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Research Paper

Optimizing Heat Management of Nanofluids' Thermal Transfer

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Abstract. Nowadays, in various industrial applications, nanofluids have been implemented to improve the rate of heat transfer. This study explores, using the lattice Boltzmann method, natural convection heat transfer optimization in different enclosures in the presence of a heated source and obstacle effects commonly used in chemical reactors, electronic cooling, heat exchangers, and thermal storage tools. The main focus is to examine the consequences of radiative mono/hybrid nanofluids, different solid nanoparticles dispersed in water, on heat transfer in an enclosure. This study deals with the effects of mono/hybrid nanofluids on natural convection and thermal transmission enhancement of the fluid in a cabinet having a local heat source in the enclosure center and two thermally insulated obstacles which are placed in the enclosure at different positions. The analysis was conducted on various parameters spanning wide ranges, including nanoparticles' volume fraction, type of nanofluid (mono or hybrid), presence of a heat source and its position, and Rayleigh numbers for hydrodynamics and thermodynamics. The influences of these factors on heat transfer, flow patterns, and velocities were analyzed. It has been revealed that an increase in the percentage of nanoparticles and the Rayleigh number improves momentum and thermal transfer rates. In addition, the hybrid nanofluid Al_2O_3 - Cu/H_2O demonstrates an extensive enhancement in natural convection heat transfer performance compared to other types of hybrid nanofluids.

Keywords: Natural convection, Mono/Hybrid nanofluid, Heat transfer, Lattice Boltzmann method, Heated source.

1. Introduction

One area that currently sparks significant interest among scholars is natural convection heat transfer. Technological advances have led to a breakthrough in heat transfer with nanofluids replacing pure fluids. Choi [1] introduced the term "nanofluids" and demonstrated that adding solid nanoparticles in dynamical fluids improves thermal and physical properties, making such fluids suitable for thermal and heat control systems [2]. Oxides like Silica SiO_2 , alumina Al_2O_3 , and titania TiO_2 and metals such as copper Cu , silver Ag and gold Au are commonly employed as solid nano-sized particles [3]. The most commonly used base fluids are water H_2O and organic fluids such as ethylene glycol (EG) [4]. By combining metal nano-sized particles with metal oxide or ceramic types, a stable solution can be created with better thermal properties [5-7].

Mono nanofluids are fluids that contain a single type of solid nanoparticle inside a base fluid [8]. Compared to mono nanofluids, a more significant kind of nanofluid is a hybrid nanofluid. A hybrid nanofluid is a sophisticated type of nanofluid that has multiple nanoparticles in the base fluid [9, 10]. Due to their enhanced thermophysical and rheological properties, they are widely employed to better meet the needs of current engineering and biomedical applications, including chemical processes, solar energy, textile industries, nuclear system cooling, and heat and transportation systems [7, 11-14]. Hashim et al. [15] performed a numerical analysis of the heat transfer optimization and sensitivity analysis of a nanofluid. The results demonstrate that the hybrid and ternary nanofluids give a clear rise in the heat transfer rate. Hafeez et al. [16] numerically examine the thermal efficiency of hybrid nanofluid flow across a stretching or shrinking wedge. They discovered that heat transmission of a hybrid nanofluid is influenced positively by melting heat transfer, magnetic fields, and thermal radiation. In addition, Tlili [17] conducted an investigation of improved heat generation and thermal radiation effects in a hybrid nanofluid. Further, the literature on hybrid nanofluid flow in various applications can be viewed in references [16-20].

Applications involving heat transfer from or to heat sources are generally met in almost all systems, such as electrical systems or heat exchangers, and they are of great importance for the overall system performance. Thus, it is very important to manage and



optimize the cooling of internal components of an apparatus or the dynamic thermal transfer to maintain fluid temperatures or control cooling/heating processes [21-23]. Many scientists are interested in how heat sources affect thermal transfer rates in a variety of physical challenges. Jubair et al. [24] explored, in ternary Carreau-Yasuda nanofluids, thermal conduction dynamics under convective boundary conditions and exponential heat source/sink influences. They found that the heat source controls the energy propagation and heat transfer of the fluid. Iqbal et al. [25] explored, in bioconvection flow of nanofluid with heat source and microorganisms, thermophoretic particle deposition. The results demonstrated that the isoline distribution behaves, in the presence of a heat source, presenting an enhancement in the thermal energy. Similarly, Pramanik et al. [26] studied the enhancement of thermal transfer in a double lid-driven trapezoidal porous enclosure with a heated obstacle via a hybrid nanofluid. The findings revealed that the thermal transfer indices and flow fields were significantly influenced by the heated block's presence and its position.

In many practical thermal management systems, including electronic cooling devices, battery thermal management systems, compact heat exchangers, chemical reactors, and thermal energy storage units [27], localized heat-generating components are frequently surrounded by structural supports or insulating elements that significantly influence the flow circulation and heat dissipation process [28-31]. Although the computational geometry considered in the present study is idealized, it represents a simplified model of these engineering configurations, allowing the fundamental mechanisms of natural convection and radiative heat transfer to be investigated under well-controlled conditions. Such simplified models are widely adopted in computational heat transfer studies because they provide valuable physical insight while reducing geometric complexity. Therefore, understanding the combined effects of heat source location, insulated obstacles, Rayleigh number, and mono/hybrid nanofluids can provide useful design guidelines for improving the thermal performance, cooling efficiency, and energy utilization of practical thermal systems.

As mentioned, there are several uses of mono/hybrid nanofluids in industry and engineering applications [14, 32-35]. However, real flows in complicated geometries are described by a collection of nonlinear equations [36-38]. As a result, numerical approaches have been employed to identify adequate and suitable choices, including finite volume (FVM), finite element (FEM), and finite difference (FDM) methods, which are macroscopic approaches. Recently, LBM, which is a mesoscopic method, has emerged, in complicated geometries, as a powerful and significant approach due to its simple mathematical algorithm for numerically solving flow problems [39, 40]. Kefayati [41] studied shallow flows of a generalized Newtonian fluid by using LBM. The results explored how the rheology of the fluid affects the evolution of waves, and they demonstrated that the proposed LBM provides a robust technique for modelling environmental engineering scenarios. Kozhanova et al. [42] presented a two-lattice Boltzmann tool hybridization of the a posteriori mood paradigm. They find that, via smooth and positivity detectors, the mood paradigm generates a hybrid LB tool enabling them to combine the greatest features of the two LB approaches into a single scheme. In addition, Liang et al. [43] investigated, by employing the bounce-immersed boundary lattice Boltzmann technique, the fluid-structure-acoustic simulation performed on the elastic airfoil trailing edge. They found that, by solid deformation in the submerged boundary approach, a strictly non-penetrating treatment technique that deals with the issue of flow penetration is induced.

The current paper's aim is to regulate fluid flow stability and optimize heat transmission enhancement of mono/hybrid nanofluids. This research contributes significantly to the scientific literature in several original and innovative ways. We propose an accurate and comprehensive model of heat exchange by natural convection using a sophisticated computational technique called the lattice Boltzmann method with single-relaxation times (SRT-LBM). Due to its greater stability and accuracy in simulating complex fluid flow issues, particularly at high Rayleigh numbers ($Ra = 10^6$), the SRT-LBM was selected over the multi-relaxation time (MRT-LBM). The SRT-LBM reduces numerical diffusion and improves the robustness of the model by providing greater control over the relaxation processes of various moments. As a result, it is better suited to capture the complex behaviors of natural convection in nanofluid-filled cavities. This method provides a numerical analysis of the performance of a hybrid nanofluid in a cavity. This method's efficacy in simulating fluid flows has been confirmed by multiple studies [44-46], which supports its use in this research. The practical applications of semi-square cavities filled with hybrid nanofluids and a heating source at various positions with obstacles include solar thermal collectors, energy-efficient building designs, and enhanced cooling systems in electronic devices. In engineering applications, the capacity to maximize heat transport in certain setups might result in more efficient thermal management systems. Our focus converges on studying the effects of nanoparticle fraction and their type in dynamic fluid flow and heat transfer inside an enclosure containing a heated source and a square obstacle. This enclosure consists of two heated corners, while the other enclosure walls are cooled. In terms of Nusselt number, isotherms, temperature and velocity distributions, and streamlines, we evaluated the influence of several solid nanoparticle types (Al_2O_3 , Cu , Fe_3O_4 , and TiO_2) on the flow and thermal transmission improvement. Our goal through this study is to significantly impact the academic community by combining an advanced numerical technique with the usage of hybrid nanofluids, semi-square cavities, and a partial shift in the location of the heating source. The findings of our investigation aim to improve our knowledge of convective heat transport mechanisms in fluid thermal exchange management, subjected to give the way for further improvements in fluid behaviors. In addition, understanding free convection heat transfer entails a thorough and in-depth examination of the ways in which many elements, like Rayleigh number, heating position, and nanoparticle composition, affect the mechanism of heat transfer. This extensive method offers a more comprehensive and sophisticated understanding of the relevant physical phenomena. It is observed that higher values of all factors result in increasing heat transmission rate in the enclosure. Therefore, this research aims to conceptually validate the following purposes:

- To generate the way in which the mono/hybrid nanofluid type affects the fluid's velocities and temperature characteristics.
- Understanding how a heat source presence and its location influence the heat transmission characteristics of mono/hybrid nanofluid.
- To optimize the thermal transmission of mono/hybrid nanofluid in this enclosure.

2. Problem Statement and Boundary Conditions

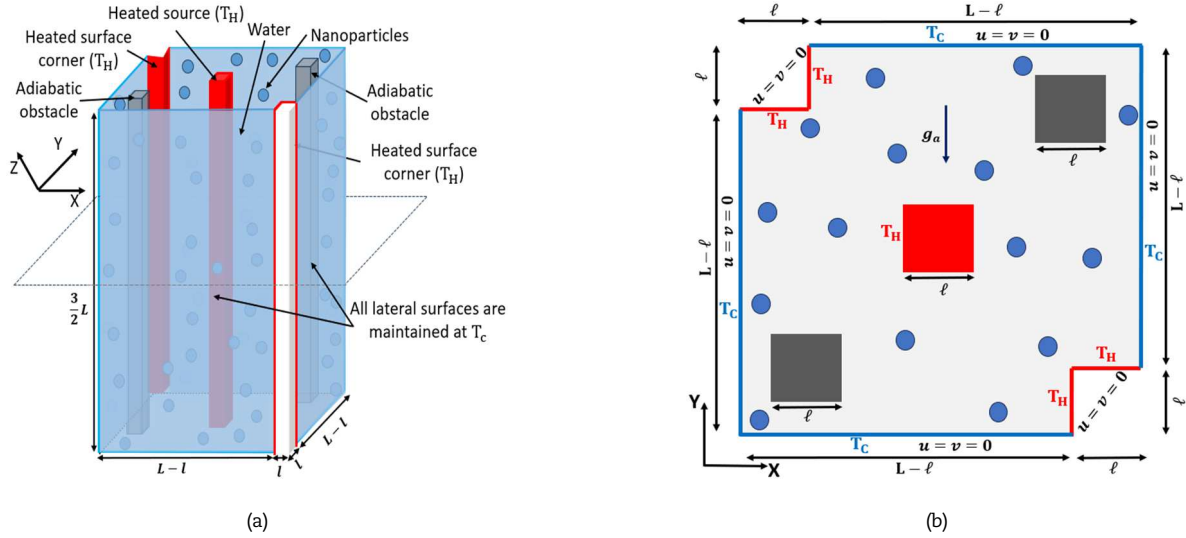
2.1. Problem statement

Thermal management in three-dimensional enclosures is crucial to improve the performance of current energy and electronics systems. Figure 1 depicts a closed 3D enclosure that is filled with a nanofluid. The geometric and thermal complexity presents substantial challenges in predicting and optimizing the thermal and fluid flow performance within the enclosure. Studying the combined impacts of nanofluid characteristics, spatial heating, and internal constraints is critical to enhance thermal performance and energy efficiency in applications. A 2D model allows a clearer visualization of flow processes, easier control of boundary conditions, and reduced computational cost, while preserving the fundamental physics of nanofluid-enhanced heat transfer and obstacle-induced flow disturbances. Such a simplification provides valuable insights and trends that can guide the design of more complex three-dimensional systems.



