eta, q) = F_0(\eta) + \sum_{k=1}^{\infty} F_k(\eta) q^k, \quad (32)$$

$$\theta(\eta, q) = \theta_0(\eta) + \sum_{k=1}^{\infty} \theta_k(\eta) q^k, \quad (33)$$

$$\phi(\eta, q) = \phi_0(\eta) + \sum_{k=1}^{\infty} \phi_k(\eta) q^k, \quad (34)$$

where,

$$F_k = \frac{1}{k!} \frac{\partial^k \psi_F(\eta, q)}{\partial \eta^k} \Big|_{q=0}, \theta_k = \frac{1}{k!} \frac{\partial^k \psi_\theta(\eta, q)}{\partial \eta^k} \Big|_{q=0}, \text{ and } \phi_k = \frac{1}{k!} \frac{\partial^k \psi_\phi(\eta, q)}{\partial \eta^k} \Big|_{q=0}. \quad (35)$$

We get the following  $k^{th}$  order deformation equations, after differentiating Eqs. (3.54) – (3.56)  $k$  times w.r.t  $q$  and dividing these by  $k!$ :

$$L_F(F_k(\eta) - \chi_{k-1} F_{k-1}(\eta)) = h_F H_F R_k^F(\eta), \quad (36)$$

$$L_\theta(\theta_k(\eta) - \chi_{k-1} \theta_{k-1}(\eta)) = h_\theta H_\theta R_k^\theta(\eta), \quad (37)$$

$$L_\phi(\phi_k(\eta) - \chi_{k-1} \phi_{k-1}(\eta)) = h_\phi H_\phi R_k^\phi(\eta), \quad (38)$$

$$F_k(0) = 0, F'_k(0) - \lambda_1 F''_{k-1}(0) - \lambda_2 F'''_{k-1}(0) = 0, f'_k(\infty) = 0, \quad (39)$$

$$\theta_k(0) - \delta \theta'_k(0) = 0, \theta(\infty) = 0, \quad (40)$$

$$Nb \phi'(0) + Nt \theta'(0) = 0. \quad (41)$$

where,

$$R_k^F(\eta) = \frac{1}{(k-1)!} \frac{\partial^{k-1} N_F}{\partial q^{k-1}} \Big|_{q=0} = \left[ (1 + 2\eta\gamma) F'''_{k-1} + 2\gamma F''_{k-1} - M^2 F'_{k-1} \right] + [Ri[\theta_{k-1} - Nr \phi_{k-1}] + \sum_{j=0}^{k-1} (F'_j F'_{k-1-j} + F_j F''_{k-1-j})], \quad (42)$$

$$R_k^\theta(\eta) = \frac{1}{(k-1)!} \frac{\partial^{k-1} N_\theta}{\partial q^{k-1}} \Big|_{q=0} = \left[ \frac{1}{Pr} \left[ (1 + 2\eta\gamma) \left( \theta''_{k-1} + \sum_{j=0}^{k-1} (Nb \theta'_j \phi'_{k-1-j} + Nt \theta'_j \theta'_{k-1-j}) \right) + 2\gamma \theta'_{k-1} \right] + (1 + 2\eta\gamma) Ec \sum_{j=0}^{k-1} F''_j F''_{k-1-j} + \sum_{j=0}^{k-1} F_j \theta'_{k-1-j} \right], \quad (43)$$

$$R_k^\phi(\eta) = \frac{1}{(k-1)!} \frac{\partial^{k-1} N_\phi}{\partial q^{k-1}} \Big|_{q=0} = \left[ (1 + 2\eta\gamma) \phi''_{k-1} + 2\gamma \phi'_{k-1} + 2 \left( \frac{Nt}{Nb} \right) \gamma \theta'_{k-1} \right] + \left[ (1 + 2\eta\gamma) \left( \frac{Nt}{Nb} \right) \theta'_{k-1} + Sc \sum_{j=0}^{k-1} F_j \theta'_{k-1-j} \right], \chi_k = \begin{cases} 0, & k \leq 1 \\ 1, & k > 1 \end{cases} \quad (44)$$

After solving the Eqs. (42) to (44) using Maple software, we get the following  $k^{th}$  term form:

$$F_k(\eta) = \zeta_k(\eta) + C_1 + C_2 \eta + C_3 e^{-\eta}, \quad (45)$$

$$\theta_k(\eta) = \vartheta_k(\eta) + C_4 + C_5 e^{-\eta}, \quad (46)$$

$$\phi_k(\eta) = \xi_k(\eta) + C_6 + C_7 e^{-\eta}, \quad (47)$$

Here  $C_1$  to  $C_7$  are the constants and  $\zeta_k(\eta)$ ,  $\vartheta_k(\eta)$ , and  $\xi_k(\eta)$  are the solutions. The constants can be calculated by using Eqs. (45) to (47). Thus, given below are the analytical series solutions of  $F(\eta)$ ,  $\theta(\eta)$  and  $\phi(\eta)$ :

$$F(\eta) = F_0(\eta) + \sum_{k=1}^{\infty} F_k(\eta), \quad (48)$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{k=1}^{\infty} \theta_k(\eta), \quad (49)$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{k=1}^{\infty} \phi_k(\eta), \quad (50)$$

The Homotopy Analysis Method (HAM) is employed in the present study because it can obtain analytical series solutions for strongly nonlinear differential equations without relying on restrictive perturbation assumptions. Unlike conventional perturbation techniques, HAM does not require a small parameter in the governing equations.