Table 1. Thermophysical properties of pure fluid and suspended nanosized particles at standard conditions.

Physical characteristics	Water [50]	Al_2O_3 [51]	Cu [52]	Fe_3O_4 [53]	TiO_2 [54]
ρ ($kg \cdot m^{-3}$)	97.1	3970	8954	5200	4250
c_p ($J/kg \cdot K$)	4179	765	383	670	86.2
λ ($W \cdot m^{-1} \cdot K^{-1}$)	0.613	40	400	6	8.954
$\beta \times 10^{-5}$ (K^{-1})	21	0.85	1.67	1.3	0.9
μ ($kg/m \cdot s$)	1.003×10^{-3}	-	-	-	-

**Fig. 1.** (a) Scheme of 3D enclosure and (b) enclosure transverse section geometry in 2D (X-Y).

This two-dimensional study examines, by adopting the Lattice Boltzmann Method (LBM), convective heat transfer in an enclosure with two hot corners (temperature T_H) and the other walls are cooled (temperature T_C). The enclosure has a square heated source and two square obstacles Fig. 1. The nanofluid concentration, its type, heated source, and nanofluid type are the relevant factors in this investigation. The nanofluid is considered to be incompressible, and the nanosized particles and base fluid are in thermal equilibrium.

Microscopically, the presence of nanosized particles in the mono/hybrid nanofluid makes its performance distinctly different from that of the pure fluid, resulting in more effective energy transport. The pure fluid (water) and nanoparticle (Al_2O_3 , Cu , Fe_3O_4 , and TiO_2) thermophysical characteristics are presented in Table 1 [47, 48]. Equations (2a)-(2d) represent the thermophysical properties of the hybrid nanofluid, which are related to the thermophysical parameters of nanosized particles and the base fluid, including its density ρ_{hnf} , specific heat $(\rho c_p)_{hnf}$, coefficient of thermal expansion $(\rho\beta)_{hnf}$, respectively [49]:

$$F_y = (\rho\beta)_{hnf}(T - T_{avg})g_a \quad (1a)$$

$$\rho_{hnf} = (1 - \gamma)\rho_f + \gamma_{np1}\rho_{np1} + \gamma_{np2}\rho_{np2} \quad (1b)$$

$$(\rho c_p)_{hnf} = (1 - \gamma)(\rho c_p)_f + \gamma_{np1}(\rho c_p)_{np1} + \gamma_{np2}(\rho c_p)_{np2} \quad (1c)$$

$$(\rho\beta)_{hnf} = (1 - \gamma)(\rho\beta)_f + \gamma_{np1}(\rho\beta)_{np1} + \gamma_{np2}(\rho\beta)_{np2} \quad (1d)$$

where $\gamma = \gamma_{np1} + \gamma_{np2}$, indicates the nanoparticle's concentration, β represents the thermal expansion coefficient, and T_m denotes the average temperature $T_{avg} = (T_H + T_C)/2$, for which subscripts np , mnf , hnf , and f refer to nanoparticles, mono nanofluid, hybrid nanofluid, and fluid, respectively.

In the case of mono nanofluid, $\gamma_{np2} = 0$. In this paper, Brinkman [55] (Eq. (2a) for mono nanofluid and Eq. (2b) for hybrid nanofluid) and Maxwell [56] (Eq. (2c) for mono nanofluid and Eq. (2d) for hybrid nanofluid) models are employed to determine the effective dynamic viscosity and effective thermal conductivity of the mono/hybrid nanofluids, respectively. The Brinkman model is applicable to dilute suspensions of small, rigid, and approximately spherical nanoparticles, while the Maxwell model is commonly used to estimate the effective thermal conductivity of well-dispersed and homogeneous nanoparticle suspensions. In the present study, these models are adopted under the assumptions of uniform nanoparticle dispersion and a dilute suspension:

$$\mu_{mnf} = \mu_f \left[\frac{1}{(1 - \gamma_{np})^{2.5}} \right] \quad (2a)$$

$$\mu_{hnf} = \mu_f \left[\frac{1}{(1 - \gamma)^{2.5}} \right] \quad (2b)$$

$$\lambda_{mnf} = \lambda_f \left[\frac{\lambda_{np} + 2\lambda_f - 2\gamma_{np}(\lambda_f - \lambda_{np})}{\lambda_{np} + 2\lambda_f + 2\gamma_{np}(\lambda_f - \lambda_{np})} \right] \quad (2c)$$



$$\lambda_{hnf} = \lambda_f \left[\frac{\lambda_{np1} + \lambda_{np2} + 2\lambda_f - 2\gamma(\lambda_f - \lambda_{np1} - \lambda_{np2})}{\lambda_{np1} + \lambda_{np2} + 2\lambda_f + \gamma(\lambda_f - \lambda_{np1} - \lambda_{np2})} \right] \quad (2d)$$

In this study, mono- and hybrid nanofluids are modeled using a single-phase approach, in which the nanoparticles and base fluid are assumed to be in thermal equilibrium and behave as a homogeneous fluid mixture. The nanoparticles are assumed to be uniformly dispersed in the base fluid, with negligible relative velocity (slip) between the nanoparticles and the carrier fluid. The effects of particle sedimentation, agglomeration, and nanoparticle–fluid slip are neglected. Thermophysical properties of the nanofluids are evaluated using the correlations presented above. Under these assumptions, the governing equations are solved for a single continuous phase, which is considered appropriate for the dilute nanoparticle concentrations investigated in the present study.

To explore the behavior and thermal transfer of the fluid, we based our approach on the lattice Boltzmann method (SRT-LBM), which is derived from the lattice gas automata [57, 58]. Fundamentally, the technique is essentially based on following a set of fictitious particles through a series of diffusion and collision operations. Two distinct distribution functions characterize the flow and temperature fields, which are defined in each computational domain lattice (cell) to depict the behavior of a collection of particles in the thermal LBM [59, 60]. The LBM, a mesoscopic methodology based on kinetic theory, has potential advantages over conventional approaches, including ease of implementation and suitability for parallel computations. In light of this, it has drawn numerous researchers from other disciplines, including those that concentrate on a range of cavity issues using nanofluids. The current LBM model implements two distribution functions to generate flow density $\tilde{F}$ and temperature $\tilde{G}$ fields. These functions' governing equations are, accordingly, as follows [61]:

$$\tilde{F}_i(x_i + c_i \Delta t, t + \Delta t) - \tilde{F}_i(x_i, t) = -\frac{1}{\tau_F} \left(\tilde{F}_i(x_i, t) - \tilde{F}_i^{eq}(x_i, t) \right) + \Delta t \tilde{F}_i, \quad i = 0 - 8 \quad (3a)$$

$$\tilde{G}_i(x_i + c_i \Delta t, t + \Delta t) - \tilde{G}_i(x_i, t) = -\frac{1}{\tau_G} \left(\tilde{G}_i(x_i, t) - \tilde{G}_i^{eq}(x_i, t) \right), \quad i = 0 - 8 \quad (3b)$$

where $\tau_{\tilde{G}}(\tau_{\tilde{F}})$ signifies the relaxation time for the temperature (flow) field, Δt denotes the lattice time step ($\Delta t = 1$), and c_i represents the discrete velocity vector of a particle in the i -direction.

$$c_i = \begin{cases} (0,0), & i = 0 \\ c(\cos[(i-1) \cdot \frac{\pi}{2}], \sin[(i-1) \cdot \frac{\pi}{2}]), & i = 1 - 4 \\ \sqrt{2}c(\cos[(i-1) \cdot \frac{\pi}{2} + \frac{\pi}{4}], \sin[(i-1) \cdot \frac{\pi}{2} + \frac{\pi}{4}]), & i = 5 - 8 \end{cases}, \quad (4)$$

The term $\tilde{G}_i^{eq}(\tilde{F}_i^{eq})$ is the temperature's (flow) local equilibrium distribution function in first (second) order, expressed as follows:

$$\tilde{F}_i^{eq} = \omega_i \rho \left[1 + 3 \frac{c_i u}{c^2} + \frac{9}{2} \frac{(c_i u)^2}{c^4} - \frac{3}{2} \frac{u^2}{c^2} \right], \quad i = 0 - 8 \quad (5a)$$

$$\tilde{G}_i^{eq} = \omega_i T \left[1 + 3 \frac{c_i u}{c^2} \right], \quad i = 0 - 8 \quad (5b)$$

with ω_i is the weight factor for temperature and for flow density. The variables T, u , and ρ indicate macroscopic temperature, velocity, and density, respectively, as follows:

$$\omega_i = \begin{cases} 4/9 & \text{for } i = 0 \\ 1/9 & \text{for } i = 1 - 4 \\ 1/36 & \text{for } i = 5 - 8 \end{cases} \quad (6)$$

In this study, we used D_2Q_9 model for both temperature and flow Fig. 2.

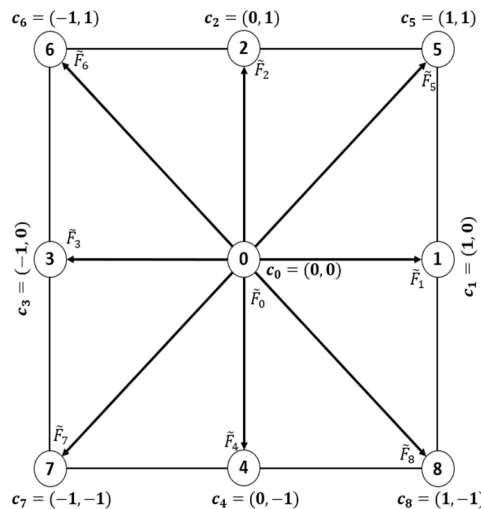


Fig. 1. Discrete velocity set of the D_2Q_9 model.



We determine the thermal diffusivity α and kinematic viscosity ν , according to this model, by relaxation times $\tau_{\tilde{G}}$ and $\tau_{\tilde{F}}$, respectively, by:

$$\alpha = \left(\tau_{\tilde{G}} - \frac{1}{2} \right) c_s^2 \Delta t \quad (7a)$$

$$\nu = \left(\tau_{\tilde{F}} - \frac{1}{2} \right) c_s^2 \Delta t \quad (7b)$$

where c_s is the sound speed. The kinematic viscosity ν is estimated by establishing the Rayleigh number (Ra), Prandtl (Pr), and Mach (Ma) numbers:

$$\nu = N \cdot Ma \cdot c_s \sqrt{\frac{Pr}{Ra}} \quad (8)$$

In the present study, the lattice Mach number is fixed at $Ma = 0.1$ for all simulations. This value is sufficiently lower than the commonly accepted limit of $Ma < 0.3$ for incompressible flows, thereby ensuring that compressibility effects remain negligible throughout the simulations.

In Eq. (8), N defines the lattice number in the y-direction. Rayleigh (Ra) and Prandtl (Pr) numbers, which are linked to thermal diffusivity and kinematic viscosity, are expressed as:

$$Pr = \frac{\nu_{hnf}}{\alpha_{hnf}} \quad (9a)$$

$$Ra = \frac{g_a (T_H - T_C) \beta_{hnf} N^3}{\nu_{hnf} \alpha_{hnf}} \quad (9b)$$

We estimated the standard velocity of flow v_{Scale} as:

$$v_{Scale} = \frac{1}{L} \sqrt{\alpha_{hnf} \nu_{hnf} Ra} \quad (10)$$

The external force expression F_i of this study, which is implemented in the collision step, as demonstrated in Eq. (3a), is generated by:

$$F_i = \frac{\omega_i}{c_s^2} c_i F_y \quad (11)$$

With $F_y = \rho \cdot \Delta t \cdot g_a \cdot \beta_{hnf}$ denotes the expression of buoyancy force, g_a represents the gravity acceleration, and Δt is expressed as $\Delta t = T - T_{avg}$, and:

$$\rho = \sum_{i=0}^8 \tilde{F}_i \quad (12a)$$

$$\rho u = \sum_{i=0}^8 \tilde{F}_i c_i \quad (12b)$$

$$T = \sum_{i=0}^8 \tilde{G}_i \quad (12c)$$

The Nusselt number is calculated using the thermal conductivity of the nanofluid and base fluids. The average Nusselt number is used to evaluate heat transmission improvement; an increase in the Nusselt number indicates better heat transmission.

We calculate the local Nusselt number Nu for each hot part wall (heated corner, heated source) and the average Nusselt number value Nu_{avg} of the enclosure, expressed respectively, in the following manner:

$$Nu_{B-R-Heated\ Corner} = -\frac{\lambda_{hnf}}{\lambda_f} \left[\left(\frac{\partial \theta}{\partial Y} \right)_{Y=0} + \left(\frac{\partial \theta}{\partial X} \right)_{X=n} \right] \quad (13a)$$

$$Nu_{L-T-Heated\ Corner} = -\frac{\lambda_{hnf}}{\lambda_f} \left[\left(\frac{\partial \theta}{\partial Y} \right)_{Y=m-l} + \left(\frac{\partial \theta}{\partial X} \right)_{X=l} \right] \quad (13b)$$

$$Nu_{Heated\ Source} = -\frac{\lambda_{hnf}}{\lambda_f} \left[\left(\frac{\partial \theta}{\partial Y} \right)_{Y=\frac{m}{2}-\frac{l}{2} \text{ and } Y=\frac{m}{2}+\frac{l}{2}} + \left(\frac{\partial \theta}{\partial X} \right)_{X=\frac{n}{2}-\frac{l}{2} \text{ and } X=\frac{n}{2}+\frac{l}{2}} \right] \quad (13c)$$

$$Nu = Nu_{B-R-Heated\ Corner} + Nu_{L-T-Heated\ Corner} + Nu_{Heated\ Source} \quad (14a)$$

$$Nu_{avg} = \frac{1}{l} \left[\int_0^l Nu \cdot dx + \int_0^l Nu \cdot dy \right] \quad (14b)$$

where θ is the normalized temperature.

2.2. Boundary conditions

In the modelling of micro-geometries, boundary conditions are extremely important. This paragraph highlights the boundary conditions employed in this work. It is implied that every wall surface has no-slip velocity and is stationary $u = v = 0$.



Table 2. Grid independence test for $Al_2O_3/water$ nanofluid, $\gamma = 2\%$, and $Ra = 10^5$ with the heater source in the center.

Value of N	Nu_{avg}
60×60	7.21533203
80×80	14.4658270
100×100	14.3311491
120×120	19.9723759
140×140	19.9671402
150×150	20.0922997

Table 3. Analysis of the average Nusselt number between our data and those cited in the previous literature for air ($Pr = 0.71$).

References	Ra			
	10^3	10^4	10^5	10^6
Present (SRT-LBM)	1.124	2.264	4.568	8.807
AlAmiri et al. [64]	N/A	N/A	4.522	8.826
De Vahl Davis [65]	1.118	2.243	4.519	8.799
Wan et al. [66]	1.117	2.254	4.592	8.976
Fusegi et al. [67]	1.105	2.302	4.646	9.012

For the flow (stream) field, the conventional bounce-back boundary condition [62, 63] was applied at the heated source and obstacle boundaries as well as the walls of the enclosure. For example, for the left wall of the enclosure, the unknowns' distributions are:

$$\begin{cases} \tilde{F}_1 = \tilde{F}_3 \\ \tilde{F}_5 = \tilde{F}_7 \\ \tilde{F}_8 = \tilde{F}_6 \end{cases} \quad (15)$$

For the temperature field, the boundary conditions employed in this investigation are established by Mohamad [62]. For the bottom side wall of the heated source and for the horizontal heated part of the enclosure top-corner, the unknowns' distributions are $\tilde{G}_2$, $\tilde{G}_5$, and $\tilde{G}_6$, are evaluated as:

$$\begin{cases} \tilde{G}_2 = \theta_H(\omega_2 + \omega_4) - \tilde{G}_4 \\ \tilde{G}_5 = \theta_H(\omega_5 + \omega_7) - \tilde{G}_7 \\ \tilde{G}_6 = \theta_H(\omega_6 + \omega_8) - \tilde{G}_8 \end{cases} \quad (16)$$

where $\theta_H = 1$. The unknowns' distributions $\tilde{G}_4$, $\tilde{G}_7$, and $\tilde{G}_8$ on the enclosure's cold top wall are determined as follows:

$$\begin{cases} \tilde{G}_4 = \theta_C(\omega_4 + \omega_2) - \tilde{G}_2 \\ \tilde{G}_7 = \theta_C(\omega_7 + \omega_5) - \tilde{G}_5 \\ \tilde{G}_8 = \theta_C(\omega_8 + \omega_6) - \tilde{G}_6 \end{cases} \quad (17)$$

where $\theta_C = 0$.

3. Results and Discussion

3.1. Mesh stabilization and code validation

In this subsection, we compute the Nu_{avg} for various meshes, from 60×60 to 150×150 . Table 2 reveals that Nu_{avg} stabilizes from 140×140 . The grid size of 140×140 was selected for the current numerical computations.

To verify the validity of the obtained results, Table 3 shows a strong agreement of the average Nusselt number between the current ones and numerical results found in the scientific literature for different Rayleigh numbers. In this validation study, the problem examined natural convection of air ($Pr = 0.71$) in a square cavity, for which the boundary conditions were selected such that the horizontal walls are adiabatically heated, the right wall is kept at a cold temperature, and the left one is heated.

Figure 3 provides additional verification of the accuracy of our numerical code by comparing the isotherms and streamlines of our results to those reported by Rashidi et al. [68] and Talebi et al. [69] for particular Rayleigh numbers ($Ra = 1.47 \times 10^4$). The comparisons reveal excellent conformity with the mentioned analysis. To demonstrate the robustness of our numerical procedure (LBM), we compared it with the experimental and numerical (FVM) results from Paroncini and Corvaro [70].

Figure 4 depicts, in terms of isotherms and streamlines, the comparison of our results with those numerical and experimental results [68]. The findings show remarkable qualitative consistency, with similar thermal patterns and flow dynamics. We consider that negative flow function values (–) in all streamline shapes indicate counterclockwise circulation within the enclosure.

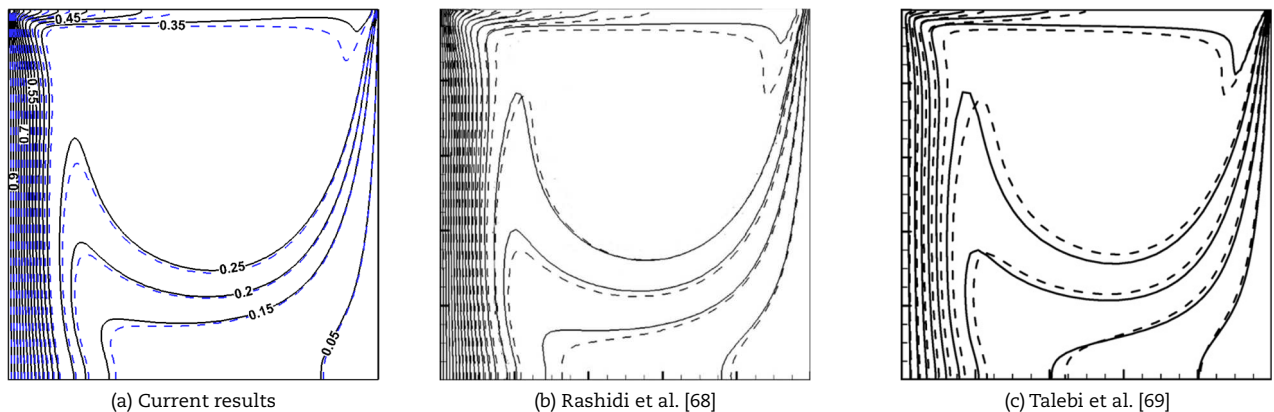


Fig. 3. Isotherms (a-c) $\gamma = 0\%$ (–) and $\gamma = 5\%$ (---) and streamlines (d and e) for $Ra = 1.47 \times 10^4$ and Cu/water nanofluid.