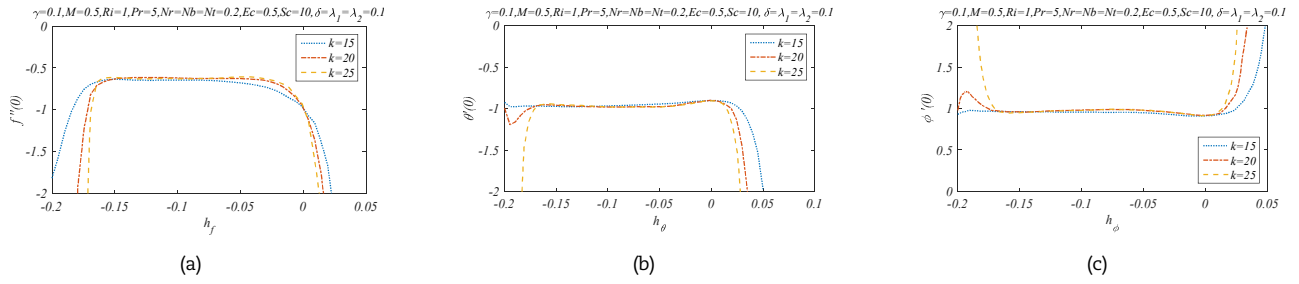


Fig. 2.  $h$  –curves, (a)  $f''(0)$ , (b)  $\theta'(0)$  and, (c)  $\phi'(0)$  for various orders of approximations  $m$ .

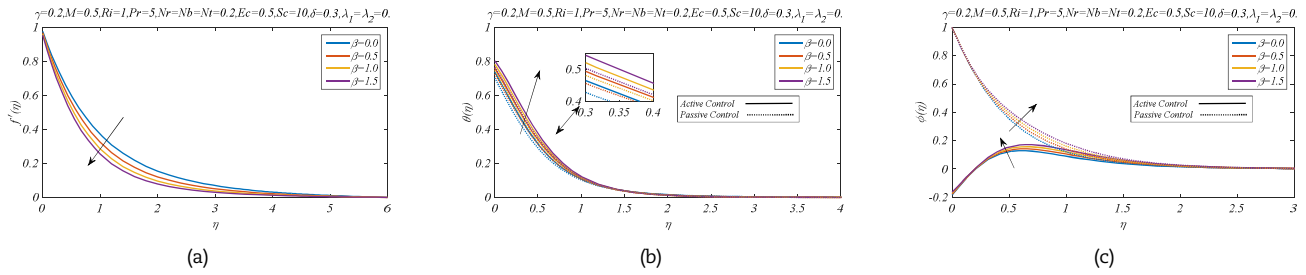


Fig. 3. Variation of thermal expansion coefficient ( $\beta$ ) on, (a)  $f'(\eta)$ , (b)  $\theta(\eta)$  and, (c)  $\phi(\eta)$ .

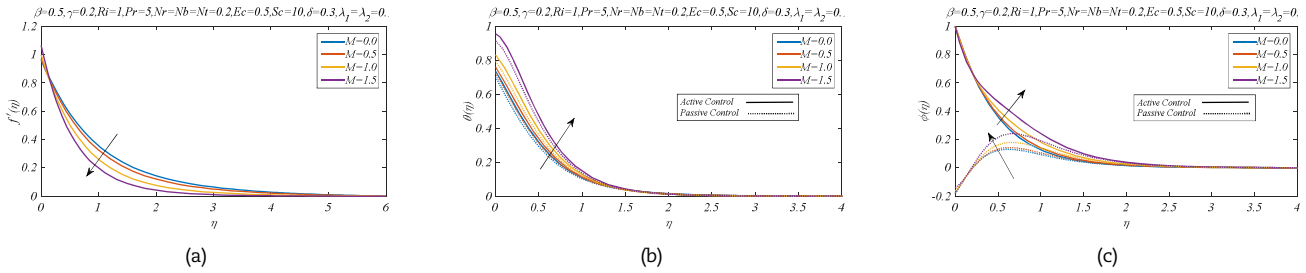


Fig. 4. Variation of Hartmann Number  $M$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .

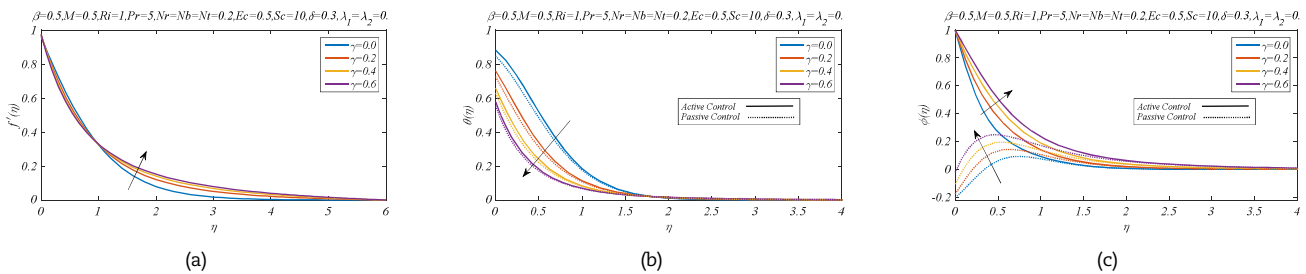


Fig. 5. Variation of  $\gamma$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .

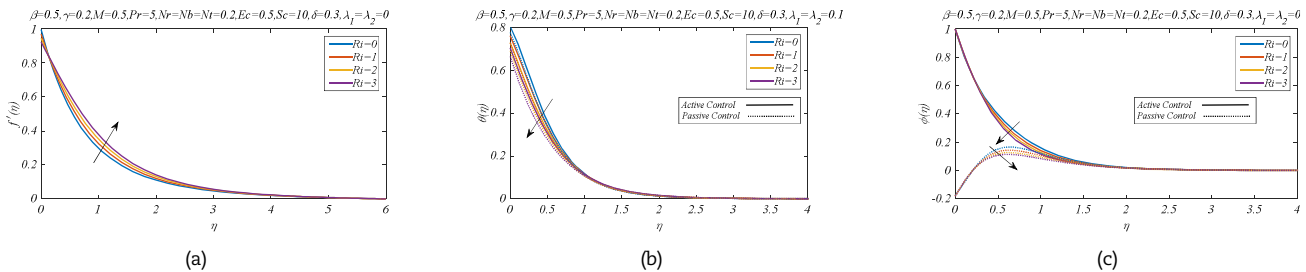


Fig. 6. Variation of  $Ri$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .

In addition, the method provides convergence-control parameters that enable systematic adjustment of the convergence region and convergence rate of the solution series. These advantages make HAM an effective and reliable approach for analyzing the coupled nonlinear behavior arising from magnetohydrodynamic effects, mixed convection, slip conditions, Brownian motion, and entropy generation in non-Newtonian nanofluid flows. The convergence of the obtained solutions is verified through the  $h$ -curves presented in Fig. 2(a–c), which establish the admissible ranges of the auxiliary parameters used throughout the analysis.



**Table 1.** Comparison of the results obtained with Shukla et al. [35] when  $Nt = Nb = Nr = 0.2$ ,  $Sc = 10$ ,  $Ri = 1$ ,  $M = 0.5$ ,  $\gamma = \delta = \lambda_1 = \lambda_2 = 0.1$ .

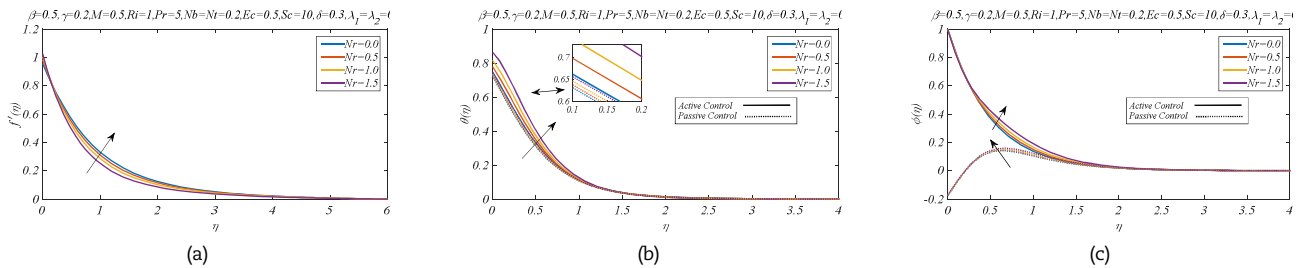
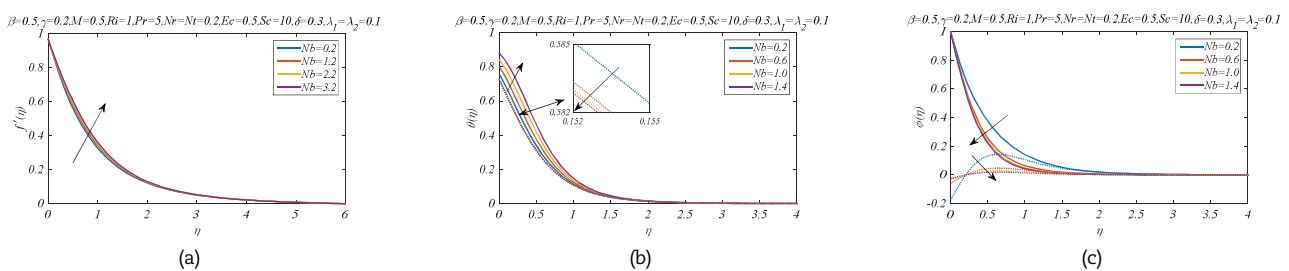
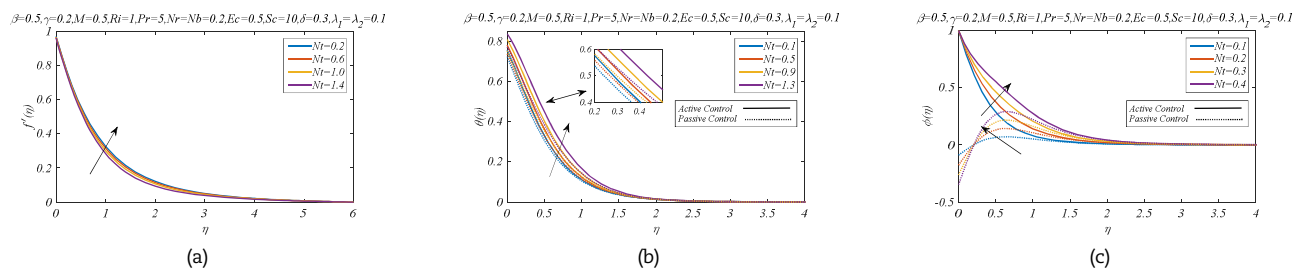
| $Ec$ | $Pr$ | $f'(0)$ |           |         | $-\theta'(0)$ |           |         |
|------|------|---------|-----------|---------|---------------|-----------|---------|
|      |      | HAM     | Numerical | Present | HAM           | Numerical | Present |
| 0.1  | 1    | 0.9725  | 0.9726    | 0.9726  | 0.5752        | 0.5750    | 0.5751  |
|      | 3    | 0.9610  | 0.9609    | 0.9609  | 0.9618        | 0.9613    | 0.9616  |
|      | 5    | 0.9572  | 0.9569    | 0.9571  | 1.2079        | 1.2075    | 1.2075  |
| 0.3  | 1    | 0.9733  | 0.9734    | 0.9734  | 0.5315        | 0.5314    | 0.5314  |
|      | 3    | 0.9623  | 0.9622    | 0.9623  | 0.8184        | 0.8177    | 0.8180  |
|      | 5    | 0.9584  | 0.9582    | 0.9582  | 0.9761        | 0.9763    | 0.9761  |
| 0.5  | 1    | 0.9741  | 0.9642    | 0.9642  | 0.4898        | 0.4896    | 0.4898  |
|      | 3    | 0.9636  | 0.9635    | 0.9635  | 0.6851        | 0.6851    | 0.6850  |
|      | 5    | 0.9636  | 0.9594    | 0.9529  | 0.7657        | 0.7663    | 0.7661  |

#### 4.1. Convergence discussion of HAM

The convergence of the Homotopy Analysis Method (HAM) solution is an important requirement for ensuring the accuracy and reliability of the obtained results. To determine the admissible ranges of the convergence-control parameters, the  $h$ -curves for the velocity function  $f(\eta)$ , temperature function  $\theta(\eta)$ , and concentration function  $\phi(\eta)$  are presented in Fig. 2. The nearly horizontal portions of these curves indicate the regions where the corresponding series solutions converge. From Fig. 2, the admissible ranges of the auxiliary parameters are found to be approximately  $h_f \in [-0.15, -0.07]$ ,  $h_\theta \in [-0.16, -0.08]$  and  $h_\phi \in [-0.16, -0.08]$ . Based on these convergence regions, the optimal values  $h_f = h_\theta = h_\phi = -0.124$  are selected for all subsequent computations. These values ensure rapid convergence and numerical stability of the HAM series solutions.

To further validate the present formulation, the computed values of the skin-friction coefficient  $f''(0)$  and local Nusselt number  $-\theta'(0)$  are compared with the results reported by Shukla et al. [35], as shown in Table 1. Excellent agreement is observed between the present results and the published HAM and numerical solutions, confirming the accuracy and validity of the current analytical approach.

Table 2 clearly demonstrates that parameters such as thermal expansion, curvature, and Prandtl number have a pronounced influence on both shear stress and heat transfer characteristics. While curvature and Prandtl number enhance both  $C_f$  and  $Nu_x$ , the thermal expansion parameter increases wall shear stress but reduces heat transfer rate. These behaviors highlight the complex coupling between momentum and thermal transport in the present nanofluid flow model.

**Fig. 7.** Variation of  $Nr$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .**Fig. 8.** Variation of  $Nb$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .**Fig. 9.** Variation of thermophoresis parameter  $Nt$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .

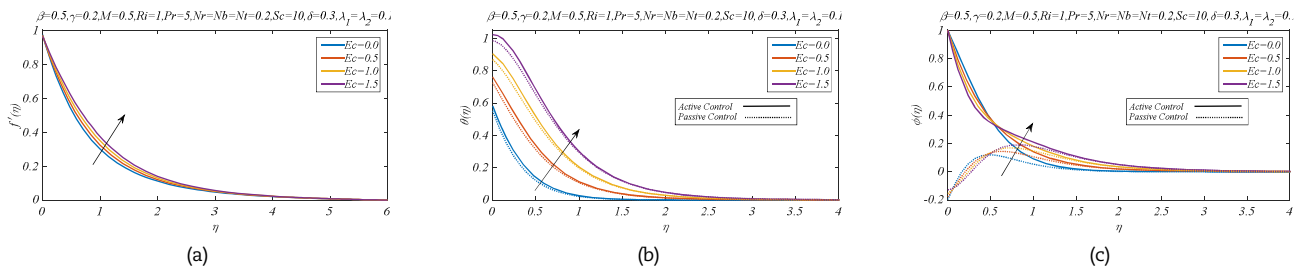


Fig. 10. Variation of Eckert number  $Ec$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .

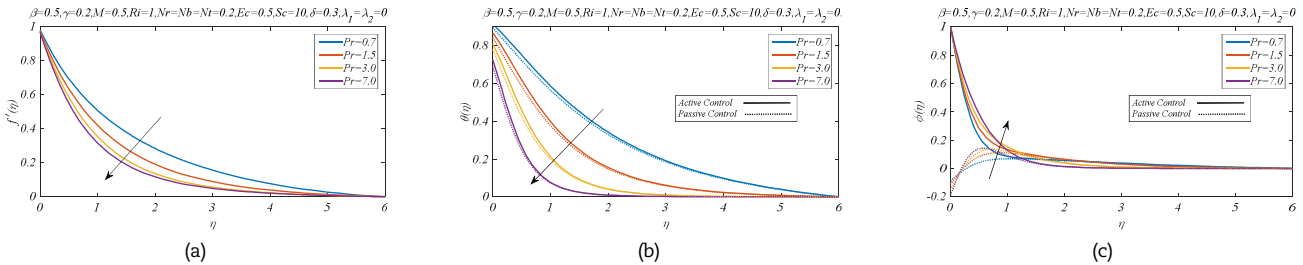


Fig. 11. Variation of Prandtl number  $Pr$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .

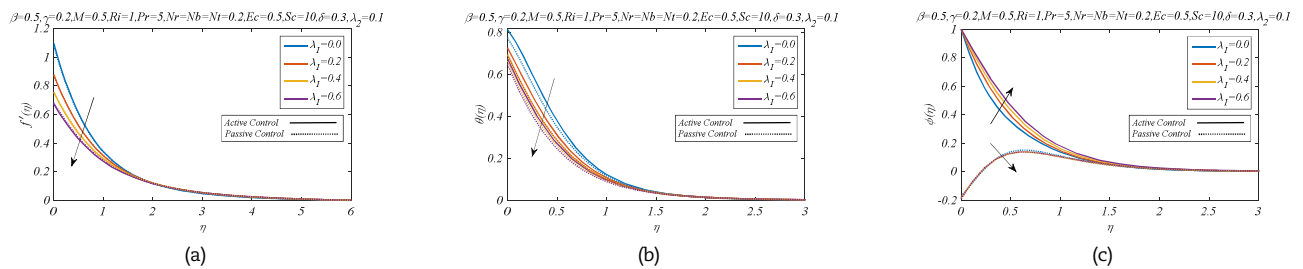


Fig. 12. Effects of the first-order velocity slip parameter  $\lambda_1$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .

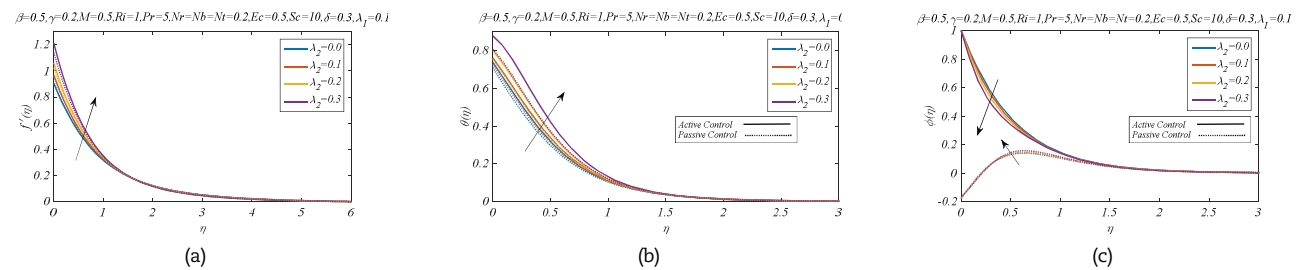


Fig. 13. Effects of the second-order velocity slip parameter  $\lambda_2$  on, (a) velocity profile  $f'(\eta)$ , (b) temperature distribution  $\theta(\eta)$  and, (c) concentration distribution  $\phi(\eta)$ .