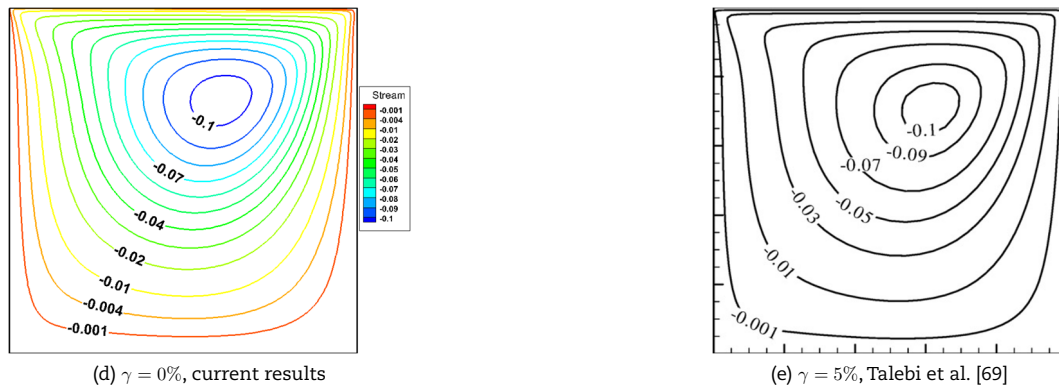
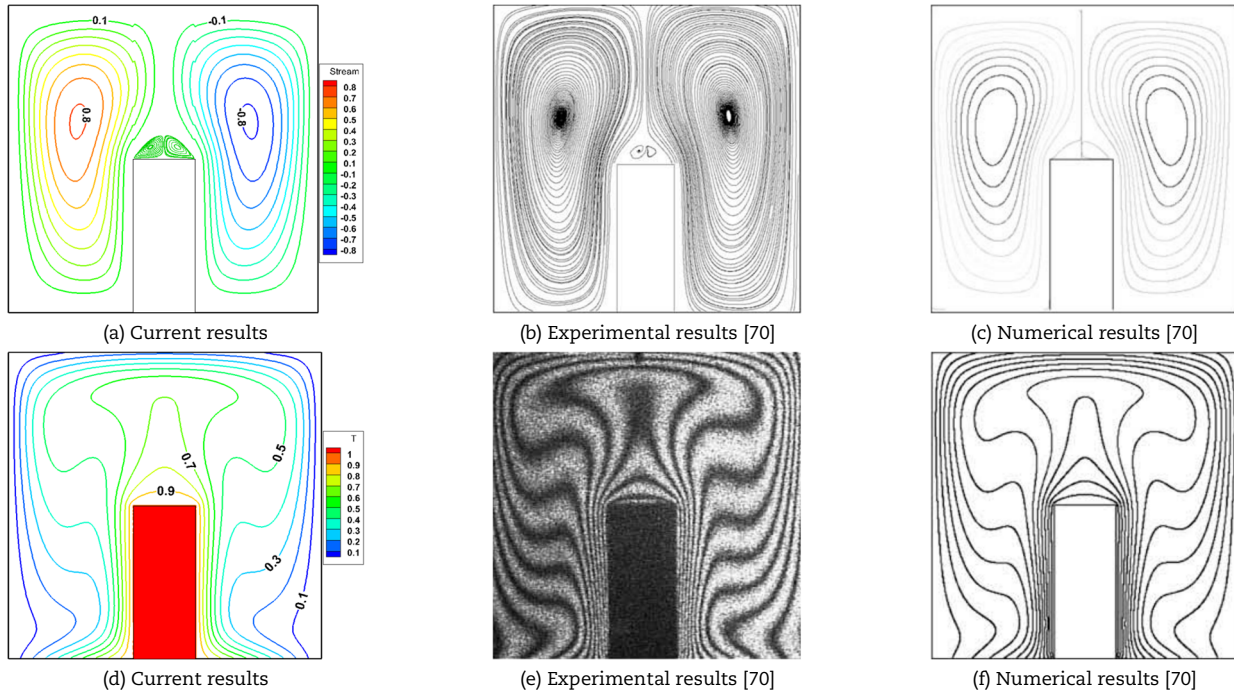
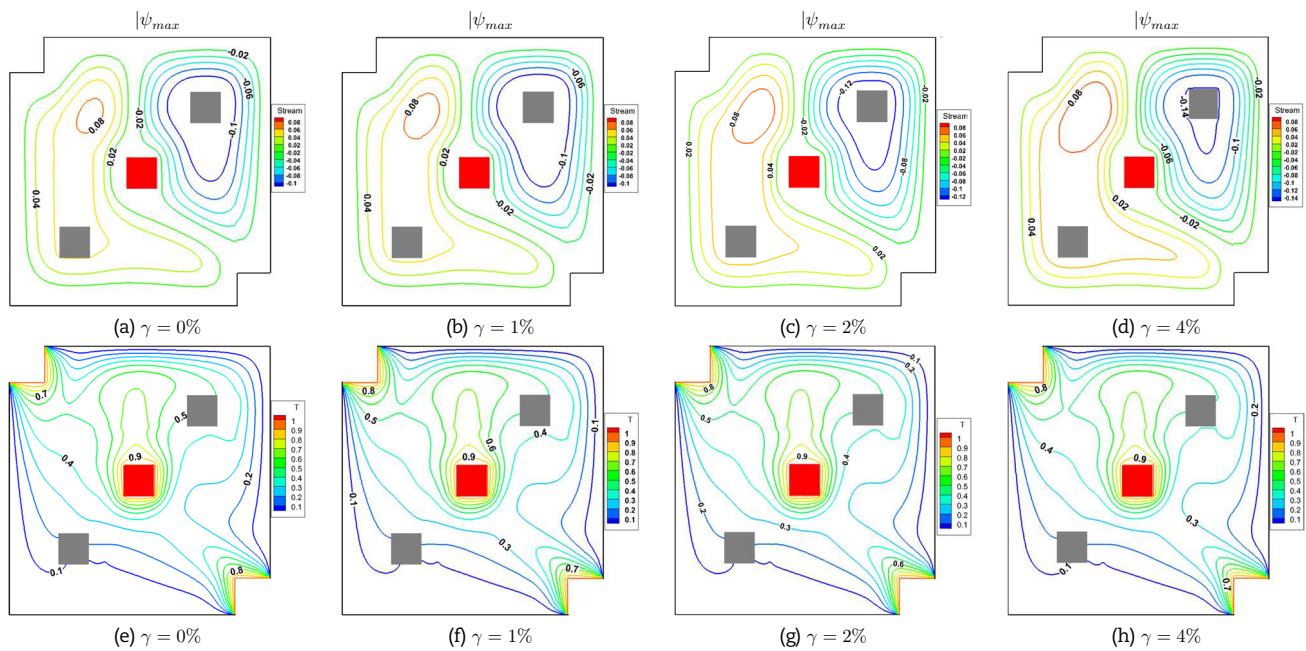


Fig. 3. Continued.

Fig. 4. Comparison of streamlines (a-c) and isotherms (d-f) between our results and those of Paroncini and Corvaro, for $Ra = 1.76 \times 10^5$ in the case of air ($Pr = 0.71$).Fig. 5. Streamlines (a-d) and isotherms (e-h) for $Ra = 10^5$, $Al_2O_3/water$, and different values of γ .

3.2. Effect of nanoparticle volume fraction γ

In this part, we discuss the influence of the nanoparticle volume fraction, $0\% \leq \gamma \leq 4\%$, on the isotherms and streamline contours for a fixed Rayleigh number, (Ra), with the heated source located at the center of the enclosure (Fig. 5).

Figure 5(a–d) depicts the streamline contours, showing two counter-rotating vortices around the heated source. The temperature difference between the heated source and the surrounding fluid generates buoyancy forces, which drive the natural-convection circulation and produce the two opposing recirculation zones. At low nanoparticle concentrations, these vortices remain relatively compact around the heat source. As the nanoparticle volume fraction increases, the effective viscosity of the nanofluid also increases, resulting in greater viscous resistance to the buoyancy-driven motion. The absolute value of the maximum stream function increases from $|\psi_{max}|$ for $\gamma = 0\%$ to $|\psi_{max}|$ for $\gamma = 4\%$. This evolution indicates that, despite the increase in viscous resistance, the modification of the thermophysical properties of the nanofluid alters the balance between buoyancy and viscous forces, leading to a more developed circulation pattern. The two counter-rotating vortices consequently expand and occupy a larger portion of the enclosure as the nanoparticle concentration increases.

For the isotherm profiles, increasing the nanoparticle volume fraction causes the hot isotherms to extend farther from the heated source and become more distributed toward the upper-left region of the enclosure (Fig. 5(e–h)). This behavior is mainly associated with the increase in the effective thermal conductivity of the nanofluid caused by the addition of nanoparticles. The enhanced thermal conductivity promotes heat diffusion away from the localized heat source and modifies the thermal boundary layer surrounding the heated region. Consequently, the thermal energy is transported more effectively through the fluid, which explains the broader distribution of high-temperature isotherms. The simultaneous modification of the flow field and temperature field demonstrates the coupled influence of nanoparticle concentration on the thermo-hydrodynamic behavior of the nanofluid.

3.3. Rayleigh number Ra effect

The influence of the Rayleigh number (Ra) for a specific nanosized particle concentration $\gamma = 2\%$ with the reference case of the heated source (middle position of the enclosure) in terms of isotherms and streamlines, is explored (Fig. 6).

Figure 6(a–d) indicates that as the Rayleigh number value Ra increases, the absolute value of the maximum flow (stream) function $|\psi_{max}|$ rises (from $|\psi_{max}|$ for $Ra = 10^3$ to $|\psi_{max}|$ for $Ra = 10^6$). This substantial increase reflects the progressive strengthening of buoyancy-driven flow as the Rayleigh number increases. Since (Ra) represents the ratio of buoyancy effects to viscous and thermal diffusion effects, increasing (Ra) promotes fluid motion and enhances the convective transport of thermal energy. Consequently, the two counter-rotating vortices become progressively stronger and occupy a larger region of the enclosure. At ($Ra = 10^6$), the two counter-rotating vortices are positioned closer to each other in the upper region of the enclosure compared with the configuration at ($Ra = 10^3$). This modification of the recirculation structure results from stronger upward motion of the heated fluid and the corresponding return flow along the surrounding walls.

Figure 6(e–h) illustrates the corresponding evolution of the isotherms with increasing (Ra). At low Rayleigh numbers ($Ra = 10^3$), buoyancy forces are relatively weak and heat transfer is mainly governed by thermal diffusion. Therefore, the isotherms remain relatively smooth and nearly parallel, with only limited distortion around the central heat source and the two obstacles. As Ra increases, the isotherms become progressively distorted and redistributed throughout the enclosure. This behavior indicates a gradual transition from conduction-dominated to convection-enhanced heat transfer. At intermediate Rayleigh numbers, the stronger buoyancy-induced circulation transports hot fluid away from the heat source and cold fluid toward the heated region, leading to greater deformation of the thermal field.

For the highest investigated Rayleigh number $Ra = 10^6$, the strong buoyancy forces generate intense recirculating flow and substantially distort the isotherms, particularly near the heated source, enclosure corners, and obstacle surfaces. The increased thermal-field deformation reflects stronger convective transport and a thinner thermal boundary layer near the heated region. Consequently, heat is removed more effectively from the source and redistributed through the enclosure, resulting in enhanced overall convective heat transfer.

3.4. Effect of heated source position

To analyze the effect of the heated source position, three configurations are considered: the top-left, middle, and bottom-right positions, for a fixed nanoparticle volume fraction of $\gamma = 2\%$ and a Rayleigh number of $Ra = 10^5$ (Fig. 7). The results demonstrate that the location of the heat source strongly modifies the buoyancy-driven circulation and the distribution of thermal energy within the enclosure. This effect results from the modification of the local temperature gradients and, consequently, the buoyancy forces generated around the heated source.

Figure 7(a–c) shows that the bottom-right position produces the highest absolute value of the maximum stream function, $|\psi_{max}|$, compared with the middle and top-left positions, for which $|\psi_{max}|$ and $|\psi_{max}|$, respectively. The stronger circulation observed for the bottom-right configuration can be attributed to the asymmetric location of the heat source, which promotes a more effective upward movement of the heated fluid and establishes a stronger recirculation path within the enclosure. The resulting flow structure favors enhanced buoyancy-driven motion and increases the intensity of the convective circulation.