Table 2. Values of skin friction coefficient  $f''(0)$ .

| $\beta$ | $\gamma$ | $Pr$ | $Nr$ | $Nt$ | $Nb$ | $Ec$ | $Cf_x$ | $Nu_x$ |
|---------|----------|------|------|------|------|------|--------|--------|
| 0.0     | 0.2      | 5.0  | 0.2  | 0.2  | 0.2  | 0.5  | 0.8803 | 1.3786 |
| 0.5     |          |      |      |      |      |      | 1.4293 | 1.2838 |
| 1.0     |          |      |      |      |      |      | 2.0422 | 1.1930 |
| 0.5     | 0.1      | 5.0  | 0.2  | 0.2  | 0.2  | 0.5  | 1.2923 | 0.9633 |
|         | 0.3      |      |      |      |      |      | 1.5562 | 1.5984 |
|         | 0.5      |      |      |      |      |      | 1.7852 | 2.1879 |
| 0.5     | 0.2      | 0.7  | 0.2  | 0.2  | 0.2  | 0.5  | 1.3256 | 0.5117 |
|         |          | 3.0  |      |      |      |      | 1.6693 | 1.4627 |
|         |          | 7.0  |      |      |      |      | 1.8403 | 2.7460 |

## 5. Results and Discussion

The governing non-linear ordinary differential equations have been solved using the HAM method. A lot of calculations have been performed to illustrate the effect of different quantities (i.e., thermal slip ( $\delta$ ), curvature ( $\gamma$ ), mixed convection ( $Ri$ ), magnetic parameter ( $M$ )) on the physical quantities (i.e., temperature profile  $\theta(\eta)$ , velocity profile  $f'(\eta)$ ). For the overall calculation, the predefined values of the parameters are chosen to be  $Ec = 0.5$ ,  $Sc = 10$ ,  $\lambda_1 = 0.1$ ,  $\lambda_2 = 0.1$ ,  $Nt = 0.2$ ,  $Nr = 0.2$ ,  $Pr = 5$ ,  $M = 0.5$ ,  $\gamma = 0.2$ ,  $Ri = 1$ ,  $\beta = 0.5$  or else mentioned.





Fig. 14. Effects of velocity slip parameter  $\delta$  on, (a) temperature distribution  $\theta(\eta)$  and, (b) concentration distribution  $\phi(\eta)$ .

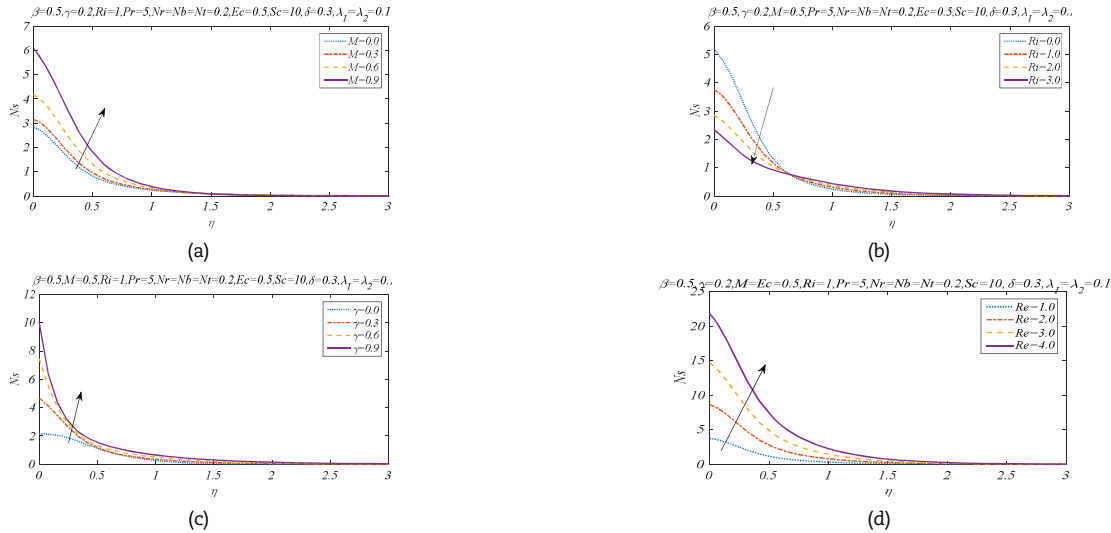


Fig. 15. Effects of entropy generation number  $N_s$  under the influence of, (a) Richardson number  $Ri$ , (b) magnetic parameter  $M$ , (c) thermal expansion parameter  $\gamma$  and, (d) Reynolds number  $Re$ .

Figure 3(a) shows that velocity decreases slowly when thermal expansion is low. When thermal expansion is high, velocity decreases rapidly. Figure 3(b-c) shows the effects of both active and passive nanoparticles for temperature  $\theta(\eta)$  and concentration  $\phi(\eta)$ . The temperature increases in both active- and passive-controlled nanoparticle conditions (Fig. 3(b)). Similarly, in Fig. 3(c) the concentration profile increases in both active- and passive controlled conditions. Figure 4(a-c) displays the behavior of the magnetic parameter  $M$  on velocity, temperature, and concentration. For higher values of the magnetic parameter, the velocity profile decreases, whereas temperature and concentration increase for both active- and passive-controlled conditions. This is because of an increase in magnetic force. This reduces fluid velocity and enhances nanoparticle concentration. A high magnetic field causes a temperature rise.

Figure 5(a-c) illustrates the effects of curvature parameters on velocity, temperature, and concentration profile. Since the curvature parameter is directly related to velocity, velocity increases with the curvature. Temperature shows an inverse relation with the curvature parameter. The concentration profile also increases the curvature with both active and passive control conditions. The Richardson number indicates natural, forced, or mixed convection. Natural convection occurred when the Richardson number is less than 1, while for  $Ri \gg 10$ , forced convection appeared. Since the study focuses on mixed convection, we use the  $Ri$  between  $[1 - 10]$ . The velocity  $f'(\eta)$  is accelerated with the enhancement of  $Ri$  (Fig. 6(a)). Temperature  $\theta(\eta)$  falls with rising mixed-convection parameter at both active and passive control conditions (Fig. 6(b)). Concentration profile  $\phi(\eta)$  shows the inverse relationship with the mixed convection parameter (Fig. 6(c)).

The influence of the buoyancy ratio parameter on velocity, temperature, and concentration is shown in Fig. 7(a-c). Velocity and temperature increase with the buoyancy ratio parameter, and the concentration profile decreases, as illustrated in Fig. 7(c). The effects of the Brownian motion parameter are shown in Fig. 8(a-c). The collision of one molecule with another increases the fluid velocity and temperature. This is why velocity and temperature increase with the Brownian motion parameter, as illustrated in Fig. 8(a-b). The concentration profile decreases as the Brownian motion parameter increases (Fig. 8(c)).

Figure 9(a-c) depicts the results of the thermophoresis parameter. Thermophoresis is the transport force due to a temperature gradient. It affects velocity and temperature, similar to the Brownian motion parameter. The temperature and velocity increase for higher values of the thermophoresis parameter. Concentration also enhances the thermophoresis parameter. The effect of Eckert number on velocity, temperature and concentration profile is visualized in Fig. 10(a-c). The graph shows that larger Eckert numbers increase velocity, temperature, and concentration. Increasing the Eckert number increases kinetic energy. Moreover, temperature is the average kinetic energy. Thus, the temperature of the fluid rises.

Figure 11(a) indicates that the velocity decreases with the increase in Prandtl number  $Pr$ . Similarly, Fig. 11(b) shows temperature decreasing as the Prandtl number increases. This is because an increasing Prandtl number increases viscosity, which then decreases the fluid's velocity and temperature. Figure 11(c) shows the direct relationship between the Prandtl number and the concentration profile. This is because an increase in viscosity increases thickness. Figures 12 and 13 show the effects of  $\lambda_1$  and  $\lambda_2$  on velocity, temperature, and concentration profiles. Velocity and temperature have opposite behavior in the first- and second-order velocity-slip parameters. The velocity and temperature decrease with the increase in the first-order velocity slip parameter, but both increase with the increase in  $\lambda_2$  with both active and passive control conditions. The concentration profile increases with the active control condition as the first-order velocity slip parameter increases, whereas the concentration decreases with the passive control condition.



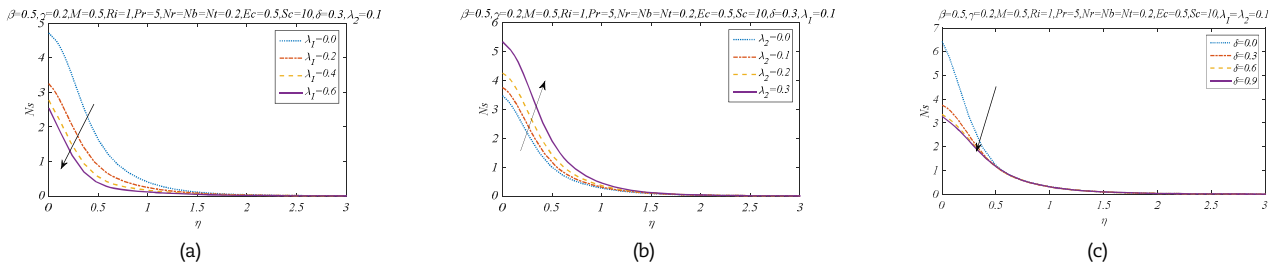


Fig. 16. Variation of entropy generation number  $N_s$  with respect to, (a) first-order velocity slip parameter  $\lambda_1$ , (b) second-order velocity slip parameter  $\lambda_2$  and, (c) velocity slip parameter  $\delta$ .

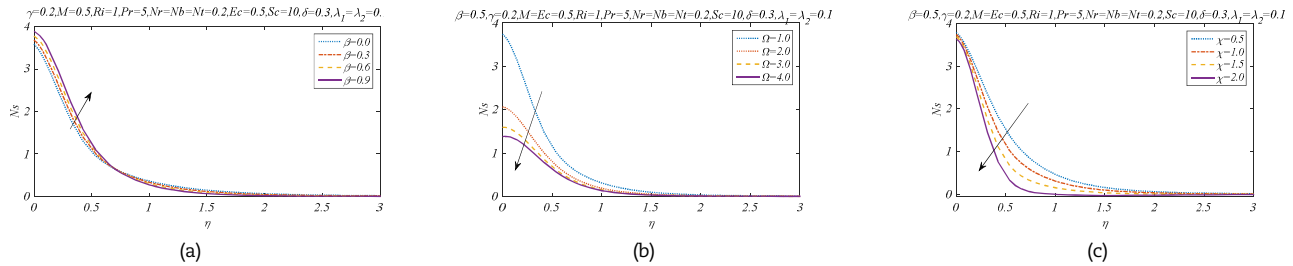


Fig. 17. Variation of entropy generation number  $N_s$  under the influence of, (a) thermal expansion parameter  $\beta$ , (b) magnetic parameter  $\Omega$  and, (c) curvature parameter  $\chi$ .

With increasing second-order velocity-slip parameter, the active-control concentration profile decreases while the passive-control concentration profile increases. The effect of the thermal parameter ( $\delta$ ) on temperature and concentration profiles is illustrated in Fig. 14(a-b). The Maxwell fluid's concentration also decreases as the thermal parameter rises.

Figures 15 and 16 show the effects of  $M$ ,  $Ri$ , curvature parameter  $\gamma$ ,  $Re$ ,  $\lambda_1$ ,  $\lambda_2$ , and thermal slip parameter  $\delta$  on the entropy generation. The entropy generation rises near the cylinder's surface. Conversely, it falls with increasing value of  $\eta$ . As the magnetic field increases, the boundary-layer temperature also increases, which enhances the entropy generation parameter (Fig. 15(a)). The increasing of  $\eta$  values enhance the entropy generation parameter at the cylinder's surface. However, a converse effect occurs when the values of  $Ri$  increase; this effect is shown in Fig. 15(b).