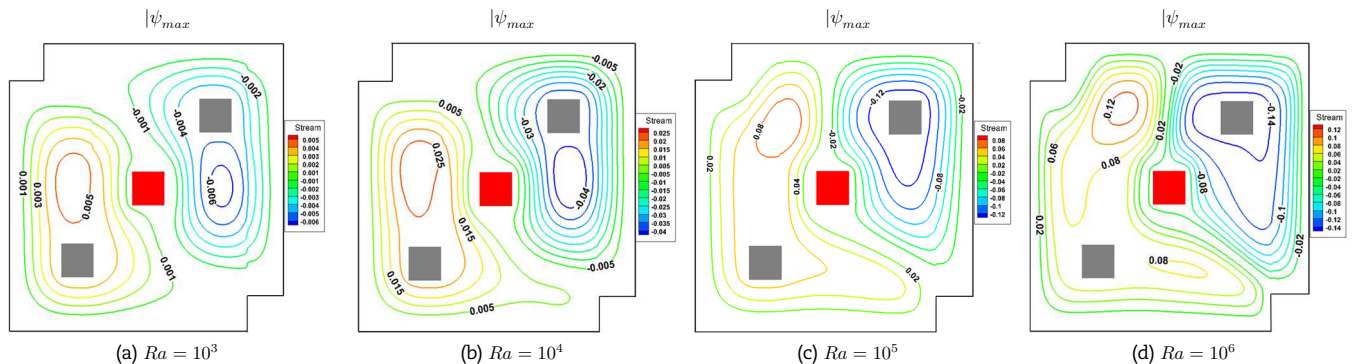


Fig. 6. Streamlines (a–d) and isotherms (e–h) for $\gamma = 2\%$, Al_2O_3 , and different values of Ra .



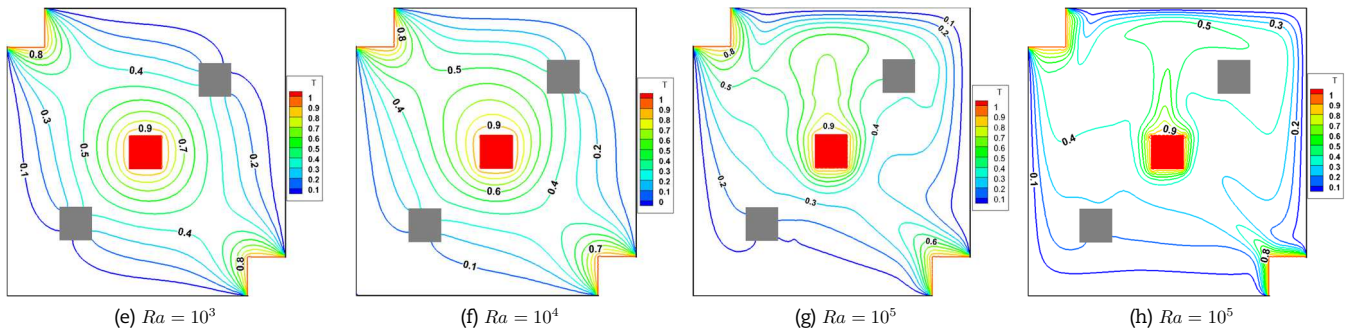
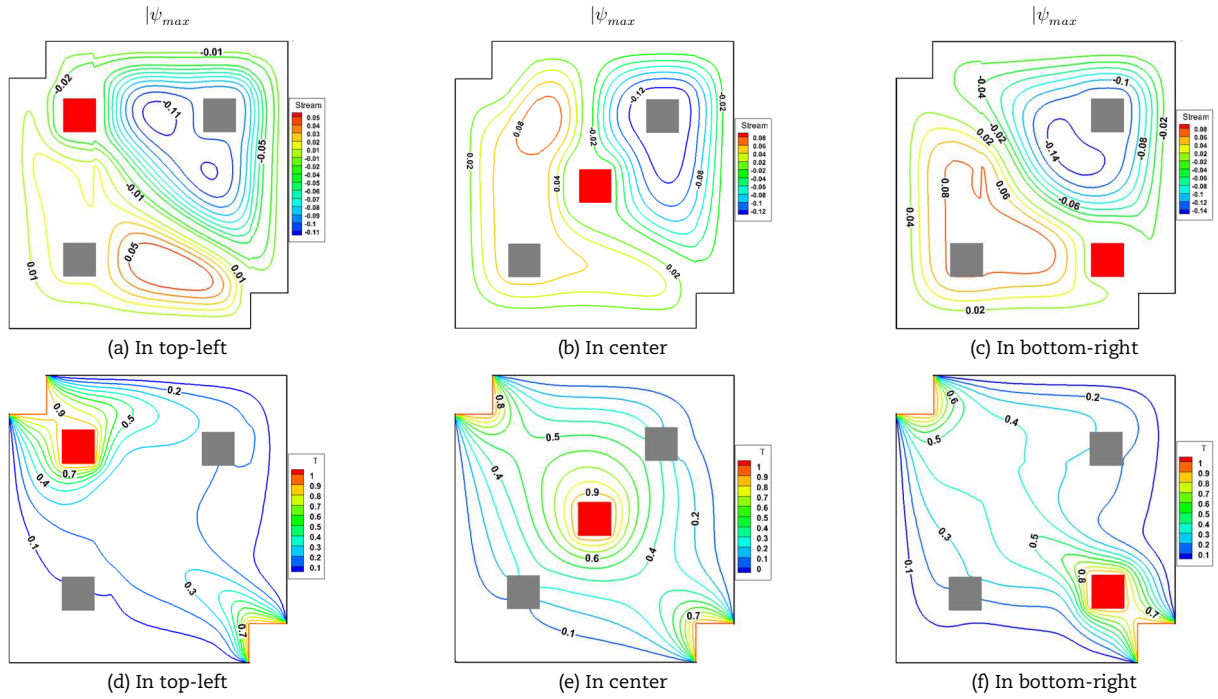
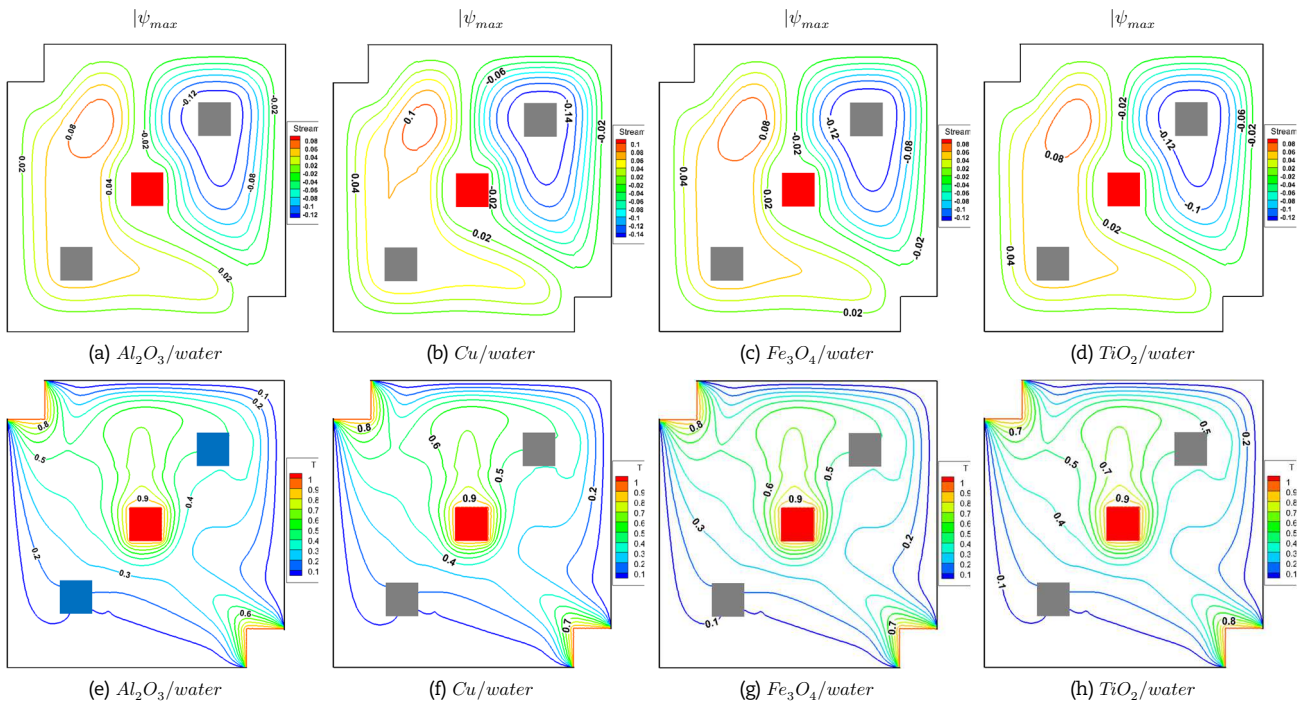


Fig. 6. Continued.

Fig. 7. Streamlines (a-c) and isotherms (d-f) for $\gamma = 2\%$, $Ra = 10^5$, $Al_2O_3/water$, and different positions of the heater source.Fig. 8. Streamlines (a-d) and isotherms (e-h) for $\gamma = 2\%$, $Ra = 10^5$, and different nanoparticle types.

For the isotherm contours shown in Fig. 7(d–f), the heat source position also substantially modifies the thermal-field distribution. The middle position leads to a broader distribution of high-temperature isotherms throughout the enclosure, whereas the other positions produce a more localized thermal field. This behavior indicates that the intensity of the circulation alone does not completely determine the thermal performance. Indeed, the interaction between the heat source location, the flow circulation, and the thermal boundary layers controls the efficiency with which heat is redistributed from the source to the surrounding fluid.

Consequently, although the bottom-right configuration generates the highest flow intensity, the central position provides a more favorable thermal distribution and promotes more effective heat transfer throughout the enclosure. This finding highlights the importance of heat-source placement as a thermal-management parameter, since an appropriate position can improve heat spreading and temperature uniformity without necessarily corresponding to the maximum flow intensity.

3.5. Effect of nanoparticle type

Figure 8 depicts the impact of nanoparticle type (Al_2O_3 , Cu , Fe_3O_4 , and TiO_2) on the streamlines and isotherms. For the streamlines, the results demonstrate that the nanofluid $Cu/water$ has stronger streaming ($|\psi_{max}|$) than nanofluids with other nanoparticles; Al_2O_3 , TiO_2 , and Fe_3O_4 , from good to low, respectively. Therefore, the nanofluid $Cu/water$ has the best convection due to slightly high viscosity (Fig. 8(a–d)). Regarding isotherms (Fig. 8(e–h)), the results demonstrate that the nanofluid $Cu/water$ has the best thermal transfer enhancement owing to the high thermal conductivity of the nanosized particles Cu more than other nanoparticles Al_2O_3 , TiO_2 , and Fe_3O_4 , successively. As a conclusion of this section, the nanofluid $Cu/water$ has the best convection stability and high heat transfer enhancement performance.