Figures 15(c-d) demonstrate that the entropy generation number  $N_s$  increases with increasing values of the Reynolds number  $Re$  and curvature parameter  $\gamma$ . Figures 16(a-b) depict the behavior  $\lambda_1$  and  $\lambda_2$  on the entropy generation  $N_s$ . The entropy generation increases with the increasing of the first-order velocity slip parameter  $\lambda_1$  whereas it decreases with the second-order velocity slip parameter  $\lambda_2$  increasing. The effect of the thermal slip parameter  $\delta$  on entropy generation is illustrated in Fig. 16(c). Since the temperature value is high at the cylinder's surface, enhancing the value  $\delta$ , the  $N_s$  increases near the cylinder's surface, whereas this effect is negligible far from the surface.

Figure 17 analyzes the influences of the thermal expansion coefficient  $\beta$ , dimensionless temperature  $\Omega$ , and diffusion constant parameter  $\chi$  on the entropy generation number  $N_s$ . Thermal expansions coefficient  $\beta$  enhances with entropy generation (Fig. 17(a)). On the contrary, dimensionless temperature  $\Omega$  and diffusion constant parameter  $\chi$  negative relation with entropy.

## 6. Conclusions

An analytical study of mixed MHD flow has been conducted with Maxwell fluid. The fluid has been passed through the vertical cylinder. The non-linear differential equation has been solved analytically with HAM. The results of various parameters like first ( $\lambda_1$ ) and second-order velocity slip, entropy, curvature, magnetic, Richardson number, Reynolds number, and thermal slip-on temperature, concentration, and velocity have been analyzed in detail. The following are a few key points that have been obtained:

- The velocity of the Maxwell fluid increases with increasing curvature parameter, indicating enhanced fluid acceleration over the cylindrical surface.
- The magnetic parameter exhibits a retarding effect; however, its reduction leads to an enhancement in the velocity field due to weakened Lorentz force.
- The mixed convection parameter strengthens the flow dynamics near the cylinder surface, resulting in accelerated fluid motion.
- Entropy generation increases with curvature parameters, thermal expansion coefficient, magnetic parameter, second-order velocity slip, and Reynolds number, indicating higher thermodynamic irreversibility under these effects.
- Entropy generation decreases with increasing mixed convection parameter, first-order velocity slip, thermal slip, dimensionless temperature, and diffusion parameter, suggesting improved system efficiency.
- The temperature profile increases with thermal expansion coefficient, magnetic parameter, buoyancy ratio, Brownian motion, thermophoresis parameter, Eckert number, and  $\lambda_2$  enhancing thermal energy transport in the system.
- The temperature profile decreases with thermal slip parameter,  $\lambda_1$ , Prandtl number, Richardson number, and curvature parameter, indicating reduced thermal boundary layer thickness.

## Author Contributions

B. Ahmed carried out the conceptualization, methodology, mathematical modeling, formal analysis, implementation of the Homotopy Analysis Method (HAM), data interpretation, visualization, manuscript drafting, and prepared the initial and final versions of the manuscript. A. Al-Radadideh provided support in refining the analysis, review of the mathematical formulation and results, manuscript review and editing and improving the presentation of the manuscript.



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## Conflict of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

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## Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

## References

- [1] Routbort, J., Wu, Y., France, D.M., Singh, D., Timofeeva, E.V., Smith, D.S., Argonne National Lab, Michellin North America, St. Gobain Corp., 2009.
- [2] Sheikholeslami, M., Mofid Gorji-Bandpy, Domiri Ganji, D., Lattice Boltzmann method for MHD natural convection heat transfer using nanofluid, *Powder Technology*, 254, 2014, 82–93.
- [3] Al-Odat, M. Q., Al-Ghamdi, A., Numerical investigation of Dufour and Soret effects on unsteady MHD natural convection flow past vertical plate embedded in non-Darcy porous medium, *Applied Mathematics and Mechanics*, 33, 2012, 195–210.
- [4] Das, R.K., Neog, B.C., MHD flow past a vertical oscillating plate with radiation and chemical reaction in porous medium, *IOSR Journal of Mathematics*, 11(1), 2015, 46–50.
- [5] Asimoni, N.R.M., Mohammad, N.F., Kasim, A.R.M., Shafie, S., MHD forced convective flow past a vertical plate: An automated solution approach, *AIP Conference Proceedings*, 1974, 2018, 020025.
- [6] Daniel, S., Tella, Y., Sani, U., An unsteady forced and free convection flow past an infinite permeable vertical plate, *International Journal of Science and Technology*, 3(3), 2013, 1251–1255.
- [7] Xu, H., Fan, T., Pop, I., Analysis of mixed convection flow of a nanofluid in a vertical channel with the Buongiorno mathematical model, *International Communications in Heat and Mass Transfer*, 44, 2013, 15–22.
- [8] Liu, C.C., Lo, C.Y., Numerical analysis of entropy generation in mixed-convection MHD flow in vertical channel, *International Communications in Heat and Mass Transfer*, 39(9), 2012, 1354–1359.
- [9] Nazar, R., Tham, L., Pop, I., Ingham, D.B., Mixed convection boundary layer flow from a horizontal circular cylinder embedded in a porous medium filled with a nanofluid, *Transport in Porous Media*, 86(2), 2011, 517–536.
- [10] Rana, P., Bhargava, R., Anwar Bég, O., Finite element modeling of conjugate mixed convection flow of Al<sub>2</sub>O<sub>3</sub>–water nanofluid from an inclined slender hollow cylinder, *Physica Scripta*, 87(5), 2013, 055005.
- [11] Grosan, T., Pop, I., Axisymmetric mixed convection boundary layer flow past a vertical cylinder in a nanofluid, *International Journal of Heat and Mass Transfer*, 54(25–26), 2011, 3139–3145.
- [12] Sheikholeslami, M., Ganji, D.D., Ferrohydrodynamic and magnetohydrodynamic effects on ferrofluid flow and convective heat transfer, *Energy*, 75, 2014, 400–410.
- [13] Yoshimura, A., Prud'homme, R. K., Wall slip corrections for couette and parallel disk viscometers, *Journal of Rheology*, 32(1), 1988, 53–67.
- [14] Hayat, T., Imtiaz, M., Alsaedi, A., Impact of magnetohydrodynamics in bidirectional flow of nanofluid subject to second order slip velocity and homogeneous–heterogeneous reactions, *Journal of Magnetism and Magnetic Materials*, 395, 2015, 294–302.
- [15] Gul, A., Khan, I., Shafie, S., Khalid, A., Khan, A., Heat transfer in MHD mixed convection flow of a ferrofluid along a vertical channel, *PloS One*, 10(11), 2015, e0141213.
- [16] Andersson, H.I., Slip flow past a stretching surface, *Acta Mechanica*, 158(1), 2002, 121–125.
- [17] Zheng, L., Zhang, C., Zhang, X., Zhang, J., Flow and radiation heat transfer of a nanofluid over a stretching sheet with velocity slip and temperature jump in porous medium, *Journal of the Franklin Institute*, 350(5), 2013, 990–1007.
- [18] Dhanai, R., Rana, P., Kumar, L., MHD mixed convection nanofluid flow and heat transfer over an inclined cylinder due to velocity and thermal slip effects: Buongiorno's model, *Powder Technology*, 288, 2016, 140–150.
- [19] Dhanai, R., Rana, P., Kumar, L., Lie group analysis for bioconvection MHD slip flow and heat transfer of nanofluid over an inclined sheet: Multiple solutions, *Journal of the Taiwan Institute of Chemical Engineers*, 66, 2016, 283–291.
- [20] Bejan, A., Entropy generation minimization, *Journal of Applied Physics*, 79(3), 1996, 1191–1218.
- [21] Aiboud, S., Saouli, S., Entropy analysis for viscoelastic magnetohydrodynamic flow over a stretching surface, *International Journal of Non-Linear Mechanics*, 45(5), 2010, 482–489.
- [22] Abolbashari, M.H., Freidoonimehr, N., Nazari, F., Rashidi, M.M., Analytical modeling of entropy generation for Casson nano-fluid flow induced by a stretching surface, *Advanced Powder Technology*, 26(2), 2015, 542–552.
- [23] Noghrehabadi, A., Pourrajab, R., Ghalambaz, M., Entropy analysis for nanofluid flow over a stretching sheet in the presence of heat generation/absorption and partial slip, *Journal of Mechanical Science and Technology*, 27(3), 2013, 927–937.
- [24] Jagadha, S., Gopal, D., Patil, N., Singh, S., Kishan, N., Abhilasha, S. K., Effect of higher order chemical reaction and Joule heating on rotational and permeable radiative MHD flow of Casson manganese ferrite fluid with heat generation/absorption, *Multiscale and Multidisciplinary Modeling, Experiments and Design*, 8(3), 2025, 164.
- [25] Kumar V., V., Sharma, R. P., Naveen Kumar, R., The role of time-varying nanofluid flow and particle aggregation on the kinetics of homogeneous and heterogeneous reactions in position-controlled squeezing porous slider, *Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems*, 2025, 23977914251338162.
- [26] Gowda, R. P., Abhilasha, S. K., Gamaoun, F., Kumar, R. V., Prasannakumara, B. C., Analysis of moment of inertia and magnetic properties under varying magnetic fields via Touchard polynomials collocation technique, *International Communications in Heat and Mass Transfer*, 165, 2025, 109092.
- [27] Kumar, V. V., Sharma, R. P., Multi-objective optimization of surface-drag phenomena in coaxial disk systems using machine learning assisted Taguchi based grey relation analysis, *International Journal of Numerical Methods for Heat & Fluid Flow*, 36(5), 2026, 1961–1992.
- [28] Kumar, R. V., Abhilasha, S. K., Banakar, V. R., Gamaoun, F., Prasannakumara, B. C., Computational analysis of wetted triangular spine fin using Lerch polynomial collocation method, *Applied Thermal Engineering*, 275, 2025, 126820.
- [29] Banakar, V. R., Mallikarjuna, A. N., Kattimani, P., Abdulrahman, A., Airfoil polynomials-based collocation approach to study time-varying fluid mechanism, *International Communications in Heat and Mass Transfer*, 171, 2026, 110105.
- [30] Varun Kumar, R. S., Abhilasha, S. K., Raghunandan, M. E., Abdulrahman, A., Naveen Kumar, R., Comprehensive analysis on the thermal performance of circular-perforated rectangular fin with internal heat generation using Chelyshkov polynomial operational matrix-based collocation approach, *International Journal of Mechanics and Materials in Design*, 21(6), 2025, 2105–2128.
- [31] Punith Gowda, R. J., Abhilasha, S. K., Patil, N., Kalia, R., Kolli, U. C., Comparison of horizontal magnetic field and uniform magnetic on the nanofluid flow in rough rotating disk system: A response surface methodology and ANOVA approach, *Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems*, 2025, 23977914251404181.
- [32] Banakar, V. R., R. N. K., Kattimani, P., Pattanayak, B., Punith Gowda, R. J., Off-Centered Stagnation Point, Particle Movement, and Accumulation in a



Chemically Reactive Fluid, *Chemical Engineering & Technology*, 49(2), 2026, e70166.

[33] Kattimani, P., Karthik, K., Arunkumar, D. T., Pattanayak, B., Analysis of Temperature-Induced and Heat-Driven Diffusion Off-Centered Stagnation Point Flow, *Heat Transfer*, 55(2), 2026, 1137–1154.

[34] Van Gorder, R.A., Optimal homotopy analysis and control of error for implicitly defined fully nonlinear differential equations, *Numerical Algorithms*, 81, 2019, 181–196.

[35] Shukla, N., Rana, P., Bég, O.A., Singh, B., Kadir, A., Homotopy study of magnetohydrodynamic mixed convection nanofluid multiple slip flow and heat transfer from a vertical cylinder with entropy generation, *Propulsion and Power Research*, 8(2), 2019, 147–162.

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