3.6. Effect of hybrid nanofluid type

In this part, the influence of hybrid nanofluid type is studied for three different hybrid nanofluids ($Al_2O_3 + Cu/water$, $Al_2O_3 + Fe_3O_4/water$, $Al_2O_3 + TiO_2/water$), $Ra = 10^5$, and $\gamma = 2\%$ (1% for each nanoparticle), in terms of streamlines and isotherms (Fig. 9). For streamlines, the results of Fig. 9(a–c) show that the hybrid nanofluid $Al_2O_3 + Cu/water$ has strong convection due to the superior conductivity of nanoparticles Cu , synergized by the stable dispersion properties of the nanoparticles Al_2O_3 than other types of nanoparticles. For isotherms, the findings demonstrate that the hybrid nanofluid $Al_2O_3 + Cu/water$ provides the strongest convection heat transfer due to the highest thermal conductivity λ_{hnf} than another hybrid nanofluid $Al_2O_3 + Fe_3O_4/water$ and $Al_2O_3 + TiO_2/water$ (Fig. 9(d–f)). Based on these results, $Al_2O_3 + Cu/water$ hybrid nanofluid offers good performance even though it presents a challenge in terms of cost.

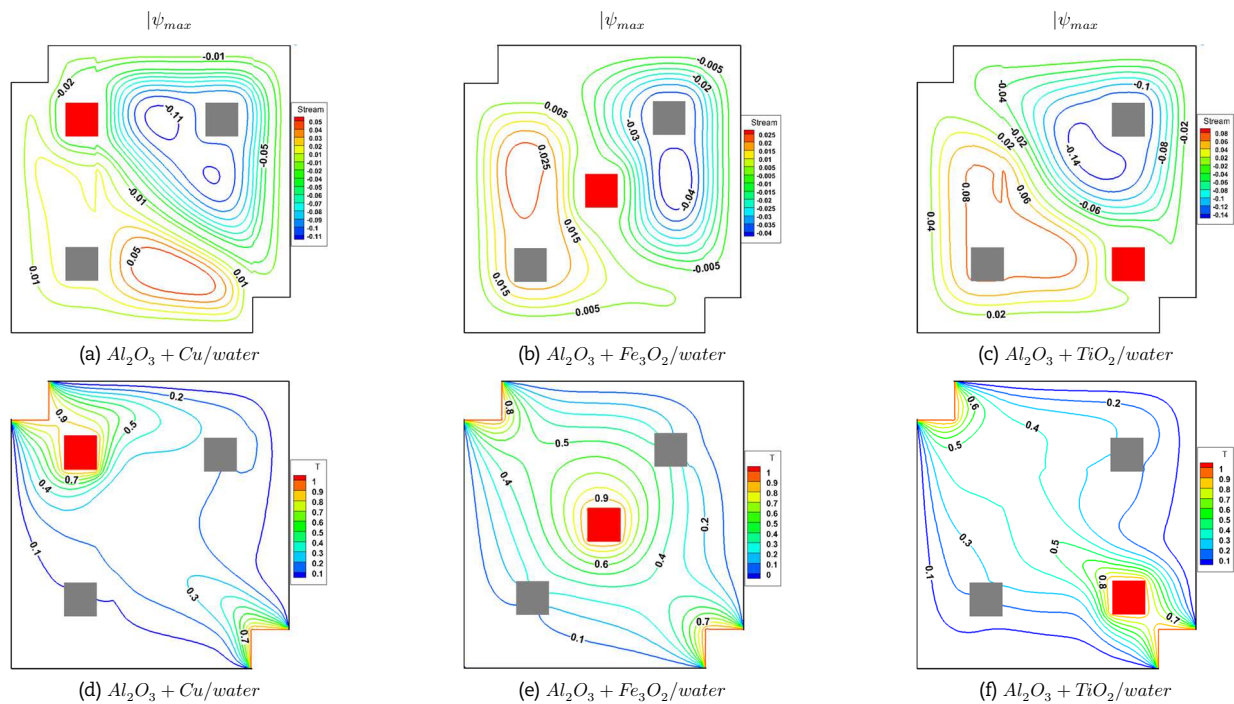


Fig. 9. Streamlines (a–c) and isotherms (d–f) for $\gamma = 2\%$, $Ra = 10^5$, and different hybrid nanofluid types.

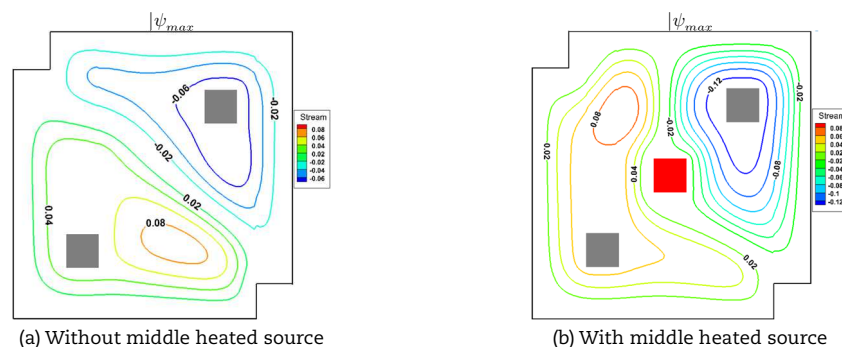


Fig. 10. Streamlines (a and b) and isotherms (c and d) for $\gamma = 2\%$, $Ra = 10^5$, and $Al_2O_3/water$.



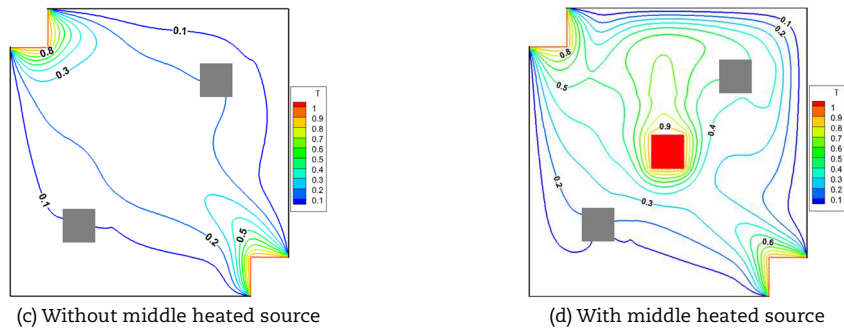
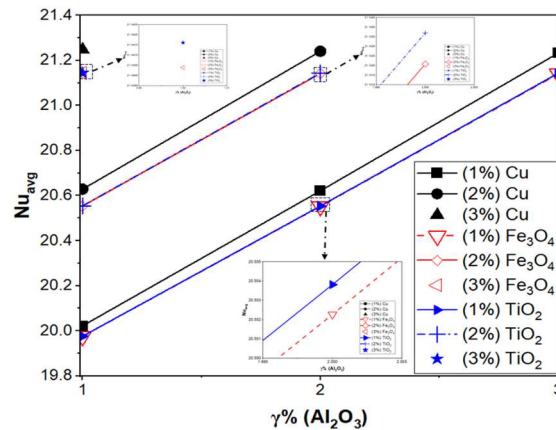
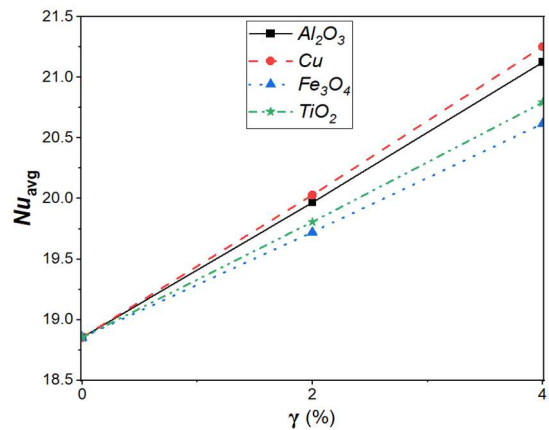
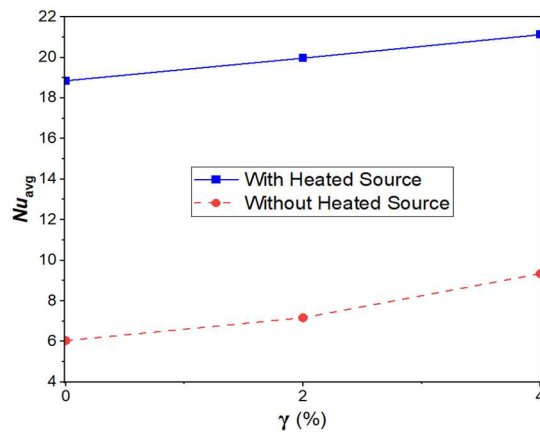
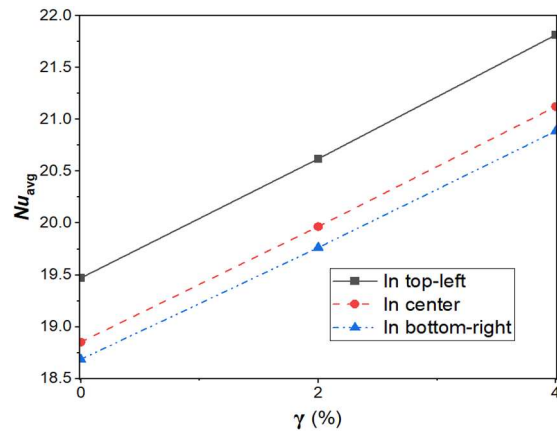
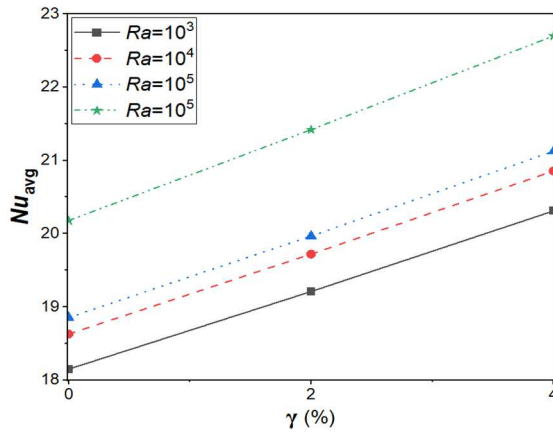


Fig. 10. Continued.

Fig. 11. Average Nusselt number variation Nu_{avg} .

That is why this type of hybrid nanofluid $Al_2O_3 + Cu/water$ is most in demand in heat exchange, high-performance cooling, and solar thermal collector applications.

3.7. Effect of middle heated source presence

The effect of heated source presence is investigated in the form of isotherms and streamlines for particular parameters; $Ra = 10^5$, $Al_2O_3/water$, and $\gamma = 2\%$ where the heated source is in the center of the enclosure (Fig. 10). The results in Fig. 10(a and b) show that with the presence of a heated source, the streamline contours are more distorted within the enclosure; as a result, the convection processes are important in the case of heated source presence; $|\psi_{max}|$ with the presence of a heated source and $|\psi_{max}|$ without a heated source. Regarding isotherms, without the heated center, the hot isotherms are limited and occur around the hot corners. In contrast, in the case of heated source presence, the hot isotherms are distributed throughout the enclosure, which means the thermal transport is important, so the heat transfer performance is strongest Fig. 10(c and d).

3.8. Overall heat transfer change

Heat transmission enhancement is typically evaluated by comparing the nanofluid average Nusselt number Nu_{avg} , where the average Nusselt Nu_{avg} is high, as the thermal transfer rate is enhanced.

Figure 11 depicts the reliance of the average Nusselt number (Nu_{avg}), computed throughout the heated surface of the enclosure, on the nanosized particle's volume percentage γ for various Rayleigh number values Ra , heated source position, nanoparticle type, and hybrid nanofluid type. The result of Fig. 11(a) shows that for all nanoparticle fractions ($\gamma = 0, 1, 2$, and 4%) values, Nu_{avg} increases as Ra rises, and the same thing for all Rayleigh numbers Ra , Nu_{avg} increases with the rise of nanoparticle volume fraction γ , i.e., thermal transfer is improved with the increase in Rayleigh number Ra and nanoparticle volume percentage γ , as we demonstrated in previous parts for isotherms and streamlines (Figs. 5 and 6). Figure 11(b) studies the effect of heated source position on Nu_{avg} . The results demonstrate that in the case where the heated source is in the center, Nu_{avg} has a maximum value greater than in other cases for all nanoparticle concentrations γ .

Figure 11(c) provides the average Nusselt number Nu_{avg} values in two cases: with and without a heated source for various nanoparticle volume fractions ($\gamma = 0, 2$, and 4%). The findings demonstrate that the Nu_{avg} is maximal and more important in the case of the heated source presence as we increase the nanoparticles volume fraction γ , where it increases faster for lower Ra values. Therefore, the presence of a heated source controls the overall heat exchange of the nanofluid.

The type of nanoparticle has an impact on stream (flow) behavior and heat transmission. We demonstrate that the nanoparticle type can control the thermal transfer rate (Fig. 11(d)). The outcomes show that the type of nanoparticle significantly influences the evolution of the average Nusselt number Nu_{avg} with increasing nanoparticle concentration. Among the commonly used metal and metal oxide nanoparticles, Cu exhibits the strongest improvement in convective heat transmission owing to its extremely high thermal conductivity. As the nanosized particle concentration increases from 0 to 4% , Cu -based nanofluids show a steep rise in Nu_{avg} , often outperforming Al_2O_3 , TiO_2 , and Fe_3O_4 . $Al_2O_3/water$ nanofluids also show a noticeable increase in Nu_{avg} with volume fraction and offer better long-term stability, making them a balanced choice for heat transfer applications. In contrast, TiO_2 and Fe_3O_4 nanofluids demonstrate only modest improvements, Fe_3O_4 being more suited to magnetic or biomedical applications rather than pure heat transfer enhancement. Overall, for maximizing convective heat transfer, Cu and Al_2O_3 nanoparticles are superior, particularly at optimized volume fractions between 2 and 4% .

The hybrid nanofluid type significantly influences the evolution of the average Nusselt number Nu_{avg} with two nanosized particle concentration values $\gamma = 2$ and 4% (Fig. 11(e)). For all nanoparticle concentration values, the results demonstrate that $(1\%)Al_2O_3 + (3\%)Cu/water$ nanofluid offers better thermal transfer enhancement due to the elevated Nu_{avg} value. In addition, as the volume percentage of Cu takes on a large value compared to Al_2O_3 ; $(1\%)Al_2O_3 + (3\%)Cu/water$, the nanofluid has strong convection heat transfer.

Regarding the stability and cost of nanofluid in industries, due to the stability, good thermal conductivity value, and the cost of nanoparticle Al_2O_3 , the hybrid nanofluid $(3\%)Al_2O_3 + (1\%)Cu/water$ is a better choice for convection flow mechanism and heat transfer than $(1\%)Al_2O_3 + (3\%)Cu/water$ due to the expansion of Cu nanoparticle.

As a resume, the selection of nanoparticle types in hybrid nanofluids significantly influences not only thermal performance, typically measured by the average Nusselt number, but also the overall cost and economic feasibility of the system. $(3\%)Al_2O_3 + (1\%)Cu/water$ hybrid nanofluids generally provide the highest convection and heat transfer among the compared mixtures, owing to the exceptional thermal conductivity of copper and stability of Al_2O_3 .

At the nanoscale, the enhancement of heat transfer in nanofluids can be associated with several physical mechanisms, including the increased effective thermal conductivity resulting from the presence of highly conductive nanoparticles, enhanced microscopic energy transport, and, depending on the nanoparticle characteristics and concentration, intensified interactions between the dispersed particles and the surrounding fluid. In hybrid nanofluids, the combination of two different nanoparticles can provide complementary thermal properties and potentially improve effective thermal conductivity compared with a mono-nanofluid. These nanoscale effects are reflected at the macroscopic scale through modifications of the effective thermophysical properties, which subsequently influence the temperature field, buoyancy-driven flow, thermal boundary layers, and ultimately the average Nusselt number. Since the present study adopts a homogeneous single-phase model, individual particle-scale trajectories and interfacial interactions are not explicitly resolved; their overall effects are incorporated through the effective-property correlations adopted in the model.

3.9. Temperature and v-velocity distributions

Figure 12(a) shows how γ affects the variation of dimensionless temperature along the horizontal centerline ($Y = 0.5$) for particular parameters; $Ra = 10^5$, Al_2O_3 nanoparticle, and heated source in the center. The results demonstrate that around the left wall, the temperature is a creasing function from the left to the heated source in the middle of the enclosure. Also, as γ increases, the temperature increases due to the thermal properties of the nanoparticle.

Figure 12(b) shows that for small Ra values (10^3 and 10^6), the variation of temperature is quasi-linear, demonstrating a conduction-dominated process, which means natural convection is relatively weak, so heat is transmitted primarily through dispersion of molecules in place of fluid motion. For $Ra \geq 10^5$, the temperature distribution becomes curved; as a result, natural convection is dominant owing to the increase in buoyancy force. With the presence of a heated source and two obstacles, the temperature variation gets more intricate, with obvious thermal gradients within the enclosure.



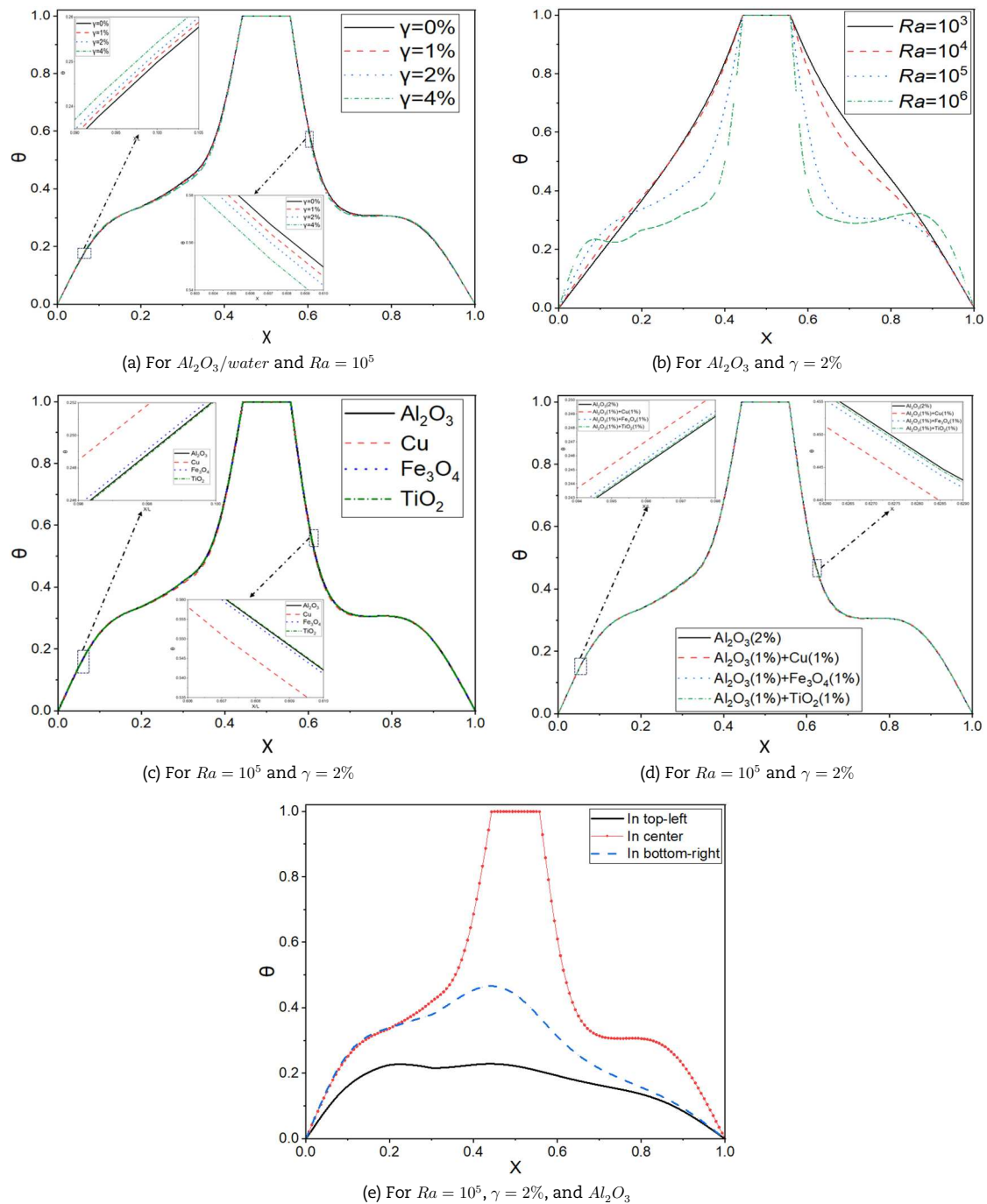


Fig. 12. Temperature profiles along the horizontal line $Y = 0.5$.

Figures 12(c and d) depict the effect of the type of nanoparticles and their hybrid. The results demonstrate that Cu has high thermal transport by diffusion of molecules due to its strong thermal conductivity. With mixing two nanoparticles to each ($Al_2O_3 + Cu$, $Al_2O_3 + Fe_3O_4$, $Al_2O_3 + TiO_2$) in the base fluid, the data prove that the temperature fluctuation is intense for the hybrid $Al_2O_3 + Cu$ more than other hybrids. We conclude that this intensity of temperature is due to the presence of nanoparticles Cu . Figure 12(e) presents the dimensionless temperature profiles for the $Y = 0.5$ lines at different positions of the heating source (in top-left, in center, in bottom-right), offering important perspectives on the mechanisms of thermal transfer in the studied enclosure. The temperature variation is more efficient and regular for the case where the heating source is in the center than in other cases of heating source positions.

Figures 13(a) and 13(b) exhibit the vertical and horizontal velocities and component plots along the horizontal line $Y = 0.5$ and the vertical one $X = 0.5$, for $Ra = 10^5$, and different nanoparticle concentrations γ . The results show that as the nanoparticle percentage γ , the maximum magnitude of v -velocity of the nanofluid diminishes dramatically owing to the rising thermal conductivity of the nanofluid λ_{nf} and viscosity.

Figures 13(c) and 13(d) demonstrate that, for $\gamma = 2\%$ and the heat source in the center position, elevating the Rayleigh number increases the amplitude of u and v -velocities elevate owing to a reduce of viscosity μ , indicating that the heat transmission improves as explored in Fig. 11(a). An insight on Figs. 13(e) and 13(f), show that the type of nanosized particles influences the fluid dynamics and thermal transfer. The result explored that the nanoparticle Cu has a crucial effect and strong intensity in fluid velocities, which means has an increasing in molecular diffusion and thus heat transfer in comparison with other nanoparticle types.



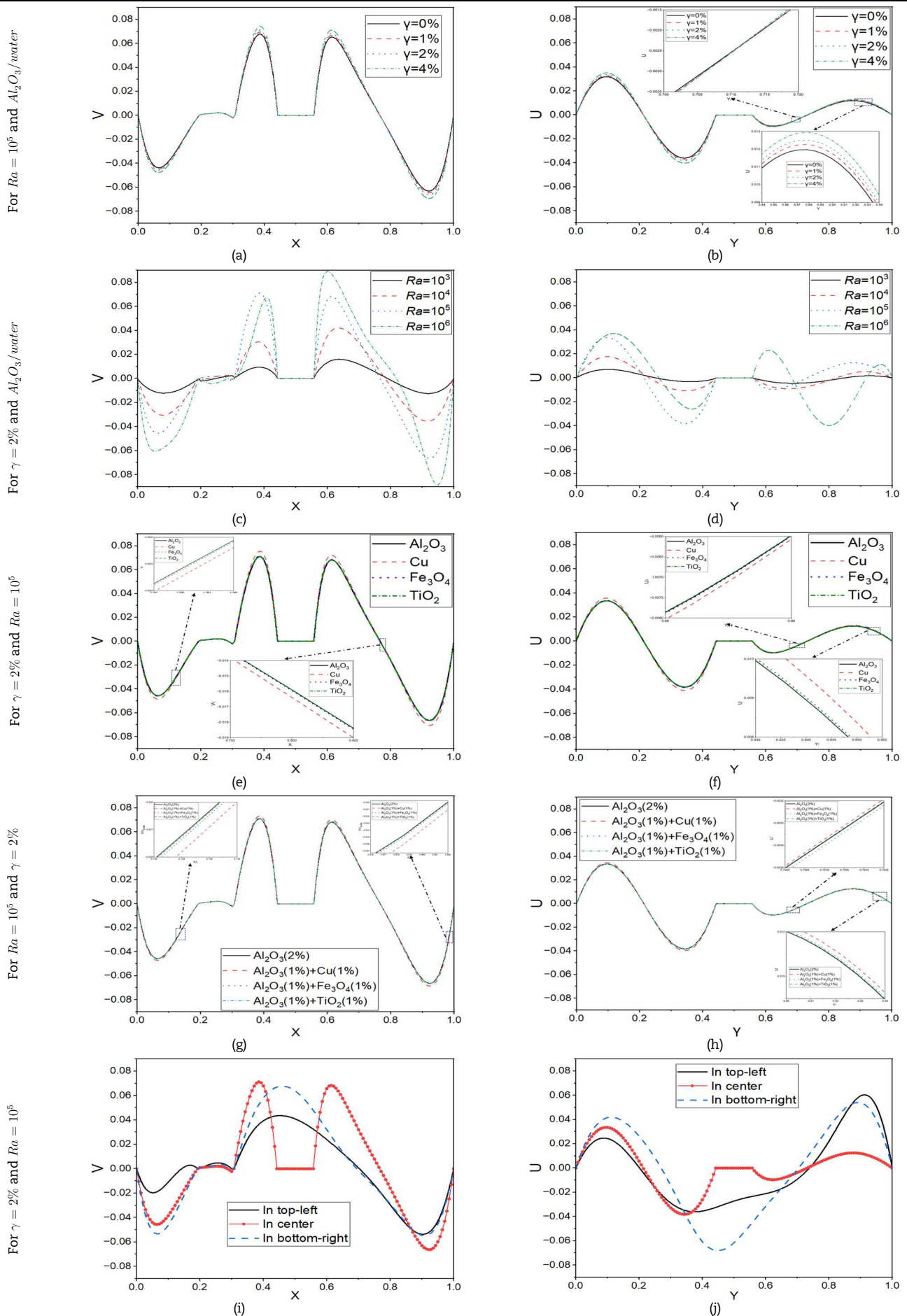


Fig. 13. u and v -velocity profiles along the horizontal and vertical lines at $Y = 0.5$ and $X = 0.5$, respectively.



As we mentioned in the literature, the thermal physical properties of hybrid nanofluid are stronger than those of mono nanofluid. Figures 13(g) and 13(h) depict the impact of hybrid nanofluid and its type on the velocities; their results demonstrate that the hybrid nanofluid ($Al_2O_3 + Cu/water$) has a higher velocity magnitude than the mono nanofluid ($Al_2O_3/water$). In addition, the findings demonstrate that the choice of nanosized particles has a crucial influence on the velocities, so in the heat transfer enhancement; the two velocities present a maximum variation using Cu nanoparticles, while using Al_2O_3 , Fe_3O_4 , and TiO_2 solid nanoparticles, their variations are almost the same.

Figure 13(i) and 13(j) show the distributions of u and v -velocities along the horizontal and vertical lines, respectively. According to these results, the maximum magnitude of the two velocities depends on the position of the heated source (in the top-left, in the middle, or in the bottom-right). The results demonstrate that the velocity is maximum in the case of the heated source in the bottom position ($V = \frac{v}{v_{Scale}} = 0.08$) for $X = 0.4$ and ($U = \frac{u}{u_{Scale}} = 0.04$) for $Y = 0.4$ more than in other positions of the heated source. The fluid dynamics are accelerated in the case of the top-left position.

4. Conclusion

The current research discussed, by employing the lattice Boltzmann method, the nanosized particles volume percentage γ , the Rayleigh numbers Ra , heated source position, nano-sized particles type (Al_2O_3 , Cu , Fe_3O_4 , and TiO_2), and type of nanofluid (mono and hybrid) effects on convection flow dynamics and thermal transfer enhancement through nanofluid. This investigation is modeled as a single-phase scheme within an enclosure including a heated source and two obstacles with different positions. The principal conclusions of this paper are outlined below:

- Although the viscosity elevates with more presence of nanosized particles, the impact remains moderate. This signifies that the thermal boundary layer is not significantly thickened, helping to maintain an effective temperature gradient at the wall and improving convective thermal transfer.
- The convection heat transfer is positively sensitive in the case where the heated source is in the bottom-right of the enclosure compared to other cases of the heated source (in the center and in the top-left of the enclosure).
- The fluid's velocities and temperature magnitude are positively influenced by high values of nanosized particle volume concentration, Rayleigh number, and the nanoparticles Cu .
- The different types of nanofluids show varied performances, highlighting the importance of nanosized particle selection for large Rayleigh number values and nanosized particle percentage. For $Ra = 10^6$, a significant enhancement in thermal transfer and stability of the fluid is demonstrated by the use of a hybrid nanofluid $Al_2O_3 + Cu/water$. In addition, a volume concentration of nanoparticles of $\gamma = 4\%$ improves the thermal-physical properties of the hybrid nanofluid without causing significant penalties in terms of viscosity. Therefore, by placing the heat source in the bottom-right of the enclosure, the convection mechanism and heat transmission of the nanofluid are enhanced.
- This hybrid nanofluid (3%) $Al_2O_3 + (1\%)Cu/water$ offers an appropriate solution for industrial thermal management. It provides high thermal transfer performance with moderate cost and pumping requirements, making it a suitable option for solar thermal collectors, heat exchangers, or cooling loops in the facilities.
- From an engineering perspective, the identified optimal configuration provides useful guidance for thermal management systems involving localized heat generation and internal obstacles. In particular, the enhanced heat transfer obtained with the selected hybrid nanofluid can be exploited in applications such as electronic cooling, compact heat exchangers, battery thermal management, solar thermal systems, and other passive cooling devices. The results also indicate that the appropriate combination of nanoparticle composition, concentration, heat-source location, and obstacle arrangement can improve thermal management performance without imposing excessive viscosity-related penalties.
- These findings demonstrate that the proposed simplified enclosure can serve as a representative computational model for engineering thermal systems in which localized heat sources and internal structures affect natural convection. The present results can therefore contribute to the preliminary design and optimization of compact thermal management configurations.
- In the future, the research could be expanded to include turbulent or transitional flow regimes at higher Rayleigh numbers, as well as various other types of nanoparticles. The model could also be extended to three-dimensional configurations and more realistic engineering geometries. Furthermore, coupling the present approach with a systematic optimization analysis could help identify optimal nanofluid compositions, heat-source locations, and obstacle configurations for specific thermal management applications.

Author Contributions

Y. Oulahou conceptualized the study, developed the methodology, performed the investigation, implemented the software, and prepared the original draft and visualizations; Y. Ighris participated in the investigation, performed the investigation, and contributed to writing-review and editing; Y. Elguennouni participated in the investigation, validated the findings, and contributed to writing-review and editing; J. Baliti provided the necessary resources, validated the findings, and contributed to writing-review and editing and examining the proofreading; M. Hssikou provided the necessary resources, participated in the investigation, validated the findings, and contributed to writing-review and editing and examining the proofreading; C. Stanciu provided the necessary resources, participated in the investigation, and contributed to writing-review and editing; M. Alaoui developed the mathematical modeling, participated in the investigation, and contributed to review. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest concerning the research, authorship, and publication of this article.



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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

LBM	Lattice Boltzmann Method	Δt	Time step [s]
c_s	Speed of sound [m. s ⁻¹]	ρ	Density [kg.m ⁻³]
c_p	Specific heat capacity at constant pressure [J. kg ⁻¹ . K ⁻¹]	α	Fluid thermal diffusivity [m ² .s ⁻¹]
c_i	Discrete vector of velocity in the i-direction [m.s ⁻¹]	μ	Dynamic viscosity [Pa. s]
$\tilde{G}$	Distribution function of temperature	κ	Thermal expansion coefficient [K ⁻¹]
$\tilde{G}^{eq}$	Equilibrium state of $\tilde{G}$	ν	Kinematic viscosity [m ² .s ⁻¹]
$\tilde{F}$	Distribution function of density	γ	Nanoparticle volume percentage [%]
$\tilde{F}^{eq}$	Equilibrium state of $\tilde{F}$	ψ	Stream (flow) function [K]
Ra	Rayleigh number	$\tau_{\tilde{G}}$	Relaxation time for temperature [s]
λ	Thermal conductivity [W. m ⁻¹ . K ⁻¹]	$\tau_{\tilde{F}}$	Relaxation time for flow [s]
Nu	Local Nusselt number	Ω_i	Weight factors for temperature and flow (D ₂ Q ₉)
Ma	Mach number	C	Cold
Pr	Prandtl number	H	Hot
θ	Normalized temperature	F	Fluid
L	Enclosure length [m]	N_p	Nanosized particles
H	Enclosure height [m]	mnf	mono nanofluid
l	Heated source and adiabatic obstacle width [m]	hnf	Hybrid nanofluid
p	Pressure [N. m ⁻²]	avg	Average
u(U. V)	Vector of velocity [m. s ⁻¹]	B – R	Bottom-Right
(x, y)	Cartesian coordinates	T – L	Top-Left
v_{Scale}	Characteristic flow velocity [m. s ⁻¹]	FVM	Finite Volume Method

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